INVESTIGATIONS BEYOND THE STANDARD MODEL

ABSTRACT

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DEPARTMENT OF PHYSICS
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The investigations carried out under this thesis can be broadly classified into two categories: (A) The impact of certain higher-dimensional operators on the predictions of SU(5) and SO(10) grand unified theories, (B) Natural seesaw mechanism for neutrino masses in a new class of models with identical parity (P)-and $SU(2)_R$-breaking scales. While the Chapters II-V have been devoted to the analysis of the type (A), Chapter VI is devoted to obtain results of the type (B). Chapter I provides a general introduction while the summary and conclusions are stated in Chapter VII.

Although superstring theories offer an exciting possibility of unification of all interactions, the extension of the original Kaluza-Klein type of unification with gravity by the introduction of extra spatial dimensions is also a very attractive one. The impact of such a unification scheme on grand unified theories (GUT's) need special attention in the context of direct and indirect experimental signatures at low or accelerator energies. When extra dimensions are compactified, besides the usual GUT Lagrangian of the effective four-dimensional theory, nonrenormalizable terms involving certain higher-dimensional operator(s) and scaled by suitable powers of the compactification mass (M_G) usually occur as residual effects of compactification. Although the presence of such terms makes the GUT Lagrangian nonrenormalizable, a very attractive feature is that they are absorbed as renormalizable terms of the Lagrangian of the effective theory at the lower scale ($\mu \ll M_U$), once the GUT

symmetry breaks spontaneously. For example, the effective theory below the unification mass in SU(5) is the standard theory $G_{st}=SU(2)_L \times U(1)_Y \times SU(3)_C$. The effect of such five-dimensional operators on SU(5) and certain SO(10) predictions have been investigated by a number of authors who have noted that such an operator might also originate from effects of quantam gravity in four dimensions especially with $M_G \simeq M_{pl}=10^{19}$GeV. We have examined the modifications caused by the five- and six-dimensional operators on SU(5) and the relevant five-dimensional operators in the effective chains of SO(10) leading to very attractive predictions for low-energy experiments not obtained earlier.

In Chapter II we show that the combined effects of the five-and six-dimensional operators lead to large enhancement of proton lifetime (τ_p) in SU(5) with $\tau_p \geq 10^{38}$yr for the $p \longrightarrow e^+\pi^o$ mode and $\sin^2\theta_W \simeq 0.22-0.24$.

In Chapter III we found that even with a single intermediate symmetry in the chain $SO(10) \xrightarrow{M_U} SU(2)_L \times U(1)_R \times SU(4)_C \xrightarrow{M_C} G_{st}$, the quark-lepton unification scale is allowed to be as low as 10^5-10^6GeV, when the effect of the relevant five-dimensional operator is included in the renormalization-group equations (RGE's). Such a scale leads to the observable rare-kaon decay ($K_L \longrightarrow \bar{\mu}$ e) and small neutrino masses.

In contrast to the chain made by Rizzo few years earlier that the low-mass $W_R^{\pm}$-gauge bosons are predicted in the scenario, $SO(10) \xrightarrow{M_U} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times P\ (\equiv G_{2213P}) \xrightarrow{M_R} G_{st}$, where P stands for parity, leading to $M_R \simeq 100$GeV, we find that the modifications to the boundary conditions of the RGE's have not been calculated correctly. When this is done, the lowest permissible value turns out to be $M_R \simeq 10^8$GeV, invalidating the earlier claim. Interestingly enough, when we adopt the method of decoupling P- and $SU(2)_R$-breakings with the same intermediate symmetry, but excluding F ($g_{2L} \neq g_{2R}$), the effect of the five-dimensional operator permits $M_R \simeq 500$GeV – few TeV. Thus a very interesting and new result of Chapter IV is the demonstration of the existence of low-mass $W_R^{\pm}$ and Z_R-gauge bosons in SO(10) without observable parity restoration.

In Chapter V we investigate the effect of the revelant five-dimensional operator on SO(10) predictions with the single $SU(2)_L \times SU(2)_R \times SU(4)_C (\equiv G_{224})$ intermediate symmetry with and without (P). In the case of G_{224P} intermediate symmetry, the quark-lepton unification mass M_C and grand unification scale M_U are found to be as large as $M_C \simeq 10^{14}$GeV and $M_U \simeq 10^{18}$GeV, respectively leading to a very stable proton and no cosmologically problematic domain walls. With G_{224} intermediate symmetry, it yields $M_C = M_{W_R} \simeq 10^5 - 10^6$GeV leading to observable predictions of $n-\bar{n}$ oscillation, and $K_L \longrightarrow \bar{\mu}$ e decay and small Majorana neutrino masses.

All investigations in Chapter VI are made without introducing any higher-dimensional operator. We find that in models of the following class,

$$SO(10) \xrightarrow{M_U} G_{2213P} \xrightarrow{M_P = M_R^+} SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C (\equiv G_{2113}) \xrightarrow{M_R^0} G_{st},$$

$$SO(10) \xrightarrow{M_U = M_R^+} G_{214} \xrightarrow{M_C} G_{2113} \xrightarrow{M_R^0} G_{st},$$

the seesaw mechanism for generating small Majorana neutrino masses can be implemented in a very natural manner in that the induced contributions are negligible for certain large value of the ratio M_R^+/M_R^0, permissible by the solutions to RGE's. In these models the low-energy gauge group could be a minimally extension of the standard group predicting the existence of a low-mass Z_R boson. At several stages of the thesis, other predictions of different model have been noted and the method of circumventing the cosmological bound on neutrino masses has been discussed.

The following papers have been produced under this thesis,

1. "Gravity-induced large grand-unification mass in SU(5) with higher-dimensional operators."
   M.K.Parida, P.K.Patra, and A.K.Mohanty, Phys.Rev.D 39, 316(1989).
2. "Spontaneous compactification effects, low-energy signatures of quark-lepton unification, and small neutrino masses in SO(10)."
   M.K.Parida and P.K.Patra, Phys.Rev.D39, 2000(1989).
3. "Spontaneous compactification effects in SO(10) with low-mass $W_R^\pm$-gauge bosons without observable parity restoration.
   M.K.Parida and P.K.Patra, Phys.Lett.234B, 45(1990).
4. "Spontaneous compactification effects on SO(10) grand unification with $SU(2)_L \times SU(2)_R \times SU(4)_C$ intermediate symmetry"
   P.K.Patra and M.K.Parida, submitted to Phys.Rev.D.
5. "Models with natural seesaw mechanism for neutrino masses with identical parity-and $SU(2)_R$-breaking scales."
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This is to certify that the thesis entitled 'Investigations beyond the standard model' submitted by Sri Prasanta Kumar Patra for the fulfilment of the degree of Doctor of Philosophy of the North-Eastern Hill University, Shillong, embodies the record of an original investigation carried out by him under my supervision. He has been duly registered and the thesis submitted is worthy of being considered for the award of the Ph.D. degree.

This work has not been submitted for any other degree to any other University or Institution.

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Finally I take this opportunity to express my heartfelt gratitude to my parents for their penance in bringing me up to what I am.

Shillong

The 22nd February, 1991

PKPatra

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CONTENTS

LIST OF PUBLICATIONS

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   M.K.Parida, P.K.Patra, and C.C.Hazra, Phys.Rev.D43, (1991).

[1]

INTRODUCTION

The standard model[1] of strong, weak, and electromagnetic interactions based upon $SU(3)_C \times SU(2)_L \times U(1)_Y$ ($\equiv G_{st}$) has been proved by numerous experimental facts.[2] From the experimental point of view, there is no compelling reason at present to go beyond the standard model. But, despite its success, the standard model has certain difficulties, which advocate that the model may not be the ultimate theory of basic interactions. Some of the arguments against the standard model and possible alternatives are discussed below.

In the standard model there are three different coupling constants, g_{3C}, g_{2L}, and g_Y associated with the gauge subgroups $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ respectively. Therefore, the theory is not truly unified. On the other hand, if the three gauge subgroups of G_{st} are allowed to emerge from a grand unified theory (GUT),[3-6] at least the three basic forces, strong, weak, and electromagnetic can be unified at a high scale ($\geq 10^{15}$GeV). Certain grand unified theories (GUT's) with N=1 supergravity manifest as effective low energy theories predicted by the superstring theory (SST)[7] which seems to be highly promising for the unification of all elementary forces including gravity. Although string theories offer very attractive possibilities, earlier, unification with gravity independent of SST has been shown to be possible by extending the Kaluza-Klein[8,9] frame work appropriately to higher dimensions. Then GUT's in four dimensions appear as a result of compactification of extra dimensions.[10,11]

The standard model does not explain the origin of parity-(P) and CP(C=charge conjugation)-violations in weak interactions. Although the observed CP-violation in weak interaction is parametrized in the standard model frame work through the Kobayashi-Maskawa (KM)[12] approach, the model does not offer an origin of CP-violations. Similarly the standard model does not explain why P-violation is confined to the weak interactions only. Besides in the KM model, one needs atleast three fermion generations to get CP-violation. On the other hand, a more interesting idea would be to start with a P or CP conserving Lagrangian and to obtain the derived violations after spontaneous symmetry breaking (SSB) leading to the standard model. Left-right-symmetric (LRS) models with or without incorporating the possibility of quark-lepton unifications, based upon the local gauge groups $SU(2)_L \times SU(2)_R \times SU(4)_C \times P(g_{2L}=g_{2R}) (\equiv G_{224P})$[5] or $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times P(g_{2L}=g_{2R}) (\equiv G_{2213P})$[13] offer such exciting possibilities where P- and CP-violations can be assigned spontaneous origins. A special interesting feature of the models as compared to the KM approach is that the CP-violation can be realized even with two fermion generations. An additional aesthetically appealing feature of the LRS models is that CP-and P-violations can be linked to each other.

The present experimental upper bound on the neutrino masses are $m_{\nu_e} \leq 18eV$, $m_{\nu_\mu} \leq 250keV$, and $m_{\nu_\tau} \leq 35MeV$[14]. There exists a limit of about 1eV on the Majorana mass[15] of ν_e. Thus it is evident from the experimental bounds that if the neutrino has a mass,it is much less than the corresponding masses of the charged lepton and quarks. On the other hand, in the standard model, the neutrino is massless and the model prediction for the neutrino magnetic moment μ_{ν_e} is much smaller as might be needed for explaining fluxes from the sun and supernova. The masslessness of the neutrino is due to the absence of right-handed neutrino (ν_R) in the standard model. However, if a ν_R is added to the standard model the neutrino gets a large Dirac mass which is of the same order as the corresponding quark or charged lepton mass. Such a large mass is ruled out by the available experimental limits and the modern big-bang cosmology. Experimental measurements involving neutrinoless double β-decay, neutrino oscillations,and the observation of neutrinos emitted from the sun and the 1987A supernova explosion are consistent with small neutrino masses. Thus, the neutrino masses, if they are nonvanishing, might provide a compelling reason for looking into gauge models beyond the standard one. The massive neutrinos can be achieved by the gauge models based upon the left-right symmetry gauge group G_{2213P} or G_{224P} and/or GUT's. In LRS models, since both left- and right-handed helicities of the neutrino are included, the neutrino can naturally have a Dirac mass. On the other hand, if neutrinos are Majorana particles, there could be suitable Higgs representation with two units of B-L, which can also generate small Majorana neutrino masses by seesaw mechanism.[16,17]

The standard model contains a large number of parameters to explain physical phenomena including fermion masses. They are[18] the 3 gauge coupling constants, 6 parameters for the 6 quarks plus three generalized Cabibbo angles, 1 CP-violating phase, 2 parameters for the Higgs potential, and either 3 or 10 mass, mixing, and phase parameters for the leptons (corresponding to massless or massive neutrinos), for a total of 18 or 25 independent parameters. In addition, there can, in principle, be 2 CP violating vacuum angles, θ_{QCD} and θ_{QFD}, associated with $SU(3)_C$ and $SU(2)_L$. But an aesthetically appealing alternative theoretical model would be the one requiring fewer number of parameters which can be achieved by invoking additional symmetries at higher mass scales. This is achieved in suitable GUT's to a certain extent.

Extensive investigation has been carried out in GUT's with a view to predicting proton lifetime (τ_P), $\sin^2\theta_W$, and other possible low energy signatures. Minimal SU(5) has been ruled out as it predicts τ_P several orders lower than the experimental limit[19] for the $p \longrightarrow e^+\pi^0$ mode. In SO(10) and other GUT's like $SU(8)_L \times SU(8)_R$, SU(16), SO(18), and E_6 etc., the renormalization group constraints easily permit a low-mass second neutral gauge boson corresponding to the existence of the minimally extended gauge group $SU(2)_L \times U(1) \times U(1)' \times SU(3)_C$ as the next higher symmetry above the electroweak unification scale (M_W). But when the conventional method of LRS breaking is adopted, where $SU(2)_R$-breaking scale is identical to the parity breaking scale, the scale of G_{2213P}-breaking or G_{224P}-breaking turns out to be very large ($\geq 10^{10}$GeV). This rules out almost all the testable low energy signatures of $W_R^{\pm}$-gauge bosons, V+A structure of charged currents, or quark-lepton unification through $SU(4)_C$-gauge-boson-mediated interactions. On the other hand if LRS has any role to play in CP-and P-violations, the $W_R^{\pm}$-gauge boson must be within

TeV range.[20] Such an objective of obtaining low G_{2213}-or G_{224}-breaking scales has been achieved by implementing the novel mechanism of decoupling P- and $SU(2)_R$-breakings[21-23] in these GUT's. Besides, the mechanism has successfully removed the well known domain-wall problem[24] while generating the observed baryon asymmetry of the universe[25]. Recently superheavy- Higgs-scalar effects have been found[26] to significantly lower the G_{2213} breaking scale in SO(10) bringing down the $W_R^{\pm}$-mass to the TeV range even with two intermediate symmetries. In all the GUT's discussed here unification of strong, weak, and electromagnetic interactions are envisaged without supersymmetry and leaving out unification with gravity. If superpartners are eventually detected at collider energies, phenomenology of SST would attract considerable global attention. In the absence of any experimentally compelling signautres in favour of a particular model beyond the standard one, it is worth while to explore more and more testable predictions in various possible interesting theories.

Extending the Kaluza-Klein framework to still higher dimensions, unification of nonabelian gauge theories with gravity has been achieved by a number of authors[11]. When extra dimensions are compactified leaving out grand unifying symmetries in four-dimensional space, certain residual effects of compactification influence the four-dimensional theory in a very significant manner. One such term contributing to important modifications of the conventional SU(5) and SO(10) predictions has been noted to be due to a five-dimensional operator[27,28] involving the gauge and Higgs fields. Although this term appears to be nonrenormalizable at the level of the GUT, an attractive feature is that, after spontaneous symmetry breaking (SSB), this term is completely absorbed into the renormalizable gauge-field-kinetic energy term of the effective standard or intermediate gauge symmetry. The five-dimensional operator bringing about significant changes might originate from effects of quantum gravity[28] in four dimensions. A major objective of this thesis in Chapters II -V is to demonstrate the possibilities of very drastic modifications of SU(5) predictions in the presence of five- and six-dimensional operators and certain scenarios of SO(10) with single intermediate symmetries in the presence of the corresponding five-dimensional operators.

Neutrino mass is a very intriguing problem that has attracted considerable attention over the decades.[29] The classic seesaw formula[16,17] for generating small Majorana neutrino masses is frequently used in model building. As a very significant development in this direction Chang and Mohapatra[30] have recently demonstrated the naturalness of the formula in the LRS models with separate P-and $SU(2)_R$-breaking scales. As a new development in this regard we demonstrate, in Chapter VI, the existence of a class of models where the seesaw mechanism is natural even though the P- and $SU(2)_R$-breaking scales are identical.

The thesis is organized in the following manner. In Chapter II we show that the SU(5) predictions are modified due to certain five-and six- dimensional operators leading to $\tau_P \geq 10^{38}$yr with $\sin^2\theta_W \simeq 0.22$-$0.24$.

In Chapter III we demonstrate that due to a five-dimensional operator in the nonrenormalizable SO(10) invariant Lagrangian, the single intermediate symmetry $SU(2)_L \times U(1)_R \times SU(4)_C (\equiv G_{214})$ can survive down to a scale as low as $10^5 - 10^6$ GeV leading to experimentally observable branching ratios for the rare-kaon decays $K_L \longrightarrow \bar{\mu} e$ and small neutrino masses. The main distinctive feature is that without the presence of the five-dimensional operator this scale is large ($M_C \geq 10^{11}$ GeV) without leaving any interesting low-energy signature for $SU(4)_C$ gauge unification.

In Chapter IV we demonstrate that the SO(10) model with the single intermediate symmetry G_{2213P} does not lead to low-mass $W_R^{\pm}$-gauge bosons in the presence of the corresponding five-dimensional operator. This observation is in contrast to the earlier claim made by Rizzo[31] implementing the conventional LRS breaking in the GUT taking into the modification introduced by the five-dimensional operator. We note that $M_{W_R^{\pm}} \geq 10^8$ GeV as against Rizzo's claim of $M_{W_R^{\pm}} \approx 100$ GeV in such a scenario. One new exciting feature of our investigation is that when the mechanism of decoupling P- and $SU(2)_R$- breakings [21] are implemented with P broken at the GUT scale ($M_P = M_U$, M_P=parity breaking scale, M_U = unification scale) the single intermediate symmetry G_{2213} ($g_{2L} \neq g_{2R}$) leads to low-mass $W_R^{\pm}$ gauge boson ($M_{W_R^{\pm}} \simeq$ 500GeV–10TeV) provided the contribution of the corresponding five-dimensional operator is included in modifying the GUT coupling constants.

In Chapter V we study the impact of the relevant five-dimensional operator on SO(10) with the G_{224P} ($g_{2L} = g_{2R}$) or G_{224} ($g_{2L} \neq g_{2R}$) intermediate symmetry. Here we note that the modification introduced by the higher-dimensional operators to $\sin^2\theta_W$ vanishes for the chain with G_{224P} intermediate symmetry whereas the modification to τ_p vanishes with G_{224} intermediate symmetry. In the former case, besides the solutions of the type obtained by Shafi and Wetterich (SW),[27] we find new solutions to the renormalization -group equations (RGE's)[32] leading to a very stable proton, large G_{224P} breaking scale circumventing the well known domain-wall problem[24] and small neutrino masses. More interesting predictions with G_{224}-breaking scale as low as 10^5-10^6GeV leading to observable predictions of $K_L \longrightarrow \bar{\mu}e$, $n-\bar{n}$ oscillation, and experimentally measurable neutrino masses are found in the other case.

As the other objective of this thesis we establish in Chapter VI that the seesaw mechanism[16,17] for generating Majorana neutrino masses is natural in a certain class of models with identical P-and $SU(2)_R$ -breaking scales. In such cases $SU(2)_R \times U(1)_{B-L}$ or $SU(2)_R \times SU(4)_C$ has to break spontaneously in more than one steps. Such models belong to a new class as the earlier investigation by Chang and Mohapatra[30] has revealed only one class of models with separate P- and $SU(2)_R$- breaking scales. The new class of models predict a low-mass right-handed neutral gauge bosons detectable at the collider energies. The investigation in this Chapter has been carried out in conventional SO(10) taking into account superheavy-Higgs-scalar effects[58,26] and without introducing additional modifications due to spontaneous compactification.

Finally the results of this thesis has been summarized and conclusions are stated in Chapter VII.

CHAPTER II

GRAVITY-INDUCED LARGE GRAND-UNIFICATION MASS IN SU(5) WITH HIGHER-DIMENSIONAL OPERATORS

II.1. Introduction

One of the most exciting consequence of grand unified theories (GUT's) is that it can predict baryon-number nonconserving processes such as proton decay. The present experimental limit on proton lifetime (τ_p) for the $p \longrightarrow e^+\pi^0$ mode and the electroweak mixing angle ($\sin^2\theta_W$) derived from neutral current data are[2,19]

$$\tau_p \geq 3\times10^{32}\ \text{yr},\quad \sin^2\theta_W = 0.230\pm0.005. \qquad (2.1)$$

Although the minimal grand unification based upon SU(5) [Ref.3] explains the quantization of electric charge and unification of coupling constants of the standard model, it is ruled out as it predicts $(\tau_p)_{max}$ about one order of magnitude less than the experimental limit[19] and $\sin^2\theta_W$ below the present world average[2].

One possible reason for such discrepancy between the minimal GUT[3] predictions and the existing experimental data could be that the gravitational forces are left out of the minimal grand unification scheme of the other three basic interactions. A very exciting possibility of unification with gravity is through the introduction of N=1 supergravity which emerges as low-energy manifestation of superstring theories. Such models need the existence of superpartners of fermions, gauge bosons, and Higgs scalars which should be detected within the TeV range at the collider energies. Leaving aside the underlying solution of the gauge hierarchy problem to some unknown deeper origin (other than supersymmetry), unification with gravity can be achieved following the conventional Kaluza-Klein approach by introducing higher dimensions. In such theories the isometries of the higher-dimensional space is related to the gauge fields of the four-dimensional effective theory. That the observed universe at present appears to be four-dimensional could be due to the possibility that the extra dimensions are compactified to very small spatial extensions not detectable by the present experiments. While carrying out compactifiction of extra dimensions leading to the effective GUT's in four dimensions, there exists certain freedom in the choice of the metric which can contain the gauge and scalar fields of the GUT. In many cases the compactification leads to the occurrence of higher-dimensional operators involving the gauge and the Higgs fields but scaled by the compactification mass,[27] as nonrenormalizable terms in the GUT Lagrangian. A highly appealing feature of such terms is that after spontaneous symmetry breaking of the GUT these are completely absorbed as the renormalizable terms by a suitable rescaling of the gauge fields corresponding to the surviving residual symmetry(e.g., the standard symmetry in the case of SU(5)). It has been argued by some authors that such higher-dimensional operators might arise as effects of quantum gravity in four dimensions[28], even without invoking the presence of higher dimensions and their compactification.

The main reason for the drastic modifications introduced by such operators is that the symmetry breaking at the GUT scale and the consequent rescaling of the relevant gauge fields changes boundary conditions to the renormalization-group equations (RGE's) at M_U. The solutions of the RGE's under such new boundary conditions then lead to very different predictions on τ_P, and $\sin^2\theta_W$.

The impact of five-dimensional operators induced by gravity on the minimal SU(5) model has been studied by a number of authors. Ellis and Gaillard[33] have examined the impact of such five-dimensional operators on the quark to lepton mass ratio m_d/m_e predicted by the minimal SU(5) model. Hill[28] has studied the effect of five-dimensional operators, scaled by Planck mass (M_{pl}), on the minimal SU(5) and supersymmetric SU(5) GUT's. In the latter case, he obtained a significant modifications to τ_P and $\sin^2\theta_W$, although such modifications in the former case are ruled out. Including five-dimensional operators, scaled by the compactification mass (M_G), in the minimal SU(5) and SO(10) GUT's, Shafi and Wetterich [27] have investigated modifications of the conventional prediction on τ_P and $\sin^2\theta_W$.

The purpose of this Chapter[36] is to investigate the impact of higher-dimensional ($d \geq 5$) operators, scaled by powers $(M_G)^{-(d-4)}$ consistent with GUT symmetry. We note that the modifications by the d=5 operator alone, taken to be making minimal SU(5) compatible with the experimental data on τ_P, are now ruled out as these solutions require $\sin^2\theta_W$ significantly below the accepted world average, even if $M_G=10^{17}$GeV. Next, we examine the modifications caused by d=5 and d=6 operators and argue that only permissible values of the unification mass (M_U) is of the order (0.1-1) M_G, where M_G could be anywhere in between 10^{17}GeV and 10^{19}

GeV, satisfying the currently accepted value of $\sin^2\theta_W \simeq 0.22-0.24$. Besides this, we find that the bare-grand-unification coupling (α_G) could be 2 orders of magnitude smaller than the earlier results. High values of $M_U \simeq (0.1-1)M_G$ and small values of the GUT coupling then lead to a very stable proton with $\tau_P \geq 10^{38}$yr. Such results have never been envisaged earlier in the context of other conceivable modification of SU(5) excluding supersymmetry. We also obtain perturbative and positivity bounds on certain parameters of the model and mention a new relation among them.

This Chapter is organized in the following manner. In Sec. II.2 we obtain general formulas for the unification mass, electroweak mixing angle, and GUT coupling constant including higher-dimensional operators in the GUT Lagrangian. In Sec.II.3 we discuss the earlier results due to Shafi and Wetterich,[27] and Hill,[28] with five-dimensional operators. In Sec.II.4 we report our numerical analysis including five- and six-dimensional operators. A brief summary of the chapter is given in Sec.II.5.

II.2. Derivation of the formulas for unification mass, electroweak mixing angle, and GUT coupling

In this section we derive the general formulas for the unification mass(M_U), electroweak mixing angle ($\sin^2\theta_W$), and grand unification coupling constant (α_G), adding nonrenormalizable terms (α_{NR}) of dimension $d \geq 5$, scaled by powers of $(M_G)^{-(d-4)}$, to the normalizable GUT Lagrangian (α_R). These nonrenormalizable higher-dimensional operators which are usually generated in theories with spontaneous compactification, contain gauge and Higgs fields. Even without invoking the idea of dimensional reduction, such operators scaled by the powers of M_{Pl} can also be present as the gravity-induced corrections to the GUT Lagrangian.[28] The form of these nonrenormalizable operators is dictated by the appropriate local and global symmetries.

In the minimal SU(5) model, the necessary stages of symmetry breaking are given by

$$SU(5) \xrightarrow[M_U]{\underline{24}} SU(3)_C \times SU(2)_L \times U(1)_Y \ (\equiv G_{st})$$

$$\xrightarrow[M_W]{\underline{5}} SU(3)_C \times U(1)_{em} (\equiv G_{13}). \qquad (2.2)$$

The first stage of symmetry breaking is achieved by giving vacuum expectation value (VEV) to the G_{st}-singlet component of the $\underline{24}$-dimensional irreducible representation (denoted by Φ) and the second stage through the VEV to the neutral component of the standard doublet contained in the $\underline{5}$-dimensional (denoted by H) representation. The SU(5)-invariant normalizable gauge boson kinetic energy term can be written as

$$\mathcal{L}_R = -\frac{1}{2}\,\mathrm{Tr}(F_{\mu\nu} F^{\mu\nu}), \qquad (2.3a)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \qquad (2.3b)$$

$$(A_\mu)^a_b = A^i_\mu (\lambda_i/2)^a_b \qquad (2.3c)$$

$$\mathrm{Tr}(\lambda_i \lambda_j) = \frac{1}{2}\,\delta_{ij}\ . \qquad (2.3d)$$

In Eq.(2.3), $F_{\mu\nu}$ is the gauge invariant field-strength tensor, A_μ is the vector gauge field, g is the SU(5) gauge coupling,

$i=1,2,\ldots,24$ is the gauge field index, $a,b=1,2,\ldots,5$ are the matrix indices, and λ_i's are the generators of SU(5) GUT.

We introduce the following SU(5)-invariant nonrenormalizable ($d\geq5$) interaction term[36]

$$\alpha_{NR} = \sum_{n=1,2\ldots} \alpha_{NR}^{(n)}, \tag{2.4a}$$

$$\alpha_{NR}^{(n)} = -\frac{1}{2}\frac{\eta^{(n)}}{M_G^n}\mathrm{Tr}(F_{\mu\nu}\,\Phi_{(24)}^n\,F^{\mu\nu}), \tag{2.4b}$$

where $\Phi_{(24)}$ denotes the Higgs 24-plet and $\eta^{(n)}$, $n=1,2,\ldots$, are the dimensionless unknown parameters. In Refs.27 and 28, the case with five-dimensional operator corresponds to $\eta^{(1)}\neq0$ and $\eta^{(n)}=0$ for $n\geq2$. It may be noted that the expression for higher-dimensional operators given in Eq.(2.4b) is not the most general one, especially when $n\geq2$. For example, with $n=2$, other gauge-invariant operators not covered by (2.4b) are $\mathrm{Tr}(\Phi_{(24)}^2)\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ and $\mathrm{Tr}(F_{\mu\nu}\Phi_{(24)})\mathrm{Tr}(F^{\mu\nu}\Phi_{(24)})$ the latter being more troublesome for computations of the physical quantities of interest. We confine to the choice (2.4b) for the sake of convenience. Thus the modified Lagrangian becomes

$$\alpha=\alpha_R+\alpha_{NR}$$
$$= -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}\,F^{\mu\nu}) - \frac{1}{2}\sum_n \frac{\eta^{(n)}}{M_G^n}\mathrm{Tr}(F_{\mu\nu}\,\Phi_{(24)}^n\,F^{\mu\nu}), \tag{2.5}$$

When the Higgs field $\Phi_{(24)}$ gets VEV

$$\langle\Phi_{(24)}\rangle=(1/15)^{1/2}\Phi_o\,\mathrm{diag}(1,1,1,-3/2,\ -3/2), \tag{2.6}$$

the grand unifying symmetry is broken and the higher-dimensional terms are absorbed in the renormalizable gauge field kinetic energy of the residual gauge group. As a consequence, we obtain the modifications in the coupling constants g_{3C}, g_{2L}, and g_Y of $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge groups, respectively, at the unification scale M_U in the following manner,

$$
\begin{aligned}
g_{3C}^2(M_U) &\longrightarrow g_{3C}^2(M_U)(1+\varepsilon_{3C}),\\
g_{2L}^2(M_U) &\longrightarrow g_{2L}^2(M_U)(1+\varepsilon_{2L}),\\
g_{Y}^2(M_U) &\longrightarrow g_{Y}^2(M_U)(1+\varepsilon_{Y}),
\end{aligned}
\qquad (2.7)
$$

where

$$\varepsilon_{3C} = \sum_{n=1,2.....} \varepsilon^{(n)} ,$$

$$\varepsilon_{2L} = -\frac{3}{2}\varepsilon^{(1)} + \frac{9}{4}\varepsilon^{(2)} - \frac{27}{8}\varepsilon^{(3)} + \ldots.,$$

$$\varepsilon_{Y} = -\frac{1}{2}\varepsilon^{(1)} + \frac{7}{4}\varepsilon^{(2)} - \frac{13}{8}\varepsilon^{(3)} + \ldots. \qquad (2.8)$$

The ellipsis in Eq.(2.8) includes the effect of operators d > 7, and

$$\varepsilon^{(n)} = \left[\frac{1}{\sqrt{15}}\ \frac{\Phi_o}{M_G}\right]^n \eta^{(n)} , \ n=1,2,.... \qquad (2.9)$$

Using $\alpha_G = g_o^2/4\pi$, where g_o is the bare-GUT coupling and the relation

$$\Phi_o = [6/(5\pi\alpha_G)]^{1/2} M_U , \qquad (2.10)$$

Eq.(2.9) can be rewritten as

$$\eta^{(n)} = \left[\left(\frac{25\pi\alpha_G}{2}\right)^{1/2} \frac{M_G}{M_U}\right]^n \varepsilon^{(n)} . \qquad (2.11)$$

In order to achieve unification of strong, weak, and electromagnetic interactions for $\mu \geq M_U$, the GUT condition is imposed through the equations

$$g_{3C}^2(M_U)(1+\varepsilon_{3C})=g_{2L}^2(M_U)(1+\varepsilon_{2L})=g_Y^2(M_U)(1+\varepsilon_Y)=g_o^2. \qquad (2.12)$$

Thus, the presence of the nonrenormalizable term (2.4b) modifies the boundary conditions of the coupling constants which are usually $g_{3C}=g_{2L}=g_Y$ at scale M_U.

With the boundary conditions modified as in Eq.(2.12), we solve the one-loop renormalization-group equations(RGE's)[32] for the Chain (2.2),

$$\frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_i(M_U)} + \frac{a_i}{2\pi}\ln\frac{M_U}{M_W}, \quad i=Y,\ 2L,\ 3C, \qquad (2.13)$$

where $\alpha_i(\mu)=g_i^2(\mu)/4\pi$, a_i is the one-loop coefficient. For the minimal number of Higgs scalars and three fermion generations $a_Y=41/10$, $a_{2L}=-19/6$, $a_{3C}=-7$. Using Eqs.(2.13) and (2.12), and the one-loop coefficients we obtain the following equations for unification mass(M_U), $\sin^2\theta_W$, and GUT coupling constant (α_G) ,

$$\ln\frac{M_U}{M_W} = \frac{1}{D} + \left\{1+\left[\varepsilon_{3C} - \frac{5\varepsilon_Y+3\varepsilon_{2L}}{3}\,\frac{\alpha}{\alpha_S}\right]\left[1-\frac{8}{3}\,\frac{\alpha}{\alpha_S}\right]^{-1}\right\}\ln\frac{M^{(5)}}{M_W}, \qquad (2.14a)$$

$$\sin^2\theta_W=\left[\sin^2\theta_W^{(5)} - \frac{19}{134}\varepsilon_{3C} + \frac{1}{67}\left(21+\frac{41}{2}\,\frac{\alpha}{\alpha_S}\right)\varepsilon_{2L} + \frac{95}{402}\,\frac{\alpha}{\alpha_S}\varepsilon_Y\right]/D, \qquad (2.14b)$$

$$\frac{1}{\alpha_G} \equiv \frac{4\pi}{g_o^2} = \frac{3}{67}\left\{\frac{11}{3\alpha_S} + \frac{7}{\alpha}\right\}/D\ , \qquad (2.14c)$$

$$D = 1 + \frac{1}{67}\,(11\varepsilon_{3C} + 21\varepsilon_{2L} + 35\varepsilon_Y)\ , \qquad (2.14d)$$

where $M^{(5)}$ and $\sin^2\theta_W^{(5)}$ denote the one-loop predictions of the minimal SU(5) model, without gravity-induced effects, including only one set of 24 + 5 of Higgs fields and three generations of fermions:

$$\ln \frac{M^{(5)}}{M_W} = \frac{6}{67\alpha}\left[1- \frac{8}{3} \quad \frac{\alpha}{\alpha_S}\right] ,$$

$$\sin^2\theta_W^{(5)} = \frac{23}{134} + \frac{109\ \alpha}{201\ \alpha_S} . \tag{2.15}$$

In Eqs.(2.14a)-(2.14d), we note that the effects of all higher-dimensional operators are contained in the parameters ε_Y, ε_{2L}, and ε_{3C} as illustrated in Eq.(2.8). Thus, Eq.(2.14) is the general formula for minimal SU(5) model when gravity-induced corrections due to higher-dimensional operators are added to the GUT Lagrangian.

II.3. Solutions with five-dimensional operators

In this section we will review the earlier solutions obtained by Shafi and Wetterich[27], and Hill[28], with d=5 operator and see how they are ruled out because of experimental constraints[2,19] on τ_P and $\sin^2\theta_W$. As Hill[28] has already reached such conclusion with $M_G \sim M_{pl}$, we, therefore, discuss Shafi and Wetterich[27] solutions with $M_G \sim 10^{17}$GeV where SU(5) has been stated to survive the then existing data. We obtain the required equations for $\ln(M_U/M_W)$, $\sin^2\theta_W$, and α_G for five-dimensional operators by using $\varepsilon^{(2)} = \varepsilon^{(3)} = ... = 0$ in our general formula (2.14). Imposition of the above conditions in Eqs.(2.9) and (2.8) yields

$$\varepsilon^{(1)} = \varepsilon = \frac{\eta\ \Phi_o}{\sqrt{15}\ M_G} , \ \varepsilon_Y = - \frac{\varepsilon}{2} , \ \varepsilon_{2L} = - \frac{3\varepsilon}{2} , \text{ and } \varepsilon_{3C} = \varepsilon . \tag{2.16}$$

Now, using Eq.(2.16) in Eqs.(2.14a)-(2.14d), we obtain

$$\ln\frac{M_U}{M_W} = \frac{6\pi}{(67-38\varepsilon)}\left[\frac{1}{\alpha} - \frac{8}{3\alpha_S} + \left\{\frac{7}{3\alpha_S} + \frac{1}{\alpha}\right\}\varepsilon\right],$$

$$\sin^2\theta_W = \frac{1}{(67-38\varepsilon)}\left[\frac{23}{2} + \frac{109}{3}\frac{\alpha}{\alpha_S} - \left\{41 + \frac{116\,\alpha}{3\,\alpha_S}\right\}\varepsilon\right], \tag{2.17b}$$

$$\frac{1}{\alpha_G} = \frac{(11\alpha_S^{-1} + 21\,\alpha^{-1})}{(67-38\varepsilon)}, \tag{2.17c}$$

$$D = 1 - \frac{38\varepsilon}{67}. \tag{2.17d}$$

The values of the M_U, $\sin^2\theta_W$, and α_G are computed by using different assumed values of the parameters ε as has been done before.Now, using Eqs. (2.11) and (2.17c), the basic parameter η which occurs in the Lagrangian becomes

$$\eta = \left(\frac{25\pi}{2}\,\frac{67-38\varepsilon}{11\alpha_s^{-1}+21\alpha^{-1}}\right)^{1/2}\frac{M_G}{M_U}\,\varepsilon\,. \tag{2.18}$$

From Eq.(2.17c), we can find out the lower and upper bound of the ε parameters by using positivity and perturbative constraints on α_G. The positivity of α_G ($\alpha_G > 0$) suggests that $\varepsilon < \frac{67}{38} = 1.76$, whereas the perturbative constraint ($\alpha_G < 1$) yields, $\varepsilon > -71.38$, with $\alpha_{3C}(M_W) = \alpha_S = 0.1088$ and $\alpha^{-1}(M_W) = 127.54$. Thus,

$$-71.38 < \varepsilon < 1.76. \tag{2.19}$$

The lower bound is dominated by $\alpha^{-1}(M_W)$ and does not vary significantly in the allowed range of α_S, corresponding to $\Lambda_{\overline{MS}}$ $= 0.160 \pm 0.100$ GeV where $\overline{MS}$ denotes the modified minimal subtraction scheme.

We compute numerical solutions for the unification mass, $\sin^2\theta_W$, α_G^{-1}, τ_P, and η for different values of the ε parameter which are presented in Table 1. For calculating η, the value of the compactification scale has been taken to be $M_G \simeq 10^{17}$GeV as before[27]. The uncertainty by a factor $10^{\pm 3}$ in τ_P arises due to the uncertainties in the matrix element for $P \longrightarrow e^+\pi^0$ and the QCD parameter[34]. From Table 1, we can see that in order to have τ_P agrees with the experimental limit, $\tau_P \geq 3\times 10^{32}$yr, it is clear that $\varepsilon > 0.02$ which needs $\sin^2\theta_W < 0.199$, even though M_G is allowed to be as low as 10^{17}GeV. Thus, the modifications with five-dimensional operators in the minimal GUT seem to be ruled out as they require $\sin^2\theta_W < 0.199$ which is much less than the recent world average[2] $\sin^2\theta_W = 0.230 \pm 0.005$, in order to yield $\tau_P \geq 3\times 10^{32}$yr for $P \longrightarrow e^+\pi^0$.

II.4. New solutions with five- and six-dimensional operators

As the modifications due to five-dimensional operators alone are ruled out, we investigate the possible modifications including other higher-dimensional operators to see whether the predictions of τ_P and $\sin^2\theta_W$ are consistent with available experimental data or not. We find that the most natural and plausible solutions for τ_P and $\sin^2\theta_W$ corresponding to the logical values of the parameters in the Lagrangian, belong to d=5 and d=6 operators. Here, the allowed values of the parameter ε, yield $M_U \sim (0.1-1)M_G \sim 10^{16}-10^{19}$GeV, and $\sin^2\theta_W = 0.22-0.24$, for each value of M_U. In this case the relation between ε parameters are

$$\varepsilon_Y = \frac{2}{5}\varepsilon_{3C} + \frac{3}{5}\varepsilon_{2L}\,, \tag{2.20a}$$

$$\varepsilon^{(1)} = \frac{9}{15}\varepsilon_{3C} - \frac{4}{15}\varepsilon_{2L}\,, \tag{2.20b}$$

$$\varepsilon^{(2)} = \frac{2}{5}\varepsilon_{3C} + \frac{4}{15}\varepsilon_{2L}\,. \tag{2.20c}$$

Table 1. Modifications of the minimal GUT predictions with d=5 operator. The parameter η has been calculated with $M_G=10^{17}$GeV.

ε	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_P (yr)	η
0.005	4.32×10^{14}	0.208	41.60	$3.58\times10^{30\pm3}$	1.10
0.010	5.80×10^{14}	0.205	41.72	$1.16\times10^{31\pm3}$	1.66
0.015	7.78×10^{14}	0.203	41.85	$3.77\times10^{31\pm3}$	1.86
0.020	1.05×10^{15}	0.199	41.95	$1.25\times10^{32\pm3}$	1.84

It may be noted that the relation (2.20a) is also valid in the d=5 case.

From Eq.(2.5), we can see that the basic parameters of the Lagrangian are the η parameters rather than the ε parameters. The theoretical constraint on the η parameters seem to be only due to the positivity and perturbative constraint on ε, as discussed in Sec.II.3. But, in order that the modified Lagrangian makes some sense, the following general criteria on the parameters need to be satisfied: (i) The magnitude of $\eta^{(n)}$, n=1,2... is not very large; (ii) Although the individual values of the η parameters may differ, one possibility is that they are of the same order; (iii) If the gravity-induced corrections might be acting as the terms in a perturbation series, for reasons hitherto unknown to us, the other possibility might be that $|\eta_2| < |\eta_1|$. Thus, we would take the accepted solutions are those, for which either criteria (i) and (ii), or (i) and (iii) are satisfied. In the earlier work[27,28] as the solutions are obtained with five-dimensional operators, there is only one parameter η. The values of η are taken differently by different authors in order to obtain allowed solutions. Shafi and Wetterich[27] have obtained the modified solutions with $\eta \sim 1$, whereas Hill[28] has investigated over a wide range of parameters η=-20-20. In Hill's case, with the maximum allowed $\eta \sim 10$, no plausible solutions have been obtained for the nonsupersymmetric minimal SU(5); but, in the supersymmetric SU(5), significant and acceptable modifications have been obtained for η=-2-2. For the d=5 operator, Shafi and Wetterich have found that $M_G \simeq 10^{17}$GeV is necessary in order to obtain $\eta \sim 1$ whereas $M_G \sim$

10^{18}–10^{19} GeV gives η=10–100, for $M_U \sim 10^{15}$GeV with the same value of ε. Thus, besides the smaller values of $\sin^2\theta_W$, the larger value of η prevents $M_G \sim M_{pl}$ for the d=5 operator in the minimal GUT. In the present case, however, we will find the criteria (i) –(iii) can clearly rule out solutions with $M_U \sim 10^{15}$ GeV, but allow only those with high unification masses which depend on the value of M_G. Here, we first compute the values of ε_{2L} and ε_{3C} and hence ε_Y, for which $M_U \geq 10^{15}$ GeV and $\sin^2\theta_W \simeq 0.22$–$0.24$, and the corresponding value of the GUT coupling α_G by using Eqs.(2.14a)–(2.14d) and (2.20a). Then by using Eqs.(2.20b) and (2.20c), we compute the numerical values of $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$. Some of these solutions are presented in Table 2 for the different values of M_U and $\sin^2\theta_W$. In the next step, to test whether all such solutions are acceptable i.e., fulfil the criteria (i)–(iii), we compute the values of the basic parameters $\eta^{(1)}$ and $\eta^{(2)}$ by using Eq.(2.11), with the calculated values of $\varepsilon^{(1)}$, $\varepsilon^{(2)}$, α_G, and M_U from Table 2 and several reasonable values of M_G existing in the literature. Our new solutions are given in Tables 3 and 4. On the basis of criteria (i)–(iii) we find that all the numerical solutions for M_U and $\sin^2\theta_W$ can be classified into the following categories.

(A) $M_U \ll M_G$

Here the inequality is used to mean values of M_U less than M_G by 2 or more orders. Since $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$ are of the same order, it is clear from Eq. (2.11) that if $M_U \ll M_G$, $|\eta^{(2)}| \gg |\eta^{(1)}|$. For example, with M_U=10^{15}GeV, we obtain $(\eta^{(1)}, \eta^{(2)}) \simeq (-4.05, -10^2)$, $(-40.5, -10^4)$, and $(-405.6, -10^6)$, for M_G= 10^{17}, 10^{18}, and 10^{19}GeV,

Table 2. Parameters ε_{3C}, ε_{2L}, ε_Y, $\varepsilon^{(1)}$, and $\varepsilon^{(2)}$ computed using one-loop renormalization-group equations and corrections due to d=5 and 6 operators. Relations among ε parameters are given in Eq.(2.20).

ε_{2L}	ε_{3C}	ε_Y	$\varepsilon^{(1)}$	$\varepsilon^{(2)}$	M_U	$\sin^2\theta_W$	α_G
−0.9841	−0.9833	−0.9838	−0.3276	−0.6558	10^{15}	0.2305	3.905×10^{-4}
−0.9913	−0.9899	−0.9907	−0.3296	−0.6603	10^{16}	0.2320	2.220×10^{-4}
−0.9945	−0.9930	−0.9939	−0.3306	−0.6624	10^{17}	0.2390	1.461×10^{-4}
−0.9957	−0.9940	−0.9950	−0.3309	−0.6631	10^{18}	0.2380	1.188×10^{-4}
−0.9964	−0.9945	−0.9956	−0.3310	−0.6635	10^{19}	0.2320	1.039×10^{-4}

Table 3. Values of the parameters $\eta^{(1)}$ and $\eta^{(2)}$ and the ratio $|\eta^{(2)}/\eta^{(1)}|$ for the class (A) solutions.

M_U (GeV)	M_G (GeV)	$\sin^2\theta_W$	$\eta^{(1)}$	$\eta^{(2)}$	$\|\eta^{(2)}/\eta^{(1)}\|$
	10^{17}		-4.056	-1.005×10^2	24.778
10^{15}	10^{18}	0.2305	-40.568	-1.005×10^4	2.477×10^2
	10^{19}		-405.68	-1.005×10^6	2.477×10^3
	10^{18}		-3.077	-57.564	18.707
10^{16}		0.232			
	10^{19}		-30.774	-5.756×10^3	1.870×10^2
10^{17}	10^{19}	0.239	-2.504	-38.00	15.1757

Table 4. Same as Table 3, but for class (B) solutions satisfying criteria (i) and (ii), or (i) and (iii) as described in the text.

M_G (GeV)	M_U (GeV)	$\sin^2\theta_W$	α_G	$\eta^{(1)}$	$\eta^{(2)}$	$\lvert\eta^{(2)}/\eta^{(1)}\rvert$
10^{17}	10^{16}	0.232	2.22×10^{-4}	-0.3077	-0.5756	1.870
	10^{17}	0.239	1.461×10^{-4}	-0.0250	-0.0038	0.1520
10^{18}	10^{17}	0.239	1.461×10^{-4}	-0.2504	-0.3800	1.5175
	10^{18}	0.238	1.188×10^{-4}	-0.0226	-0.0030	0.1327
10^{19}	10^{18}	0.238	1.188×10^{-4}	-0.2260	-0.3094	1.369
	10^{19}	0.232	1.039×10^{-4}	-0.0211	-0.0027	0.1279

respectively. Further, the combination $(M_U, M_G)=(10^{16}, 10^{18})$GeV, $(10^{17}, 10^{19})$GeV, and $(10^{16}, 10^{19})$GeV, correspond to the basic parameter values $(\eta_1, \eta_2)\simeq(-3.07, -57.56)$, $(-2.50, -38)$, and $(-30.7, -5.7\times10^3)$, respectively. Thus, from the above value, we can see that if M_U is lower than M_G by more than one order, the magnitude of the basic parameters become large and so also their ratio $|\eta^{(2)}/\eta^{(1)}|$, which are not allowed in order to make the Lagrangian some sense. Also, the values of $(\eta^{(1)}, \eta^{(2)})$ and their ratio $|\eta^{(2)}/\eta^{(1)}|$ are further magnified if $M_G \sim M_{pl}$ and $M_U \sim 10^{15}$–10^{16}GeV. The reason for such large values of the ratio can be seen from Eq.(2.11), which gives $|\eta^{(2)}/\eta^{(1)}| \propto M_G/M_U$. Such large values of the ratio clearly violate criteria (ii) and (iii) when M_U is several orders less than M_G. Thus, from the above facts, we conclude that if the addition of five- and six- dimensional operators are going to make some sense, the solutions with M_U several order smaller than M_G are not acceptable. Again for the values of $M_G \sim M_{pl}$, as used by Hill[28], the lower compactification masses, $M_U \sim 10^{15}$–10^{17}GeV, are clearly ruled out.

(B) $M_U \sim (0.1-1)M_G$

In constrast to class (A) solutions, Table 3 contains other solutions which satisfy either $|\eta_2/\eta_1| \sim 1$ or $|\eta_2/\eta_1| \sim 0.1$. When $M_U \sim 0.1M_G$, we find that the criteria (i) and (ii) are satisfied. For example, for $M_U \sim 0.1\ M_G \sim 10^{16}$, 10^{17}, and 10^{18}GeV, the values of the basic parameters correspond to $(\eta_1, \eta_2)\simeq(-0.307, -0.575)$, $(-0.25, -0.38)$, and $(-0.226, -0.309)$ and the corresponding ratio $|\eta_2/\eta_1|\simeq1.87$, 1.51, and 1.36, respectively. Thus, combining criteria (i) and (ii) yields allowed values of high unification masses $M_U \sim 0.1M_G$. But, as mentioned earlier, criterion (iii), an alternative to (ii), could also be possible, if the nonrenormalizable terms act like perturbation on the normalizable Lagrangian. We can see that such solutions satisfying criteria (i) and (iii) *are possible on the minimal SU(5) model by including* gravity-induced effects. For example, high unification mass $M_U=M_G=10^{17}$, 10^{18}, and 10^{19}GeV correspond to $(\eta^{(1)},\eta^{(2)}) \simeq(-0.025,-3.8\times10^{-3})$, $(-0.0226,-3.09\times10^{-3})$, and $(-0.0211, -2.7\times10^{-3})$, and the ratio $|\eta^{(2)}/\eta^{(1)}| \simeq 0.152$, 0.133, and 0.128 respectively. For every allowed value of $M_U \sim 0.1M_G$, satisfying criteria (i) and (ii), or $M_U \sim M_G$, satisfying (i) and (iii), the value of $\sin^2\theta_W$ is found to be in the range 0.22–0.24. Some of our allowed solutions belonging to class (B) and satisfying criteria (i) and (ii), or (i) and (iii) are presented in Table 4. From Table 4, we can see that for the possible values of $M_G=10^{17}-10^{19}$GeV, the high values of unification mass are found to cover a wider range, $M_U \sim (0.1-1)\ M_G \simeq 10^{16}-10^{19}$GeV. Another interesting new feature of the present solutions is the smallness of the bare GUT coupling constant, $\alpha_G \sim 10^{-4}$, as compared to all earlier results existing in the literature. Such a small numerical value of α_G can be understood from Eqs.(2.14c)–(2.14d) and (2.20a) by noting that

$$\alpha_G=(67+25\varepsilon_{3C}+42\varepsilon_{2L})/(11\alpha_S^{-1}+21\alpha^{-1}) \ , \qquad (2.21)$$

where the numerator tends to be small as $\varepsilon_{3C}\simeq \varepsilon_{2L} \longrightarrow -1$. The small value of α_G decreases the proton decay rate resulting in a very significant increase in τ_p. Thus, according to our present observations, the enhancement in τ_P occurs due to two factors: (a) largeness of the unification mass, and (b) smallness of the GUT coupling. The minimum value of τ_P corresponding to the lowest allowed $M_U \simeq 10^{16}$GeV, turns out to be $\tau_P \sim 10^{38}$yr, where a factor of 10^4 enhancement due to smallness of α_G and a factor of $10^{\pm 3}$ uncertainty due to proton decay matrix element and the QCD parameter[34] have been included. In the present case, for the most general expectation value of the compactification scale $M_G \sim M_{Pl} = 10^{19}$GeV, the GUT does not seem to have unification significantly below $M_U \sim 10^{18}$GeV.

It is evident from Table 2 that $\varepsilon^{(2)}/\varepsilon^{(1)} \sim 2$, where $\varepsilon^{(n)}$ is related to the basic parameter $\eta^{(n)}$ by Eq.(2.11). This might give the impression that the expression for ε_Y, ε_{2L}, and ε_{3C} expressed in Eq.(2.8) are not converging. But, in this chapter, we have used only the first two terms corresponding to the five- and six-dimensional operators out of a large number of terms in the series in Eq.(2.8) to show they fully account for the available data on $\sin^2\theta_W$, and large values of M_U. This implies that, so far as the available values of $\sin^2\theta_W$ and allowed values of τ_P are concerned, $\varepsilon^{(n)} \sim 0$ for $n > 2$, thus guaranteeing convergence of the series and the self-consistency of the method for $M_U \sim (0.1-1)M_G$.

Another way of looking into the convergence of expansions is the following. Since the first two terms are capable of explaining the available experimental values of $\sin^2\theta_W$, for M_U in the range 10^{16}–10^{19}GeV, it is certainly true that at least the same values of $\sin^2\theta_W$ and M_U are possible by taking large number of terms such that $|\varepsilon^{(n+1)}| \ll |\varepsilon^{(n)}|$, for $n \geq 2$; this guarantees convergence of the series and self-consistency of the method used.

II.5. Summary and Conclusion

The minimal SU(5) model is ruled out as it predicts proton lifetime about one order of magnitude less than the observed experimental lower limit[19] and $\sin^2\theta_W \simeq 0.21$ which is much less than the present world average[2] $\sin^2\theta_W = 0.230 \pm 0.005$. By introducing five-dimensional operators, induced by gravity, scaled by the compactification mass although Hill[28] obtained quite lower values of $\sin^2\theta_W$, and, hence, ruled out any modifications with $M_G \sim M_{pl}$, Shafi and Wetterich[27] found that the minimal GUT could be saved by such operator provided M_G is 2 order less than $M_{pl} = 10^{19}$GeV, which is atleast possible within certain Kaluza-Klein-type theories[35]. But, as we have noted here, these solutions can be consistent with experiments provided $\sin^2\theta_W < 0.199$ which seems to disagree with the present world average[2].

In order to obtain the improved solutions in the minimal SU(5) model by spontaneous compactification effects, we have investigated with five-and six-dimensional operators, which are allowed in principle, atleast, and in the same spirit. Out of atleast, three different possible forms for the six-dimensional operator, we have chosen only one form for the sake of simplicity and convenience and obtained modifications to GUT predictions within the constraint expressed by Eq.(2.20a). Although our computation in Table 2 indicates $\sin^2\theta_W \simeq 0.23-0.24$, we have checked that with slight change of ε_{3C} and ε_{2L}, the allowed range is $\sin^2\theta_W \simeq 0.22-0.24$. In Table 2 it seems, as if all numerical solutions with M_U= 10^{15}-10^{19} GeV and $\sin^2\theta_W$=0.22-0.24 are allowed. But when we computed the basic parameter $\eta^{(1)}$ and $\eta^{(2)}$ and their ratio $|\eta^{(2)}/\eta^{(1)}|$, where $\eta^{(1)}$ ($\eta^{(2)}$) occurs as the coefficient of the five-(six-) dimensional operators ,we found that $|\eta^{(2)}|\gg|\eta^{(1)}|$ for those solutions for which $M_U \leq 10^{-2}M_G$, with M_G=10^{17}-10^{19}GeV. As the Lagrangian does not make any sense with such parameters, the corresponding solutions with low unification masses $M_U \simeq 10^{15}$GeV are ruled out. This criterion, ruling out $M_U \simeq 10^{15}$GeV is found to be strongly vaild if the compactification occurs at the most generally acceptable scale, M_G=M_{pl}. Next, we examined the other solutions following the criteria (i)-(ii), which are stated in Sec.II.4 and found such allowed solutions are possible when $M_U \sim (0.1-1)M_G \sim 10^{16}$-$10^{19}$GeV with $\sin^2\theta_W \simeq 0.22$-0.24 for every M_U. In such a case the values of the η parameters are found to be small and the ratio $|\eta_2/\eta_1| \sim 1$ for those solutions with $M_U \sim 0.1M_G$, but $|\eta_2/\eta_1| \sim 0.1$ for others with $M_U \sim M_G$. Although such parameters with $|\eta_2/\eta_1| \sim 1$ are generally expected in unified gauge theories, the other values with $|\eta_2/\eta_1| \sim 0.1$ suggest that the successive terms containing higher-dimensional operators might be acting as perturbation upon the renormalizable Lagrangian. Using the most general value, $M_G \simeq M_{pl}$, we found that the solutions with $M_U \simeq 10^{15}$-10^{17}GeV are ruled out and the gravity-induced effects permit only $M_U \sim 10^{18}$-10^{19}GeV. For the first time, we found a grand unified theory with a GUT coupling constant as small as $\alpha_G \sim 10^{-4}$. In our case, the enhancement of the proton lifetime occurs due to two factors (a) largeness of the unification mass and (b) smallness of the GUT coupling constant. Thus, if the addition of five-and six- dimensional operators to the GUT Lagrangian is going to make sense, the predictions of minimal SU(5) with an unstable proton and $\sin^2\theta_W < 0.215$ are modified to be consistent with an extremely stable proton ($\tau_p > 10^{38}$yr) and $\sin^2\theta_W \simeq 0.22-0.24$.

CHAPTER III

SPONTANEOUS COMPACTIFICATION EFFECTS, LOW-ENERGY SIGNATURE OF QUARK-LEPTON UNIFICATION, AND SMALL NEUTRINO MASSES IN SO(10)

III.1. Introduction

Including gravity-induced contributions, although the SU(5) model can be consistent with a very stable proton[36] and the accepted values of $\sin^2\theta_W \simeq 0.22-0.24$, there exists a grand desert in between the electroweak unification scale $M_W \simeq 10^2$GeV and the grand unification scale $M_U \geq 10^{15}$GeV. However, if some new physics are discovered between these energy gaps in foreseeable future, the SU(5) model can be ruled out. Also neutrino is massless in the SU(5) model discussed in Chapter II. Any experimental evidence for non vanishing neutrino mass would also question the validity of the model. A very attractive GUT, which can provide new physics populating the grand desert is the SO(10) model[4]. Some of the attractive features of the SO(10) GUT compare to many other GUT's are described below. It is the minimal left-right-symmetric extension of SU(5), and contains all known fermions (plus the right-handed neutrino) of one generation in a single spinorial representation. It can explain the origins of parity-(P) and CP-violations which arise spontaneously as a result of symmetry breaking. It can provide massive neutrinos over a wide range of values in order to solve the solar neutrino puzzle via the so called Mickhayev-Smirnov-Wolfenstein (MSW) mechanism[37] or eV-keV-MeV energy spectrum of neutrinos, near their present experimental limits[14]. Using the mechanism of decoupling P- and $SU(2)_R$-

breakings[21], it is posible to have a natural solution to the domain-wall problem[24]. The introduction of one or more intermediate symmetries in SO(10) promises experimental verification of interesting theoretical ideas such as the quark-lepton unification based upon $SU(4)_C$ and left-right symmetry[22], besides explaining the observed proton stability.

After the discovery of SO(10) model[4], several attempts have been made to obtain a low $SU(4)_C$-breaking scale (M_C) in the presence of the gauge groups $SU(2)_L \times SU(2)_R \times SU(4)_C (\equiv G_{224})$ or $SU(2)_L \times U(1)_R \times SU(4)_C$ ($\equiv G_{214}$). Such a low $M_C \sim 10^5-10^6$GeV, is expected to provide the low-energy signature of quark-lepton unification through rare-kaon decays ($K_L \longrightarrow \bar{\mu}e$) and small Majorana neutrino masses. In addition the existence of such scale for G_{224}-breaking would predict the experimental observation of phenomenon such as $n-\bar{n}$ oscillation[38]. Due to the nonavailability of the free neutron sources, as the neutron oscillation experiments are very difficult, it might be useful to search a GUT where the consequences of $SU(4)_C$-breaking can be testified by a relatively easier class of experiments, such as the rare-kaon decays[39] only. Even with two intermediate symmetries[65] in the SO(10) GUT,

$$SO(10) \xrightarrow{M_U} G_{214} \xrightarrow{M_C} G_{2113} \xrightarrow{M_R^0} G_{st} \xrightarrow{M_W} G_{13} \,, \qquad (3.1)$$

where $G_{2113} \equiv SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C$, it has not been possible to obtain $M_C = 10^5-10^6$GeV, for the presently accepted values[2] of $\sin^2\theta_W = 0.230 \pm 0.005$. Although, two or more intermediate symmetries populating the grand desert provide possibilities of richer physical structure, the prediction with single intermediate symmetry are very appealing because of the minimal nature of the GUT scenario.

In this Chapter[40] we note that the conventional SO(10) GUT, with the single G_{214} intermediate symmetry, is ruled out as it predicts a proton lifetime lower than the Irvine-Michigan-Brookhaven (IMB) limit[19] for the $p \longrightarrow e^+\pi^o$ mode. On the other hand for the first time we find that, when the effects of a five-dimensional operator[27,28] is included in the GUT Lagrangian corresponding to the single intermediate symmetry, in the chain

$$SO(10) \xrightarrow[M_U]{\underline{54}+\underline{45}} G_{214} \xrightarrow[M_C]{\underline{126}} G_{st} \xrightarrow[M_W]{\underline{10}} G_{13}\,, \qquad (3.2)$$

it is possible to have $M_C \sim 10^5$–10^{11}GeV, $M_U \sim 10^{15}$–10^{17}GeV, for $\sin^2\theta_W \simeq 0.22$–$0.24$. For $M_C \sim 10^5$–10^6GeV, corresponding to observable rare-kaon decays[39], some of the Majorana neutrino masses could be measured in the laboratory. In this chain the proton lifetime is found to be significantly larger than the IMB limit[19] ($\tau_p(p \longrightarrow e^+\pi^o) \geq 3\times10^{32}$yr) depending upon the values of $\sin^2\theta_W$ and M_C. For still larger values of $M_C \sim 10^7$–10^{11}GeV, corresponding to undetectable rare-kaon decays, Majorana neutrino masses decrease further and the proton lifetime also decreases, saturating the IMB limit for $M_C \sim 10^{11}$GeV.

This Chapter is organized in the following manner. In Sec.III.2 we derive modified GUT-boundary conditions, formulas for unification mass, electroweak mixing angle, and the GUT coupling constant including the effects of the five-dimensional operator.

In Sec.III.3 we discuss the solutions with single G_{214} intermediate symmetry without gravity-induced effect. In Sec.III.4 we report our new results with single G_{214} intermediate symmetry by including the effects of the five-dimensional operator. Majorana neutrino masses of various ranges have been predicted in Sec.III.5. A brief summary and conclusion of the Chapter are stated in Sec.III.6.

III.2. Derivation of the formulas for unification mass, electroweak mixing angle, and GUT coupling

In this section we derive the formulas for the unification mass (M_U), electroweak mixing angle ($\sin^2\theta_W$), and grand unification coupling (α_G). For chain (3.2), it is usually stated that the vacuum expectation value (VEV) of the Higgs field $\chi(1,0,1) \subset \underline{45} \subset SO(10)$, where the transformation property of χ is under G_{214}, might achieve the spontaneous symmetry breaking (SSB) at the first stage. But, according to the observations made by Yasue[41] several years ago, both $\underline{54}$ and $\underline{45}$ are needed to break $SO(10) \longrightarrow G_{214}$. In this case, as $\underline{45}$ is antisymmetric, it does not contribute to the gravity-induced corrections through the five-dimensional operator; but the necessary presence of $\underline{54}$ is sufficient to induce significant modifications to the GUT predictions through the five-dimensional operators. Following the similar techniques of Refs.27 and 28, the nonrenormalizable five-dimensional operator can be written as

$$\alpha_{NR} = -\frac{\eta}{2M_G} \mathrm{Tr}\,[F_{\mu\nu}\Phi_{(54)}F^{\mu\nu}], \qquad (3.3)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\,[A_\mu, A_\nu],$$
$$(A_\mu)^a_b = A^i_\mu (\lambda_i)^a_b,$$
$$\mathrm{Tr}(\lambda_i \lambda_j) = (1/2)\delta_{ij} \,. \qquad (3.4)$$

In Eqs. (3.3)-(3.4) A_μ's are the gauge field matrices, λ_i's are the SO(10) generators, η is an unknown parameter, M_G is the compactification scale, and $\Phi_{(54)}$ is the scalar field $\underline{54} \subset SO(10)$. When $\Phi_{(54)}$ acquires a nonzero VEV

$$\langle\Phi_{(54)}\rangle = \frac{\Phi_o}{(30)^{1/2}} \mathrm{diag}(1,1,1,1,1,1,-3/2,-3/2,-3/2,-3/2), \qquad (3.5)$$

the presence of the nonrenormalizable term (3.3) modifies the usual gauge kinetic energy terms,

$$\alpha_R = (-1/2)\ \mathrm{Tr}(F_{\mu\nu}\ F^{\mu\nu}), \qquad (3.6)$$

of the $SU(2)_L$, $U(1)_R$, and $SU(4)_C$ gauge fields which can be written as

$$\alpha = -\frac{1}{2}\left[1-\frac{3}{2}\varepsilon\right] \mathrm{Tr}(F^{(2L)}_{\mu\nu}\ F^{(2L)\mu\nu}) - \frac{1}{4}\left[1-\frac{3}{2}\varepsilon\right] F^{(1R)}_{\mu\nu} F^{(1R)\mu\nu}$$
$$-\frac{1}{2}(1+\varepsilon)\mathrm{Tr}(F^{(4C)}_{\mu\nu}\ F^{(4C)\mu\nu}) \ , \qquad (3.7)$$

where

$$\varepsilon = \frac{1}{(30)^{1/2}} \frac{\eta\ \Phi_o}{M_G} \,. \qquad (3.8)$$

The superscripts (2L), (1R), and (4C) stand for the $SU(2)_L$, $U(1)_R$, and $SU(4)_C$, respectively. Now rescaling of the gauge fields changes their coupling constants as

$$g^2_{2L}(M_U) \longrightarrow g^2_{2L}(M_U)\left[1-\frac{3}{2}\varepsilon\right],\ g^2_{1R}(M_U) \longrightarrow g^2_{1R}(M_U)\left[1-\frac{3}{2}\varepsilon\right],$$
$$g^2_{4C}(M_U) \longrightarrow g^2_{4C}(M_U)\ (1+\varepsilon), \qquad (3.9)$$

where $g_{2L}(M_U)$, $g_{1R}(M_U)$, and $g_{4C}(M_U)$ denote the coupling constants of $SU(2)_L$, $U(1)_R$, and $SU(4)_C$, respectively, without gravity-induced corrections. As the three coupling constants are equal at the scale $\mu \geq M_U$ for achieving unification, the GUT condition is imposed through the equations

$$g_{2L}^2(M_U)\left[1-\frac{3}{2}\varepsilon\right]=g_{1R}^2(M_U)\left[1-\frac{3}{2}\varepsilon\right]=g_{4C}^2(M_U)\ (1+\varepsilon)=g_o^2, \quad (3.10)$$

where g_o is the bare-GUT coupling constant. The one-loop RGE's[32] for the chain (3.2) are given by

$M_W \leq \mu \leq M_C$:

$$\frac{1}{\alpha_i(M_W)}=\frac{1}{\alpha_i(\mu)}+\frac{a_i}{2\pi}\ln\frac{\mu}{M_W}, \quad i=Y,\ 2L,\ 3C, \qquad (3.11)$$

$M_C \leq \mu \leq M_U$:

$$\frac{1}{\alpha_j(M_C)}=\frac{1}{\alpha_j(\mu)}+\frac{a'_j}{2\pi}\ln\frac{\mu}{M_C}, \quad j=2L,\ 1R,\ 4C, \qquad (3.12)$$

where $a_i(a'_j)$ is the one-loop coefficient in the lower (higher) scale. Confining to the minimal fine-tuning conditions the Higgs scalars, needed for SSB in the two different mass ranges are $M_W \leq \mu \leq M_C$, $\Phi(1,2,1)$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$, $M_C \leq \mu \leq M_U$, $\Phi(1,1/2,1)$ and $\Delta_R(1,1,10)$ under $SU(2)_L \times U(1)_R \times SU(4)_C$. With the minimal number of Higgs scalars and the three fermion generations, the values of the one-loop coefficients are $a_Y=41/10$, $a_{2L}=-19/6$, $a_{3C}=-7$, $a'_{2L}=-19/6$, $a'_{1R}=15/2$, and $a'_{4C}=-29/3$.

Using Eqs. (3.10)-(3.12) and the combinations $\alpha^{-1}(M_W)-(8/3)\alpha_{3C}^{-1}(M_W)$, $\alpha^{-1}(M_W)-(8/3)\ \alpha_{2L}^{-1}(M_W)$, $\alpha^{-1}(M_W)=(5/3)\alpha_Y^{-1}(M_W)+\alpha_{2L}^{-1}(M_W)$, we obtain the following equations for the unification mass, $\sin^2\theta_W$, and the GUT coupling constant ($\alpha_G = g_o^2/4\pi$):

$$\ln\frac{M_U}{M_W}=\frac{6\pi}{71-74\varepsilon}\left[\frac{1}{\alpha}-\frac{8}{3\alpha_S}+\left\{\frac{7}{3\alpha_S}+\frac{1}{\alpha}\right\}\varepsilon\right]+\left[\frac{4-36\varepsilon}{71-74\varepsilon}\right]\ln\frac{M_C}{M_W}, \qquad (3.13)$$

$$\sin^2\theta_W=\frac{1}{71-74\varepsilon}\left[\left\{\frac{39}{2}+(19-\frac{38}{3}\varepsilon)\frac{\alpha}{\alpha_S}-53\varepsilon\right\}-\frac{\alpha}{\pi}\left\{\frac{245}{3}-170\varepsilon\right\}\ln\frac{M_C}{M_W}\right], \tag{3.14}$$

$$\frac{1}{\alpha_G}=\frac{1}{71-74\varepsilon}\left[\frac{29}{\alpha}-\frac{19}{3\alpha_S}-\frac{226}{3\pi}\ln\frac{M_C}{M_W}\right], \tag{3.15}$$

where $\alpha_S=\alpha_{3C}(M_W)=g^2_{3C}(M_W)/4\pi$ and $\alpha(M_W)=e^2(M_W)/4\pi$.

III.3. Solutions with G_{214} intermediate symmetry without five-dimensional operator

From Eq.(3.10), it is clear that $\varepsilon=0$ corresponds to the absence of gravity-induced effects. Substituting $\varepsilon = 0$ in Eqs.(3.13)-(3.15), we obtain

$$\ln\frac{M_U}{M_W}=\frac{6\pi}{71}\left\{\frac{1}{\alpha}-\frac{8}{3\alpha_S}\right\}+\frac{4}{71}\ln\frac{M_C}{M_W}, \tag{3.16}$$

$$\sin^2\theta_W=\frac{1}{71}\left[\left\{\frac{39}{2}+19\frac{\alpha}{\alpha_S}\right\}-\frac{245\alpha}{3\Pi}\ln\frac{M_C}{M_W}\right], \tag{3.17}$$

$$\frac{1}{\alpha_G}=\frac{1}{71}\left[\frac{29}{\alpha}-\frac{19}{3\alpha_S}-\frac{226}{3\pi}\ln\frac{M_C}{M_W}\right]. \tag{3.18}$$

Now using $\alpha_s=0.1088$ ($\Lambda_{\overline{MS}}$ = 160 MeV), $\alpha^{-1}(M_W)$ = 127.54, we compute *the numerical solutions for* M_C, M_U, *and* $\sin^2\theta_W$. *Some of our* solutions are presented in Table 5. It is clear from Table 5 that with a purely renormalizable Lagrangian (3.6), the chain (3.2) yields a maximum $M_U\simeq3\times10^{14}$GeV for the allowed value of $\sin^2\theta_W$=0.22–0.24. For the maximum M_U, the corresponding proton lifetime $\tau_p\simeq10^{29\pm3}$yr, which is significantly less than the IMB limit[19]. Here the uncertainty $10^{\pm3}$ in τ_p arises due to the *uncertainties in the proton decay matrix element and the* QCD parameter[34]. Thus, purely renormalizable SO(10) model with single

Table 5. One-loop solutions for SO(10) with single intermediate symmetry, $SU(2)_L \times U(1)_R \times SU(4)_C$, in the absence of gravity-induced corrections.

M_C (GeV)	M_U (GeV)	$\sin^2\theta_W$
10^5	9.2×10^{13}	0.273
10^7	1.2×10^{14}	0.260
10^9	1.5×10^{14}	0.247
10^{11}	2.0×10^{14}	0.233
10^{13}	2.57×10^{14}	0.220
10^{14}	2.95×10^{14}	0.214

G_{214} intermediate symmetry is ruled out.

III.4. Solutions with G_{214} intermediate symmetry with five-dimensional operator

To see whether the effects of the five-dimensional operator improves the predictions, we compute the solutions using Eqs.(3.13)-(3.15) with the same input parameters as in Sec.II.3. In this case, interesting solutions are obtained for allowed regions for $M_C(\equiv 10^{n_C})$ and ε within the available experimental constraint (Refs.2 and 19) on M_U and $\sin^2\theta_W$. Some of our allowed solutions obtained with $\varepsilon > 0$ are presented in Tables 6 and 7 and Figs. 1-3. At first, Fig.1 is plotted using Eq.(3.13), and Fig.2 using Eq.(3.14). In Fig.1 the horizontal lines are the IMB and the Planck limits on the unification mass. The projection of the line PQ onto Fig.2 has been denoted as the IMB limit in the latter. The horizontal lines in Fig.2 represent the 2σ limits of the world average[2], $\sin^2\theta_W=0.230\pm0.005$. The projection of the Planck limit from Fig.1 onto Fig.2 does not provide any useful boundary for the allowed region. But, a much better limit exists[39] from the experimentally observed bounds on the rare-kaon decay mode, $K_L\to\bar{\mu}e$, corresponding to $M_C\geq 3\times10^5$GeV. Specifying the four sides of the quadrilateral in Fig.2 in this fashion, the allowed solutions are shown by the shaded area.

The numerical value of M_C, ε, M_U, $\sin^2\theta_W$, and α_G^{-1} are shown in Table 6 for $M_C=10^5-10^6$GeV and, in Table 7 for $M_C=10^7-10^{11}$GeV. For chain (3.2), we find that the modification due to five-dimensional operator permit $10^5\leq M_C\leq 10^{11}$GeV for allowed values of M_U and $\sin^2\theta_W$. For every M_C, the parameter ε and the

Table 6. Predictions for M_U, $\sin^2\theta_W$, and values of M_C, corresponding to observable rare-kaon decay, as a function of ε, in the presence of gravity-induced corrections, for the same chain as Table 5. The proton lifetime is for the $P \longrightarrow e^+\pi^0$ mode excluding uncertainties.

M_C (GeV)	ε	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_P (yr)
	0.10	1.3×10^{17}	0.225	54.56	4.8×10^{40}
10^5	0.09	5.9×10^{16}	0.230	53.93	2.0×10^{39}
	0.08	2.7×10^{16}	0.235	53.32	8.6×10^{37}
	0.09	6.0×10^{16}	0.224	53.08	2.1×10^{39}
	0.08	2.8×10^{16}	0.229	52.47	9.6×10^{37}
10^6	0.07	1.3×10^{16}	0.234	51.88	4.4×10^{36}
	0.06	6.3×10^{15}	0.239	51.30	2.4×10^{35}

Table 7. Same as Table 6, but for larger values of M_C.

M_C (GeV)	ε	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_P (yr)
	0.07	1.3×10^{16}	0.228	51.04	4.2×10^{36}
10^7	0.06	6.7×10^{15}	0.233	50.48	2.9×10^{35}
	0.05	3.3×10^{15}	0.238	49.92	1.7×10^{34}
	0.06	7.2×10^{15}	0.227	49.65	3.8×10^{35}
10^8	0.05	3.6×10^{15}	0.232	49.10	2.3×10^{34}
	0.04	1.8×10^{15}	0.236	48.57	1.4×10^{33}
10^9	0.05	3.8×10^{15}	0.225	48.28	2.8×10^{34}
	0.04	1.9×10^{15}	0.230	47.75	1.7×10^{33}
10^{10}	0.04	2.1×10^{15}	0.223	46.94	2.4×10^{33}
10^{11}	0.03	1.2×10^{15}	0.221	45.63	2.0×10^{32}

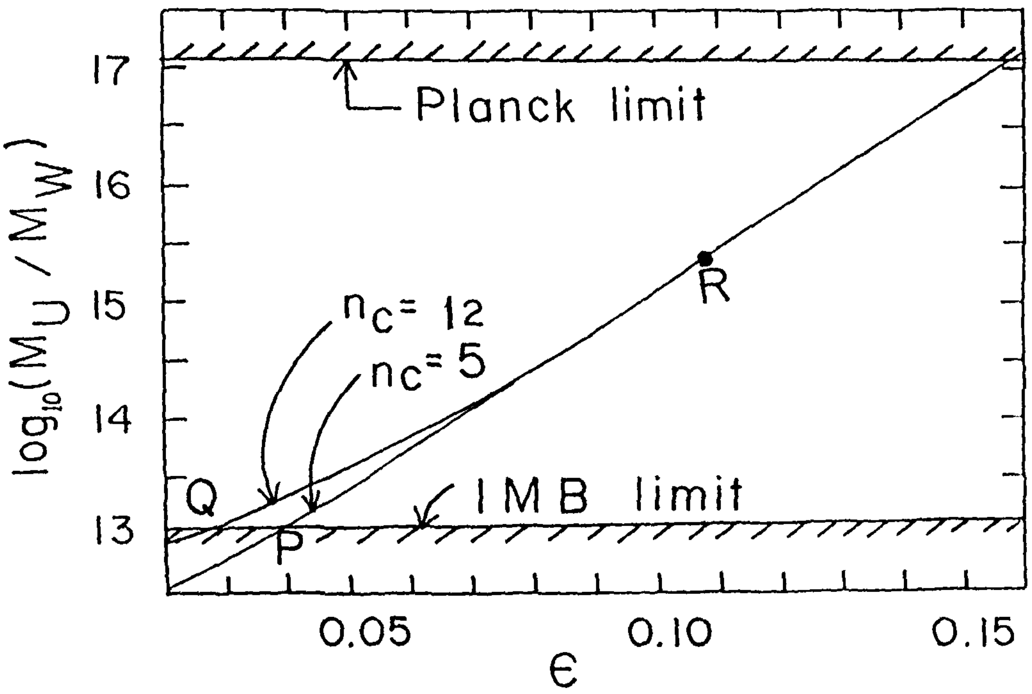

Fig.1. Solutions of one-loop renormalization-group equations for M_U as a function of ε, and for $M_C = 10^{n_c}$, n_c = 5-12. The horizontal lines are the IMB (lower) and the Planck (upper) limits. The allowed upper limit, for n_c = 5 shown as point R is obtained as the projection of the corresponding point in Fig.2.

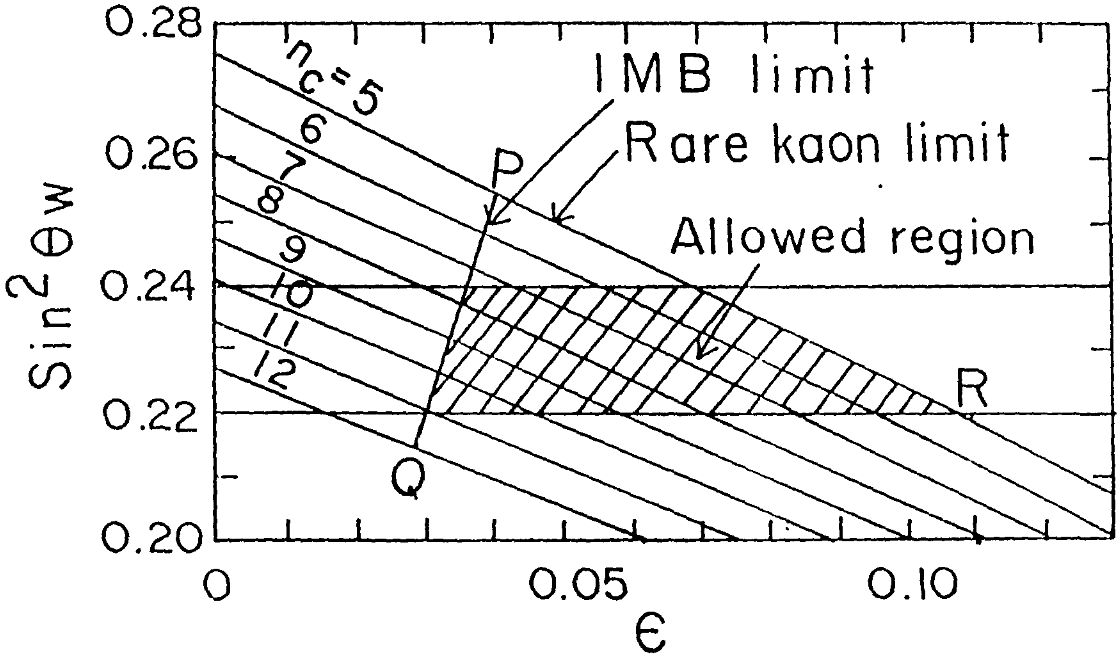

Fig.2. Solutions of one-loop renormalization-group equations for $\sin^2\theta_W$ as a function of M_C and ε.

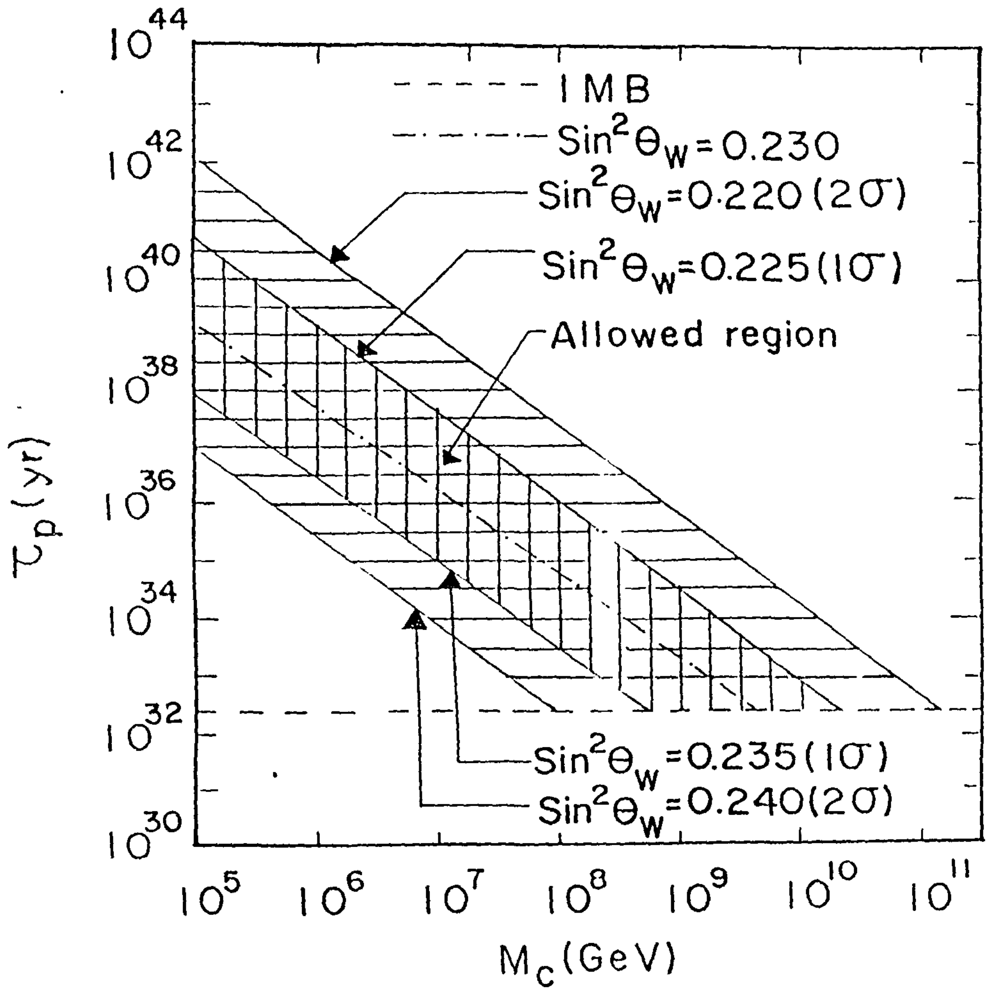

Fig.3. Predictions on proton lifetime for the $p \rightarrow e^+\pi^0$ mode with $\Lambda_{\overline{MS}}$ = 160 MeV, as a function of M_C and $\sin^2\theta_W$. The solid lines are for values of $\sin^2\theta_W$ corresponding to 1σ and 2σ limits. The dot-dashed line is for $\sin^2\theta_W$ = 0.230. The IMB limit is shown by the dashed line.

unification mass M_U are allowed over a wider range depending upon the 2σ or 1σ limit of $\sin^2\theta_W$. The solutions with smaller (larger) values of $\sin^2\theta_W$ are associated with larger (smaller) values of M_U and τ_P. In Table 6, we have presented the solutions for the values of $M_C \sim 10^5$–10^6GeV which predict rare-kaon decays to be observable for any value of $\sin^2\theta_W$ in the range of 0.22–0.24. The highest value of $M_U \simeq 3\times10^{17}$GeV is possible for $M_C=10^5$GeV and $\sin^2\theta_W=0.22$. This has been shown by the point R in Fig.1 which has been obtained by the projection of the corresponding point in Fig.2.

It is clear from Table 6 and 7 that for the increase in the value of M_C, the value of M_U for a fixed value of $\sin^2\theta_W$, and the proton lifetime for the $p \longrightarrow e^+\pi^0$ mode decreases. This has been shown in Fig.3 for the 1σ and 2σ boundaries and the central value of $\sin^2\theta_W=0.230$. For $M_C > 10^8$GeV, the allowed range of τ_P also decreases being restricted by the IMB limit[19] from below. The IMB limit[19] is found to be saturated nearly at $M_C \sim 10^{10}(10^{11})$GeV if the values of $\sin^2\theta_W$ is allowed to be 0.225 (0.220).

The order of magnitude of the compactification scale M_G that makes these gravity-induced corrections important can be calculated by using $\eta=(40\pi\alpha_G)^{1/2}\varepsilon M_G/M_U$. Our estimation depends, crucially, on the assumption that $|\eta| \sim 1$ as in the Shafi and Wetterich[27] case. Solutions having $\varepsilon \simeq 0.03$–0.05 are found to be associated with lower values of the unification mass, $M_U \sim 10^{15}$GeV which require M_G to be nearly 2 orders of magnitude smaller than M_{Pl}. The other class of solutions found in this model are associated with $\varepsilon \simeq 0.07$–0.10, and $M_U \sim 10^{16}$–10^{17}GeV which require $M_G \sim 10^{17}$–10^{18}GeV. In particular the observable predictions for rare-kaon decay corresponding to $M_C \simeq 10^5-10^6$ GeV are found to be possible with $\sin^2\theta_W \simeq 0.22-0.23$, $\varepsilon \simeq 0.1$, and $M_U \sim 10^{17}$GeV, requiring $M_G \sim 10^{18}$GeV. This scale is generally expected from the Kaluza-Klein- type compactification, where $M_G = M_{pl}/2\pi \simeq 1.6\times10^{18}$GeV. If, on the other hand η is allowed to be $|\eta| \sim 0.1(10)$, our estimation would require M_G 1 order less (more) for every value of ε. For example, with $M_C \sim 10^5$GeV, and $M_U \sim 10^{17}$GeV, consistency of the solutions with $\varepsilon \simeq 0.1$ requires $M_G \sim 10^{17}(10^{19})$GeV, if $|\eta| \sim 0.1(10)$, instead of $|\eta| \sim 1$.

III.5. Predictions on Majorana neutrino masses

At the second stage of the chain (3.2), the scalar representation $\underline{126} \subset SO(10)$ is used to break the intermediate gauge symmetry spontaneously to the standard group. The scalar representation $\underline{126}$ contains right-handed $SU(2)_R$ triplet $\Delta_R(1,3,10)$ under $SU(2)_L\times SU(2)_R\times SU(4)_C$, which carries 2 units of lepton number. Therefore,when VEV is given to Δ_R, lepton number is broken by 2 units, as a result of which Majorana neutrino masses are generated at this stage fulfilling the formula[16,17]

$$m_{\nu_i} \simeq m_i^{D^2}/M_C, \qquad i=1,\ 2,\ 3, \qquad (3.19)$$

where m_{ν_i} is the neutrino mass of the ith generation. Gell-Mann, Ramond, and Slansky[16] have identified the Dirac mass m_i^D with the up quark mass($m_1^D = m_u$, $m_2^D = m_c$, $m_3^D = m_t$), where as Mohapatra and Senjanovic[17] and others have taken the Dirac mass as the charged lepton mass ($m_1^D = m_e$, $m_2^D = m_\mu$, $m_3^D = m_\tau$) of the corresponding generation. Using quark masses, $m_u = 5$ MeV, $m_c = 1.25$GeV, and $m_t \simeq 100$GeV in formula (3.19), the Majorana neutrino masses for different generations corresponding to the allowed range of $M_C \simeq 10^5-10^{11}$GeV are given by

$m_{\nu_e} \sim (2.5\times10^{-7}-0.25)\,eV$, $\quad m_{\nu_\mu} \sim (1.5\times10^{-2}eV-15.6keV)$, and

$$m_{\nu_\tau} \sim (100eV-100MeV), \tag{3.20}$$

where the lower (upper) limit corresponds to $M_C=10^{11}\,(10^5)\,GeV$. But the same formula with the charged lepton masses, $m_e=0.51$ MeV, $m_\mu=106MeV$, $m_\tau=1.78$ GeV, and the same values of $M_C \simeq 10^5-10^{11}GeV$ provides

$m_{\nu_e} \sim (2.6\times10^{-9}-2.6\times10^{-3})\,eV$, $\quad m_{\nu_\mu} \sim (1.1\times10^{-4}-112)\,eV$, and

$$m_{\nu_\tau} \sim (3.2\times10^{-2}eV-31.7keV). \tag{3.21}$$

Out of these, the masses for ν_μ and ν_τ obtained in Eqs.(3.20) and (3.21) are measurable by laboratory experiments. However, the mass of ν_μ and ν_τ corresponding to $M_C \simeq 10^5GeV$ clearly violate the cosmological bound[42] which states that $\sum_{i=e,\mu,\tau} m_{\nu_i} \leq 65eV$, where the sum is over the stable and light neutrino species. One possible way to evade the cosmological bound is to make these ν_μ and ν_τ unstable with respect to Majoron emission[43] which is a massless scalar carrying 2 units of lepton number, and is created by introducing an additional $U(1)_l$-global symmetry (l=lepton number), and breaking it spontaneously at a scale $M \gg M_W$.

III.6. Summary and Conclusion

In the absence of gravity-induced corrections, the SO(10) GUT with single G_{214} intermediate symmetry is ruled out as it predicts proton lifetime significantly below the IMB limit[19]. However, we found that such a model can provide a stable proton ($\tau_p \geq 3\times10^{32}yr$ for $p \longrightarrow e^+\pi^0$ mode) with the allowed values of $\sin^2\theta_W \simeq 0.22-0.24$, when five-dimensional operator, induced by gravity, scaled by the compactification mass, is included. Such higher-dimensional nonrenormalizable operators are usually present in theories with Kaluza-Klein type unification with gravity. With five-dimensional operator, the $SU(4)_C$-breaking is found to be permitted over a wide range $M_C \sim 10^5-10^{11}$GeV. Such allowed value of $M_C \sim 10^5-10^{11}$GeV provide two different values of Majorana neutrino masses (i)$m_{\nu_e} \sim (2.5\times10^{-7}-0.25)$eV, $m_{\nu_\mu} \sim (1.5\times10^{-2}eV-15.6keV)$, $m_{\nu_\tau} \sim$ (100eV$-$100MeV) and (ii) $m_{\nu_e} \sim (2.6\times10^{-9}-2.6\times10^{-3})$eV, $m_{\nu_\mu} \sim (1.1\times10^{-4}-112)$eV, $m_{\nu_\tau} \sim (3.2\times10^{-2}eV-31.7$ keV$)$, depending upon whether the Dirac masses are taken as quarks or charged lepton masses respectively. Out of them, although ν_e mass is too small compared to ν_μ and ν_τ masses, these masses could be measured by laboratory experiments depending upon M_C and the choice of the Dirac mass.

For the first time, with single G_{214} intermediate symmetry, we have obtained interesting SO(10) predictions, for the observable $SU(4)_C$-breaking by rare-kaon decay modes at low energies with $M_C \sim 10^5-10^6$GeV, and any value of $\sin^2\theta_W$ in the range 0.22-0.24. For such lower values of M_C, the proton lifetime is larger depending upon the value of $\sin^2\theta_W$. For larger values of $M_C > 10^8$GeV, the allowed range of τ_P decrease with the increasing M_C. For a fixed $\sin^2\theta_W$, τ_P decreases with M_C and the IMB limit[19] is saturated when $M_C \sim 10^{11}$GeV. The order of magnitude of the compactification scale, estimated in this model, is found to be in the range $10^{17}-10^{18}$GeV, unless the parameter in the nonrenormalizable term has the value $|\eta| \sim 10$, or larger.

In this model, as the mass of the right-handed neutral gauge boson $M_{Z_R} = M_C \geq 10^5$GeV, and that of the charged gauge bosons $M_{W_R^\pm} = M_U \sim 10^{15}-10^{17}$GeV, there is negligible contribution to the V+A structure of charged and neutral currents. Similarly the K_L-K_S mass difference and other CP-violating parameters have, essentially, the same prediction as the standard model. At low energies, this model does not seem to predict any other detectable signatures, except rare-kaon decays and neutrino masses.

CHAPTER IV

SPONTANEOUS COMPACTIFICATION EFFECTS IN SO(10) WITH LOW-MASS $W_R^{\pm}$-GAUGE BOSONS WITHOUT OBSERVABLE PARITY RESTORATION

IV.1. Introduction

In Chapter III we found[40] that the effect of a five-dimensional operator could bring down the scale of spontaneous symmetry breaking of the intermediate gauge group $SU(2)_L \times U(1)_R \times SU(4)_C$ ($\equiv G_{214}$) in the SO(10) model to a value ($M_C \simeq 10^5-10^6$ GeV) which might manifest in the observation of rare-kaon decay $K_L \longrightarrow \bar{\mu}e$ and small neutrino masses. Although with the single intermediate symmetry the second neutral gauge boson also occurs around the same mass ($M_C = M_{Z_R} \simeq 10^5-10^6$ GeV), it can be made lighter by breaking the G_{214} symmetry in two steps. In any case the $W_R^{\pm}$ mass in the model is as high as the unification mass ($M_{W_R^{\pm}} \simeq M_U$) leaving no testable signatures of left-right symmetry through V+A strucure of charged currents or spontaneous CP-violation through $K^0-\bar{K}^0$ and $B^0-\bar{B}^0$ mixings. In this Chapter[50] we show how to obtain low-mass $W_R^{\pm}$ and Z_R gauge bosons in SO(10) when the effects of a suitable five-dimensional operator is included in the GUT Lagrangian.

Several attempts have been made during the past years in left-right-symmetric (LRS) models[5,13] based upon the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ ($\equiv G_{2213}$) to obtain a low-mass right-handed scale (M_R) in GUT's after the spontaneous symmetry breaking (SSB) of the intermediate symmetry G_{2213P} including the left-right-symmetry (P=parity, when $g_{2L} = g_{2R}$) or excluding it ($g_{2L} \neq g_{2R}$). Such a low $M_R \sim$ TeV is expected to reveal new physics beyond the standard model, such as V+A structure of weak-charged and neutral currents, masses of the right-handed gauge bosons ($W_R^{\pm}$, Z_R), and neutrino masses etc. Following the conventional approach of breaking both P and $SU(2)_R$ at the same scale, such a model when embedded in a GUT like SO(10) yields $M_R = M_P \geq 10^{12}$GeV for $\sin^2\theta_W \simeq 0.230 \pm 0.005$ (M_P=P-violating scale), which rules out any possibility of observing low-energy signatures of right-handed currents. On the other hand if $M_R < 10^{12}$GeV, several cosmological difficulties such as inadequate baryon asymmetry of the universe[44], and the presence of undesirable domain walls[24] occur in the model. In order to avoid such difficulties, a new approach to left-right gauge theories have been suggested by Chang, Mohapatra, and Parida[21], where the breakings of P and $SU(2)_R$ are decoupled from each other. In such a case, it has been pointed out that there is an element of the SO(10) gauge group, called D-parity, where $D = \Sigma_{23}\Sigma_{67}$ (Σ_{ij}, i,j=1,2,..., 10 represent the totally antisymmetric generators of SO(10)), which plays the role similar to charge conjugation (C) or the parity (P) operator. Under special circumstances, D can be identified with C by demanding that $\Delta_L \xrightarrow{C} \Delta_R^*$ and with P when all couplings in the Lagrangian are real which takes $\psi_L \longrightarrow \psi_R$. But, in general it cannot be identified with either. For instance, D changes a fermion (quark or lepton) $\psi_L \longrightarrow (\psi^C)_L = (C\,\bar{\psi}^T)_L$ which has charge opposite to ψ_L but it has also opposite helicity. This is clearly different from the usual C operator of transforming a particle to its antiparticle. It has been found[21] that the neutral components $\chi_o(1,1,15) \subset \underline{45} \subset SO(10)$ under G_{2213} and $\eta(1,1,1) \subset \underline{210} \subset SO(10)$

under G_{224} are odd under D and hence, under P; but the corresponding components $\xi(1,1,15) \subset \underline{210}$ under G_{2213} and $\eta'(1,1,1) \subset \underline{54}$ under G_{224} are even. Thus, it is possible to descend down to G_{2213P} (G_{224P}) with P unbroken through the Higgs representation $\underline{210}$ ($\underline{54}$). Similarly, by assigning VEV along the direction odd under D, it is possible to break the discrete symmetry D and hence, P at the GUT scale without breaking $SU(2)_R$. In such cases P-and $SU(2)_R$-breakings are decoupled and one can descend down to G_{2213}(G_{224}) with P broken at the GUT scale through the Higgs representation $\underline{45}$ ($\underline{210}$). When such a parity breaking scale $M_P > 10^{12}$GeV, cosmological walls[24] do not cause problem and so also adequate baryon asymmetry of the universe[44] is generated if $M_P \sim M_U \gg M_R$ in the model with G_{2213} intermediate symmetry. With the mechanism of decoupling P- and $SU(2)_R$-breakings one interesting symmetry-breaking chain SO(10) has been noted [22] to be

$$SO(10) \xrightarrow[M_U]{\underline{54}} G_{224P} \xrightarrow[M_P]{\underline{210}} G_{224} \xrightarrow[M_R^+]{\underline{210}} G_{2113} \xrightarrow[M_R^o]{\underline{126}} G_{st} , \quad (4.1)$$

which provides $M_R^o = M_{Z_R} \simeq 500-1000$GeV and $M_R^+ \simeq M_C = 10^5$GeV offering the possibility of detection of a low-mass Z_R at the super colliders, measurement of signatures of quark-lepton unification through $K_L \longrightarrow \bar{\mu}e$, $n-\bar{n}$ oscillation and Majorana neutrino masses ($m_{\nu_e} \sim$ 1eV) with proton lifetime barely within the reach of ongoing experiments. The chain (4.1) has three intermediate symmetries. Because of their minimal character, GUT predictions with one or two intermediate symmetries might be more appealing. Even with two intermediate symmetries,[45]

$$SO(10) \xrightarrow{M_U} G \xrightarrow{M_R^+} G_{2113} \xrightarrow{M_R^o} G_{st}, \quad (4.2)$$

although a low-mass right-handed neutral gauge boson is found to be permitted with $M_R^0 \sim$ 500GeV whether $G \equiv G_{2213}$ or G_{2213P}, the predicted value of the charged $W_R^{\pm}$-gauge boson mass is large $M_R^{\pm} \geq 10^{10}$GeV[45] ruling out any observed low energy phenomenon by right-handed charged gauge bosons such as CP violations, and neutrinoless double beta decay etc.

Introducing five-dimensional operator[27,28] in the SO(10) GUT with single G_{2213P} intermediate symmetry, Rizzo[31] has obtained a low-mass right-handed scale $M_R \sim$ 100GeV with observable parity restoration. In contrast to the above conclusion, we note that low-mass right-handed gauge bosons ($M_R \sim$ 100GeV) are ruled out as the parity restoring gauge group G_{2213P} is not allowed at low-mass scales.[50] On the other hand, by using the mechanism of decoupling P- and $SU(2)_R$-breakings[21] and a different possible way of descending down to G_{2213} through $\underline{210} \subset$ SO(10) without observable parity restoration, we find that the low-mass right-handed gauge bosons ($M_R \sim$ 100GeV–10TeV) are permitted when five-dimension operator scaled by the compactification mass (M_G) is included[50].

This Chapter is organized in the following manner. In Sec.IV.2 we obtain modifications of gauge coupling constants and GUT boundary conditions in SO(10) with single $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ intermediate gauge group including the left-right symmetry ($g_{2L} = g_{2R}$) or excluding it ($g_{2L} \neq g_{2R}$). In Sec.IV.3 we derive formulas for unification mass, electroweak mixing angle, and GUT coupling constants with five-dimensional operator, exploiting the extended survival hypothesis under the natural assumption that all superheavy masses are the same as M_U. Our new solution with single G_{2213P} and G_{2213} intermediate symmetries are presented in Secs.IV.4 and IV.5, respectively. Predictions on neutrino masses for low M_R are stated in Sec.IV.4. Phenomenological constraints on the $W_R^{\pm}$ mass scales are discussed in Sec.IV.5. Finally, a brief summary and conclusion of the Chapter are given in Sec.IV.6.

IV.2. Modifications of gauge-coupling constants and GUT boundary conditions

In this section we derive the modifications in gauge couplings and GUT boundary conditions in the presence of a five-dimensional operator in the following cases,

$$\text{(A)}\quad SO(10) \xrightarrow[M_U]{\underline{210}} G_{2213P} \xrightarrow[M_R]{\underline{126}} G_{st} \xrightarrow[M_W]{\underline{10}} U(1)_{em}\times SU(3)_C \;,$$

$$\text{(B)}\quad SO(10) \xrightarrow[M_U]{\underline{210}} G_{2213} \xrightarrow[M_R]{\underline{126}} G_{st} \xrightarrow[M_W]{\underline{10}} U(1)_{em}\times SU(3)_C \;.$$

The case (A) has been considered by Rizzo in Ref.31. As the modifications to the GUT boundary conditions obtained in Ref.31, are different from ours, we derive them in both cases. In all the cases, the nonrenormalizable SO(10)-invariant five-dimensional operator [27,28] can be written as

$$\alpha_{NR}=(C/M_G)\,Tr(F_{\mu\nu}\Phi_{(210)}F^{\mu\nu}), \qquad (4.3)$$

where $\Phi_{(210)}$ is the four-index antisymmetric Higgs scalar representation $\underline{210} \subset SO(10)$, $F_{\mu\nu}$ is the gauge invariant field tensor and C is a constant. Rizzo's case corresponds to $M_G=M_{pl}=2\times10^{19}$GeV. In Eq. (4.3),

$$F_{\mu\nu}=\partial_\mu W_\nu-\partial_\nu W_\mu+ig[W_\mu,\ W_\nu],$$

$$W_\mu=(1/4)\sum_{i,j=1}^{10} \sigma^{ij} W_\mu^{ij}, \qquad C=(-1/8)\eta, \qquad (4.4)$$

where (1/2) $\sigma^{ij}(W_\mu^{ij})$, with i,j=1,2...,10 denote the 45 generators (gauge bosons)[46] of the SO(10) and the constant C has been reparametrized. In this notation every gauge boson appears in more than one element of the 16x16 matrix $W_{\mu\nu}$ and thus, the renormalized kinetic energy term (α_R) is $(-1/8)Tr(F_{\mu\nu}F^{\mu\nu})$ instead of $(-1/2)Tr(F_{\mu\nu}F^{\mu\nu})$(Ref.47). We follow the convention[46,48] in which i,j=1,2,...,6(7,8,9,10) are the SO(10) (SO(4)) indices. When the GUT symmetry is broken spontaneously, in all such cases, α_{NR} is absorbed in the renormalizable kinetic energy term (α_R) of the gauge fields as a results of which the coupling constants and hence, the boundary conditions are modified. In both cases (A) and (B), the modified boundary conditions can be expressed in a general form

$$\alpha_{2L}(M_U)(1+\varepsilon_{2L})=\alpha_{2R}(M_U)(1+\varepsilon_{2R})=\alpha_{BL}(M_U)(1+\varepsilon_{BL})=\alpha_{3C}(M_U)(1+\varepsilon_{3C})=\alpha_G, \qquad (4.5)$$

where $\alpha_G=g_o^2/4\pi$, g_o being the bare coupling of the GUT Lagrangian and ε_i(i=2L, 2R, BL, 3C) is the nonrenormalized term.

IV.2.1 Modifications with G_{2213P} intermediate symmetry

In Ref.31 the following values of the coefficients ε_i occurring in Eq.(4.5) have been taken,

$$\varepsilon_{2L}=\varepsilon_{2R}=\frac{15A}{\sqrt{2}}, \quad \varepsilon_{BL}=\frac{A}{\sqrt{2}}, \quad \varepsilon_{3C}= -\frac{20A}{\sqrt{2}} \qquad A=\frac{C\, M_U}{2(64\pi\alpha_G)^{1/2}M_{pl}}, \qquad (4.6)$$

such that all the gauge coupling constants at boundary $\mu = M_U$ get modified according to Eq.(4.5). In this case, when <u>210</u> acquires the VEV as

$$\langle\Phi_{(210)}\rangle = \frac{\Phi_0}{4\sqrt{6}}\,(-\Gamma_1\Gamma_2\Gamma_3\Gamma_4+\Gamma_1\Gamma_2\Gamma_5\Gamma_6+\Gamma_3\Gamma_4\Gamma_5\Gamma_6), \qquad (4.7)$$

SO(10) breaks to G_{2213P} by allowing the parity breaking scale to survive down to the W_L-scale[31]. In Eq.(4.7) $-\langle\Phi^{1234}\rangle=\langle\Phi^{1256}\rangle=\langle\Phi^{3456}\rangle$, where $\Phi_{(210)}=(1/4!)\Gamma_i\Gamma_j\Gamma_k\Gamma_l\times\Phi^{ijkl}$, Φ^{ijkl} being antisymmetric in ijkl. Noting that,

$$-\Gamma_1\Gamma_2\Gamma_3\Gamma_4+\Gamma_1\Gamma_2\Gamma_5\Gamma_6+\Gamma_3\Gamma_4\Gamma_5\Gamma_6$$
$$=\mathrm{diag}[-1,-1,-1,3,-1,-1,-1,3,-1,-1,-1,3,-1,-1,-1,3], \qquad (4.8)$$

we use the VEV given by (4.7) in Eq.(4.3). After a suitable rescaling of the gauge fields in the usual fashion we find that (4.3) gets absorbed in the kinetic energy terms for $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$, and $SU(3)_C$ gauge fields resulting in

$$\varepsilon_{2L} = \varepsilon_{2R} = 0, \qquad (1/2)\varepsilon_{BL} = -\varepsilon_{3C} = \varepsilon ,$$

$$\varepsilon = \frac{\eta\Phi_0}{4\sqrt{6}M_G} = \frac{\eta\, M_U}{2(32\pi\alpha_G)^{1/2}M_G} . \qquad (4.9)$$

From Eq.(4.9), it is clear that the modification to the boundary conditions occurs only for the coupling constants $\alpha_{3C}(M_U)$ and $\alpha_{BL}(M_U)$, but not for $\alpha_{2L}(M_U)$ or $\alpha_{2R}(M_U)$. Thus, Eq.(4.3) does not contribute to the modifications of the $SU(2)_{L,R}$ kinetic energies. This can be further checked by using i, j=7,8,9,10 in Eq.(4.3) and verifying that $\mathrm{Tr}(F^{(L,R)}_{\mu\nu}\langle\Phi_{(210)}\rangle F^{(L,R)\mu\nu})=0$. Therefore, we find the boundary conditions (4.9) for the case (A), which is substantially different from that used in Ref.31.

IV.2.2. Modifications with G_{2213} intermediate symmetry

In the previous Sec.IV.2.1, we have seen that when 210 acquires the VEV as (4.7), SO(10) breaks to G_{2213P}. Here we discuss yet another possibility of decoupling P- and $SU(2)_R$-breakings through 210 that breaks SO(10) to G_{2213} with P-breaking at the unification scale. This can be explained in the following way. The scalar representation 210 contains a singlet $\eta(1,1,0,1)$ under G_{2213} which is odd under P. When the VEV is assigned along this direction such that,

$$\langle\Phi_{(210)}\rangle \sim \Phi_o(\Gamma_7\Gamma_8\Gamma_9\Gamma_{10}) , \qquad (4.10)$$

P breaks at the GUT scale. Thus, while Eq.(4.7) gives the parity invariant vacuum with G_{2213P} gauge symmetry, addition of (4.7) and (4.10) with

$$\langle\Phi_{(210)}\rangle = \frac{\Phi_o}{8\sqrt{2}} (-\Gamma_1\Gamma_2\Gamma_3\Gamma_4+\Gamma_1\Gamma_2\Gamma_5\Gamma_6+\Gamma_3\Gamma_4\Gamma_5\Gamma_6+\Gamma_7\Gamma_8\Gamma_9\Gamma_{10}), \qquad (4.11)$$

yields the parity-violating vacuum having G_{2213} gauge symmetry. Now, noting

$$-\Gamma_1\Gamma_2\Gamma_3\Gamma_4+\Gamma_1\Gamma_2\Gamma_5\Gamma_6+\Gamma_3\Gamma_4\Gamma_5\Gamma_6+\Gamma_7\Gamma_8\Gamma_9\Gamma_{10}$$
$$=\mathrm{diag}[-2,-2,-2,2,-2,-2,-2,2,\ 0,\ 0,\ 0,4,\ 0,\ 0,\ 0,4], \qquad (4.12)$$

and using Eq. (4.11) in (4.3), we compute the ε_i parameters contributing to the modification of the boundary conditions,

$$\varepsilon_{2R}=-\varepsilon_{2L}=-\varepsilon_{3C}=\frac{1}{2}\varepsilon_{BL}=\varepsilon,$$

$$\varepsilon = \frac{\eta\Phi_o}{8\sqrt{2}\ M_G} = \frac{\eta\ M_U}{16M_G}\left(\frac{3}{2\pi\alpha_G}\right)^{1/2} , \qquad (4.13)$$

where we have used the relation between the superheavy gauge boson masses (M_U) and Φ_o, $M_U=(4\pi\alpha_G/3)^{1/2}\Phi_o$. Compared to case (A), we note that, in case of (B), ε_{2R} and ε_{2L} are opposite in sign. Such a difference in boundary conditions in two cases reflect in the resulting solutions.

IV.3. Formulas for electroweak mixing angle, unification mass, and GUT coupling

The one-loop renormalization group equations[32] for the gauge coupling constants $\alpha_i(\mu)=g_i^2(\mu)/4\pi$ in both cases (A) and (B), are

$M_W \le \mu \le M_R$:

$$\frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_i(M_R)} + \frac{a_i}{2\pi}\ln\frac{M_R}{M_W}, \quad i=Y,\ 2L,\ 3C, \qquad (4.14)$$

$M_R \le \mu \le M_U$:

$$\frac{1}{\alpha_j(M_R)} = \frac{1}{\alpha_j(M_U)} + \frac{a'_j}{2\pi}\ln\frac{M_U}{M_R}, \quad j=2L,\ 2R,\ BL,\ 3C, \qquad (4.15)$$

where a_i (a'_j) is the one-loop coefficient in the lower (higher) scale. In case (A) P is left unbroken down to the M_R scale while in case (B) P breaks at the unification scale M_U.

Using the RGE's (4.14)-(4.15) and the boundary conditions in the general form (4.5), we follow the standard procedure for obtaining formulas for $\ln M_U/M_W$, $\sin^2\theta_W$, and α_G^{-1} through the combinations $\alpha^{-1}(M_W)-(8/3)\alpha_{3C}^{-1}(M_W)$, $\alpha^{-1}(M_W)-(8/3)\alpha_{2L}^{-1}(M_W)$, $\alpha^{-1}(M_W)=(5/3)\alpha_Y^{-1}(M_W)+\alpha_{2L}^{-1}(M_W)$, and $\alpha_Y^{-1}(M_R)=(3/5)\alpha_{2R}^{-1}(M_R)+(2/5)\alpha_{BL}^{-1}(M_R)$,

$$\ln\frac{M_U}{M_W} = \frac{2\pi}{D}\left[\frac{\frac{8}{3}+\varepsilon_{2L}+\varepsilon_{2R}+\frac{2}{3}\varepsilon_{BL}}{\alpha_{3C}(M_W)} - \frac{1+\varepsilon_{3C}}{\alpha(M_W)}\frac{a_{3C}}{a'_{3C}} + \right.$$

$$\left.\frac{1}{2\pi}\left\{(1+\varepsilon_{3C})(a_{2L}+\frac{5}{3}a_Y-a'_{2L}-a'_{2R}-\frac{2}{3}a'_{BL})+(\frac{8}{3}+\varepsilon_{2L}+\varepsilon_{2R}+\frac{2}{3}\varepsilon_{BL})(a'_{3C}-a_{3C})\right\}\ln\frac{M_R}{M_W}\right],$$

(4.16)

$$\sin^2\theta_W = \frac{1}{D}\left[(1+\varepsilon_{2L})a'_{3C} - \frac{a'_{2L}a_{3C}}{a'_{3C}}(1+\varepsilon_{3C}) + \frac{\alpha(M_W)}{\alpha_{3C}(M_W)}\left\{a'_{2L}(\frac{5}{3}+\varepsilon_{2R}+\frac{2}{3}\varepsilon_{BL})\right.\right.$$

$$\left.-(1+\varepsilon_{2L})(a'_{2R}+\frac{2}{3}\ a'_{BL})\right\}+\frac{\alpha(M_W)}{2\pi}\left[(\frac{5}{3}+\varepsilon_{2R}+\frac{2}{3}\varepsilon_{BL})(a'_{3C}a_{2L}-a'_{2L}a_{3C})+(1+\varepsilon_{2L})\times\right.$$

$$\left.\left.\left\{a_{3C}(a'_{2R}+\frac{2}{3}\ a'_{BL})-\frac{5}{3}a'_{3C}a_Y\right\}+(1+\varepsilon_{3C})\left\{\ \frac{5}{3}a'_{2L}a_Y-a_{2L}(a'_{2R}+\frac{2}{3}\ a'_{BL})\right\}\right]\ln\frac{M_R}{M_W}\right], \tag{4.17}$$

$$\frac{1}{\alpha_G} = \frac{1}{D}\left[\frac{a'_{3C}}{\alpha(M_W)} - \frac{a'_{2L}+a'_{2R}+\frac{2}{3}a'_{BL}}{\alpha_{3C}(M_W)} + \frac{1}{2\pi}\left\{a_{3C}(a'_{2L}+a'_{2R}+\frac{2}{3}\ a'_{BL})-a'_{3C}(a'_{2L}+\frac{5}{3}a_Y)\right\}\right.$$

$$\left.\times\ln\frac{M_R}{M_W}\right], \tag{4.18}$$

where

$$D=a'_{3C}(\frac{8}{3}+\varepsilon_{2L}+\varepsilon_{2R}+\frac{2}{3}\varepsilon_{BL})-(1+\varepsilon_{3C})(a'_{2L}+a'_{2R}+\frac{2}{3}\ a'_{BL}). \tag{4.19}$$

IV.4. New predictions with G_{2213P} intermediate symmetry

In Eqs.(4.16)-(4.19), the case considered by Rizzo[31] corresponds to $M_R=M_W$, with $a_{2L}=a_{2R}=a'_{2L}=a'_{2R}=(4/3)n_g + (1/3)(D+2T)-22/3$, $a_{3C}=a'_{3C}=(4/3)n_g-11$, $a'_{BL}=(4/3)n_g+3T$, where n_g is the fermion generation number, and D(T) is the number of light Higgs doublets (triplets).

As our boundary conditions (4.9) are quite different from (4.6) that is used in Ref. 31, we compute numerical solutions to Eqs.(4.16)-(4.19) under the condition (4.9) and with $M_R=M_W$, $M_G=2M_{pl}=2\times10^{19}$GeV, $\alpha_{3C}(M_W)\simeq0.1$, and $\alpha_{em}^{-1}(M_W)$ = 128 as has been used in Ref. 31. In contrast to Ref.31 we note that in all cases of D=T, $\sin^2\theta_W$ given by Eq.(4.17) is independent of ε. This happens due to the fact that the ε-dependence occurring in the first factor (D^{-1}) in Eq. (4.19) gets exactly cancelled by the same dependence in the numerator of the second factor. For other combinations of D≠T the ε-dependence of $\sin^2\theta_W$ is weak. Some of our numerical solutions for D=T=1,2; D=2, T=1; and D=1, T=2 are shown in Table 8. Here the entry in the last column is the parameter $C=(-1/8)\eta$, that has been computed using formula (4.9) and different values for the compactification scale (M_G). It can be seen from Table 8 that the lowest value of $\sin^2\theta_W$ corresponds to the case D=1, T=2 and is found to be 0.266 with $\varepsilon=-0.208$ for which $M_U=2M_{pl}$. For all other values of $M_U< 2M_{pl}$, $\sin^2\theta_W$ >0.266. In this case, when we attempt to decrease $\sin^2\theta_W$ further with $\varepsilon < -0.208$, M_U exceeds $2M_{pl}$ making the solutions unacceptable. Thus, the possibilty of low-mass right-handed gauge bosons with $M_R \sim M_W$ accompanied by observable low-mass parity restoration through G_{2213P} intermediate symmetry needs $\sin^2\theta_W \simeq 0.266$, which is far too large as compared to the present world average of $\sin^2\theta_W \simeq 0.230\pm0.005$ obtained from neutral-current data[2].

In order to obtain the lowest allowed value of M_R under the boundary conditions (4.9) we allow $M_R \gg M_W$ in Eqs.(4.16)-(4.19). Using the same input parameters as in Table 8, we compute the value of M_U, M_R, $\sin^2\theta_W$, α_G^{-1}, and C for different values of ε, which are presented in Table 9. In the case D=T=1 (D=1, T=2) whenever we attempt to decrease $\sin^2\theta_W$ by decreasing $\varepsilon < -0.12$ ($\varepsilon < -0.2$), M_U exceeds $2M_{pl}$ which rules out the possibility of M_R

Table 8. Prediction of the SO(10) model with parity-restoring left-right gauge group at low-mass scales, $M_G=2\times10^{19}$GeV, and $M_R \sim 100$GeV.

Number of D and T	ε	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	$C=(-1/8)\eta$
D=T=1	−0.05	2.6×10^{18}	0.274	49.0	0.14
	−0.08	2.0×10^{19}	0.274	49.8	0.03
D=2,T=1	−0.08	4.4×10^{18}	0.282	48.2	0.13
	−0.10	2.0×10^{19}	0.282	48.7	0.04
D=1,T=2	−0.18	4.4×10^{18}	0.266	44.14	0.31
	−0.208	2.0×10^{19}	0.266	44.50	0.08
D=T=2	−0.20	2.9×10^{18}	0.274	43.03	0.51
	−0.238	2.0×10^{19}	0.274	43.44	0.09

Table 9. Prediction of the SO(10) model with the parity-restoring gauge group as an intermediate symmetry and $M_G=2\times10^{19}$GeV.

Number of D and T	ε	M_R (GeV)	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	$C=(-1/8)\eta$
D=T=1	−0.12	10^{10}	4.6×10^{18}	0.233	46.5	0.19
	−0.12	10^{9}	7.9×10^{18}	0.238	47.1	0.11
D=1, T=2	−0.2	10^{9}	9.3×10^{18}	0.234	44.1	0.16
	−0.2	10^{8}	9.7×10^{18}	0.238	44.1	0.15

below 10^9GeV. Here, in order to have $\sin^2\theta_W$ <0.235, we find $M_R > 10^9$GeV. Thus, the low-mass right-handed gauge bosons with observable parity restoration are ruled out in this model. Besides, even with such high values of $M_R < 10^{12}$GeV the model gives rise to stable domain walls[24] and negligible baryon asymmetry of the universe[44] which are unacceptable to the modern big-bang cosmology.

IV.5. New predictions with G_{2213} intermediate symmetry

In the absence of five-dimensional operator, such a model[22] provides $M_R > 10^{12}$GeV for $\sin^2\theta_W \simeq 0.230 \pm 0.005$, which rules out the possibility of observing the desired low-energy signatures. Here we investigate the effect of the corresponding five-dimensional operator in such a model. At first, we confine to the minimal number of Higgs particles D=T=1, needed for the SSB of the gauge symmetries. The Higgs contributions in the two different mass ranges are $M_W \le \mu \le M_R$, $\Phi(1,2,1)$; $M_R \le \mu \le M_U$, $\Phi(2,2,0,1)+\Delta_R(1,3,2,1)$, where the transformation properties in the lower (higher) mass ranges are under G_{st} (G_{2213}). We use the constraint of minimal fine-tuning of parameters where the left-handed triplet is made superheavy with its mass $\sim M_U$ and does not contribute to the RGE's of the coupling constants. In the minimal case the coefficients occurring in Eqs.(4.16)-(4.19) are

$$a_Y = 41/10,\ a_{2L} = -19/6,\ a_{3C} = -7,$$

$$a'_{2L} = -3,\ a'_{2R} = -7/3,\ a'_{BL} = 11/2, \text{ and } a'_{3C} = -7. \qquad (4.20)$$

Now, using the modified boundary conditions (4.13), we solve Eqs.(4.16)-(4.19) to obtain values of M_U, $\sin^2\theta_W$, and α_G for certain values of M_R as a function of ε. Some of our solutions are presented in Table 10. Here, the entry in the last column $C=(-1/8)\eta$ has been computed using formula (4.13) for different values of M_G. From Table 10, it is clear that low-mass right-handed gauge bosons with $M_R \sim$ 100GeV–10TeV are permitted with $|C|\simeq 0.2-3$ provided the compactification scale M_G is in the range of $\sim 10^{17}-10^{18}$GeV.[27] We have also calculated the value of parameter C corresponding to the compactification of the fifth dimension on a circle in the Kaluza-Klein model where $M_G=10^{19}\text{GeV}/2\pi=1.6\times10^{18}$GeV. It may be noted that in Kaluza-Klein theories, M_G could be made 2 orders of magnitude smaller than M_{Pl}[35]. If we use $M_G=M_{Pl}$, the parameter C increases by a factor 10–100 making it unacceptably large for the minimal choice of Higgs representations (D=T=1).

We have carried out a similar analysis in the case D=1 and T=2 corresponding to the nonminimal choice of Higgs representations; the results are also reported in Table 10. In this case low-mass $W_R^{\pm}$-gauge bosons are favoured with $M_G \sim 10^{17}-10^{18}$GeV when $|C|\simeq$ 0.2–0.8. Contrary to the case D=T=1, here however $M_G=M_{pl}$ could be permitted provided C is allowed to be in the range 3–10.

Thus, we find that low-mass $W_R^{\pm}$ - bosons are favoured in the SO(10) model with single G_{2213} intermediate symmetry which might be appearing as an *effective gauge theory* in four-dimensions resulting from compactification of extra dimension in some basic higher-dimensional theory.[11,15]

IV.6. Predictions on neutrino masses

Since the scalar representation <u>126</u> $\subset$ SO(10) is used in both

Table 10. Prediction of the SO(10) model with parity-violating left-right gauge group at lower mass scales ($M_R=10^2-10^4$GeV).

Number of D and T	ε	M_R (GeV)	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	M_G (GeV)	$C=(-1/8)\eta$
	0.05	10^4	1.6×10^{16}	0.234	48.3	1.6×10^{18}	-2.1
	0.06	10^4	10^{16}	0.232	48.1	10^{17}	-0.28
D=T=1	0.08	10^3	4.4×10^{15}	0.229	48.2	10^{17}	-0.76
	0.08	10^2	8.2×10^{15}	0.233	49.0	10^{17}	-0.4
	0.07	10^2	1.6×10^{16}	0.235	49.3	1.6×10^{18}	-2.8
	0.01	10^4	2.0×10^{16}	0.236	46.5	1.6×10^{18}	-0.34
D=1,T=2	0.02	10^3	1.7×10^{16}	0.236	46.8	1.6×10^{18}	-0.81
	0.04	10^2	7.4×10^{15}	0.232	46.9	10^{17}	-0.23

cases (A) and (B), to break the intermediate gauge symmetry spontaneously to the standard group, Majorana neutrino masses are generated. Further the Higgs representation 10 contains the standard Higgs doublet that generates the Dirac mass term for all fermions including the neutrino since the right-handed neutrino occurs among the fermion representation $\underline{16} \subset SO(10)$. The neutrino mass matrix is 2x2 with the diagonal (nondiagonal) components representing the Majorana (Dirac) mass terms. Diagonalization of the mass matrix then yields the seesaw formula[17]

$$m_{\nu_i} \simeq \frac{m_{l_i}^2}{M_R}, \quad i = e, \mu, \tau, \qquad (4.21)$$

where m_{ν_i} (m_{l_i}) is the neutrino (charged-lepton) mass of the ith generation. As has been noted in Chapter III earlier and Chapter IV subsequently, there is an alternative choice to use the up quark mass of the ith generation instead of the charged lepton mass in the right-hand side (R.H.S) of Eq (4.21). This would yield much larger masses for ν_μ and ν_τ. The computation of these masses and the corresponding procedure for evading the cosmological bound can be carried out in the same manner as the ones followed here with the charged lepton masses in the R.H.S of Eq.(4.21). With low right-handed scale $M_R \sim 1$TeV, the SO(10) model with G_{2213} intermediate symmetry predicts,

$$m_{\nu_e} \sim \text{eV}, \; m_{\nu_\mu} \sim 10\text{keV}, \text{ and } m_{\nu_\tau} \sim 4\text{MeV} . \qquad (4.22)$$

Such masses if testified by laboratory measurements would be in conflict with the cosmological bound[42] according to which the sum of stable neutrino masses should be less than 65eV. In order to satisfy the cosmological bounds, one of the mechanism that applies here is the decay of unstable heavier neutrinos to the stable light neutrinos like ν_e by the emission of a Majoron (χ)[43]. Such a Majoron is a massless Goldstone boson which is created when an additional global $U(1)_l$ symmetry is broken spontaneously by the VEV of a Higgs scalar carrying lepton number l=2 at a higher scale. The introduction of such an additional global symmetry does not affect the GUT predictions as described in this Chapter.

IV.7. Constraints on $W_R^{\pm}$-mass

Besides the constraints obtained from RGE's including spontaneous compactification effects, there are several phenomenological constraints[49] on the $W_R^{\pm}$-mass. The most stringent theoretical constraint on their mass comes from their contribution to K_L-K_S mass difference which sets a lower bound[49] of about 2.5TeV in manifestly left-right-symmetric G_{2213P} models where the two gauge coupling constants, and the fermion mixing angles in the left-and the right-handed sectors are equal $(g_{2L}=g_{2R},\ \theta_{2L}=\theta_{2R})$. This lower bound can be decreased substantially in the asymmetric models $(g_{2L}\neq g_{2R},\ \theta_{2L}\neq\theta_{2R})$.[66]

Another constraint comes from the experimental data on the Majorana neutrino masses. With the Majorana neutrino mass $m_{\nu_e} \sim$ 1-2eV, available experimental data are consistent with a low W_R-mass $\sim$ 3-4 TeV[51]. Besides these, the electric dipole moment of the neutron, a manifestation of CP-and P-violations, close to the experimental limit $d_n^e \leq 10^{-26}$e cm[49,52] predicts $M_R \sim$ 10-20 TeV.

IV.8. Summary and Conclusion

In this section, we briefly summarize and state our conclusions relating to the investigations reported in this

Chapter. The main result of this Chapter[50] is that the low-mass right-handed gauge bosons in SO(10) with single G_{2213P} intermediate symmetry, proposed by Rizzo[31] are ruled out as the parity restoring gauge group G_{2213P} is not allowed to survive at low-mass scales. In such a case we noted that $\sin^2\theta_W < 0.235$ constrains $M_R > 10^9$GeV. On the other hand, with the conventional mechanism of decoupling P- and $SU(2)_R$-breakings, we found a low-mass right-handed gauge boson ($M_R \sim$ 100GeV-10TeV) in SO(10) with G_{2213} intermediate symmetry, without parity restoration, when five-dimensional operator, scaled by the compactification mass is included. Such low-mass right-handed ($W_R^{\pm}$, Z_R) gauge bosons, in addition to being detected at the super colliders in the near future, could manifest in low-energy experiments with detectable V+A structure of weak charged and neutral currents, CP-violations in $K^0 \longrightarrow \bar{K}^0$ mixings, neutrino masses, and a host of other processes. If $W_R^{\pm}$ -boson masses are within a few TeV, they might manifest in neutrinoless double β-decay, muon decay, $\mu \longrightarrow 3e$, and muonium-antimuonium transitions at low energies if the accuracy of such experiments are improved in near future. One of the spectacular signatures of low mass $W_R^{\pm}$ -bosons at SSC energy would be through the decay modes

$$W_R^{\pm} \longrightarrow e^{+}N_R \longrightarrow e^{\pm} \text{ (jets)},$$

where N_R is the right-handed Majorana neutrino. Detailed investigations have shown that the detection limit for $W_R^{\pm}$ in this case is nearly 8.6TeV.[53]

It has been shown[30] that the seesaw mechanism for generating Majorana neutrino mass operates in a profound manner when the mechanism of decoupling P- and $SU(2)_R$-breakings is employed, as compared to the conventional methods[49]. With $M_R \sim 1$ TeV, the G_{2213} model predicts $m_{\nu_e} \sim$ eV, $m_{\nu_\mu} \sim$ 10keV, and $m_{\nu_\tau} \sim$ 4MeV. Such massses are measurable by laboratory experiments. Out of these, ν_μ and ν_τ masses would be in conflict with the cosmological bound according to which the sum of stable neutrino masses should not exceed 65eV[42]. In order to satisfy cosmological bound, in such cases, these heavier neutrinos can be made unstable with respect to decay into the lighter ones ν_e by the emission of a Goldstone boson, called the Majoron[43] that arises as a result of spontaneous breaking of a global lepton number associated with the G_{2213} gauge symmetry. In case (B), since the parity breaks at the unification scale $M_P \sim M_U \gg 10^{12}$GeV, there are no problems due to undesirable domain walls[24] or inadequate baryon number generation[44].

CHAPTER V

SPONTANEOUS COMPACTIFICATION EFFECTS ON SO(10) GRAND UNIFICATION WITH $SU(2)_L \times SU(2)_R \times SU(4)_C$ INTERMEDIATE SYMMETRY

V.1. Introduction

In Chapter III[40] we have found that in SO(10) model the intermediate scale(M_C) for the occurrence of the gauge group $SU(2)_L \times U(1)_R \times SU(4)_C$ can be brought down to $M_C \simeq 10^5-10^6$GeV leading to the possible low-energy signature of quark-lepton unification through rare-kaon decays($K_L \longrightarrow \bar{\mu} e$), but in this case $W_R^{\pm}$-gauge boson mass is high ($M_{W_R^{\pm}} = M_U$) ruling out possible low-energy manifestations of left-right symmetry (LRS) breaking through V+A charged currents or CP-violation. In Chapter IV[50] on the other hand we demonstrated the possibility of bringing down the $W_R^{\pm}$-and Z_R-masses to 500 GeV-few TeV leading to possible low-energy signatures of V+A charged and neutral currents,and CP-violations; but in this case $M_C \simeq M_U$ leaving no testable signature for quark-lepton unification. In this chapter we show how the single intermediate gauge group G_{224} can survive down to the scale $M_C \simeq 10^5-10^6$GeV so that $W_R^{\pm}$-masses can occur around the same scale corresponding to the manifestation of quark-lepton unification through rare-kaon decays[39] and $n-\bar{n}$ oscillation[38] ($M_C = M_{W_R^{\pm}} \simeq 10^5-10^6$GeV).

In this chapter[56] the impact of five-dimensional operator[27,28] which might originate from compactification of extra dimensions is investigated on SO(10) grand unification with Pati-Salam (PS)[5] intermediate gauge group, $SU(2)_L \times SU(2)_R \times SU(4)_C$ including left-right symmetry ($g_{2L}=g_{2R}$) or excluding it ($g_{2L} \neq g_{2R}$). The corresponding symmetry breaking chains with suitable SO(10) Higgs representations are

$$\text{I} \quad SO(10) \xrightarrow[M_U]{\underline{54}} G_{224P} \xrightarrow[M_C]{\underline{126}} G_{st} \xrightarrow[M_W]{\underline{10}} U(1)_{em} \times SU(3)_C ,$$

$$\text{II} \quad SO(10) \xrightarrow[M_U]{\underline{210}} G_{224} \xrightarrow[M_C]{\underline{126}} G_{st} \xrightarrow[M_W]{\underline{10}} U(1)_{em} \times SU(3)_C ,$$

where $G_{224P} \equiv SU(2)_L \times SU(2)_R \times SU(4)_C \times P \ (g_{2L} = g_{2R})$,

$G_{224} \equiv SU(2)_L \times SU(2)_R \times SU(4)_C \ (g_{2L} \neq g_{2R})$,

and P is parity, the left-right discrete symmetry. In the absence of higher-dimensional operators, such models cannot provide low $SU(4)_C$-breaking scale $M_C \sim 10^5$–10^6 GeV, thus, ruling out the possibility of observing low-energy phenomena, such as n–$\bar{n}$ oscillation, rare-kaon decays, and small neutrino masses. By including five-dimensional operator[27,28] scaled by the compactification mass (M_G)[27], we find that the chain II can provide such interesting phenomena corresponding to the low intermediate scale $M_C \sim 10^5$–10^6 GeV. In this case we find the equations for the unification mass (M_U) and GUT coupling (α_G) are independent of the parameter (ε) of the nonrenormalizable Lagrangian, although the electroweak mixing angle $(\sin^2\theta_W)$ does depend upon it. The unification mass is found to be large and the solutions are consistent with large compactification scale. On the other hand, when the PS is left-right-symmetric, the resulting equations for $\sin^2\theta_W$ is noted to be independent of ε, but M_U and α_G do depend upon it. For this case, besides the solutions obtained by Shafi and Wetterich (SW)[28], the new predictions with much larger M_G have been found out.

This Chapter is organized in the following manner. In Sec.V.2 we obtain modifications of gauge coupling constants and boundary conditions in SO(10) with $SU(2)_L \times SU(2)_R \times SU(4)_C$ intermediate symmetry including or excluding left-right symmetry. In Sec.V.3 we derive formulas for $\ln(M_U/M_W)$, $\sin^2\theta_W$, and α_G with the five-dimensional operator. New predictions with single G_{224P} and single G_{224} intermediate symmetries have been discussed in Secs.V.4 and V.5 respectively. The summary, discussion, and conclusion of this Chapter are stated in Sec.V.6.

V.2. Modifications of gauge coupling constants and GUT boundary conditions

For chain I, the modified boundary conditions have been derived by Shafi and Wetterich[27]. For chain II, we derive the corresponding new modifications in the presence of five-dimensional operator. In all such cases, when the GUT symmetry is broken spontaneously the nonrenormalized Lagrangian (α_{NR}) containing five-dimensional operator is absorbed in the renormalized kinetic energy term (α_R) modifying the gauge couplings at the unification scale, which can be expressed in general form,

$$\alpha_{2L}(M_U)(1+\varepsilon_{2L})=\alpha_{2R}(M_U)(1+\varepsilon_{2R})=\alpha_{4C}(M_U)(1+\varepsilon_{4C})=\alpha_G , \qquad (5.1)$$

where $\alpha_G=g_o^2/4\pi$, g_o being the bare GUT coupling constant.

V.2.1. Modifications with G_{224P} intermediate symmetry

Shafi and Wetterich[27] have already considered this case including the effects of the five-dimensional operator. They have used the nonrenormalizable five-dimensional operator,

$$\alpha_{NR}=-(\eta/2M_G)\,\mathrm{Tr}[F_{\mu\nu}\phi_{(54)}F^{\mu\nu}], \qquad (5.2)$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu-ig[A_\mu, A_\nu]$,

$$(A_\mu)^a_b = A^i_\mu (\lambda_i)^a_b,$$

$$\mathrm{Tr}(\lambda_i \lambda_j) = (1/2)\delta_{ij} \qquad (5.3)$$

In Eqs. (5.2)-(5.3), η is an unknown parameter, $F_{\mu\nu}$ is an antisymmetric 10x10 matrix, $\Phi_{(54)}$ denotes a symmetric traceless 10x10 matrix, A_μ is the gauge field, and λ_i's are the SO(10) generators. Using the vacuum expectation value (VEV),

$$\langle \Phi_{(54)} \rangle = \frac{\Phi_0}{\sqrt{30}} \,\mathrm{diag}\,[1,1,1,1,1,1,-3/2,\ -3/2,\ -3/2,\ -3/2], \qquad (5.4)$$

necessary for spontaneous symmetry breaking (SSB) at the first stage of the chain I. they obtained the following values of ε_{2L}, ε_{2R}, ε_{4C}, occurring in Eq.(5.1),

$$\varepsilon_{2L} = \varepsilon_{2R} = -(3/2)\varepsilon, \qquad \varepsilon_{4C} = \varepsilon, \quad \varepsilon = \frac{\eta\Phi_0}{\sqrt{30}M_G}. \qquad (5.5)$$

V.2.2. Modifications with G_{224} intermediate symmetry

Now we derive the modifications of the GUT boundary conditions for the chain II. In this case, the first stage of symmetry breaking is obtained by the antisymmetric tensor 210. As described in Chapter IV, we follow the convention[46,48] in which i,j=1,2,3...,6(7,8,9,10) denote the SO(6)(SO(4)) indices, and use he representation of generators by 16x16 matrices.[46] Using Γ_i, =1,2....,10, as the matrices defined in Ref.46, the 45 generators are given by $(1/2)\sigma^{ij} = (1/4i)\,[\Gamma_i, \Gamma_j]$, i,j=1,2,...10. Representing the 45-gauge bosons by the two-index antisymmetric tensor, W^{ij}_μ, he gauge boson matrix is 16x16,

$$W_\mu = (1/4) \sum_{i,j=1}^{10} \sigma^{ij} W^{ij}_\mu, \qquad (5.6)$$

where every gauge boson occurs repeatedly in more than one matrix element. This is in contrast to the SU(N)-gauge boson matrix where every boson occurs in only one matrix element of the corresponding NxN matrix. Following the usual definition,

$$F_{\mu\nu}=\partial_{\mu}W_{\nu}-\partial_{\nu}W_{\mu}-ig[W_{\mu}, W_{\nu}], \qquad (5.7)$$

with W_{μ} given by Eq.(5.6) and the expression for the kinetic energy,

$$\alpha_R = -(1/4) \sum_{m=1}^{45} \left[V_{\mu\nu}^{(m)} V^{(m)\mu\nu}\right],$$

$$V_{\mu\nu}^{(m)}=\partial_{\mu}V_{\nu}^{(m)}- \partial_{\nu}V_{\mu}^{(m)}-ig[V_{\mu}^{(m)}, V_{\nu}^{(m)}], \qquad (5.8)$$

where $V_{\mu}^{(m)}$ (m=1,2,...,45) represents 45 components of W_{μ}^{ij}, the corresponding expression in terms of $F_{\mu\nu}$ that reduces to (5.8) in the case of SO(10) is[47]

$$\alpha_{NR}=-(1/8)\ Tr[F_{\mu\nu}F^{\mu\nu}] \ . \qquad (5.9)$$

To the renormalizable Lagrangian given in (5.9) we add the nonrenormalizable term containing the five-dimensional operator ,

$$\alpha_{NR}= - \frac{\eta}{8M_G}\ Tr[F_{\mu\nu}\phi_{(210)}F^{\mu\nu}], \qquad (5.10)$$

where

$$\Phi_{(210)}=(1/4!) \sum_{ijkl} \Gamma_i\Gamma_j\Gamma_k\Gamma_l\ \Phi^{ijkl},\ i,j,k,l=1,2,..,10.$$

Wetterich[54] has shown how light generations of fermions can be obtained from a six-dimensional SO(12) gauge theory which might originate from pure gravity in eighteen dimensions coupled to Majorana-Weyl spinors. In this theory the Higgs representaion <u>210</u>, <u>126</u>, and <u>10</u> of SO(10) necessary for spontaneous symmetry breaking at different stages of the chain II have been demonstrated to emerge from suitable SO(12) representations possessing nonvanishing coupling to spinors[54]. With SO(10) gauge symmetry preserved after compactification of extra dimensions at a scale M_G, the five-dimensional operator in Eq.(5.10) is expected to occur as a nonrenormalizable term in the Lagrangian. When SO(10) $\xrightarrow{M_U}$ G_{224}, with P-broken at $\mu \sim M_U$, Φ^{ijkl} assumes VEV in the direction $\langle\Phi^{78910}\rangle=\Phi_o\neq 0$. Using the normalized VEV,

$$\langle\Phi_{(210)}\rangle=\frac{\Phi_o}{\sqrt{32}}\,\Gamma_7\Gamma_8\Gamma_9\Gamma_{10}\langle\Phi^{78910}\rangle$$

$$=\frac{\Phi_o}{\sqrt{32}}\mathrm{diag}[-1,-1,-1,-1,-1,-1,-1,-1,1,1,1,1,1,1,1,1] \tag{5.11}$$

in Eq.(5.10), we compute the values of ε_{2L}, ε_{2R}, and ε_{4C}, occurring in Eq.(5.1) ,

$$\varepsilon_{2L}=-\varepsilon_{2R}=\varepsilon, \qquad \varepsilon_{4C}=0, \qquad \varepsilon=\frac{\Phi_o}{4\sqrt{2}M_G}=\frac{\eta}{8}\left(\frac{3}{2\pi\alpha_G}\right)^{1/2}\frac{M_U}{M_G}. \tag{5.12}$$

Here, we have used the relation between the superheavy gauge boson masses (M_U) and Φ_o, $M_U=(4\pi\alpha_G/3)^{1/2}\Phi_o$. From Eq. (5.12), it is clear that the contributions due to ε_{2L} and ε_{2R} are equal and opposite, whereas that due to ε_{4C} vanishes. When ε_i parameters given by Eqs.(5.5) and (5.12) are used in Eq.(5.1), the GUT boundary conditions in the two cases are significantly different. Such a difference in the boundary conditions provide substantially different solutions to the unification mass and the electroweak mixing angle.

V.3. Formulas for unification mass, electroweak mixing angle, and GUT coupling

The one-loop gauge coupling constant $g_i(\mu)$ $(\alpha_i(\mu)=g_i^2(\mu)/4\pi)$

in the chains I and II satisfy the following forms of the renormalization-group equations[32] in the two mass ranges,

$M_W \le \mu \le M_C$:

$$\frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_i(\mu)} + \frac{a_i}{2\pi}\ln\frac{\mu}{M_W}, \quad i=Y,\ 2L,\ 3C, \qquad (5.13)$$

$M_C \le \mu \le M_U$:

$$\frac{1}{\alpha_j(M_C)} = \frac{1}{\alpha_j(\mu)} + \frac{a'_j}{2\pi}\ln\frac{\mu}{M_C}, \quad j=2L,\ 2R,\ 4C, \qquad (5.14)$$

where $a_i(a'_j)$ is the one loop coefficient at the lower(higher) mass scale. In chain I, P is left unbroken down to scale M_C, whereas in chain II, P is broken at the unification scale. In both cases, the Higgs scalar contributions at the lower mass range $M_W \le \mu \le M_C$ is $\Phi(2,1,1)$ under $SU(2)_L \times U(1)_Y \times SU(3)_C$. With three fermion generations and minimum number of Higgs, the one-loop coefficients occurring in Eq.(5.13) are

$$a_Y=41/10, \qquad a_{2L}=-19/6, \qquad a_{3C}=-7. \qquad (5.15)$$

For the higher mass range $M_C \le \mu \le M_U$, the Higgs scalars contributions are different. For the chain I, the Higgs scalar maintains left-right symmetry having the decomposition under G_{224P} as $\Phi(2,2,1) + \Delta_L(3,1,\overline{10}) + \Delta_R(1,3,\overline{10})$ and contributes the following values of the one-loop coefficients occurring in Eq. (5.14),

$$a'_{2L}=a'_{2R}=11/3, \qquad a'_{4C}=-14/3. \qquad (5.16)$$

For the chain II, the Higgs scalar multiplets contributing to the RGE's in the mass range $M_C \le \mu \le M_U$ are $\Phi(2,2,1) + \Delta_R(1,3,\overline{10})$ under G_{224} leading to the values of the coefficients occurring in Eq.(5.14),

$$a'_{2L}=-3, \quad a'_{2R}=11/3, \qquad a'_{4C}=-23/3 \qquad (5.17)$$

Using Eqs.(5.13)-(5.14) and the boundary conditions in the generalized form (5.1), we obtain the formulas for $\ln(M_U/M_W)$, $\sin^2\theta_W$ and α_G^{-1} through the following combinations $\alpha^{-1}(M_W)-(8/3)\alpha_{3C}^{-1}(M_W)$, $\alpha^{-1}(M_W)-(8/3)\alpha_{2L}^{-1}(M_W)$, and $\alpha^{-1}(M_W)=(5/3)\alpha_Y^{-1}(M_W)+\alpha_{2L}^{-1}(M_W)$,

$$\ln\frac{M_U}{M_W}=\frac{2\pi}{D}\left[\left\{2+\varepsilon_{2L}+\varepsilon_{2R}+\frac{2}{3}(1+\varepsilon_{4C})\right\}\alpha_S^{-1}-(1+\varepsilon_{4C})\alpha^{-1}\right]+$$
$$\frac{1}{D}\left\{(1+\varepsilon_{4C})(a_{2L}+\frac{5}{3}a_Y-\frac{2}{3}a_{3C}-a'_{2L}-a'_{2R})-(2+\varepsilon_{2L}+\varepsilon_{2R})(a_{3C}-a'_{4C})\right\}\ln\frac{M_C}{M_W}, \tag{5.18}$$

$$\sin^2\theta_W=\frac{1}{D}\left[(1+\varepsilon_{2L})a'_{4C}-(1+\varepsilon_{4C})a'_{2L}+\frac{\alpha}{\alpha_S}\left\{a'_{2L}(1+\varepsilon_{2R})\right.\right.$$
$$\left.+(1+\varepsilon_{4C})\frac{2}{3}a'_{2L}-(a'_{2R}+\frac{2}{3}a'_{4C})(1+\varepsilon_{2L})\right\}+\frac{\alpha}{2\pi}\left[\left\{a_{3C}(a'_{2R}+\frac{2}{3}a'_{4C})-\frac{5}{3}a'_{4C}a_Y\right\}\times\right.$$
$$\left.(1+\varepsilon_{2L})+(1+\varepsilon_{2R})(a_{2L}a'_{4C}-a'_{2L}a_{3C})+\left\{a'_{2L}(\frac{5}{3}a_Y-\frac{2}{3}a_{3C})-a_{2L}a'_{2R}\right\}(1+\varepsilon_{4C})\right]\times$$
$$\left.\ln\frac{M_C}{M_W}\right], \tag{5.19}$$

$$\frac{1}{\alpha_G}=\frac{1}{D}\left[\frac{a'_{4C}}{\alpha}-\frac{a'_{2L}+a'_{2R}+\frac{2}{3}a'_{4C}}{\alpha_S}-\frac{1}{2\pi}\left\{a'_{4C}(a_{2L}+\frac{5}{3}a_Y)-a_{3C}(a'_{2L}+a'_{2R}+\frac{2}{3}a'_{4C})\right\}\times\right.$$
$$\left.\ln\frac{M_C}{M_W}\right], \tag{5.20}$$

where

$$D=a'_{4C}(2+\varepsilon_{2L}+\varepsilon_{2R})-(1+\varepsilon_{4C})(a'_{2L}+a'_{2R}). \tag{5.21}$$

V.3.1. Formulas with G_{224P} intermediate symmetry

Using the coefficients from Eqs.(5.15)-(5.16), and the values of ε_{2L}, ε_{2R}, and ε_{4C} from Eq.(5.5) in Eqs.(5.18)-(5.21) we obtain

D=-(50/3)(1-2ε/5) and the following formulas,

$$\ln\frac{M_U}{M_W} = \frac{3\pi}{(25-10\varepsilon)}\left[\frac{1}{\alpha} - \frac{8}{3\alpha_S} + \left\{\frac{7}{3\alpha_S} + \frac{1}{\alpha}\right\}\varepsilon\right] - \frac{(17-18\varepsilon)}{(50-20\varepsilon)}\ln\frac{M_C}{M_W}, \qquad (5.22)$$

$$\sin^2\theta_W = \frac{1}{2} - \frac{\alpha}{3\alpha_S} - \frac{11\,\alpha}{3\pi}\ln\frac{M_C}{M_W}, \qquad (5.23)$$

$$\frac{1}{\alpha_G} = \frac{1}{(50-20\varepsilon)}\left[\frac{14}{\alpha} + \frac{38}{3\alpha_S} + \frac{56}{3\pi}\ln\frac{M_C}{M_W}\right]. \qquad (5.24)$$

From the above equations it is clear that $\sin^2\theta_W$ is independent of ε whereas $\ln(M_U/M_W)$ and α_G are dependent upon it.

V.3.2. Formulas with G_{224} intermediate symmetry

Using the coefficients from Eqs.(5.15) and (5.17), and the values of ε_{2L}, ε_{2R}, and ε_{4C} from Eq.(5.12) in Eqs.(5.18)-(5.21) we obtain D to be independent of ε, D=-16,

$$\ln\frac{M_U}{M_W} = \frac{\pi}{8}\left(\frac{1}{\alpha} - \frac{8}{3\alpha_S}\right) - \frac{19}{48}\ln\frac{M_C}{M_W}, \qquad (5.25)$$

$$\sin^2\theta_W = \frac{7}{24} + \frac{2\alpha}{9\alpha_S} - \left(\frac{23}{48} - \frac{5}{18}\frac{\alpha}{\alpha_S}\right)\varepsilon - \frac{\alpha}{2\pi}\left(\frac{193}{72} - \frac{533}{144}\varepsilon\right)\ln\frac{M_C}{M_W}, \qquad (5.26)$$

$$\frac{1}{\alpha_G} = \frac{1}{48}\left[\frac{23}{\alpha} - \frac{40}{3\alpha_S} - \frac{533}{6\pi}\ln\frac{M_C}{M_W}\right]. \qquad (5.27)$$

It is clear that $\ln(M_U/M_W)$ and α_G are independent of ε but $\sin^2\theta_W$ does depend upon it.

V.4. New predictions with G_{224P} intermediate symmetry

Using Eqs.(5.22)-(5.24), $\alpha^{-1}(M_W)=127.54$, $\alpha_S(M_W)=g_3^2(M_W)/4\pi=0.11$, ($\Lambda_{\overline{MS}}$ =160 MeV where $\overline{MS}$ denotes the modified subtraction scheme), and M_W=83 GeV, we compute the possible allowed values of

M_C and M_U as a function of the parameter ε, keeping view the constraint $M_U \geq 10^{15}$GeV and $\sin^2\theta_W \simeq 0.22-0.24$. Some of our solutions are presented in Table 11.

From Table 11, it is clear that the value of $\sin^2\theta_W$ is not affected by the parameter ε and it decreases below (increases beyond) 0.22 (0.24) when $M_C > 10^{14}(<10^{13})$GeV. For a fixed value of M_C in the range $10^{13}-10^{14}$GeV, M_U is controlled by ε. In this case, we find that the solutions already obtained by Shafi and Wetterich[27], corresponding to the enhancement of τ_p by a factor 10-100, over the conventional SO(10) predictions, occur for $\varepsilon \simeq$ 0.01-0.02 and are consistent with lower values of the compactification scale $M_G \sim 10^{17}$GeV.[27] With such GUT predictions, proton decay in the $p \longrightarrow e^+\pi^0$ mode might be observable in near future by low-energy experiments with improved accuracy[55].

Besides SW[27] type of solutions, here we obtain the new class of solutions with larger values of $M_U \sim 10^{16}-10^{18}$GeV, and $M_G \sim 10^{17}-10^{19}$GeV. As reported in Table 11, for a fixed value of $M_C \simeq 4\times10^{13}$GeV, the allowed value of M_U varies from 1.5×10^{16}GeV to 2.2×10^{18}GeV for ε=0.04-0.10. In this case, the values of η, computed using the relation $\eta=2(10\pi\alpha_G)^{1/2}\times(M_G/M_U)\varepsilon$ are found to be consistent with the values of $\eta\simeq0.2-4$ with $M_G=10^{17}-10^{19}$GeV. In particular, solutions with a very stable proton and lifetime, accessible to low-energy measurements in foreseeable future, are found to be possible with $M_U \simeq 10^{17}-10^{18}$GeV, consistent with the larger compactification scale $M_G \simeq 10^{18}-10^{19}$GeV.

As noted by Kibble, Lazaridis, and Shafi[24] several years ago, the domain-wall problem could have been severe if the intermediate

Table 11. Solutions of renormalization group equations in SO(10) with left-right symmetric G_{224P} intermediate gauge group (chain I), in the presence of the five-dimensional operator.

M_C (GeV)	$\sin^2\theta_W$	ε	M_U (GeV)	α_G^{-1}	M_G (GeV)	η
4×10^{13}	0.230	0.01	1.4×10^{15}	41.4	10^{17}	1.21
		0.02	3.2×10^{15}	41.6	10^{17}	1.10
		0.04	1.5×10^{16}	41.9	10^{17}	0.44
					10^{18}	4.47
		0.06	7.8×10^{16}	42.2	10^{17}	0.13
					10^{18}	1.32
		0.08	4.1×10^{17}	42.6	10^{18}	0.33
					10^{19}	3.38
		0.10	2.2×10^{18}	42.9	10^{19}	0.79
10^{14}	0.221	0.01	1.1×10^{15}	41.5	10^{17}	1.65
		0.02	2.3×10^{15}	41.6	10^{17}	1.50
		0.04	1.1×10^{16}	42.0	10^{17}	0.61
					10^{18}	6.05
		0.06	5.8×10^{16}	42.3	10^{17}	0.17
					10^{18}	1.78
		0.08	3.0×10^{17}	42.7	10^{18}	0.45
					10^{19}	4.55
		0.10	1.6×10^{18}	43.1	1.6×10^{18}	0.17
					10^{19}	1.06

scale M_C were less than 10^{12}GeV. But, with the renormalization group permitting $M_C \simeq 10^{14}$GeV, the problem does not exist in the present model. The Majorana neutrino masses are governed by the seesaw formula for the three generations[16-17],

$$m_{\nu_i} \simeq m_i^2/M_C, \qquad i=1,2,3, \tag{5.28}$$

where $m_{\nu_1}=m_{\nu_e}$, $m_{\nu_2}=m_{\nu_\mu}$, and $m_{\nu_3}=m_{\nu_\tau}$. As stated in Chapters III and IV, two different choices for m_i exist in the literature. While up quark mass of the ith generation has been used by Gell-Mann, Ramond, and Slansky[16], others have used the corresponding charged lepton mass.[17] Using $M_C \simeq 10^{13}-10^{14}$GeV, the allowed range from Table 11, and $m_1=m_u=$ 5MeV, $m_2=m_c=1.25$GeV, and $m_3=m_t \simeq 100$GeV, the model predicts,

$m_{\nu_e} \simeq (2.5\times10^{-10}-2.5\times10^{-9})$eV, $\quad m_{\nu_\mu} \simeq (1.5\times10^{-5}-1.5\times10^{-4})$eV, and

$m_{\nu_\tau} \simeq (0.1-1)$eV.

(5.29a)

But using the charged lepton masses, $m_1=m_e$, $m_2=m_\mu$, and $m_3=m_\tau$, the model has prediction,

$m_{\nu_e} \simeq (2.6\times10^{-12}-2.6\times10^{-11})$eV, $\quad m_{\nu_\mu} \simeq (1.1\times10^{-7}-1.1\times10^{-6})$eV, and

$m_{\nu_\tau} \simeq (3.2\times10^{-5}-3.2\times10^{-4})$eV.

(5.29b)

Although such neutrino masses are too small to be observed in the laboratory, but they might be compatible with the value needed to understand the solar neutrino puzzle by the so-called MSW mechanism.[37]

V.5. New predictions with G_{224} intermediate symmetry

In the absence of higher-dimensional operators such a model does not provide interesting solutions as for $\sin^2\theta_W=0.22(0.24)$, $M_C\simeq5\times10^{13}(10^{11})$GeV and $M_U\simeq10^{15}(6\times10^{15})$GeV at the one-loop level[16]. Using the same parameter as in chain I, we compute values of M_C and M_U as a function of ε from Eqs.(5.25) and (5.26), imposing the constraint $M_U \geq 10^{15}$GeV and $\sin^2\theta_W\simeq 0.22$–$0.24$. Some of our solutions with lower (higher) values of M_C are presented in Table 12 (Table 13).

From both the tables, it is clear that for a fixed value of M_C, the value of $\sin^2\theta_W$ is controlled by the parameter ε. In this case, for the allowed value of M_U and $\sin^2\theta_W$, we obtain the predicted value of M_C which varies over a wider range $M_C\simeq10^5-10^{11}$GeV. For simplicity of explanation we divide the solutions into 3 categories.

The first category of solutions belong to $M_C=10^{11}-10^{12}$GeV, for which the model predicts $M_U=(3-8)\times10^{15}$GeV with $\tau_p\simeq10^{33\pm3}-10^{35\pm3}$yr for the $p \longrightarrow e^+\pi^0$ mode. The uncertainty by a factor $10^{\pm3}$, in τ_p arises due to uncertainties in the proton decay matrix element and the QCD parameter.[34] Such an observable proton decay can be verified by high precision low-energy experiments.[55] These solutions are similar to the SW[27] type as obtained in chain I with the lower values of $M_G \simeq 10^{17}$GeV. In this case the neutrino masses are two orders of magnitude larger than the values given in Eq.(5.29). For example, with $M_C\simeq10^{11}$GeV and up quark masses for m_i, Eq.(5.28) gives $m_{\nu_e} \simeq 2.5\times10^{-7}$eV, $m_{\nu_\mu} \simeq 1.5\times10^{-2}$eV and $m_{\nu_\tau}\simeq100$eV, when charged lepton masses are used for m_i, although the predicted neutrino masses are too small, they could still be compatible with values needed to understand the solar neutrino puzzle.

Table 12. Same as Table 11, but for Chain II, and lower intermediate scales (M_C).

M_C (GeV)	M_U (GeV)	ε	$\sin^2\theta_W$	α_G^{-1}	M_G (GeV)	η
10^5	1.9×10^{18}	0.12	0.233	54.4	2×10^{18}	0.20
					10^{19}	1.00
		0.14	0.224	54.4	2×10^{18}	0.23
					10^{19}	1.17
10^6	7.6×10^{17}	0.10	0.235	53.0	10^{18}	0.21
					10^{19}	2.11
		0.12	0.226	53.0	10^{18}	0.25
					10^{19}	2.53
10^7	3.0×10^{17}	0.08	0.236	51.6	10^{18}	0.42
					10^{19}	4.25
		0.10	0.228	51.6	10^{18}	0.53
					10^{19}	5.32
		0.12	0.220	51.6	10^{18}	0.63
					10^{19}	6.38
10^8	1.2×10^{17}	0.06	0.237	50.3	10^{18}	0.80
					10^{19}	8.04
		0.08	0.229	50.3	10^{18}	1.07
					10^{19}	10.72
		0.10	0.221	50.3	10^{18}	1.34
					10^{19}	13.41

Table 13. Same as Table 12, but for higher intermediate scales.

M_C (GeV)	M_U (GeV)	ε	$\sin^2\theta_W$	α_G^{-1}	M_G (GeV)	η
10^9	4.9×10^{16}	0.04	0.238	48.9	10^{17}	0.13
					10^{18}	1.35
		0.06	0.230	48.9	10^{17}	0.20
					10^{18}	2.03
		0.08	0.222	48.9	10^{17}	0.27
					10^{18}	2.71
10^{10}	1.9×10^{16}	0.02	0.238	47.6	10^{17}	0.17
					10^{18}	1.71
		0.04	0.230	47.6	10^{17}	0.34
					10^{18}	3.41
		0.06	0.223	47.6	10^{17}	0.51
					10^{18}	5.12
10^{11}	7.9×10^{15}	0.01	0.234	46.2	10^{17}	0.21
					10^{18}	2.15
		0.02	0.230	46.2	10^{17}	0.43
					10^{18}	4.31
		0.04	0.223	46.2	10^{17}	0.64
					10^{18}	6.46
10^{12}	3.2×10^{15}	0.01	0.226	44.9	10^{17}	0.54
					10^{18}	5.43
		0.02	0.223	44.9	10^{17}	1.08
					10^{18}	10.87

The second category of solutions belong to $M_C \simeq 10^8-10^{10}$GeV for which M_U varies in the range $M_U \simeq 2\times10^{16}-10^{17}$GeV with $\tau_P \simeq 10^{36\pm3}-10^{39\pm3}$yr. The neutrino masses corresponding to such range of M_C are 4-5 orders larger compared to Eq.(5.29). Using the quark masses for m_i we obtain $m_{\nu_e} \simeq (2.5\times10^{-6}-2.5\times10^{-4})$eV, $m_{\nu_\mu} \simeq (0.15-15)$eV $m_{\nu_\tau} \simeq (1-10^2)$keV and; but the model predicts $m_{\nu_e} \simeq (2.6 \times 10^{-8} -2.6 \times10^{-6})$eV, $m_{\nu_\mu} \simeq (1.1\times 10^{-3}-0.11)$eV, and $m_{\nu_\mu} \simeq (0.3-32)$eV, when the charged lepton masses are used for m_i. Thus the ν_μ and ν_τ masses are within the detectable range and the solutions in this class are consistent with the compactification scale $M_G \simeq 10^{17}-10^{18}$GeV.

The last category of solutions which belong to $M_C \simeq 10^5-10^6$GeV with $\sin^2\theta_W \simeq 0.22-0.24$ are most interesting. Such lower values of M_C predict observable signatures of quark-lepton unification which can be verified by low-energy experiments through $n-\bar{n}$ oscillation[38] with $\tau_{n-\bar{n}} \simeq 10^8-10^9$s, and rare-kaon decays, $K_L \longrightarrow \bar{\mu}e$ with branching ratio $7\times(10^{-8}-10^{-12})$. With $M_C \simeq 10^5-10^6$GeV the predicted neutrino masses are in the range

$$m_{\nu_e} \simeq (2.6\times10^{-4}-2.6\times10^{-3})\text{eV}, \quad m_{\nu_\mu} \simeq (11.2-112)\text{eV}, \quad \text{and}$$

$$m_{\nu_\tau} \simeq (3.2-31.7)\text{keV}, \qquad (5.30)$$

when charged lepton masses are used for m_i; but $m_{\nu_e} \simeq (0.02-0.25\text{eV})$, $m_{\nu_\mu} \simeq (1.5-15.6)$keV and $m_{\nu_\tau} \simeq (10-100)$MeV when the up quark masses are used for m_i. Thus the seesaw formula with m_i as the quark masses forbids $M_C \simeq 10^5$GeV as the predicted m_{ν_μ} and m_{ν_τ} violate the existing laboratory limits[14] ($m_{\nu_\mu} \leq$ 250keV, $m_{\nu_\tau} \leq$ 35MeV). This implies that $M_C > 10^6$GeV, BR $(K_L \longrightarrow \bar{\mu}e) \leq 7\times10^{-12}$ and $\tau_{n-\bar{n}} \geq 10^9$s. But the seesaw formula with m_i as the charged lepton masses allows $M_C \simeq 10^5-10^6$GeV since the ν-masses do not violate the existing laboratory limits. The unification mass for $M_C \simeq 10^6(10^5)$ GeV is high, $M_U \simeq 10^{17}(10^{18})$GeV predicting a very stable proton with lifetime $\tau_p \simeq 10^{42}(10^{44})$yr in the $P \longrightarrow e^+\pi^o$ mode. Such high unification masses are consistent with the five-dimensional operator scaled by high compactification masses $M_G \sim 10^{18}-10^{19}$GeV. It may be noted that in the simplest Kaluza-Klein model leading to the four-dimensional space-time as a result of compactification of extra dimension on a circle yields $M_G = M_{pl}/2\pi = 1.6\times10^{18}$GeV. Most of our solutions with observable low-energy signatures of quark-lepton unification are compatible with $\eta \sim 1$ and such a high compactification scale. Since the left-right discrete symmetry (P) is broken at the GUT scale along with SO(10) gauge symmetry, the model does not in principle possess the domain-wall problem.[24]

In a number of predictions for the neutrino masses in the chain II the cosmological bound $\sum_i m_{\nu_i} \leq 65$eV[42] seems to be violated. This happens, for example, for $M_C \simeq 10^5-10^8$GeV for m_{ν_μ} and m_{ν_τ} using up quark masses for m_i in Eq. (5.28). One procedure to evade the cosmological bound is to make the heavier neutrinos unstable with respect to Majoron emission and decay into the lightest neutrino (ν_e)[43]. The Majoron is generated by breaking spontaneously an additional global $U(1)_l$ (l=lepton number) symmetry which must be introduced along with SO(10) to start with and broken at a scale M $\gg M_W$.

V.6. Summary, Discussion, and Conclusion

Higher-dimensional operators in specific forms involving gauge and Higgs fields might appear as nonrenormalizable terms in the GUT Lagrangian in four dimensions as a result of compactification of extra dimensions in higher-dimensional theories.[8-9,54] or as effects of quantum gravity[28]. It has been shown that[27,28] such terms can be absorbed in the renormalizable -gauge-fields-kinetic energy of the residual gauge group when the grand unifying symmetry is broken spontaneously by the VEV of the Higgs field occurring in the higher-dimensional operator(s). In such cases the gauge coupling constants at the GUT scale are usually modified resulting in the modifications of M_U and $\sin^2\theta_W$. We have demonstrated in this Chapter[56] that although the gauge couplings are modified, in certain cases, either M_U, or $\sin^2\theta_W$ might remain unaffected by the introduction of higher-dimensional operator.

Including five-dimensional operator on SO(10) with Pati-Salam intermediate symmetry, Shafi and Wetterich[27] obtained a factor of 10-100 enhancement in τ_P over the conventional SO(10) predictions and the $SU(4)_C$-breaking scale $M_C \simeq 10^{13}$GeV, consistent with $M_G \simeq 10^{17}$GeV (chain I). With such large value of M_C, besides proton decay, no other GUT signatures is predicted to be observable by low-energy experiments. Examining formulas obtained as solutions of RGE's in this case, we found that $\sin^2\theta_W$ is independent of the parameter ε. Besides the GUT predictions of the SW[27] type, we found that the model also predict a very stable proton corresponding to large values of M_U consistent with higher

$M_G \simeq 10^{18}-10^{19}$GeV. As for this chain, the allowed values of $M_C \simeq 10^{13}-10^{14}$GeV, the model does not possess the domain-wall problem[24]. Corresponding to such high values of M_C, although the predicted value of neutrino masses are small, they might be still compatible with the values needed to understand the solar neutrino puzzle via the so-called MSW mechanism[37].

As the primary objective of this Chapter we examined the impact of five-dimensional operators in SO(10) with single G_{224} intermediate symmetry when parity (P) is broken at the GUT scale (chain II), keeping view the constraint on $M_U \geq 10^{15}$GeV and $\sin^2\theta_W \simeq 0.22-0.24$. Since P is broken at the GUT scale the model does not posses the well known domain-wall problem[24]. The five-dimensional operator is expected to be present as a nonrenormalizable term in the Lagrangian after compactification of extra dimensions in the higher-dimensional model of Wetterich[54]. In the absence of higher-dimensional operators, for allowed values of $\sin^2\theta_W$, such model predicts $10^{11} \leq M_C \leq 5\times10^{13}$GeV, and $6\times10^{15} \geq M_U \geq 10^{15}$GeV at one-loop level such that, except proton decay, there is no other possibilities of GUT signatures at low-energies. Including the appropriate five-dimensional operator, we found that the unification mass M_U and α_G are independent of the parameter ε, although $\sin^2\theta_W$ does depend upon it.

As the primary distinguishing feature in the structural form of the equations in the two cases we note that, for a fixed M_C, the values of $\ln(M_U/M_W)$ $(\sin^2\theta_W)$ are controlled by the parameter in chain I (chain II). In the chain II the solutions of renormalization-group equations are classified into three categories: (a) solutions of SW type with $M_C \simeq 10^{11}-10^{12}$GeV and $M_G \simeq 10^{17}$GeV, predicting observable τ_P and neutrino masses 1-3 orders larger than the chain I, (b) $M_C \simeq 10^{8}-10^{10}$GeV, τ_P at least 4 orders larger than the experimental lower limit, and neutrino masses 4-5 orders larger than the chain I, (c) $M_C \simeq 10^{5}-10^{6}$GeV with a very stable proton but experimentally observable $n-\bar{n}$ oscillation, rare-kaon decays, and Majorana neutrino masses consistent with higher values of the compactification scales, $M_G \simeq 10^{18}-10^{19}$GeV. The cosmological bound in appropriate cases can be evaded by making the heavier neutrino unstable with respect to decay into the lighter neutrino by the emission of a Majoron[43]. The Majoron can be be generated by invoking an additional global symmetry $U(1)_l$ (l=lepton number) and breaking it spontaneously at a scale $M \gg M_W$.

Finally, we conclude that the impact of the five-dimensional operator, which might arise as a result of compactification of extra dimensions from some deeper higher-dimensional theory, causes drastic but very attractive modifications of SO(10) predictions with single Pati-Salam intermediate symmetry when parity- and $SU(2)_R$-breakings are decoupled[21].

CHAPTER VI

MODELS WITH NATURAL SEESAW MECHANISM FOR NEUTRINO MASSES WITH IDENTICAL P- AND $SU(2)_R$-BREAKING SCALES

VI.1. Introduction

In Chapters II-V we have investigated possible modifications of SU(5) and SO(10) GUT's due to higher-dimensional operators and found drastic changes from the conventional results. In this Chapter[64] *we explore a very interesting aspect of Majorana* neutrino masses without invoking the effects of spontaneous compactification or quantum gravity.

One of the most attractive suggestions to obtain small Majorana neutrino masses, is through the seesaw mechanism[16,17] has been widely exploited in partially unified or grand unified theories (GUT's) of strong, weak, and electromagnetic interactions. The mechanism explains the smallness of neutrino masses and universality of weak interactions simultaneously. Recently Chang and Mohapatra[30,49] have made an important observation on general validity of the mechanism as a viable theory for neutrino masses. They found that the implementation of the mechanism in left-right-symmetric (LRS) model or SO(10) GUT needs a wide separation of parity- (P) and $SU(2)_R$- breaking scales. It has been found that a wide separation of P- and $SU(2)_R$-breaking scales in LRS models or GUT's such as SO(10), SU(16), or $SU(8)_L \times SU(8)_R$, is possible in the presence of suitable Higgs representations,[21] or specific spontaneous symmetry breaking (SSB) patterns.[23] In the case when the P- and $SU(2)_R$- breaking scales are identical, the present bound on neutrino masses does not permit the right-handed gauge bosons to be light ($M_{W_R} = M_{Z_R} = M_R = M_P > 10^8$–$10^{10}$GeV), thus leaving no other testable signatures at lower energies. In the latter situation the mechanism[16,17] has no role in explaining neutrino masses. In this Chapter[64] we demonstrate that the seesaw mechanism is natural in certain models even if the two scales are identical. In such cases $SU(2)_R \times U(1)_{B-L}$ or $SU(2)_R \times SU(4)_C$ has to break spontaneously to $U(1)_Y$ in more than one steps. Here, we provide two examples in SO(10) GUT where we predict proton lifetime within the observable limit of the second generation experiments and V+A structure of neutral currents corresponding to a low mass Z_R-boson manifesting as a result of SSB of the minimally extended gauge group based upon $SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C$ ($\equiv G_{2113}$). With G_{2113} as one of the intermediate symmetry, a different form of the seesaw formula has been derived in Ref.60, where the left-handed triplet Δ_L carrying B-L = 2 has been taken to be light for the sake of convenience, which spoils the naturalness of the mechanism. In all models leading to G_{2113} considered in this Chapter,[64] the condition of minimal fine-tuning of parameters requires all the components of Δ_L are much heavier than the $U(1)_{B-L}$ breaking scale which renders the mechanism to be natural.

This Chapter is organized in the following manner. In Sec. VI.2 we review the work of Chang and Mohapatra illustrating the naturalness of the seesaw mechanism in gauge models with a wide separation between P- and $SU(2)_R$- breaking scales. In Sec. VI.3 we derive the new naturalness condition in order that the seesaw mechanism provides a meaningful theory for Majorana neutrino masses and show how the naturalness criterion operates with identical P- and $SU(2)_R$- breaking scales using the LRS model and partial unification scheme. In Secs.VI.4 and VI.5 we show how such models can be embedded in two different scenarios of SO(10) grand unification. A brief summary and conclusion of this Chapter are stated in Sec.VI.6.

VI.2. Natural seesaw mechanism with separate P-and $SU(2)_R$-breaking scales

In this section we summarize the work of Chang and Mohapatra[30] establishing the naturalness of the seesaw mechanism in left-right gauge models and GUT's with a wide separation between P- and $SU(2)_R$-breaking scales[21]. For convenience we discuss the conventional mechanism in the context of LRS models based upon the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times P$ ($\equiv G_{2213P}$, $g_{2L} = g_{2R}$), that breaks down to the standard group in the following manner

$$G_{2213P} \xrightarrow[M_R]{\langle\Delta_R^0\rangle = V_R} G_{st} \xrightarrow[M_W]{\langle\phi^0\rangle = k} G_{13} . \qquad (6.1)$$

The quarks (Q_L, Q_R) and leptons(ψ_L, ψ_R) of each generation, and Higgs scalars (Φ, Δ_L, Δ_R), possess the following transformation properties under G_{2213} : $Q_L(2, 1, 1/3, 3)$, $Q_R(1, 2, 1/3, 3)$, $\Psi_L(2,1,-1,1)$, $\Psi_R(1,2,-1, 1)$, $\Phi(2,2,0,1)$, $\Delta_L(3,1,2,1)$, and $\Delta_R(1,3,2,1)$. In order to obtain SSB at various stages of the chain (6.1), the Higgs scalars are assigned the following vacuum expectation values (VEV's):

$$\langle\Delta_L\rangle = \begin{bmatrix} 0 & 0 \\ V_L & 0 \end{bmatrix}, \quad \langle\Delta_R\rangle = \begin{bmatrix} 0 & 0 \\ V_R & 0 \end{bmatrix}, \qquad \Phi = \begin{bmatrix} k & 0 \\ 0 & k' \end{bmatrix}, \qquad (6.2)$$

which lead to the neutrino mass term in the Lagrangian :

$$\alpha^{(\nu)}_{mass}=(\nu^T\ N^T)\begin{bmatrix} m_{LL} & m_{LR} \\ m_{LR} & m_{RR} \end{bmatrix} \tau_2 \begin{pmatrix} \nu \\ N \end{pmatrix}, \tag{6.3}$$

where $N=C(\bar{\nu}_R)^T$, $m_{LL}=h_3V_L$, $m_{RR}=h_3V_R$, $m_{LR}=h_1k+h_2k'$, and h_i's are Yukawa couplings. Imposition of the constraint $V_R \gg k \gg V_L$, k' and the diagonalization of the mass matrix leads to small (large) mass eigen values of the left-(right-) handed neutrinos $\nu_i (N_i)$:

$$m_{\nu_i} \simeq \frac{m_i^{D^2}}{M_R},\ m_{N_i}=M_R,\ i=1,2,3, \tag{6.4}$$

where m_i^D is the Dirac mass and M_R is the mass of $W_R^{\pm}$ and Z_R bosons. In this case P and $SU(2)_R$ break at the same scale M_R. Taking the Dirac-mass equals to the charged-lepton mass[17] and $M_R \sim$ TeV, the formula (6.4) leads to

$$m_{\nu_e} \simeq 0.26\text{eV},\ m_{\nu_\mu} \simeq 11.2\text{keV},\ \text{and}\ m_{\nu_\tau} \simeq 3.17\text{MeV} \tag{6.5}$$

On the other hand, using the Dirac mass as the quark mass[16] with $m_t \simeq 100$GeV and $M_U=M_R=10^{15}$GeV, the Gell-Mann-Ramond-Slansky-type spectrum is

$$m_{\nu_e} \simeq 10^{-11}\text{eV},\ m_{\nu_\mu} \simeq 10^{-6}\text{eV},\ \text{and}\ m_{\nu_\tau} \simeq 10^{-2}\text{eV}\ . \tag{6.6}$$

Such a feature of the mechanism as obtaining small ν_i masses simultaneously with small mixing angles was considered very natural until Chang and Mohapatra[30,49] observed that the presence of the terms,

$$V_I = \sum_{i,j=1}^{2} \lambda_{ij}\ \text{Tr}\ (\Delta_L^+\ \Phi_i\ \Delta_R\ \tau_2\ \Phi_j^+\ \tau_2\ ), \tag{6.7}$$

in the Higgs potential, where $\Phi_1 = \Phi$ and $\Phi_2 = \tau_2\ \Phi^*\ \tau_2$, leads to much larger induced values of $\langle\Delta_L^0\rangle$ and the left-handed Majorana mass through Fig.4,

$$m_{LL}^{(I)}=\lambda\ h_3\frac{k^2\ V_R}{M_\Delta^2}, \tag{6.8}$$

even though one has $\langle\Delta_L^o\rangle = 0$ to start with. Here λ is a function of scalar coupling and M_Δ is the mass of Δ_L. Thus, the seesaw mechanism[16,17] meant to explain small neutrino masses holds provided the seesaw masses (SSM) obtained by Eq.(6.4) dominate over the induced masses ($m_{LL}^{(I)}$) given by (6.8). This condition requires,

$$\frac{\lambda\ V_R^2}{M_\Delta^2} \ll \left[\frac{h_1}{h_3}\right]^2. \tag{6.9}$$

Using extended survival hypothesis (ESH)[57], the maximum values of $M_\Delta=M_P=M_R$, obtained for $V_L=0$. Thus, the fine-tuning needed to satisfy (6.9), or $\lambda\ll(h_1/h_3)^2$,is arbitrary since the standard model Yukawa coupling $h_1\simeq 10^{-5}$, and there is no reason for h_3 to be small. Without arbitrary fine-tuning, Eq.(6.8) dominates over Eq.(6.4) and the bound on neutrino masses needs $M_\Delta=M_P=M_R\simeq10^{10}-10^{11}$GeV consistent with $m_{\nu_i}\simeq1-10$eV, i=1,2,3. In such a situation the proposed mechanism[16,17], does not explain neutrino masses.

In order to provide a natural explanation for neutrino masses by the seesaw mechanism even for low values of M_R, $m_{LL}^{(I)}$ should be made negligible compared to SSM. This is possible by decoupling P-and $SU(2)_R$-breaking scales[21] with $M_\Delta \simeq M_P \gg M_R$. A number of symmetry breaking patterns including two-loop effects have been worked out in SO(10) and found to be consistent with $M_P \gg M_R$. In

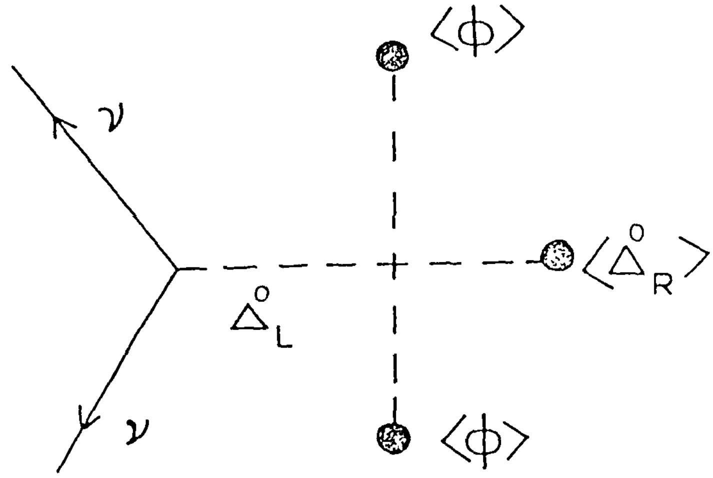

Fig. 4. Induced values of left-handed Majorana mass term that spoils the seesaw mechanism.

these new SO(10) models the seesaw mechanism is natural.[21,22]

VI.3. Natural seesaw mechanism in models with identical P- and $SU(2)_R$- breaking scales

In this section we demonstrate how the mechanism is natural in some gauge models even if P and $SU(2)_R$ break at the same scale. The popular models involving P-and $SU(2)_R$- breaking scales are the LRS models associated with the gauge group G_{2213P} or G_{224P} Besides, these gauge subgroups are contained in GUT's like SO(10), E_6, $SU(8)_L \times SU(8)_R$, or SU(16). In models having G_{224P} gauge group, B-L forms a diagonal generator of $SU(4)_C$. In the alternate class of models exhibiting a natural seesaw mechanism, although P and $SU(2)_R$ break at the same scale, $SU(2)_R \times U(1)_{B-L}$ or $SU(2)_R \times SU(4)_C$ breaks to $U(1)_Y$ in more than one steps. In the first step $U(1)_{B-L}$ or $SU(4)_C$ must remain unbroken but $SU(2)_R \longrightarrow U(1)_R$ to generate wide separation between M_P and M_{B-L}, where M_{B-L} is the breaking scale of $U(1)_{B-L}$. This is achieved through the following chains in the two models:

$$\text{(i)} \quad G_{2213P} \xrightarrow[\langle\chi^0\rangle\neq 0]{M_P=M_R^+} G_{2113} \xrightarrow[\langle\Delta_R^0\rangle\neq 0]{M_R^0} G_{st}\ ,$$

$$\text{(iia)} \quad G_{224P} \xrightarrow[\langle\chi^0\rangle\neq 0]{M_P=M_R^+} G_{214} \xrightarrow[\langle\xi^0\rangle\neq 0]{M_C} G_{2113} \xrightarrow[\langle\Delta_R^0\rangle\neq 0]{M_R^0} G_{st}\ ,$$

$$\text{(iib)} \quad G_{224P} \xrightarrow[\langle\sigma^0\rangle\neq 0]{M_P=M_R^+=M_C} G_{2113} \xrightarrow[\langle\Delta_R^0\rangle\neq 0]{M_R^0} G_{st}\ ,$$

where $G_{214} \equiv SU(2)_L \times U(1)_R \times SU(4)_C$. In order to understand the naturalness of the seesaw mechanism it is necessary to know the order of the masses of Higgs scalars occurring as left-handed and right-handed triplets which carry B-L = 2. In case (i) the first stage of symmetry breaking, $G_{2213P} \longrightarrow G_{2113}$ is obtained by giving VEV to the neutral component of the right-handed Higgs scalar triplet with B-L = 0 transforming as $\chi(1,3,0,1)$ under G_{2213P}. The second stage of symmetry breaking $G_{2113} \longrightarrow G_{st}$ can be achieved by assigning VEV to the neutral component of the right-handed Higgs scalar triplet $\Delta_R^0(1,-1,2,1)$ under G_{2113} carrying B-L=2. Thus, at the second stage Majorana neutrino masses are generated. By extended survival hypothesis[57], $W_R^{\pm}$ gauge boson masses are of order M_R^+ whereas the right-handed neutral gauge boson mass $\simeq M_R^0$. It may be noted that in the first stage, both P and $SU(2)_R$ break at the same scale ($\mu=M_P=M_R^+$) but $U(1)_R \times U(1)_{B-L}$ remain unbroken, which at subsequent stage break to form $U(1)_Y$ at the lower scale $M_R^0 = M_{B-L} \ll M_R^+ = M_P$. In case (iia), the first stage of symmetry breaking is obtained by giving VEV to the neutral component of the Higgs scalar transforming as $\chi(1,3,1)$ under G_{224} whereas the second stage of breaking is possible by assigning VEV to the neutral component of the Higgs scalar transforming as $\xi(1,1,15)$ under G_{224} with $M_R^+ \gg M_C$. In the case (iib), the first stage of symmetry breaking is achieved by giving VEV to the neutral component of the Higgs scalar transforming as $\sigma(1,3,15)$ under G_{224}. In all cases (i) and (ii) the final stage of SSB is achieved by giving the VEV to the neutral component of the standard doublet of Higgs scalars whereas the SSB of $G_{2113} \longrightarrow G_{st}$ is possible by assigning VEV to the neutral component of the right-handed Higgs scalar triplet $\Delta_R^0(1,-1,2,1)$ under G_{2113} carrying B-L=2 as a consequence of which Majorana neutrino masses are generated at this stage. Besides, since LRS is maintained at scales $\mu \gg M_R^+$, the Higgs sector must be left-right symmetric for such values of μ. Using ESH[57], the neutral component of the standard Higgs scalar acquires mass $M_{\Phi^o} \simeq M_W$ and the neutral component of the right-handed triplet acquires mass $M_{\Delta_R^o} \simeq M_R^o$. Similarly the charged components $\Delta_R^{\pm}$, Δ_R^{++}, and Δ_R^{--} in the triplet Δ_R (1,3,2,1) under G_{2213} in case (i) acquire masses of order M_R^+. The left-handed counterpart of Δ_R i.e., $\Delta_L(3,1,2,\bar{1})$ under G_{2213} does not contribute to the SSB at any stage. Its role is to maintain LRS, and according to ESH masses of all the components in Δ_L is of the order M_R^+. In the cases (iia) and (iib), the right-handed triplet is contained in the G_{224} representation $\Delta_R(1,3,10)$, whereas the left-handed triplet is contained in $\Delta_L(3,1,\overline{10})$. Only the neutral component of Δ_R^o acquires a mass $\simeq M_R^o$ but all other components of Δ_R and Δ_L have masses of order M_R^+. In the case (iia) all other components of $\xi(1,1,15)$ under G_{224} have masses $\simeq M_R^+$ except the neutral component which acquires mass $M_{\xi^o} \simeq M_C$. All the components of $\chi(1,3,1)$ under G_{224} have masses $\simeq M_R^+$. In the case (iib) all the components of $\sigma(1,3,15)$ under G_{224} have masses $M_R^+ = M_C$.

Now using the seesaw mechanism and adding the induced mass term due to Fig.4, we obtain for the neutrino mass of ith generation,

$$m_{\nu_i} = \frac{\lambda\, h_3^{(i)}}{g^3}\; \frac{M_W^2\, M_R^o}{M_R^{+2}} - \frac{(m_i^D)^2}{M_R^o}, \qquad i=e,\ \mu,\ \tau, \tag{6.10}$$

where the first (second) term is the induced (seesaw mechanism) contribution and g is the appropriate gauge coupling. The naturalness criterion requires the dominance of the second term over the first, i.e.,

$$m_i^D \gg \frac{M_W \, M_R^0}{M_R^+} ,$$

or

$$\rho m_i^D \gg M_W , \quad i=e, \mu, \tau, \qquad (6.11)$$

where $\rho = M_R^+/M_R^0$ and $\frac{\lambda \, h_3^{(i)}}{g^3} \simeq 1$. Thus Eq.(6.11) is our new naturalness condition in order that the seesaw mechanism provides a meaningful theory for Majorana neutrino masses. When the Dirac mass is taken as the charged lepton mass, the first generation gives the condition $\rho \gg 10^5$ which is automatically guarantees naturalness for the second and third generations since $m_\tau \gg m_\mu \gg m_e$. For a low $M_R^0 \sim 1$ TeV, the mechanism is natural provided $M_R^+ = M_P \gg 10^8$GeV. An interesting common feature of the new class of models specified in (i) and (ii) is the naturalness of the mechanism with the minimally extended gauge group G_{2113} at lower energies. By using RGE's[32] it is easy to satisfy the condition $M_R^+ \gg M_R^0$ in the case (i) and the constraint arising out of $K_L - K_S$ mass difference. In fact RGE's do not constrain $M_P = M_R^+$ as there are three unknown gauge coupling constants $g_{2L} = g_{2R}$, g_{BL}, and g_{3C} for $\mu \geq M_R^+ = M_P$. But in cases (iia) and (iib) there are two unknown gauge coupling constants, $g_{2L} = g_{2R}$ and g_{4C} for $\mu \geq M_R^+ = M_P$, one of which can be eliminated by using the fine-structure constant matching at $\mu = M_W$. For the case (iia) the relation between $\sin^2\theta_W$ and the mass scales can be expressed including one - loop corrections as

$$\sin^2\theta_W = \frac{1}{2} - \frac{\alpha}{3\alpha_S} - \frac{8\alpha}{3\pi}\ln\frac{M_P}{M_C} - \frac{23\alpha}{6\pi}\ln\frac{M_C}{M_R^0} - \frac{11\alpha}{3\pi}\ln\frac{M_R^0}{M_W}, \qquad (6.12)$$

where $\alpha \equiv \alpha(M_W)=e^2(M_W)/4\pi$ and $\alpha_S=g_3^2(M_W)/4\pi$. The corresponding equation for case (iib) is obtained from Eq.(6.12) by using $M_R^+=M_P=M_C$. Some of our solutions for cases, (iia) and (iib) corresponding to $\Lambda_{\overline{MS}} \simeq 0.2$ GeV ($\overline{MS}$ denotes the modified minimal subtraction scheme), are presented in Tables 14 and 15 respectively for $\sin^2\theta_W \simeq 0.22-0.24$ and for values $M_R^0 \simeq (3\times10^2-10^5)$GeV. For the case (iia) we find $7\times10^{13}\text{GeV} \le M_R^+ = M_P \le 2\times10^{17}$GeV for $10^{10}\text{GeV} \gg M_C \gg 10^5$ GeV. In this case, in addition to predicting the low energy gauge group to be G_{2113}, beyond the standard model, the rare-kaon decays are also predicted to be observable, corresponding to $M_C \simeq 10^5$GeV. In the case (iib) the solutions are consistent: $M_P = M_C = M_R^+ \simeq 10^{12}\text{GeV} - 5\times10^{13}$GeV with a low mass Z_R boson. The parameter $\rho = M_R^+/M_R^0 \ge 10^7$ in both cases and is found to guarantee the naturalness condition. The neutrino mass spectrum for lower values of $M_R^0 \simeq$ 300GeV– 1TeV is of the type eV-keV-MeV for the three generations. In such cases m_{ν_μ} and m_{ν_τ} would violate the cosmological bound. One method of evading the cosmological bound is to make ν_μ and ν_τ unstable against Majoron emission[43] by adding $U(1)_l$ global symmetry (l=lepton number).

VI.4. Implementation in SO(10) with G_{2213P} as an intermediate symmetry

In this section we show how the new seesaw mechanism operates in an SO(10) model with G_{2213P} and G_{2113} as the two intermediate symmetries. Such a GUT scenario predict $M_P=M_R^+ \ll 10^{11}$GeV for the

Table 14. Some predictions of the partial unification model $G_{224P} \xrightarrow[M_R^+]{} G_{214} \xrightarrow[M_C]{} G_{2113}$ with $M_R^0 = 1$ TeV as described in the text.

M_C (GeV)	$M_R^+=M_P$ (GeV)	$\sin^2\theta_W$
10^5	5×10^{16}	0.230
	10^{17}	0.225
10^6	8×10^{16}	0.220
	2×10^{16}	0.230
	8×10^{15}	0.235

Table 15. Some predictions of the partial unification scheme

$$G_{224P} \xrightarrow[M_P]{} G_{2113} \xrightarrow[M_R^o]{} G_{213}.$$

M_R^o (GeV)	$M_R^+=M_P=M_C$ (GeV)	$\sin^2\theta_W$
10^3	8×10^{12}	0.235
	1.4×10^{13}	0.230
10^5	10^{13}	0.235
	1.6×10^{13}	0.230

allowed value of $\sin^2\theta_W \simeq 0.22-0.24$ as a consequence the model gives rise to stable domain walls[24] and negligible baryon asymmetry[44] of the universe unacceptable to modern big bang cosmology. On the other hand if $M_P=M_R^+ \geq 10^{11}$GeV the baryon asymmetry is compatible with the observed value and the domain walls created in the early universe might have been removed by inflation. Our analyses in this chapter demonstrate that the RGE's permit such solutions when the renormalization effects on gauge coupling constants upto two - loops and superheavy-Higgs-scalar effects[34, 58-59] are included. We discuss the embeddings of these groups in SO(10) and find solutions to the unification mass (M_U), $\sin^2\theta_W$, and intermediate scales. The case (i) mentioned in Sec.VI.3 can be embedded in SO(10) grand unification as follows:

$$SO(10) \xrightarrow[M_U]{\underline{210}} G_{2213P} \xrightarrow[M_P=M_R^+]{\underline{45}} G_{2113} \xrightarrow[M_R^0=M_{B-L}]{\underline{126}} G_{st} \xrightarrow[M_W]{\underline{10}} G_{13}\ , \tag{6.13}$$

where the Higgs scalars mentioned in (i) are contained in the respective SO(10) representations : $\chi \subset \underline{45}$, $\Delta_R \subset \underline{126}$, $\Phi \subset \underline{10}$. In addition, the GUT symmetry breaks down to G_{2213P} when the neutral component of the Higgs scalar transforming as $(1,1,0,15) \subset \underline{210}$ under G_{2213P} acquires VEV $\simeq M_U$. In order to make GUT predictions using the effective gauge theory approach[58,59] the superheavy components in different Higgs representations needed for SSB in case (i) are noted below along with their masses and transformation properties G_{2213}:[59]

$\underline{10} \supset M_{H_1}\,(1,1,\ \sqrt{3/2}\ 1/3,\ 3)+M_{H_2}\,(1,1,-\sqrt{3/2}\ 1/3,\ \bar{3})$,

$$\underline{126} \supset M'_{H_1}(3,1,\sqrt{3/2}\,1/3,\bar{3})+M'_{H_2}(3,1,-\sqrt{3/2}\,1/3,\bar{6})$$
$$+M'_{H_3}(1,3,\sqrt{3/2}\,1/3,3)+M'_{H_4}(1,3,-\sqrt{3/2}\,1/3,6)$$
$$+M'_{H_5}(1,1,\sqrt{3/2}\,1/3,3)+M'_{H_6}(1,1,-\sqrt{3/2}\,1/3,\bar{3})$$
$$+M'_{H_7}(2,2,0,1)+M'_{H_8}(2,2,-\sqrt{3/2}\,2/3,3)$$
$$+M'_{H_9}(2,2,\sqrt{3/2}\,2/3,\bar{3})+M'_{H_{10}}(2,2,0,8),$$
$$\underline{45} \supset M_{S_1}(1,1,-\sqrt{3/2}\,2/3,3)+M_{S_2}(1,1,\sqrt{3/2}\,2/3,\bar{3})$$
$$+M_{S_3}(1,1,0,8)+M_{S_4}(2,2,\sqrt{3/2}\,1/3,3)$$
$$+M_{S_5}(2,2,-\sqrt{3/2}\,1/3,\bar{3}),$$
$$\underline{210} \supset M'_{S_1}(3,1,-\sqrt{3/2}\,2/3,3)+M'_{S_2}(3,1,\sqrt{3/2}\,2/3,\bar{3})$$
$$+M'_{S_3}(3,1,0,8)+M'_{S_4}(1,3,-\sqrt{3/2}\,2/3,3)+M'_{S_5}(1,3,\sqrt{3/2}\,2/3,\bar{3})$$
$$+M'_{S_6}(1,3,0,8)+M'_{S_7}(2,2,\sqrt{3/2},1)+M'_{S_8}(2,2,\sqrt{3/2}\,1/3,3)$$
$$+M'_{S_9}(2,2,-\sqrt{3/2}\,1/3,6)+M'_{S_{10}}(2,2,\sqrt{3/2},\bar{1})$$
$$+M'_{S_{11}}(2,2,\sqrt{3/2}\,1/3,\bar{3})+M'_{S_{12}}(2,2,-\sqrt{3/2}\,1/3,\bar{6}). \qquad (6.14)$$

Among the components not explicitly mentioned in (6.14), some are singlets under G_{2213} which do not contribute the desired modifications. Others are either absorbed as would-be Goldstone components of appropriate gauge bosons, or they are light, and the corresponding contributions are included in one-and two-loop coefficients of the β-function in the usual manner.[22] If the component masses are taken to be arbitrarily nondegenerate, the model loses its predictive power on proton lifetime (τ_P) and $\sin^2\theta_W$. We examine their impact on GUT predictions by assuming the masses to be (a) degenerate, (b) nondegenerate but not arbitrary as they are constrained by Coleman-Weinberg[67] type mechanism according to which the mass of only one scalar component in the larger Higgs representation of a GUT be within a factor of 10 of their vector masses. Here we make a reasonably stringent assumption that all superheavy-component masses differ from M_U by a factor of 10 or 30. In all cases τ_p ($p \longrightarrow e^+\pi^o$) is predicted near the observable limit. In order to constrain masses under the condition (b), we maximize τ_p using the RGE for ln (M_U/M_W), which leads to

$$M'_{H_1}=M'_{H_7}=M'_{H_8}=M'_{H_9}=M_{S_4}=M_{S_5}=M'_{S_1}=M'_{S_2}=M'_{S_7}=M'_{S_8}=M'_{S_{10}}=M'_{S_{11}}=M^{(+)},$$
$$M_{H_1}=M_{H_2}=M'_{H_2}=M'_{H_5}=M'_{H_6}=M'_{H_{10}}=M'_{S_3}=M'_{S_9}=M'_{S_{12}}=M_{S_1}=M_{S_2}=M_{S_3}=M^{(-)}. \quad (6.15)$$

Using minimal number of Higgs scalars and three fermion generations we have computed the one- and two-loop coefficients in the equations for $\ln(M_U/M_W)$ and $\sin^2\theta_W$ given below:

$$\ln\frac{M_U}{M_W} = \frac{3\pi}{29}\left[\frac{1}{\alpha} - \frac{8}{3\alpha_S}\right] - \frac{11}{58}\ln\frac{M_R^+}{M_W} + \frac{1}{29}\ln\frac{M_R^o}{M_W}$$
$$-\frac{3}{116}\left[-27\ln X_{2L}^U - 27\ln X_{2R}^U + \frac{64}{7}\ln X_{BL}^U - \frac{96}{7}\ln X_{3C}^U + \frac{32}{9}\ln X_{BL}^+ + \right.$$
$$2\ln X_{1R}^+ + \frac{10}{19}\ln X_{2L}^+ - \frac{96}{7}\ln X_{3C}^+ + \frac{46}{41}\ln X_Y^o + \frac{10}{19}\ln X_{2L}^o$$
$$\left. -\frac{96}{7}\ln X_{3C}^o\right] + \frac{1}{29}\left[21\ln\frac{M^{(+)}}{M_U} - 28\ln\frac{M^{(-)}}{M_U}\right], \quad (6.16)$$

$$\sin^2\theta_W=\frac{15}{58}+\frac{9\alpha}{29\alpha_S}-\frac{92\alpha}{87\pi}\ln\frac{M_R^+}{M_W}+\frac{5\alpha}{58\pi}\ln\frac{M_R^o}{M_W}$$

$$-\frac{\alpha}{29\pi}\left[\frac{1017}{16}\ln X_{2L}^U-\frac{897}{16}\ln X_{2R}^U+\frac{103}{28}\ln X_{BL}^U+\frac{191}{14}\ln X_{3C}^U\right.$$

$$+\frac{91}{18}\ln X_{BL}^+ +\frac{193}{36}\ln X_{1R}^+ +\frac{307}{76}\ln X_{2L}^+ +\frac{191}{14}\ln X_{3C}^+ +\frac{403}{164}\ln X_Y^o$$

$$\left.+\frac{307}{76}\ln X_{2L}^o+\frac{191}{14}\ln X_{3C}^o\right]-\frac{\alpha}{174\pi}\left[33\ln\frac{M^{(+)}}{M_U}-44\ln\frac{M^{(-)}}{M_U}\right], \qquad (6.17)$$

where

$$\chi_i^U=\frac{\alpha_i(M_U)}{\alpha_i(M_R^+)},\qquad \chi_i^+=\frac{\alpha_i(M_R^+)}{\alpha_i(M_R^o)},\qquad \chi_i^o=\frac{\alpha_i(M_R^o)}{\alpha_i(M_W)},\ \text{and}\ \alpha_i(\mu)=g_i^2(\mu)/4\pi.$$

Using an iterative convergence procedure that ensures fine-structure constant matching[59] at $\mu=M_W$ we have computed M_U, τ_P, and $\sin^2\theta_W$ as a function of M_R^+ for the degenerate and nondegenerate cases as shown in Figs. 5 and 6, respectively, while keeping Z_R light ($M_R^o \simeq$ 1TeV), where $\eta^{(\pm)}=\ln(M^{(\pm)}/M_U)$. Some interesting solutions are summarized in Table 16.

Excluding superheavy-scalar effects, at the one-loop level, the model predicts $M_U\simeq 10^{15}$GeV and $\sin^2\theta_W\simeq 0.225$ for $\Lambda_{\overline{MS}} \simeq$ 250MeV and $M_P=M_R^+ \simeq 10^{11}$GeV. This is consistent with $(\tau_P)_{max} \simeq 10^{35}$yr, where we have included an uncertainty factor of $10^{\pm 3}$ in τ_P arising out of uncertainties in the estimation of the proton decay matrix elements, branching ratios and $\Lambda_{\overline{MS}}$[61,62]. Including contributions upto two loops and no superheavy-Higgs-scalar effects, τ_P decreases by two orders corresponding to the curve $\eta^{(+)}=\eta^{(-)}=0$ in

Table 16. Some predictions of the model $SO(10) \xrightarrow{M_U} G_{2213P} \xrightarrow{M_P} G_{2113}$ on $\sin^2\theta_W$ and τ_P with M_R^0=1 TeV, $\Lambda_{\overline{MS}}$ =0.16 GeV, and different values of the parity violating scale (M_P), including superheavy-Higgs-scalar effects.

$\eta^{(+)}$	$\eta^{(-)}$	$M_R^+=M_P$ (GeV)	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_P (Yr)
-2.30	-2.30	8×10^{8}	2×10^{15}	0.240	36.1	$1.2\times10^{32\pm3}$
		5.6×10^{9}	1.4×10^{15}	0.235	35.7	$3\times10^{31\pm3}$
		2.5×10^{11}	7×10^{14}	0.225	32.0	$1.5\times10^{30\pm3}$
-3.45	-3.45	4.5×10^{9}	2×10^{15}	0.235	32.4	$10^{32\pm3}$
		2×10^{11}	1.2×10^{15}	0.226	31.6	$10^{31\pm3}$
-2.3	-4.6	1.6×10^{11}	2.4×10^{15}	0.230	31.1	$2\times10^{32\pm3}$
		10^{12}	1.8×10^{15}	0.225	30.8	$10^{32\pm3}$
		6.3×10^{12}	1.3×10^{15}	0.221	30.4	$1.4\times10^{31\pm3}$

Fig.5 for which $\Lambda_{\overline{MS}}$ = 160MeV. Including the superheavy-Higgs scalars lighter than M_U by a factor 10(32) increases the two-loop computation of τ_P by 2(3) orders for $\Lambda_{\overline{MS}}$ = 0.16GeV, and the decrease in $\sin^2\theta_W$ is only 0.0015. Allowing the possibility of $\Lambda_{\overline{MS}}$ $\simeq$ 0.25GeV and the superheavy scalars lighter by a factor 32 from M_U, we find $(\tau_P)_{max} \simeq 10^{34}-10^{36}$yr, with $M_P = M_R^+ \geq 10^{11}$GeV and $\sin^2\theta_W$ = 0.220–0.227 as shown in Fig. 5 and Table 16. Increasing M_R^0 from 1TeV to 100 TeV does not have a significant impact on the GUT predictions. In the case of nondegenerate superheavy components, restricting $M_P = M_R^+ > 10^{11}$GeV and $\sin^2\theta_W \simeq 0.22-0.23$, τ_p is found to increase over the one-loop predictions by nearly two orders if $M^{(+)} = M_U$ and $M^{(-)} = M_U/10$. In this case $\tau_P \simeq 10^{32\pm3}-10^{34\pm3}$yr, with M_R^+ = M_P = $10^{11}-10^{12}$GeV and $\sin^2\theta_W \simeq$ 0.22–0.225. For larger values of nondegeneracy factor, τ_P could be larger as shown in Fig. 6. The allowed values of the low mass of the Z_R– boson (300 GeV – 1 TeV) are consistent with the eV – keV – MeV type of mass spectrum for the neutrinos of the three generations when we choose $m_1^D = m_e$, $m_2^D = m_\mu$, and $m_3^D = m_\tau$, as a consequence of natural seesaw mechanism. Out of these the masses of the order of keV and MeV for ν_μ and ν_τ violate the cosmological bound. The difficulty is removed by making them unstable with respect to decay into ν_e by the emission of a Majoron, which is obtained by introducing an additional global $U(1)_l$ (l= lepton number) symmetry in the theory and breaking it spontaneously at a scale $M \gg M_W$. The RGE's also permit solutions with larger values of $M_R^0 = M_{Z_R} \simeq 10^5-10^6$GeV. When $M_P=M_R^+$ $\simeq 10^{11}-10^{12}$GeV, $\rho \simeq 10^6-10^7$ for such larger values of M_R^0 ,which satisfies the naturalness criterion. In this case $m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau} \simeq$

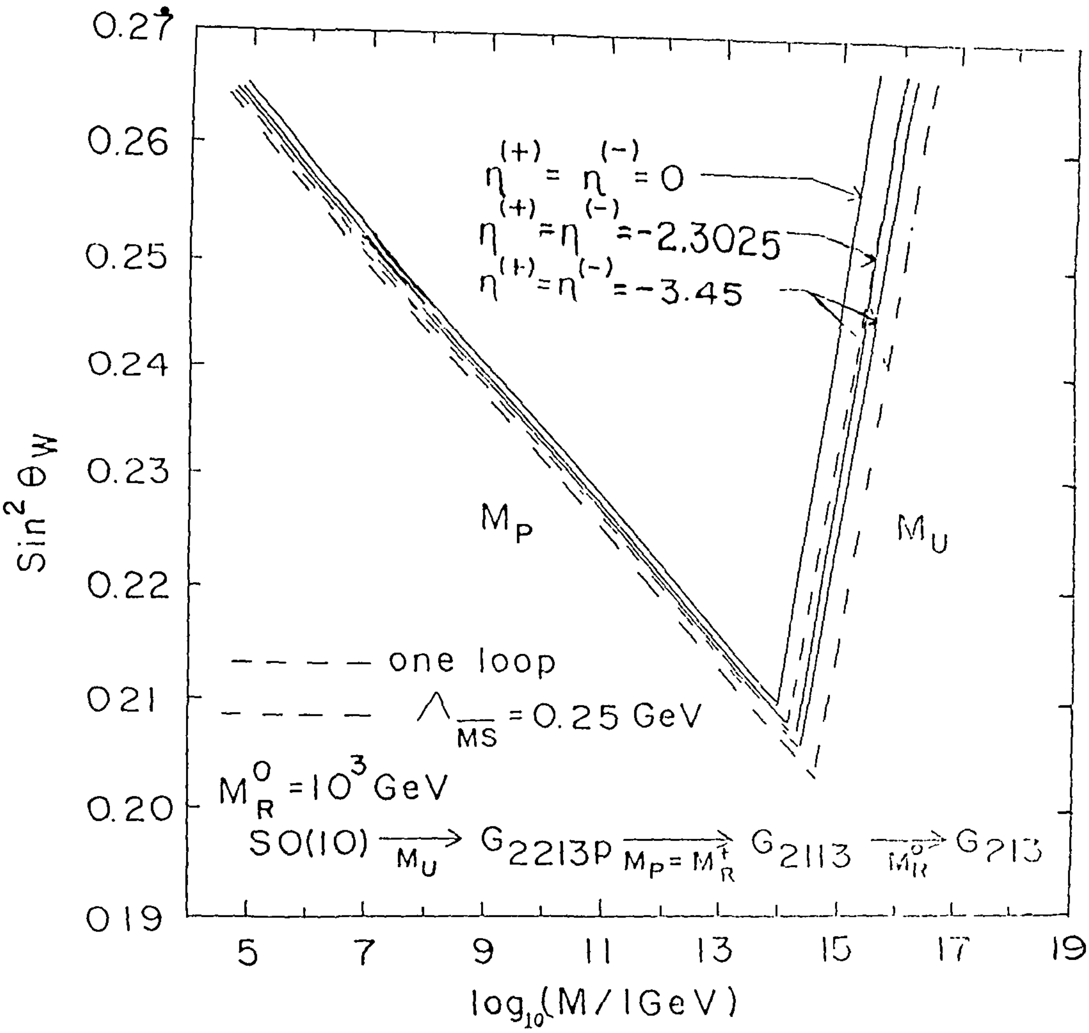

Fig.5. Predictions of the symmetry-breaking pattern $SO(10) \longrightarrow G_{2213P} \longrightarrow G_{2113}$ as described in the text with $M_{Z_R} \simeq 1$ TeV with and without degenerate superheavy-Higgs-scalar contributions. The dot-dashed curve is for $\Lambda_{\overline{MS}} \simeq 0.250$ GeV, others are for $\Lambda_{\overline{MS}} = 0.160$ GeV.

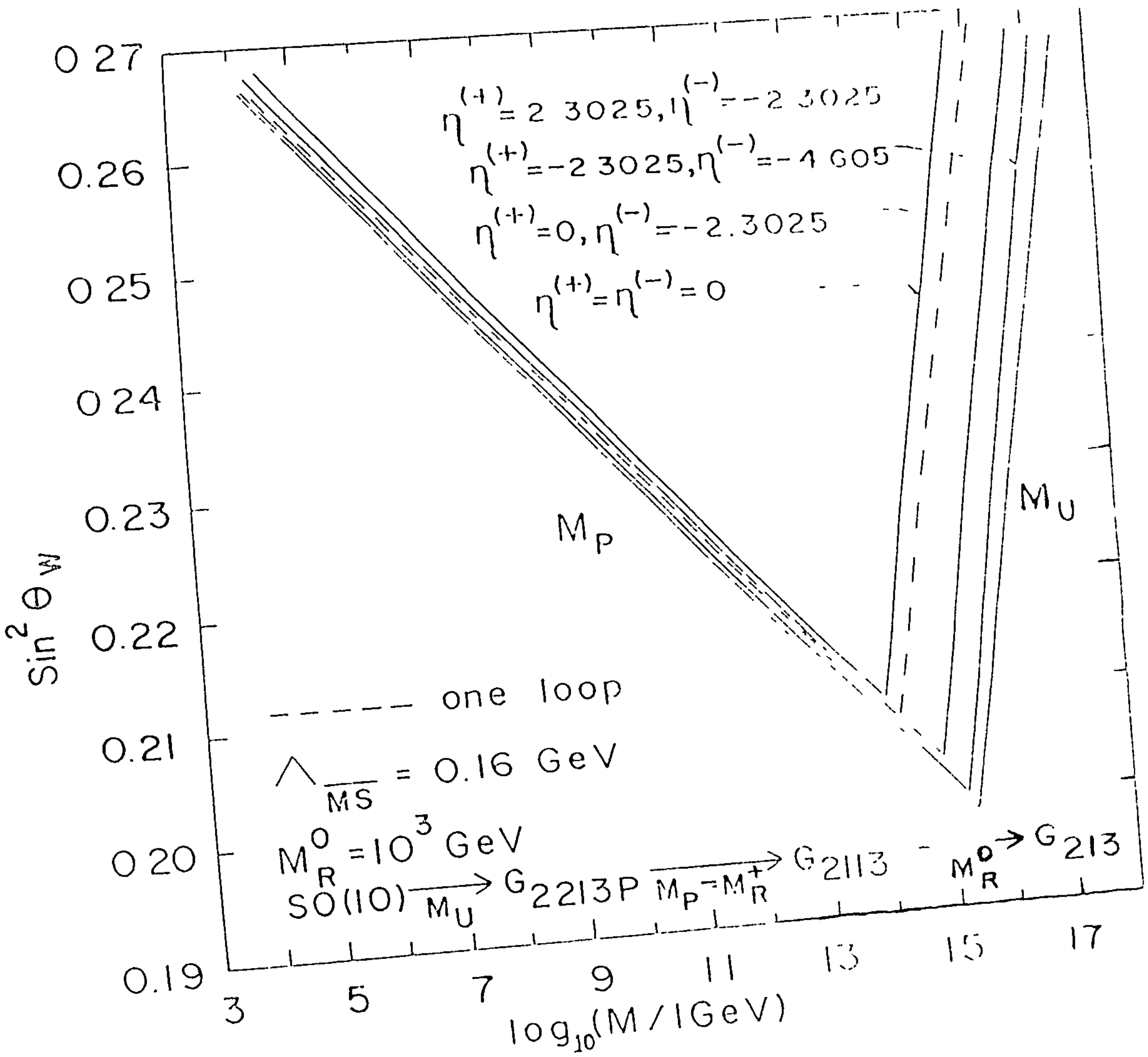

Fig.6. Same as Fig.5, but for nondegenerate superheavy scalar masses under a Coleman-Weinberg-type constraint, and for $\Lambda_{\overline{MS}}$ = 0.160 GeV.

1-10eV and there is no conflict with the cosmological bound.

VI.5. Implementation in SO(10) with G_{214} as an intermediate symmetry

In this section we show how the seesaw mechanism can be implemented naturally with G_{214} as one of the two intermediate symmetries occurring in an SO(10) scenario. In this case both $SU(2)_R$ and P break at the GUT scale $M_U \geq 10^{15}$ GeV as a result of which there is no question of domain-wall problem. If $M_U \simeq 10^{15}$GeV, the proton decay rate could be close to the observable limit. For case (iib) we have found the unification mass too low to be allowed by proton lifetime measurements unless additional fine-tuning is permitted. On the other hand, case (iia) is promising in the context of the GUT scenario:

$$SO(10) \xrightarrow[M_U]{\underline{54}+\underline{45}_1} G_{214} \xrightarrow[M_C]{\underline{45}_2} G_{2113} \xrightarrow[M_R^o]{\underline{126}} G_{213} . \qquad (6.18)$$

The Higgs scalars mentioned in sec. VI.3 for case (iia) are contained in various SO(10) representations : $\chi(1,3,1) \subset \underline{45}_1$, $\xi(1,1,15) \subset \underline{45}_2$, $\Delta_R(1,3,10) \subset \underline{126}$, $\Phi(2,2,1) \subset \underline{10}$ where the transformation properties mentioned are under G_{224}. Note that both $\underline{54}$ and $\underline{45}_1$ are needed for the SSB at $\mu \sim M_U$. The masses of superheavy components of different Higgs representations needed for SSB in the case (6.18) are noted below with their transformation properties under G_{214} :

$\underline{10} \supset M_{H_1}(2,-1/2,1)+M_{H_2}(1,0.6)$,

$\underline{126} \supset M'_{H_1}(1,0,6)+M'_{H_2}(3,0,\overline{10})+M'_{H_3}(1,0,10)+M'_{H_4}(1,-1,10)$
$+M'_{H_5}(2,1/2,15) + M'_{H_6}(2,-1/2,15)$,

$\underline{45}_1 \supset M'_{S_1}(3,0,1)+M'_{S_2}(1,0,15)$,

$$\underline{45_2} \supset M_{S_1}(3,0,1)+M_{S_2^\bullet}(1,1,1)+M_{S_3}(1,0,1,)+M_{S_4}(1,-1,1)+M_{S_5}(2,1/2,6)$$
$$+M_{S_6}(2,-1/2,6),$$
$$\underline{54} \supset M''_{S_1}(3,1,1)+M''_{S_2}(3,0,1)+M''_{S_3}(3,-1,1,)+M''_{S_4}(1,0,20)+M''_{S_5}(2,1/2,6)$$
$$+M''_{S_6}(2,-1/2,6). \qquad (6.18)$$

As mentioned earlier, the components which are singlets under G_{214} or those which are either absorbed as would-be Goldstone components of appropriate gauge bosons or lights do not contribute to the desired contributions, are not included. Maximization of τ_P leads to the following constraint on the superheavy-component masses:

$$M_{H_1}=M'_{H_2}=M'_{H_4}=M_{S_1}=M_{S_2}=M_{S_4}=M_{S_5}=M_{S_6}=M'_{S_1}=M''_{S_1}=M''_{S_2}=M''_{S_3}=M''_{S_5}=M''_{S_6}=M^{(+)},$$
$$M_{H_2}=M'_{H_1}=M'_{H_3}=M'_{H_5}=M'_{H_6}=M'_{S_2}=M''_{S_4}=M^{(-)}. \qquad (6.19)$$

Using three generations of fermions with masses $\mu < M_W$, minimal number of Higgs scalars at various stages of SSB, and the superheavy-Higgs- scalar effects near $\mu \simeq M_U$ we compute $\ln(M_U/M_W)$ and $\sin^2\theta_W$ upto two loops as

$$\ln\frac{M_U}{M_W} = \frac{6\pi}{67}\left(\frac{1}{\alpha} - \frac{8}{3\alpha_S}\right) - \frac{2}{67}\ln\frac{M_C}{M_W} + \frac{2}{67}\ln\frac{M_R^o}{M_W}$$
$$-\frac{3}{134}\left(\frac{10}{19}\ln X_{2L}^U + \frac{34}{15}\ln X_{1R}^U - \frac{224}{9}\ln X_{4C}^U + \frac{10}{19}\ln X_{2L}^C + 2\ln X_{1R}^C\right.$$
$$+\frac{32}{9}\ln X_{BL}^C - \frac{96}{7}\ln X_{3C}^C + \frac{46}{41}\ln X_Y^o + \frac{10}{19}\ln X_{2L}^o$$
$$\left.-\frac{96}{7}\ln X_{3C}^o\right) + \frac{1}{67}\left[28\ln\frac{M^{(+)}}{M_U} - 29\ln\frac{M^{(-)}}{M_U}\right], \qquad (6.21)$$

$$\sin^2\theta_W = \frac{35}{134} + \frac{61\alpha}{201\alpha_S} - \frac{437\alpha}{402\pi}\ln\frac{M_C}{M_W} + \frac{35\alpha}{402\pi}\ln\frac{M_R^o}{M_W}$$

$$- \frac{\alpha}{67\pi}\left[\frac{711}{76}\ln X_{2L}^U + \frac{4473}{128}\ln X_{1R}^U - \frac{13187}{44}\ln X_{4C}^U + \frac{711}{76}\ln X_{2L}^C\right.$$

$$+\frac{449}{36}\ln X_{1R}^C + \frac{71}{6}\ln X_{BL}^C + \frac{433}{14}\ln X_{3C}^C + \frac{939}{164}\ln X_Y^o + \frac{711}{76}\ln X_{2L}^o$$

$$\left.+ \frac{433}{14}\ln X_{3C}^o\right] + \frac{\alpha}{804\pi}\left[-1633\ln\frac{M^{(+)}}{M_U} + 928\ln\frac{M^{(-)}}{M_U}\right],$$

where (6.22)

$$\chi_i^U = \frac{\alpha_i(M_U)}{\alpha_i(M_C)}, \quad \chi_i^C = \frac{\alpha_i(M_C)}{\alpha_i(M_R^o)}, \quad \text{and } \chi_i^o = \frac{\alpha_i(M_R^o)}{\alpha_i(M_W)}.$$

Following the iterative convergence approach to solve two-loop renormalization-group equations and using plausible values of superheavy component masses, our solutions for the intermediate scale and M_U for $M_R^o \simeq 1$ TeV are shown in Figs. 7 and 8 for $\Lambda_{\overline{MS}} \simeq 0.160$ GeV, and 0.350 GeV respectively (Ref.62) where $\eta^{(\pm)} = \ln(M^{(\pm)}/M_U)$. Some of the interesting solutions are also presented in Table 17. At the one-loop level with $\Lambda_{\overline{MS}} \simeq 0.350$ GeV, the predicted value of τ_P is found to be very close to the observed experimental limit for $M_C = 10^{11}$ GeV and $\sin^2\theta_W \simeq 0.235$, but τ_P is found to be 1-2 orders less than the experimental limit for $\Lambda_{\overline{MS}} \simeq 0.160$ GeV. When superheavy Higgs-scalar effects are included in two-loop calculations, we find $\tau_P \simeq 10^{32\pm3}-10^{34\pm3}$yr, $\sin^2\theta_W \simeq 0.230$, $M_C \simeq 10^7$ GeV with $\Lambda_{\overline{MS}} \simeq 0.160$ GeV if the heavier (*lighter*) *components differ by a factor 10 from the unification*

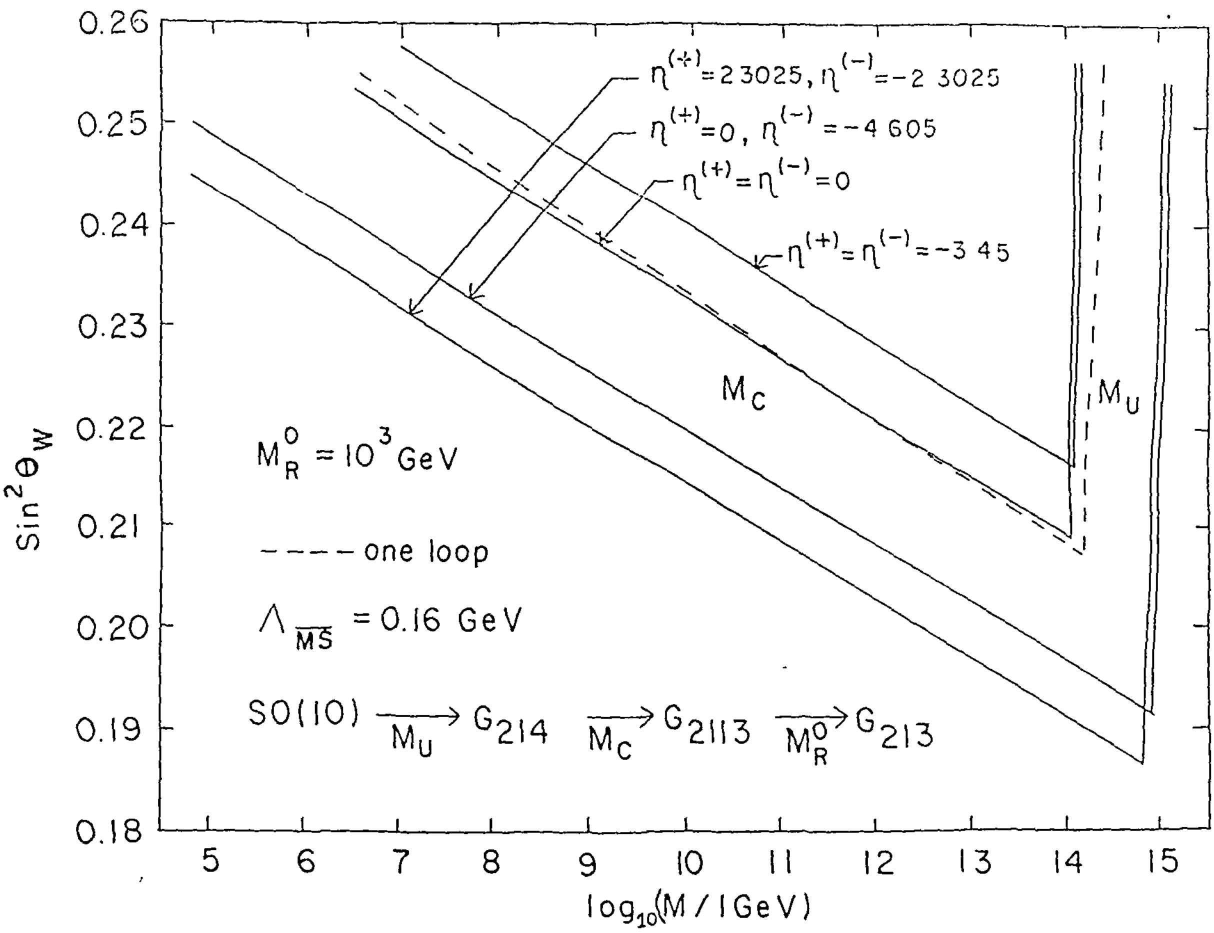

Fig.7. Predictions of the symmetry-breaking pattern $SO(10) \longrightarrow G_{214} \longrightarrow G_{2113}$ as described in the text including superheavy-Higgs-scalar effects for $M_{Z_R} \simeq 1\,TeV$ and $\Lambda_{\overline{MS}} = 0.16$ GeV.

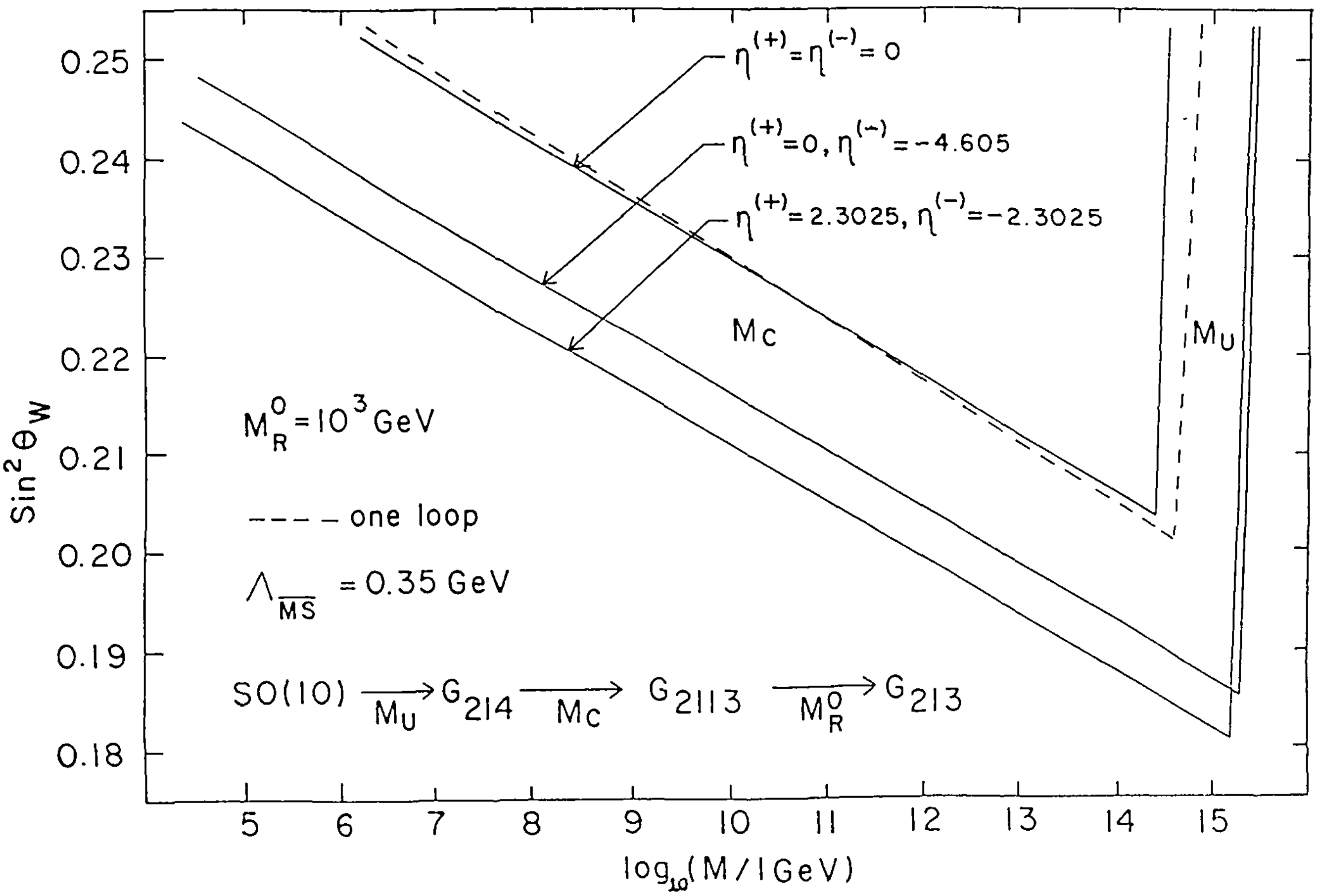

Fig.8. Same as Fig.7, but for $\Lambda_{\overline{MS}}$ = 0.35GeV.

Table 17. Some predictions of the model $SO(10) \xrightarrow{M_U} G_{214} \xrightarrow{M_C} G_{2113}$ for two values of $\Lambda_{\overline{MS}}$ on $\sin^2\theta_W$ and τ_P with M_R^o=1 TeV and different M_C including superheavy-Higgs-scalar effects as described in the text.

$\Lambda_{\overline{MS}}$ (GeV)	$\eta^{(+)}$	$\eta^{(-)}$	M_C (GeV)	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_P (yr)
0.16	2.3	-2.3	4×10^5	10^{15}	0.240	48.3	$1.7\times10^{31\pm3}$
			3.0×10^6	10^{15}	0.235	47.6	$1.4\times10^{31\pm3}$
	0	-4.6	3×10^6	1.3×10^{15}	0.240	42.5	$2.5\times10^{31\pm3}$
			1.6×10^8	1.1×10^{15}	0.230	40.8	$1.5\times10^{31\pm3}$
0.35	2.3	-2.3	10^5	2.4×10^{15}	0.240	48.8	$4.3\times10^{32\pm3}$
			5.6×10^6	2.2×10^{15}	0.230	47.4	$2.9\times10^{32\pm3}$
	0	-4.6	5.6×10^6	2.7×10^{15}	0.235	41.7	$5\times10^{32\pm3}$
			4×10^7	2.5×10^{15}	0.230	41.0	$3.8\times10^{32\pm3}$

mass. For larger values of $\Lambda_{\overline{MS}}$ or nondegeneracy factors, τ_P is found to increase further. We find that this SO(10) model permits observable rare-kaon decays corresponding to $M_C \simeq 10^5$ GeV provided $\sin^2\theta_W \simeq 0.24$. In all allowed solutions in this model $M_R^+ = M_U = M_P \simeq 10^{15}$GeV. With $M_R^0 \simeq$ 1TeV, $\rho \simeq 10^{12}$, the naturalness criterion is easily satisfied. As in Sec. VI.2, the low-mass Z_R boson yields the neutrino mass spectrum as eV – keV – MeV for the three generations. The violation of the cosmological bound by the ν_μ and ν_τ masses is avoided by making these neutrinos unstable against Majoron emission through the introduction of an additional global lepton-number symmetry $U(1)_1$.[43] But the RGE's also permit $M_R^0 \simeq 10^5 - 10^6$GeV as the Z_R-boson mass for which $m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau} \simeq$ 1-10eV as a consequence of the natural seesaw mechanism with $\rho = 10^9 - 10^{10}$, and this is consistent with the cosmological bound with stable neutrinos. In this case the predicted weak – interaction phenomenology at low energy cannot be distinguished from the standard model predictions.

VI.6. Summary and Discussion

Chang and Mohapatra[30] have observed that the seesaw mechanism[16,17] is spoiled in a large class of models where P- and $SU(2)_R$- breaking scales are identical and suggested that, in order to implement the mechanism in LRS or SO(10) models with B-L symmetry-breaking scale, the parity-and $SU(2)_R$-breaking scales must be widely separated. However, in this Chapter[64] we have *suggested the new possibilty* that the seesaw mechansim for neutrino masses could be natural in the context of the left-right-symmetric gauge group, partial unification scheme, and

GUT's even if the scales of P- and $SU(2)_R$- breakings are identical. In our case the P- breaking scale is the same as the $W_R^{\pm}$ gauge boson mass ($M_P = M_R^+$) and $U(1)_{B-L}$-breaking scale is the same as the Z_R-boson mass (M_R^o). The criterion which guarantees naturalness has been derived and is found to depend upon the largeness of the ratio $\rho = M_R^+/M_R^o \gg 10^5$. At the critical value of the ratio $\rho \simeq 10^5$, the induced and the seesaw mechanism contributions are comparable, but for larger values of ρ the induced neutrino mass becomes smaller.

In the LRS model based upon the gauge group G_{2213P}, it is very easy to implement the mechanism as there is not much restriction on $M_P = M_R^+$. In the partial unification scheme with one intermediate symmetry G_{2113}, the RGE permits $M_P = M_R^+ \simeq M_C = 10^{12}$–$5\times10^{13}$GeV with $M_R^o = M_{Z_R} = 300$ GeV – 10^6 GeV (case (iib)). However with two intermediate symmetries G_{214} and G_{2113} (case (iia)) the solutions allow $M_P = M_R^+ \simeq 7\times10^{13}$– 10^{17}GeV for 10^{10}GeV $> M_C >$ 10^5GeV, predicting rare-kaon decays to be observable by low-energy experiments besides a low - mass Z_R boson.

In the SO(10) model, we found that the natural seesaw mechanism can be implemented with parity (P) surviving down to an intermediate scale $M_P = M_R^+ \simeq 10^{11}$GeV or broken at the GUT scale $M_P = M_U = M_R^+ \geq 10^{15}$GeV. With G_{2213P} and G_{2113} intermediate symmetries, we also found an intermediate P- breaking scale $M_P \simeq 10^{11}$–10^{12}GeV, observable proton decay by the second generation of experiments with $\tau_P \simeq 10^{33}$–10^{35} yr, and a low - mass Z_R boson ($M_R^o \simeq 300$ – 10^3GeV) by including renormalization effects on gauge coupling constants upto two loops and superheavy-Higgs-scalar masses lighter than M_U by a factor of 10-32. In this case there is the possibility that the domain walls[24] created in the early universe might have been removed by inflation. In this context it is to be noted that the large P-violating scale can be associated with the breaking of Peccei-Quinn symmetry invoked to solve the strong CP problem and can be generated by the principle of geometric hierarchy from $M_{pl} \simeq 10^{19}$GeV and $M_{Z_R} \simeq 10^3$ GeV, or M_W. Further, it has been observed that while embedding a LRS gauge group as an intermediate symmetry in SO(10), the generation of an adequate baryon asymmetry of the universe needs such a large P-violating scale[44]. In the other interesting SO(10) scenario with G_{214} and G_{2113} as the two intermediate symmetries, superheavy-Higgs-scalar masses differing by a factor 10 (lighter or heavier) from M_U allow $(\tau_P)_{max} \sim 10^{35}$yr with the possibility of observable rare-kaon decays and a low mass Z_R boson. In the two SO(10) models discussed here G_{2113} is allowed to be the gauge symmetry beyond the standard model with the permitted values of Z_R boson mass varying over a wider range : 300 - 10^5GeV.

With the G_{2113} model, it is possible to have a low- mass Z_R boson ($M_R^o \simeq$ 300GeV-1TeV) which yields fits to the neutral and charged- current data similar to the standard model predictions[63]. When such values of M_R^o are used in the natural seesaw mechanism, it gives eV-keV-MeV type of mass sepectrum for the first-, second-, and third-generation neutrinos, respectively, out of which the masses of the order keV and MeV for ν_μ and ν_τ violate the cosmological bound. The difficulty can be removed by making ν_μ and ν_τ unstable with respect to the emission of a Majoron which is created when an additional global symmetry $U(1)_l$ (l = lepton number), attached to the models, breaks spontaneously[43]. With the other allowed possibility, $M_R^0 \simeq 10^5$GeV , $m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau} \simeq 1-10$eV, there is no violation of the cosmological bound. The weak-interaction phenomenology at lower energies is then indistinguishable from the standard model predictions within the available experimental accuracies. However, one novel feature in the partial unification scheme and SO(10) model with G_{214} and G_{2113} intermediate symmetries is the prediction of observable rare-kaon decays such as $K_L \longrightarrow \bar{\mu}e$. The analyses carried out here in SO(10) can be easily implemented in other GUT's like SO(2N) (N > 5), E_6, and SU(16) with similar predictions. However, in $SU(8)_L \times SU(8)_R$ while all other low-energy predictions are similar, it is possible to have a more stable proton since the gauge-boson-mediated interaction corresponding to the proton decay is absent.

Finally from the investigations carried out in this Chapter we conclude that, (1) Natural seesaw mechanism explaining small neutrino masses, scenarios different from those discussed by Chang and Mohapatra in Ref.30 is realizable in LRS models, partial unification schemes, and GUT's with identical P-and $SU(2)_R$-braking scales; (ii) The necessary condition for naturalness is wide separation between P-and $U(1)_{B-L}$-breaking scales; (iii) In the present model since $U(1)_R$ and $U(1)_{B-L}$ breaking at the same scale ($M_{Z_R} = M_{B-L}$), the naturalness criterion guarantees a low-mass Z_R boson.

CHAPTER VII

SUMMARY AND CONCLUSION

In this Chapter we briefly summarize the results of investigations reported in this thesis. The main results obtained are of two types. In the first part, from Chapters II–V, we investigated the impact of higher-dimensional operators on SU(5) and SO(10) GUT's. In second part, in Chapter VI, we have successfully investigated the naturalness of the seesaw mechanism in the context of the left-right-symmetric gauge groups or GUT's with identical parity-(P) and $SU(2)_R$-breaking scales.

In earlier investigations, the effect of the five-dimensional operator has been analysed on nonsupersymmetric minimal SU(5) model, but the predictions are inconsistent with either proton lifetime, or $\sin^2\theta_W$. Including the effects of the five-and six-dimensional operators, scaled by suitable powers of the compactification scale, or the Planck scale, we find[36] that the renormalization group constraints permit only high unification mass, $M_U \sim 10^{16}$–10^{19}GeV and smaller GUT coupling constant leading to $\tau_P \geq 10^{38}$yr for the $p \longrightarrow e^+\pi^0$ mode even with $\sin^2\theta_W \simeq 0.22$–$0.24$. A notable result, also obtained for the first time, is the smallness of the GUT coupling for $\mu > M_U$ (i.e.,$\alpha_G \simeq 10^{-4}$). Such smaller values of α_G can be reconciled with the standard model gauge couplings $O(\alpha)$ at $\mu \simeq M_U$ due to the operator threshold effects. The enhancement of the proton lifetime is caused due to two factors: (i) Increase in the unification mass, and (ii) Decrease in the GUT-coupling constant.

In Chapter III[40] we have noted that the SO(10) predictions with single G_{214} intermediate symmetry in the absence of the gravity-induced corrections, is ruled out as the prediction yields τ_p significantly below the IMB limit[19]. Including the effects of the five-dimensional operator in the nonrenormalizable SO(10) invariant Lagrangian, we found that the G_{214} intermediate symmetry can survive down to a lower scale $M_C \simeq 10^5-10^6$ GeV which has the prospect of being experimentally verified by rare-kaon decays (e.g., $K_L \longrightarrow \bar{\mu} e$). In addition to the lower values of M_C, the solutions to RGE's also permit higher values of M_C, covering the wide range $M_C \simeq 10^5-10^{11}$GeV where the solutions with higher values of M_C predict observable proton decay with $\tau_p \gtrsim 3\times10^{32\pm3}$yr for the $p \longrightarrow e^+\pi^0$ mode. A broad range of M_C reflects in the wider range of Majorana neutrino masses:

(a) $m_{\nu_e} \sim (2.5\times10^{-7}-0.25)$eV, $m_{\nu_\mu} \sim (1.5\times10^{-2}eV-15.6$keV),

$m_{\nu_\tau} \sim$ (100eV$-$100MeV),

(b) $m_{\nu_e} \sim (2.6\times10^{-8}-2.6\times10^{-3})$eV, $m_{\nu_\mu} \sim (1.1\times10^{-4}-112)$eV,

$m_{\nu_\tau} \sim (3.2\times10^{-2}eV-31.7$keV),

where the masses in (a) ((b)) have been obtained using the up quark (charged lepton) masses in the seesaw formula. In both the above cases, the lowest (highest) value corresponds to $M_C=10^{11}(10^5)$ GeV. Out of these the neutrino masses in the range 100eV$-$100MeV violate the cosmological bound, according to which the sum of the stable neutrino masses should be less than 65 eV[42]. The cosmological bound can be evaded by making the heavier neutrinos unstable with respect to decay into the lighter neutrinos by the emisssion of a Majoron[43]. Such a Majoron is a massless Goldstone boson carrying 2 units of lepton number and is created when an additional $U(1)_l$-global symmetry (l=lepton number) is broken spontaneously by the VEV of a Higgs scalar carrying lepton number l=2 at a higher scale $M \gg M_W$. The introduction of such an additional global symmetry does not affect the GUT predictions as described in the text. In this model, we found that the predicted proton lifetime is closer to the observable limit for $M_C \simeq 10^{10}-10^{11}$GeV but it is larger for $M_C \simeq 10^{5}-10^{7}$GeV. For a fixed $\sin^2\theta_W$, τ_P decreases with increasing M_C and IMB limit[19] is saturated when $M_C \sim 10^{11}$GeV. The order of magnitude of the compactification scale, estimated in this model is found to be in the range $10^{17}-10^{18}$GeV.

In Chapter IV[50], as against the conclusion of Rizzo[31], we note that a low-mass right-handed gauge boson ($W_R^{\pm}$) in SO(10) with the single G_{2213P} intermediate symmetry as the parity restoring gauge group, is not allowed. On the other hand using the mechanism of decoupling P- and $SU(2)_R$ -breakings, we found that such a low-mass $W_R^{\pm}$-gauge bosons are possible in SO(10) model with single G_{2213} ($g_{2L} \neq g_{2R}$) intermediate symmetry without parity restoration. In this case a low $W_R^{\pm}$-mass, $M_R \sim 500$GeV to 10TeV is allowed. Such a low-mass $W_R^{\pm}$ provides the possibility of observing the V+A structure of charged currents, CP-violation through $K^0 \longrightarrow \bar{K}^0$ mixing, and small neutrino masses. With $M_R \sim 1$TeV, the model predicts $m_{\nu_e} \sim$ eV, $m_{\nu_\mu} \sim$ 10keV, and $m_{\nu_\tau} \sim$ 4MeV when charged lepton masses are used in the seesaw formula. Again the ν_μ and ν_τ masses violate the cosmological bound which is evaded in the manner already described..

In Chapter V[56] we have discussed the effect of five-dimensional operator in SO(10) model with the single G_{224} intermediate symmetry including parity ($g_{2L}=g_{2R}$) or excluding it ($g_{2L}\neq g_{2R}$). Here, we note that when G_{224} is left-right symmetric, the corresponding equation for $\sin^2\theta_W$ is independent of the parameter ε of the nonrenormalizable Lagrangian whereas the equations for unification mass (M_U) and the GUT-coupling constant (α_G) do depend upon it. In this case a new type of solutions are found which is consistent with $M_C \sim 10^{14}$GeV for $\sin^2\theta_W \simeq 0.22$–$0.24$ and the experimentally observable proton lifetime $\tau_P \geq 3\times10^{32\pm3}$yr. The value fo M_C, being one order larger than the one obtained by Shafi and Wetterich[27], makes the model less problematic from cosmological domain walls[24]. On the contrary, using the mechanism of decoupling P-and $SU(2)_R$-breakings[21], we found that the equations for M_U and α_G are independent of ε wheras $\sin^2\theta_W$ does depend upon it. In such a case a low $SU(4)_C$-breaking scale $M_C \sim 10^5$–10^6GeV is allowed which can manifest in low-energy signatures of quark-lepton unification through rare-kaon decays (e.g.,$K_L \longrightarrow \bar{\mu}e$) and n-$\bar{n}$ oscillation. Such low values of $M_C \simeq 10^5$–10^6GeV yield higher values of Majorana neutrino masses noted in (a) and (b). In these cases proton seems to be very stable with $\tau_P \geq 3\times10^{42\pm3}$yr.

In the first part of the Chapter VII[64] the work of Chang and Mohapatra[30] on the naturalness of the seesaw mechanism for Majorana neutrino masses is reviewed. They have pointed out that the implementation of the mechanism explaining small neutrino masses in left-right symmetric or SO(10) models in a natural manner requires wide separation between P-and $SU(2)_R$-breaking scales ($M_P \gg M_R$). We have found the new possibility that the seesaw mechanism for neutrino masses could be also natural in the context of the left-right symmetric gauge group, partial unification schemes, and GUT's even if the P-and $SU(2)_R$-breaking scales are identical. In our models the general criterion is wide separation betweeen P and $U(1)_{B-L}$ breaking scales. In such cases $SU(2)_R \times U(1)_{B-L}$ or $SU(2)_R \times SU(4)_C$ has to break spontaneously in more than one steps. In all the models discussed here, the gauge group immediately preceeding the standard model emerges to be one of its minimal extensions based upon $SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C$ with a second neutral Z_R-boson mass $M_R^0 \simeq 300\text{-}10^5$GeV. An embedding in the partial unification scheme leads to observable rare-kaon decays. In the two symmetry breaking chains investigated in SO(10) with parity broken either at the unification scale ($M_P = M_U$) or at an intermediate scale ($M_P \geq 10^{11}$GeV), proton decay is predicted with lifetime nearer the observable limit; but, in the former case, rare-kaon decays ($K_L \longrightarrow \bar{\mu} e$) are also predicted near the observable limit when an intermediate gauge group $SU(2)_L \times U(1)_R \times SU(4)_C$ survives down to the scale $M_C \simeq 10^5$GeV provided $\sin^2\theta_W \simeq 0.24$.

In conclusion we find that the presence of higher-dimensional operators, which might appear in the GUT Lagrangian either as a result of compactification of extra dimensions through specific forms of the metric tensor, or as effects of quantum gravity can cause drastic changes to the conventional GUT predictions made earlier ignoring such effects. Very interesting physical predictions testable by low energy experiments or at the collider energies are possible in SO(10) with certain single intermediate symmetries. Although the higher-dimensional operators introduce nonrenormalizable terms into the GUT Lagrangian, they are absorbed as the renormalizable terms in the effective gauge theories after the spontaneous symmetry breaking of the GUT's. In the analyses carried out here on the effects of higher-dimensional operators, all superheavy masses have been taken to be same as M_U. If they are different from M_U, the corresponding changes in the GUT predictions have to be included.

A very interesting observation is the alternative class of models with identical parity and $SU(2)_R$ breaking scales leading to natural seesaw mechanism for Majorana neutrino masses. The extra Z_R boson and its effects predicted in the range of few hunderd GeV to few TeV would be observed at the collider energies. Currently there has been intensive studies for identification of extra Z-bosons emerging from grand unified theories, superstring unification, or otherwise.

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Gravity-induced large grand-unification mass in SU(5) with higher-dimensional operators

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Following the recent method due to Hill, Shafi, and Wetterich, we investigate the impact of higher-dimensional operators ($d \geq 5$) induced by gravity and compactification of extra dimensions on the minimal SU(5) grand unified theory. Modifications caused by the $d=5$ operator alone seem to be ruled out, even if the compactification scale M_C is as low as 10^{1} GeV, as they require $\sin^2\theta_W \leq 0.203$ in conflict with the present world average. The addition of a six-dimensional operator is found to allow only high unification masses $M_U \sim (0.1-1)M_C$ with $M_C \sim 10^{17}-10^{1}$ GeV and $\sin^2\theta_W \simeq 0.22-0.24$. The grand unification coupling constant is also found to be significantly smaller.

I. INTRODUCTION

Gauge theories of the Kaluza-Klein[1] type offer the exciting possibility of unification with gravity through the introduction of higher dimensions leading to the four-dimensional structure of the present Universe as a result of the compactification of extra dimensions.[1-3] Attempts have been made to generate effective four-dimensional gauge theories, such as the standard model and grand unified theories (GUT's) from the isometry group of the compactified manifold, to identify the observed fermions as the chiral representations of the effective gauge theories, and to compute the gauge couplings in terms of the characteristic length scales of extra spatial dimensions.[1-5] Although superstring theory[6] is expected to provide a realistic gauge unification of all basic interactions, a lot of interest still remains in conventional GUT's,[7-9] with and without gravity-induced effects. Although most of the GUT's with intermediate symmetries can satisfy the experimentally observed constraints on the proton lifetime (τ_p) for the $p \to e^+\pi^0$ mode and $\sin^2\theta_W$ (Ref. 10),

$$\tau_p \geq 3\times 10^{32} \text{ yr}, \quad \sin^2\theta_W = 0.230 \pm 0.005 \quad (1)$$

the minimal SU(5) model[7] and certain other GUT's with a grand desert are ruled out as they predict significantly lower values.[11-13] If the masses of other superheavy gauge bosons in SO(10) are nondegenerate and differ from the $X^{\pm 4/3}$ and $Y^{\pm 1/3}$ gauge boson mass, the model can be made consistent with (1), even with a grand desert.[12] In a nonminimal SU(5) model,[14] however, τ_p and $\sin^2\theta_W$ can satisfy (1) by including one-loop contributions of nondegenerate superheavy scalars from additional representations such as 10, 15, 45, and 50. Since the grand unification occurs at a high scale ($M_U > 10^{15}$ GeV), it is natural to suppose that there could be significant modification to the GUT predictions by gravity-induced corrections. It is the purpose of this paper to compute such modifications to the minimal GUT predictions.

Recently, the impact of five-dimensional operators scaled by the compactification scale (M_C) has been investigated to calculate the modification caused by the minimal and other GUT predictions. Similar nonrenormalizable operators induced by gravity with dimensions $d>5$ and scaled by powers $(M_C)^{4-d}$ are subject only to the symmetries of low-energy theory and are known to occur, for example,[15] in the presence of gravitational instantons for $M_C \sim M_{Pl}$. The five-dimensional operators are seen to arise naturally as a result of compactification of extra dimensions in a Kaluza-Klein-type theory.[3] The impact of such an operator on the quark-to-lepton mass ratio m_d/m_e predicted by the minimal SU(5) model was examined by Ellis and Gaillard.[16]

In the case of a supersymmetric SU(5) GUT, significant modifications to τ_p and $\sin^2\theta_W$ have been noted[17] with $M_C \sim M_{Pl} = 10^{19}$ GeV. In the SO(10) GUT, with $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ as intermediate symmetry, the masses of the W_R gauge bosons have been brought down to the order of the W scale leading to the possible observation of low-mass parity restoration in the future.[18] In the minimal SU(5) GUT, if the compactification scale is allowed to be about two orders lower than M_{Pl}, which is possible within certain Kaluza-Klein-type theories,[19] Shafi and Wetterich[20] have observed a very significant increase of τ_p so as to be compatible with the experimental limit for the $p \to e^+\pi^0$ mode.

In this paper we use the method due to Shafi and Wetterich[20] and Hill[17] to compute modifications of the minimal SU(5) predictions in the presence of $d \geq 5$ operators, especially in view of the recent measurements on $\sin^2\theta_W$. We note that the modifications of the $d=5$ operator alone, taken to be making the minimal GUT compatible with the experimental data on τ_p, are now ruled out as these solutions require $\sin^2\theta_W$ significantly *below the accepted world average, even if* $M_C = 10^{17}$ GeV. As our main result, we then examine the modifications caused by adding a $d=6$ operator in the Lagrangian. We find that the only permissible values of the unification mass should be of the order $(0.1-1)\,M_C$, where M_C could be anywhere between 10^{17} and 10^{19} GeV. Interestingly enough, the allowed values of the electroweak mixing angle can be made consistent with the currently available world average with $\sin^2\theta_W \simeq 0.22-0.24$ for every value of the unification mass. Another interesting aspect of the present analysis is that the bare-grand-unification coupling α_G turns out to be nearly 2 orders of magnitude smaller than the earlier results. We also obtain perturbative and positivity bounds on certain parameters and mention a new relation among them.

In Sec. II we obtain general formulas for the unification mass and the electroweak mixing angle including five-dimensional operators and particular forms of still higher-dimensional operators in the Lagrangian. In Sec. III we discuss earlier results with five-dimensional operators. In Sec. IV we report numerical analysis including five- and six-dimensional operators. Our conclusions are stated in Sec. V.

II. FORMULAS FOR GAUGE COUPLINGS, UNIFICATION MASS, AND $\sin^2\theta_W$

As has been emphasized earlier, gravitational effects could induce nonrenormalizable operators of dimension $d \geq 5$, scaled by powers of $M_C^{-(d-4)}$, into the normalizable Lagrangian, but only the impact of $d=5$ operators have been examined so far on the unification mass M_U and $\sin^2\theta_W$. Such operators are restricted only by the symmetries of the theory at lower energies. Denoting ϕ as the scalar in the adjoint representation $24 \subset SU(5)$, the effect of the nonrenormalizable operators can be included in the following manner in the SU(5)-invariant Lagrangian:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{NR}, \tag{2a}$$

$$\mathcal{L}_{NR} = \sum_{n=1,2,\ldots} \mathcal{L}_{NR}^{(n)}, \tag{2b}$$

$$\mathcal{L}_{NR}^{(n)} = -\frac{1}{2}\frac{\eta^{(n)}}{M_C^n}\,\mathrm{Tr}(F_{\mu\nu}\phi^n F^{\mu\nu}), \tag{2c}$$

$$\mathcal{L}_0 = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}), \tag{2d}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \tag{2e}$$

$$(A_\mu)^a_b = A^i_\mu\left[\frac{\lambda_i}{2}\right]^a_b, \tag{2f}$$

$$\mathrm{Tr}(\lambda_i\lambda_j) = \tfrac{1}{2}\delta_{ij}. \tag{2g}$$

In Eq. (2), A^i_μ is the ith component of the gauge field, λ_i is the corresponding generator, and $\eta^{(n)}$, $n = 1, 2, \ldots$, are the unknown parameters. In Refs. 17 and 20 the case with the five-dimensional operator corresponds to $\eta^{(1)} \neq 0$ and $\eta^{(n)} = 0$ for $n \geq 2$. It may be noted that the expression (2c) for higher-dimensional operators given in Eq. (2c) is not the most general one, especially when $n \geq 2$. *For example, with* $n=2$, *other gauge-invariant operators* not covered by (2c) are $\mathrm{Tr}(\phi^2)\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ and $\mathrm{Tr}(F_{\mu\nu}\phi)\mathrm{Tr}(F^{\mu\nu}\phi)$, the latter being more troublesome for computations of the physical quantities of interest in this paper. We confine to the choice (2c) for the sake of convenience and obtaining modifications to M_U and $\sin^2\theta_W$ with a constraint on the parameters as shown in Eq. (14a) in Sec. IV. Using the vacuum expectation value of 24 as

$$\langle\phi\rangle = \left(\tfrac{1}{15}\right)^{1/2}\phi_0\,\mathrm{diag}\left(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2}\right), \tag{3}$$

denoting g_3, g_2, and g_1 as the $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ coupling constants, respectively, and the gravity induced changes as

$$\begin{aligned} g_3^2(M_U) &\to g_3^2(M_U)(1+\epsilon_C), \\ g_2^2(M_U) &\to g_2^2(M_U)(1+\epsilon_I), \\ g_1^2(M_U) &\to g_1^2(M_U)(1+\epsilon_Y), \end{aligned} \tag{4}$$

we obtain, using Eqs. (2)–(4),

$$\begin{aligned} \epsilon_C &= \sum_{n=1,2} \epsilon^{(n)}, \\ \epsilon_I &= -\tfrac{3}{2}\epsilon^{(1)} + \tfrac{9}{4}\epsilon^{(2)} - \tfrac{27}{8}\epsilon^{(3)} + \cdots, \\ \epsilon_Y &= -\tfrac{1}{2}\epsilon^{(1)} + \tfrac{7}{4}\epsilon^{(2)} - \tfrac{13}{8}\epsilon^{(3)} + \cdots, \end{aligned} \tag{5}$$

where the ellipsis in (5) includes the effect of operators $d > 7$ and

$$\epsilon^{(n)} = \left[\frac{1}{\sqrt{15}}\frac{\phi_0}{M_C}\right]^n \eta^{(n)}, \quad n = 1, 2, \ldots \tag{6}$$

Using $\alpha_G = g_0^2/4\pi$, where g_0 is the bare GUT coupling and the relation

$$\phi_0 = [6/(5\pi\alpha_G)]^{1/2} M_U, \tag{7}$$

Eq. (6) can be rewritten as

$$\eta^{(n)} = \left[\left[\frac{25\pi\alpha_G}{2}\right]^{1/2}\frac{M_C}{M_U}\right]^n \epsilon^{(n)}. \tag{8}$$

Imposing the condition of equality of the three coupling constants at M_U,

$$g_3^2(1+\epsilon_C) = g_2^2(1+\epsilon_I) = g_1^2(1+\epsilon_Y) = g_0^2, \tag{9}$$

the one-loop renormalization group equations[24]

$$\frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_i(M_U)} + \frac{a_i}{2\pi}\ln\frac{M_U}{M_W}, \quad i = 1, 2, 3 \tag{10}$$

are solved with $a_1 = \frac{41}{10}$, $a_2 = -\frac{19}{6}$, $a_3 = -7$ to yield

$$\ln\frac{M_U}{M_W}=\frac{1}{D}\left[1+\left(\epsilon_C-\frac{5\epsilon_Y+3\epsilon_L}{3}\frac{\alpha}{\alpha_s}\right)\left(1-\frac{8}{3}\frac{\alpha}{\alpha_s}\right)^{-1}\right]\times\ln\frac{M^{(5)}}{M_W}\,, \tag{11a}$$

$$\sin^2\theta_W=\left[\sin^2\theta_W^{(5)}-\tfrac{19}{134}\epsilon_C+\tfrac{1}{67}\left(21+\frac{41}{2}\frac{\alpha}{\alpha_s}\right)\epsilon_L+\frac{95}{402}\frac{\alpha}{\alpha_s}\epsilon_Y\right]\Big/D\,, \tag{11b}$$

$$\frac{1}{\alpha_G}\equiv\frac{4\pi}{g_0^2}=\frac{3}{67}\left(\frac{11}{3\alpha_s}+\frac{7}{\alpha}\right)\Big/D\,, \tag{11c}$$

$$D=1+\tfrac{1}{67}(11\epsilon_C+21\epsilon_L+35\epsilon_Y)\,, \tag{11d}$$

where $M^{(5)}$ and $\sin^2\theta^{(5)}$ denote the one-loop predictions of the minimal model, without gravity induced effects, including only one set of 24+5 of Higgs fields and three generations of fermions

$$\ln\frac{M^{(5)}}{M_W}=\frac{6}{67\alpha}\left(1-\frac{8}{3}\frac{\alpha}{\alpha_s}\right),$$

$$\sin^2\theta_W^{(5)}=\frac{23}{134}+\frac{109}{201}\frac{\alpha}{\alpha_s} \tag{12}$$

In Eqs (11) and (12), $\alpha^{-1}(M_W)=127.54$ and $\alpha_s=g_3^2(M_W)/4\pi=0.1088$ *corresponding to* $\Lambda_{\overline{MS}}=160$ MeV, where $\overline{MS}$ denotes the modified minimal subtraction scheme Solutions in the similar forms including only the five-dimensional operators have been obtained in *Refs 17 and 20 Here we note that the effects of all* higher-dimensional operators are contained in the parameters ϵ_C, ϵ_L, and ϵ_Y as illustrated in Eq (5) Thus, Eq (11) through Eq (5), can, in principle, account for all gravity induced corrections due to higher-dimensional operators

III SOLUTIONS WITH FIVE-DIMENSIONAL OPERATORS

In this section we briefly review the earlier solutions obtained with $d=5$ operator noting that they are ruled out because of experimental constraints on τ_p and $\sin^2\theta_W$ Such a conclusion was already reached by Hill[17] with $M_C\sim M_{Pl}=10^{19}$ GeV We, therefore, discuss Shafi-Wetterich[20] solutions with $M_C\sim10^{17}$ GeV where SU(5) has been stated to survive the then existing experimental data Using $\epsilon^{(2)}=\epsilon^{(3)}=\cdots=0$ and $\epsilon^{(1)}=\epsilon=\eta\phi^0/(\sqrt{15}M_C)$ in Eqs (5) and (11) yields $\epsilon_C=\epsilon$, $\epsilon_L=-3\epsilon/2$, and $\epsilon_Y=-\epsilon/2$ and $D=1-38\epsilon/67$, as in Ref 20, leading to

$$\frac{1}{\alpha_G}=\frac{11\alpha_s^{-1}+21\alpha^{-1}}{67-38\epsilon}\,, \tag{13a}$$

$$\ln\frac{M_U}{M_W}=\frac{6\pi}{67-38\epsilon}\left[\alpha^{-1}-\tfrac{8}{3}\alpha_s^{-1}+(\tfrac{7}{3}\alpha_s^{-1}+\alpha^{-1})\epsilon\right], \tag{13b}$$

$$\sin^2\theta_W=\frac{1}{67-38\epsilon}\left[\frac{23}{2}+\frac{109}{3}\frac{\alpha}{\alpha_s}-\left(41+\frac{116}{3}\frac{\alpha}{\alpha_s}\right)\epsilon\right] \tag{13c}$$

For different assumed values of the parameter ϵ the gauge coupling constant α_G, unification mass M_U, and $\sin^2\theta_W$ are computed as has been done before The basic parameter η occurring in the Lagrangian is calculated using the relation

$$\eta=\left[\frac{25\pi}{2}\frac{67-38\epsilon}{11\alpha_S^{-1}+21\alpha^{-1}}\right]^{1/2}\frac{M_C}{M_U}\epsilon \tag{13d}$$

It is worth mentioning the new additional fact that the ϵ parameter can be bounded from above and below, in this case, using the positivity and perturbative constraints on α_G From Eq (13a), the positivity of α_G suggests that $\epsilon<\frac{67}{38}=1.76$, whereas the perturbative constraint ($\alpha_G<1$) yields, with $\alpha_s=0.1088$ and $\alpha^{-1}=127.54$, $\epsilon>-70$, i e,

$$-70<\epsilon<1.76$$

The lower bound is dominated by α^{-1} and does not vary significantly in the allowed range of $\alpha_s(M_W)$ corresponding to $\Lambda_{\overline{MS}}=0.160\pm0.100$ GeV

Numerical solutions for the unification mass, τ_p, $\sin^2\theta_W$, α_G^{-1}, and η for different values of the ϵ parameter are presented in Table I For calculating η, the value of the compactification scale has been taken to be $M_C=10^{17}$ GeV as before[20] The uncertainty by a factor $10^{\pm2}$ in τ_p arises due to the uncertainty in $\Lambda_{\overline{MS}}$ and the matrix elements for $p\to e^+\pi^0$ In order that τ_p agrees with the experimental limit, $\tau_p\gtrsim3\times10^{32}$ yr, it is clear that $\epsilon>0.015$ which needs $\sin^2\theta_W<0.203$, even though M_C is allowed to be as low as 10^{17} GeV Such lower values of $\sin^2\theta_W$ in the modified solutions, needed for the stability of the proton, are clearly in contradiction with the recent world average $\sin^2\theta_W=0.23\pm0.005$

TABLE I Modifications of the minimal GUT predictions with $d=5$ operator The parameter η has been calculated with $M_C=10^{17}$ GeV

ϵ	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_p (yr)	η
0.005	4.32×10^{14}	0.208	41.60	$3.58\times10^{30\pm2}$	1.10
0.010	5.80×10^{14}	0.205	41.72	$1.16\times10^{31\pm2}$	1.66
0.015	7.78×10^{14}	0.203	41.85	$3.77\times10^{31\pm2}$	1.86
0.020	1.05×10^{15}	0.199	41.95	$1.25\times10^{[illegible]\pm2}$	1.84

IV. NEW SOLUTIONS WITH FIVE- AND SIX-DIMENSIONAL OPERATORS

As we mentioned in Sec III, modifications with $d=5$ operator in the minimal GUT seem to be ruled out as they require $\sin^2\theta_W < 0\ 203$, in order to yield $\tau_p \geq 3\times 10^{32}$ yr But, following the similar philosophy as in Refs 17 and 20, we investigate whether inclusion of still higher-dimensional operators could predict τ_p and $\sin^2\theta_W$ consistent with the available experimental data As the next economic choice we include both $d=5$ and $d=6$ operators in $\mathcal{L}_{NR}$ and find that the most natural and plausible solutions which correspond to logical values of the parameters in the Lagrangian, yield $M_U \sim (0\ 1-1)M_C \sim 10^{16}-10^{19}$ GeV, and $\sin^2\theta_W = 0\ 22-0\ 24$, for each value of M_U In this case the relations between ϵ parameters are

$$\epsilon_Y = \tfrac{2}{5}\epsilon_C + \tfrac{3}{5}\epsilon_L \ , \qquad (14a)$$

$$\epsilon^{(1)} = \tfrac{9}{15}\epsilon_C - \tfrac{4}{15}\epsilon_L \ , \qquad (14b)$$

$$\epsilon^{(2)} = \tfrac{2}{5}\epsilon_C + \tfrac{4}{15}\epsilon_L \qquad (14c)$$

Note that the relation (14a) is also valid in the $d=5$ case

The basic parameters of the Lagrangian are the η parameters, rather than the ϵ parameters Except the positivity and perturbative constraints on ϵ, as already discussed in this paper, there seems to be no theoretical constraint on the η parameters But, in order that the modified Lagrangian makes some sense, the following general criteria on the parameters need to be satisfied, and we treat the solutions as acceptable when either criteria (i) and (ii) or (i) and (iii) are satisfied (i) The magnitude of $\eta^{(n)}$, $n=1,2,$ is not very large, (ii) although the individual values of the η parameters may differ, one *possibility is that they are of the same order* (iii) If the gravity-induced corrections might be acting as the terms in a perturbation series, for reasons hitherto unknown to us, the other possibility might be that $|\eta_2| < |\eta_1|$

Now we discuss briefly how far criterion (i) has been satisfied by earlier solutions with $d=5$ operator where there is only one parameter, $\eta(1)\equiv\eta$ Shafi and Wetterich[20] have obtained modifications with $\eta \sim 1$ Although Hill[17] has investigated over the range of parameters $\eta = -20-20$, i e , with maximum allowed $\eta \sim 10$, no plausible solutions have been obtained for the nonsupersymmetric minimal GUT, but, in the supersymmetric SU(5), significant and acceptable modifications have been observed for $\eta = -2-2$ In the SO(10) grand unification, with Pati-Salam gauge group as the intermediate symmetry, solutions with $\eta \sim 1$ have been found[21] to yield τ_p and $\sin^2\theta_W$ consistent with Eq (1), while, with $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{R-L} (g_{2L} = g_{2R})$ as the proposed and possible low-energy symmetry $(M_{W_R} \sim M_{W_L})$, acceptable solutions have been obtained with $\eta \simeq -1$, with $\sin^2\theta_W \simeq 0\ 22$ and $\tau_p \gtrsim 10^{40}$ yr It is worth mentioning how the first criterion controls the value of the compactification scale in relation to the unification mass With only $d=5$ operator, Shafi and Wetterich have found that $M_C = 10^{17}$ GeV is necessary in order to obtain $\eta \sim 1$ On the other hand if $M_C \sim 10^{18}-10^{19}$ GeV, $\eta \sim 10-100$, for $M_U \sim 10^{15}$ GeV with the same values of ϵ Thus, largeness of η, besides the smallness of $\sin^2\theta_W$, prevents having $M_C \sim M_{Pl}$, with only the $d=5$ operator for the minimal GUT In the present case, however, we will find that criteria (i)–(iii) can clearly rule out solutions with $M_U \sim 10^{15}$ GeV, but allow only those with high unification masses which depend on the compactification scale

Using Eqs (11a)–(11d) and (14a) we first compute values of ϵ_C and ϵ_L such that $M_U \gtrsim 10^{15}$ GeV and $\sin^2\theta_W \simeq 0\ 22-0\ 24$, and the corresponding value of the GUT coupling, α_G Using Eqs (14b) and (14c) we then compute the numerical values of $\epsilon^{(1)}$ and $\epsilon^{(2)}$ Some of these solutions are presented in Table II for different values of the unification mass In the second step, to test whether all such type of solutions are acceptable we compute the values of the basic parameters $\eta^{(1)}$ and $\eta^{(2)}$ using Eq (8), with the knowledge of $\epsilon^{(1)}$, $\epsilon^{(2)}$, α_G, and M_U *from Table II, and several reasonable values of* M_C existing in the literature Such computations are shown in Tables III and IV On the basis of criteria (i)–(iii) we find that all the numerical solutions for M_U and $\sin^2\theta_W$, can be classified into the following categories

(A) $M_U \ll M_C$ Here the inequality is used to mean values of M_U less than M_C by 2 or more orders Since $\epsilon^{(1)}$ and $\epsilon^{(2)}$ are of the same order, it is clear from Eq (8) that if $M_U \ll M_C$, $|\eta^{(2)}| \gg |\eta^{(1)}|$ For example, with $M_U = 10^{15}$ GeV, we obtain $(\eta^{(1)},\eta^{(2)}) = (-4\ 05,\ -10^2)$ $(-40\ 5,\ -10^4)$, and $(-405\ 6,\ -10^6)$, for $M_C = 10^{17}$, 10^{18}, and 10^{19} GeV, respectively Further, the combination $(M_U,\ M_C) = (10^{16},\ 10^{18})$ GeV, $(10^{17},\ 10^{19})$ GeV, and $(10^{16},\ 10^{19})$ GeV, correspond to the parameter values $(\eta_1,\eta_2) = (-3\ 07,\ -57\ 56)$, $(-2\ 50\ \ -38)$, and $(-30\ 7$

TABLE II Parameters ϵ_C, ϵ_L, ϵ_Y, $\epsilon^{(1)}$, and $\epsilon^{(2)}$ computed using one loop renormalization group *equations and corrections due to $d=5$ and 6 operators* Relations among ϵ parameters are given in Eq (14)

ϵ_L	ϵ_C	ϵ_Y	$\epsilon^{(1)}$	$\epsilon^{(2)}$	M_U (GeV)	$\sin^2\theta_W$	α_G
−0 9841	−0 9833	−0 9838	−0 3276	−0 6558	10^{15}	0 2305	$-3\ 905\times 10^{-4}$
−0 9913	−0 9899	−0 9907	−0 3296	−0 6603	10^{16}	0 232	$2\ 22\times 10^{-4}$
−0 9945	−0 9930	−0 9939	−0 3306	−0 6624	10^{17}	0 239	$1\ 461\times 10^{-4}$
−0 9957	−0 9940	−0 9950	−0 3309	−0 6631	10^{18}	0 238	$1\ 188\times 10^{-4}$
−0 9964	−0 9945	−0 9956	−0 331	−0 6635	10^{19}	0 232	$1\ 039\times 10^{-4}$

TABLE III Values of the parameters $\eta^{(1)}$ and $\eta^{(2)}$ and the ratio $|\eta^{(2)}/\eta^{(1)}|$ for the class (A) solutions

M_U (GeV)	M_C (GeV)	$\sin^2\theta_W$	$\eta^{(1)}$	$\eta^{(2)}$	$\lvert\eta^{(2)}/\eta^{(1)}\rvert$
10^{15}	10^{17}	•	4 056	$1\ 005\times10^{3}$	24 776
	10^{18}	0 2305	$-40\ 568$	$-1\ 005\times10^{4}$	$2\ 477\times10^{2}$
	10^{19}		$-405\ 68$	$-1\ 005\times10^{5}$	$2\ 477\times10^{3}$
10^{16}	10^{18}	0 232	$-3\ 077$	$-57\ 564$	18 707
	10^{19}		$-30\ 774$	$-5\ 756\times10^{2}$	$1\ 870\times10^{2}$
10^{17}	10^{19}	0 239	$-2\ 504$	$-38\ 00$	15 1757

$-5\ 7\times10^{3}$), respectively Thus, if M_U is lower than M_C by more than 1 order, besides the magnitudes of the parameters being large, their ratio $|\eta^{(2)}/\eta^{(1)}|$ becomes far too large for the Lagrangian to make any sense The values $(\eta^{(1)}, \eta^{(2)})$ and the ratio $|\eta^{(2)}/\eta^{(1)}|$ are further magnified if $M_C \sim M_{Pl}$ and $M_U \sim 10^{15}-10^{16}$ GeV The reason for such large values of the ratio is due to the fact that $|\eta^{(2)}/\eta^{(1)}| \propto M_C/M_U$ Such large values of the ratio clearly violate criteria (ii) and (iii) when M_U is several orders less than M_C Because of these highly undesirable values of the parameters and their ratio, we conclude that if the addition of a six-dimensional operator is going to make any sense, in the presence of a five-dimensional operator, the solutions with M_U several orders smaller than M_C are not acceptable For the most general expectation of the compactification scale,[17] $M_C \sim M_{Pl}$, the lower unification masses, $M_U \sim 10^{15}-10^{17}$ GeV, are clearly ruled out

(B) $M_U \sim (0\ 1-1)M_C$ In contrast with the class (A) solutions, it is evident from Table III that there are other solutions which satisfy either $|\eta_2/\eta_1| \sim 1$, or $|\eta_2/\eta_1| \sim 0\ 1$ When $M_U \sim 0\ 1M_C$, criteria (i) and (ii) seem to be satisfied For example, $M_U \sim 0\ 1M_C \sim 10^{16}$, 10^{17}, and 10^{18} GeV correspond to the parameter; values $(\eta_1,\eta_2) \simeq (-0\ 307,\ -0\ 575)$, $(-0\ 25\ \ -0\ 38)$, and $(-0\ 226,\ -0\ 309)$ and the ratio $|\eta_2/\eta_1| \simeq 1\ 87$, 1 51, and 1 36, respectively Thus, combining criteria (i) and (ii) yields allowed values of high unification masses $M_U \sim 0\ 1M_C$ But, as mentioned earlier, criterion (iii), an alternative to (ii), could also be possible, if the nonrenormalizable terms act like perturbation on the normalizable Lagrangian The gravity-induced effects on SU(5) do permit such solutions satisfying criteria (i) and (iii) For example, high unification masses $M_U = M_C = 10^{17}$, 10^{18}, and 10^{19} GeV correspond to $(\eta^{(1)},\eta^{(2)}) \simeq (-0\ 025,\ -3\ 8\times10^{-3})$, $(-0\ 022,\ -3\ 09\times10^{-3})$, and $(-0\ 0211,\ -2\ 707\times10^{-3})$, and the ratio $|\eta^{(2)}/\eta^{(1)}| \sim 0\ 152$, 0 132, and 0 128, respectively For every allowed value of $M_U \sim 0\ 1M_C$, satisfying criteria (i) and (ii), or $M_U \sim M_C$, satisfying (i) and (iii), the value of $\sin^2\theta_W$ is found to be in the range 0 22–0 24 Some allowed solutions belonging to class (B) and satisfying criteria (i) and (ii) or (i) and (iii) are presented in Table IV With the possible values of the compactification scale, $M_C = 10^{17}-10^{19}$ GeV, the high values of the unification mass are found to cover rather a wider range, $M_U \sim (0\ 1-1)M_C \simeq 10^{16}-10^{19}$ GeV Another interesting new feature of the present solutions is the smallness of the bare GUT coupling constant, $\alpha_G \sim 10^{-4}$, as compared to all earlier results existing in the literature Such a small numerical value of α_G is understood by noting that

$$\alpha_G = \frac{67+25\epsilon_C+42\epsilon_L}{3\left[\dfrac{11}{3\alpha_s}+\dfrac{7}{\alpha}\right]}, \tag{15}$$

where the numerator tends to be small as $\epsilon_C \sim \epsilon_L \to -1$ The small value of α_G decreases the proton decay rate resulting in a very significant increase in τ_p Thus, according to the present observations, the enhancement in τ_p occurs due to two sources largeness of M_U and smallness of α_G Introducing a factor of $10^{\pm2}$ uncertainty, the minimum value of τ_p corresponding to the lowest allowed $M_U \sim 10^{16}$ GeV turns out to be $\tau_p \sim 10^{38}$ yr, where a factor of 10^4 enhancement due to smallness of α_G has been included If, on the other hand, we confine to the most general expectation, $M_C \sim M_{Pl} = 10^{19}$ GeV, the GUT does not seem to have unification significantly below $M_U \sim 10^{18}$ GeV

Before closing this section it might be necessary to

TABLE IV Same as Table III, but for class (B) solutions satisfying criteria (i) and (ii) or (i) and (iii) as described in the text

M_C (GeV)	M_U (GeV)	$\sin^2\theta_W$	α_G	$\eta^{(1)}$	$\eta^{(2)}$	$\lvert\eta^{(2)}/\eta^{(1)}\rvert$
10^{17}	10^{16}	0 232	$2\ 22\times10^{-4}$	$-0\ 3077$	$-0\ 5756$	1 870
	10^{17}	0 239	$1\ 461\times10^{-4}$	$-0\ 0250$	$-0\ 0038$	0 1520
10^{18}	10^{17}	0 239	$1\ 461\times10^{-4}$	$-0\ 2504$	$-0\ 3800$	1 5175
	10^{18}	0 238	$1\ 188\times10^{-4}$	$-0\ 0226$	$-0\ 003$	0 1327
10^{19}	10^{18}	0 238	$1\ 188\times10^{-4}$	$-0\ 226$	$-0\ 3094$	1 369
	10^{19}	0 232	$1\ 039\times10^{-4}$	$-0\ 0211$	$-0\ 0027$	0 1279

clarify certain points regarding the self-consistency of the treatment of compactification effects through an expansion in higher-dimensional operators. As evident from Table II, $\epsilon^{(2)}/\epsilon^{(1)} \sim 2$, where $\epsilon^{(n)}$ is related to $\eta^{(n)}$ by Eq. (8). This might give the impression that the expansions for ϵ_C, ϵ_L, and ϵ_Y expressed in Eq. (5) are not converging. But we have taken only the first two out of a large number of terms in the series in Eq. (5) to show that they fully account for the available data on $\sin^2\theta_W$, and large values of M_U. This implies that, so far as the available values of $\sin^2\theta_W$ and allowed values of τ_p are concerned, $\epsilon^{(n)} \sim 0$ for $n > 2$, thus guaranteeing convergence of the series and the self-consistency of the method for $M_U \sim (0.1-1)M_C$. Another way of looking into the convergence of expansions is the following. Since the first two terms are capable of explaining the available experimental values of $\sin^2\theta_W$, for M_U in the range $10^{16}-10^{19}$ GeV, it is certainly true that at least the same values of $\sin^2\theta_W$ and M_U are possible by taking larger number of terms such that $|\epsilon^{(n+1)}| << |\epsilon^{(n)}|$, for $n >> 2$, this guarantees convergence of the series and self-consistency of the method adopted.

V. SUMMARY, CONCLUSION, AND DISCUSSION

The minimal SU(5) model predicts a proton lifetime about 2–3 orders less than the observed experimental lower limit and $\sin^2\theta_W \simeq 0.21$. Including gravity-induced effects through $d = 5$ operators, scaled by the compactification mass, although Hill[17] obtained quite lower values of $\sin^2\theta_W$, and, hence, ruled out any modification with $M_C \sim M_{Pl}$, Shafi and Wetterich[20] found that the minimal GUT could be saved by such compactification effects if $M_C = 10^{17}$ GeV. But, as we have noted here, these solutions can be consistent with experiments provided $\sin^2\theta_W < 0.203$ which seem to disagree with the present world average $\sin^2\theta_W = 0.230 \pm 0.005$.

To investigate further, whether the SU(5) predictions can be improved by spontaneous compactification effects, we have investigated the combined role of both five- and six-dimensional operators, which is allowed, in principle, at least, and in the same spirit. Out of, at least, three different possible forms for the six-dimensional operator, we have chosen only one for the sake of simplicity and convenience, and also for obtaining modifications to GUT predictions within the constraint expressed by Eq. (14a). Although our computation in Table II indicates $\sin^2\theta_W \simeq 0.23-0.24$, we have checked that with slight change of ϵ_C and ϵ_I the allowed range is $\sin^2\theta_W \simeq 0.22-0.24$. It seems, at first sight, as if all numerical solutions with $M_U = 10^{15}-10^{19}$ GeV and $\sin^2\theta_W = 0.22-0.24$, as shown in Table II, are allowed. But when we compute the basic parameters $\eta^{(1)}$ and $\eta^{(2)}$ and their ratio $|\eta^{(2)}/\eta^{(1)}|$, where $\eta^{(1)}(\eta^{(2)})$ occurs as the coefficient of the five- (six-) dimensional operators, we find that $|\eta^{(2)}| >> |\eta^{(1)}|$ for those solutions for which $M_U/M_C \lesssim 10^{-2}$, with $M_C = 10^{17}-10^{19}$ GeV. As the Lagrangian does not make sense with such parameters, the corresponding solutions with low unification masses $M_U \simeq 10^{15}$ GeV are ruled out. This criterion, ruling out $M_U \simeq 10^{15}$ GeV is found to be strongly valid if the compactification occurs at the most generally acceptable scale, $M_C = M_{Pl}$.

The present analysis reveals that the gravity-induced corrections with $d = 5$ and 6 operators permit high unification mass, $M_U \sim (0.1-1)M_C \sim 10^{16}-10^{19}$ GeV, if $M_C = 10^{17}-10^{19}$ GeV, with $\sin^2\theta_W \simeq 0.22-0.24$ for every M_U. The values of the η parameters are never found to be large for such solutions belonging to class (B), and the ratio $|\eta_2/\eta_1| \sim 1$ for those solutions with $M_U \sim 0.1M_C$, but $|\eta_2/\eta_1| \sim 0.1$ for others with $M_U \sim M_C$. Although such parameters with $|\eta_2/\eta_1| \sim 1$ are generally expected in unified gauge theories, the other values with $|\eta_2/\eta_1| \sim 0.1$ suggest that the successive terms containing higher-dimensional operators might be acting as perturbation upon the renormalizable Lagrangian. Using the most general value, $M_C \simeq M_{Pl}$, we find that solutions with $M_U \simeq 10^{15}-10^{17}$ GeV are ruled out and the gravity-induced effects permit only $M_U \sim 10^{18}-10^{19}$ GeV. For the first time we find here a grand unified theory with the GUT coupling constant as small as $\alpha_G \sim 10^{-4}$. The enhancement of the proton lifetime occurs due to two factors: largeness of M_U and smallness of α_G. Thus, if the addition of five- and six-dimensional operators to the GUT Lagrangian is going to make sense, the predictions of minimal SU(5) with an unstable proton and $\sin^2\theta_W < 0.215$ are modified to be consistent with an extremely stable proton ($\tau_p > 10^{38}$ yr) and $\sin^2\theta_W \simeq 0.22-0.24$.

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Spontaneous compactification effects, low-energy signature of quark-lepton unification, and small neutrino masses in SO(10)

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Signatures of quark-lepton unification can be experimentally verified by rare-kaon-decay processes provided the $SU(4)_C$-breaking scale $M_C \sim 10^5$–10^6 GeV. With the single intermediate symmetry, $SU(2)_L \times U(1)_R \times SU(4)_C$ in SO(10), we find, for the first time, that such a scale and small Majorana neutrino masses are allowed when gravity-induced corrections due to a five-dimensional operator, arising out of spontaneous compactification of extra dimensions, are included. For such lower values of M_C, the predicted proton lifetime is large depending upon the value of $\sin^2\theta_W$ in the range 0.22–0.24. For still larger values of M_C, neutrino masses and proton lifetime decrease, and the latter saturates the Irvine-Michigan-Brookhaven limit when $M_C \sim 10^{11}$–10^{12} GeV.

I. INTRODUCTION

Two of the most important theoretical developments in particle physics have been the Kaluza-Klein-type unification with gravity[1,2] and grand unified theories[3–5] (GUT's) of strong, weak, and electromagnetic interactions. Originally proposed to unify gravitation with electromagnetism, the Kaluza-Klein method[1] has been applied with the standard, or grand unifying symmetries,[2] in higher dimensions with a view to unifying all basic forces of nature. In such cases, effective gauge theories in four-dimensional space-time emerge as a result of compatification of extra dimensions. In theories employing spontaneous compactification, nonrenormalizable higher-dimensional operators involving gauge and Higgs fields are always generated and the coefficients of these operators are scaled by the powers of the compactification scale (M_G) (Refs. 2 and 6). Even without invoking the idea of dimensional reduction, such operators scaled by powers of the Planck mass ($M_{\rm Pl}=10^{19}$ GeV) can also be present as the gravity-induced corrections to the GUT Lagrangian.

Compared to many other GUT's, SO(10) has many attractive features.[5] It is the minimal left-right-symmetric extension of SU(5), and contains all known fermions (plus the right-handed neutrino) of one generation in a single spinorial representation. It can attribute the origins of parity (*P*) and *CP* violations[7] as arising out of spontaneous symmetry breaking. It can explain neutrino masses over a wide range of values. With the decoupling of parity (*P*) and $SU(2)_R$-breaking scales, the new SO(10) model[8] provides a natural solution to the domain-wall problem.[9] With one or more intermediate symmetries, besides explaining the observed proton stability, SO(10) promises experimental verification of interesting theoretical ideas such as the quark-lepton unification based upon $SU(4)_C$, and left-right symmetry.[10] To date, possible low-energy signatures of $SU(4)_C$ breaking in SO(10), SU(16), and $SU(8)_L \times SU(8)_R$ GUT's have been predicted,[8,10,11] within the context of decoupling *P* and $SU(2)_R$ breakings, in the presence of the gauge group $SU(2)_L \times SU(2)_R \times SU(4)_C (\equiv G_{224})$ with $g_L \neq g_R$, as one of the intermediate symmetries, at a scale $M_C \sim 10^5$–10^6 GeV, such that both free neutron oscillations[12] and rare-kaon decays are expected to be experimentally observable. Since the first class of experiments are very difficult, because of nonavailability of free neutron sources, it might be useful to search for a grand unified theory where the consequences of $SU(4)_C$ breaking can be testified by a relatively easier class of experiments, such as the rare-kaon decays only. Even with two intermediate symmetries in the SO(10) GUT (Ref. 13),

$$\begin{aligned}
\mathrm{SO}(10) &\xrightarrow{M_U} SU(2)_L \times U(1)_R \times SU(4)_C (\equiv G_{214}) \\
&\xrightarrow{M_C} SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C \\
&\xrightarrow{M_R^0} SU(2)_L \times U(1)_Y \times SU(3)_C (\equiv G_{213}) , \qquad (1)
\end{aligned}$$

it has not been possible to obtain $M_C \simeq 10^5$–10^6 GeV, for the presently accepted values[14] of $\sin^2\theta_W = 0.23 \pm 0.005$. Two or more intermediate symmetries populating the grand desert provide possibilities of richer physical structure; but predictions with a single intermediate symmetry are very appealing because of the minimal nature of the underlying GUT scenario.

In this paper we note that, with the single intermediate symmetry $SU(2)_L \times U(1)_R \times SU(4)_C (\equiv G_{214})$, the SO(10) GUT is ruled out as it predicts a proton lifetime lower than the Irvine-Michigan-Brookhaven (IMB) limit[15] for the $p \to e^+ \pi^0$ mode. But, when the twin ideas underlying grand unification and Kaluza-Klein theory are combined together, gravity-induced corrections by a five-dimensional operator[6,16] allow the chain

$$\mathrm{SO}(10) \underset{M_U}{\overset{54+45}{\longrightarrow}} G_{214} \underset{M_C}{\overset{126}{\longrightarrow}} G_{213} \qquad (2)$$

with $M_C \sim 10^5$–10^{11} GeV, $M_U \sim 10^{15}$–10^{17} GeV, and $\sin^2\theta_W = 0.22$–0.24. For $M_C \sim 10^5$–10^6 GeV, corresponding to observable rare-kaon decays, ν_e mass is negligible; but ν_μ and ν_τ masses could be measured in the laboratory. The proton lifetime is significantly larger than the IMB limit depending upon the values of $\sin^2\theta_W$ and M_C. For still larger values of $M_C \sim 10^7$–10^{11} GeV, corresponding to undetectable rare-kaon decays, ν masses decrease further and the proton lifetime also decreases, saturating the IMB limit for $M_C \sim 10^{11}$ GeV.

This paper is organized in the following manner. In Sec. II we summarize earlier contributions in SO(10) where gravity-induced corrections have been included and discuss their implications in the context of cosmological domain-wall problem and neutrino masses. In Sec. III our new results are reported. The paper is summarized in Sec. IV.

II. MODIFICATIONS IN SO(10) WITH $SU(2)_L \times SU(2)_R \times SU(4)_C$ AND $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ INTERMEDIATE SYMMETRIES

In this section we summarize earlier contributions in SO(10) grand unified theory including gravity-induced corrections and discuss their significance in the context of domain walls in the early Universe and neutrino masses. Hill[16] and Shafi and Wetterich[6] (SW) introduced five-dimensional operators, involving gauge and Higgs fields, and scaled by the Planck mass ($M_{\rm Pl} = 10^{19}$ GeV) (Ref. 16) or the compactification scale ($M_G < M_{\rm Pl}$) (Ref. 6), with a view to obtain modified predictions on τ_p and $\sin^2\theta_W$. Detailed analysis in the minimal GUT has been made by them and others.[17] Besides SU(5), SW investigated[6] possible changes in SO(10) predictions in the presence of Pati-Salam intermediate symmetry:

$$SO(10) \underset{M_U}{\overset{54}{\longrightarrow}} SU(2)_L \times SU(2)_R \times SU(4)_C \underset{M_C}{\overset{126}{\longrightarrow}} G_{213} \ . \tag{3}$$

In the absence of a $d=5$ operator, purely renormalizable interactions permit $M_C \simeq 10^{13}$ GeV, $M_U \simeq 10^{15}$ GeV for $\sin^2\theta_W \simeq 0.23$. When a nonrenormalizable term

$$\mathcal{L}_{\rm NR} = -\frac{\eta}{2M_G}{\rm Tr}(F_{\mu\nu}\Phi_{54}F^{\mu\nu}) \tag{4}$$

is added, the following changes were noted:

$$\sin^2\theta_W \simeq 0.22, \quad \tau_p(p \to e^+\pi^0) \gtrsim (10\text{–}100)\tau_p({\rm IMB}) \ , \tag{5}$$

where τ_p(IMB) is the IMB limit.[15] In Eq. (4),

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \ , \\ (A_\mu)^a_b &= A^i_\mu(\lambda_i)^a_b \ , \\ {\rm Tr}(\lambda_i\lambda_j) &= \tfrac{1}{2}\delta_{ij} \ , \end{aligned} \tag{6}$$

and A_μ's are the gauge field matrices, λ_i's are the SO(10) generators, η is an unknown parameter, M_G is the compactification scale, and Φ_{54} is the scalar field $54 \subset SO(10)$. As in SU(5), SW noted that their modifications are consistent with $M_G \simeq 10^{-2}M_{\rm Pl}$, compatible with the parameter $\epsilon \simeq 0.01$–0.02 where

$$\epsilon = \frac{1}{\sqrt{30}}\frac{\eta\Phi_0}{M_G} \tag{7}$$

and Φ_0 is related to the vacuum expectation value

$$\langle \Phi_{54} \rangle = \frac{\Phi_0}{\sqrt{30}}{\rm diag}(1,1,1,1,1,1,-\tfrac{3}{2},-\tfrac{3}{2},-\tfrac{3}{2},-\tfrac{3}{2}) \ . \tag{8}$$

Without introducing the idea of spontaneous compactification, as in Hill's approach,[16] Rizzo[18] investigated the possibility of low-mass purity restoration in SO(10),

$$SO(10) \underset{M_U}{\overset{210}{\longrightarrow}} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \underset{M_R}{\overset{126}{\longrightarrow}} G_{213} \tag{9}$$

using a $d=5$ operator scaled by the Planck mass:

$$\mathcal{L}_{\rm NR} = \frac{C}{2M_{\rm Pl}}{\rm Tr}(F_{\mu\nu}\Phi_{210}F^{\mu\nu}) \ . \tag{10}$$

With a purely renormalizable Lagrangian, $M_R \sim M_W$ in the chain requires[19] $\sin^2\theta_W \simeq 0.27$–$0.28$, which are much larger than the accepted world average. Addition of (10) has been found to allow $M_R \sim M_W$ with $C \simeq -0.5$ and $\sin^2\theta_W$ within acceptable limits, for different combinations of Higgs triplets and doublets.

In the symmetry-breaking chain (3), the parity-violating scale $M_P = M_C \simeq 10^{13}$ GeV, and in (9) $M_P = M_R \sim M_W$. Several years before, it was noted by Kibble, Lazarides, and Shafi,[9] that in a model such as (3), where parity breaks at a scale lower than M_U, domain walls, bounded by strings, are created in the early Universe. Such domain walls contribute to the mass density of the Universe, much larger than observed values, unless $M_P = M_C \gtrsim 10^{13}$ GeV. With gravity-induced corrections,[6] or otherwise,[9] $M_P = M_C \simeq 10^{13}$ GeV, such that problematic domain walls are likely to be absent in chain (3). But in (9), since $M_P = M_R \sim M_W \ll 10^{11}$ GeV, the domain walls created are supposed to be extremely problematic.

Since the scalar representation $126 \subset SO(10)$ is used in the chains (3) and (9), to break the intermediate gauge symmetry spontaneously to the standard group, Majorana neutrino masses are generated by a seesaw mechanism satisfying the formula[20]

$$m_{\nu_i} \simeq \frac{m_i^2}{M_{W_R}}, \quad i = e, \mu, \tau \ , \tag{11}$$

where m_i is the charged-lepton mass of ith generation and M_{W_R} is the $W_R^\pm$ gauge-boson mass. In case (3), investigated by SW (Ref. 6) $M_{W_R} = M_C \simeq 10^{13}$ GeV, such that

$$m_{\nu_e} \sim 10^{-11}\ {\rm eV}, \quad m_{\nu_\mu} \sim 10^{-6}\ {\rm eV}, \quad m_{\nu_\tau} \sim 10^{-4}\ {\rm eV} \ . \tag{12}$$

Such neutrino masses are too small to be detected by laboratory experiments, although they might be compatible with the solution to the solar-neutrino puzzle. In model (9), observable low-mass parity-restoration requires $M_{W_R} \sim M_{W_L}$, leading to rather larger values of Majorana neutrino masses:

$$m_{\nu_e} \sim 1 \text{ eV}, \quad m_{\nu_\mu} \sim 100 \text{ keV}, \quad m_{\nu_\tau} \sim 10 \text{ MeV} . \qquad (13)$$

In this case, although the masses could be measured in the laboratory, ν_μ and ν_τ masses might be too large.

In the next section we study the possibility of SO(10) grand unification with single G_{214} intermediate symmetry. Since $SU(2)_R$ breaks at the GUT scale, there is no domain-wall problem in this model. When the effects of the $d=5$ operator are included, the allowed solutions for M_C are such that we obtain neutrino masses larger than (12) but smaller than (13) by 3–9 orders of magnitude.

III. GRAVITY-INDUCED CORRECTIONS WITH $SU(2)_L \times U(1)_R \times SU(4)_C$ INTERMEDIATE SYMMETRY

In this section we study the modifications caused by the $d=5$ operator in the predictions of the symmetry-breaking pattern (2), with G_{214} intermediate symmetry. It is usually stated that the vacuum expectation value of the Higgs field $\chi(1,0,1) \subset 45 \subset SO(10)$, where the transformation properties of χ are under G_{214}, might achieve the spontaneous symmetry breaking at the first stage of the chain (2). But, according to observations made by Yasue[21] several years ago, both 54 and 45 are needed to break $SO(10) \rightarrow G_{214}$. As 45 is antisymmetric, it does not contribute to the gravity-induced corrections through the $d=5$ operator disussed in Sec. II; but the necessary presence of 54 is sufficient to induce significant modifications to the GUT predictions through the $d=5$ operator of Eq. (4). Following the techniques of Refs. 6 and 16, and using Eqs. (4)–(8), the Lagrangian $\mathcal{L} = \mathcal{L}_R + \mathcal{L}_{NR}$, with $\mathcal{L}_R = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$, is at first decomposed into kinetic energies of the $SU(4)_C$, $SU(2)_L$, and $U(1)_R$ gauge fields:

$$\mathcal{L} = -\tfrac{1}{2}(1+\epsilon)\mathrm{Tr}(F^{(C)}_{\mu\nu}F^{(C)\mu\nu}) - \tfrac{1}{2}(1-\tfrac{3}{2}\epsilon)\mathrm{Tr}(F^{(L)}_{\mu\nu}F^{(L)\mu\nu}) - \tfrac{1}{4}(1-\tfrac{3}{2}\epsilon)F^{(R)}_{\mu\nu}F^{(R)\mu\nu} , \qquad (14)$$

where the superscripts (C), (L), and (R) stand for the $SU(4)_C$, $SU(2)_L$, and $U(1)_R$, respectively. Now rescaling of the gauge fields changes their coupling constants as

$$g_C^2(M_U) \rightarrow g_C^2(M_U)(1+\epsilon), \quad g_L^2(M_U) \rightarrow g_L^2(M_U)(1-\tfrac{3}{2}\epsilon) ,$$
$$g_R^2(M_U) \rightarrow g_R^2(M_U)(1-\tfrac{3}{2}\epsilon) ,$$

where $g_C(M_U)$, $g_L(M_U)$, and $g_R(M_U)$ denote the coupling constants of $SU(4)_C$, $SU(2)_L$, and $U(1)_R$, respectively, without gravity-induced corrections. In order to achieve unification of strong, weak, and electromagnetic interactions for $\mu \geq M_U$, the GUT condition is imposed through the equations

$$g_C^2(M_U)(1+\epsilon) = g_L^2(M_U)(1-\tfrac{3}{2}\epsilon) = g_R^2(M_U)(1-\tfrac{3}{2}\epsilon) = g_0^2 , \qquad (15)$$

where g_0 is the bare-GUT coupling constant. With the boundary conditions modified as in (15), we solve one-loop renormalization group equations for the chain (2).

$M_W \leq \mu \leq M_C$:

$$\frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_i(\mu)} + \frac{a_i}{2\pi}\ln\frac{\mu}{M_W}, \quad i = Y, L, 3C \qquad (16)$$

with

$$\alpha_i(\mu) = g_i^2(\mu)/4\pi, \quad a_Y = \tfrac{41}{10}, \quad a_L = -\tfrac{19}{6}, \quad a_{3C} = -7 .$$

$M_C \leq \mu \leq M_U$:

$$\frac{1}{\alpha_i(M_C)} = \frac{1}{\alpha_i(\mu)} + \frac{a_i}{2\pi}\ln\frac{\mu}{M_C}, \quad i = R, L, 4C , \qquad (17)$$

with $a_R = \frac{15}{2}$, $a_L = -\frac{19}{6}$ and $a_{4C} = -\frac{29}{3}$.

Note that we have been confined to the minimal fine-tuning condition and used three fermion generations, and the minimal number of Higgs scalars, needed for spontaneous symmetry breaking. The Higgs scalars used in the two different mass ranges are $M_W \leq \mu \leq M_C$, $\Phi(1,2,1)$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$, $M_C \leq \mu \leq M_U$, $\Phi(2,\frac{1}{2},1)$ and $\Delta_R(1,1,10)$ under $SU(2)_L \times U(1)_R \times SU(4)_C$. These are present in 10 and 126 of SO(10). Using Eqs. (15)–(17) and the combinations, $e^{-2}(M_W) - \frac{8}{3}g_3^{-2}(M_W)$, $e^{-2}(M_W) - \frac{8}{3}g_L^{-2}(M_W)$, $e^{-2}(M_W) = \frac{5}{3}g_Y^{-2}(M_W) + g_L^{-2}(M_W)$, yields the following constraints on the unification mass, $\sin^2\theta_W$, and the GUT coupling constant ($\alpha_G = g_0^2/4\pi$)·

$$\ln\frac{M_U}{M_W} = \frac{6\pi}{71-74\epsilon}\left[\frac{1}{\alpha} - \frac{8}{3\alpha_s} + \left[\frac{7}{3\alpha_s} + \frac{1}{\alpha}\right]\epsilon\right] + \left[\frac{4-36\epsilon}{71-74\epsilon}\right]\ln\frac{M_C}{M_W} , \qquad (18)$$

$$\sin^2\theta_W = \frac{1}{71-74\epsilon}\left[\left[\tfrac{19}{2} + (19 - \tfrac{38}{3}\epsilon)\frac{\alpha}{\alpha_s} - 53\epsilon\right] - \frac{\alpha}{\pi}(\tfrac{245}{3} - 170\epsilon)\ln\frac{M_C}{M_W}\right] , \qquad (19)$$

$$\frac{1}{\alpha_G} = \frac{1}{71-74\epsilon}\left[\frac{29}{\alpha} - \frac{19}{3\alpha_s} - \frac{226}{3\pi}\ln\frac{M_C}{M_W}\right] . \qquad (20)$$

In Eqs. (18)–(20), $\alpha_s = g_3^2(M_W)/4\pi$ and $\alpha = e^2(M_W)/4\pi$.

For the chain (3), or (9), where formula (11) is applicable, $SU(2)_R$ breaks along with $U(1)_R$ at the same scale such that $M_{W_R} = M_{Z_R}$. But, in the present case, $SU(2)_R$ breaks at $M_{W_R} = M_U$, keeping $U(1)_R \times SU(4)_C$ unbroken down to $\mu = M_C$. It is at the second stage of the spontaneous symmetry breaking that the Majorana neutrino mass is generated when $\Delta_R(1,1,\overline{10})$ under G_{214} acquires vacuum expectation value. In this case $M_{Z_R} = M_C \ll M_{W_R} = M_U$, and the corresponding seesaw mechanism, worked out by Parida and Hazra,[22] yields a different formula for the Majorana neutrino mass.

$$m_{\nu_i} \simeq \frac{m_i^2}{M_C}, \quad i = e, \mu, \tau . \qquad (21)$$

Using Eqs. (18)–(20), $\alpha_s=0.1088$ ($\Lambda_{\overline{\rm MS}}=160$ MeV, where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme), $\alpha^{-1}=127.54$, we compute numerically allowed regions for $M_C(\equiv 10^{n_c})$ and ϵ within the available experimental constraints (Ref. 14) on M_U and $\sin^2\theta_W$. Note that $\epsilon=0$ corresponds to the absence of gravity-induced effects and such solutions are presented in Table I. It is clear that, with a purely renormalizable Lagrangian, chain (2) is ruled out as it yields a maximum $M_U=3\times10^{14}$ GeV and the corresponding proton lifetime $\tau_p\simeq10^{29\pm2}$ yr which is significantly less than the IMB limit.[15]

Interesting solutions are obtained when $\epsilon>0$ and are presented in Figs. 1–3, and Tables II and III. At first, Fig. 1 is plotted using Eq. (18), and Fig. 2 using Eq. (19). In Fig. 1 the horizontal lines are the IMB and the Planck limits on the unification mass. The projection of the line PQ onto Fig. 2 has been denoted as the IMB limit in the latter. The horizontal lines in Fig. 2 represent the 2σ limits of the world average,[14] $\sin^2\theta_W=0.230\pm0.005$. The projection of the Planck limit from Fig. 1 onto Fig. 2 does not provide any useful boundary for the allowed region. But, a much better limit exists[23] from the experimentally observed bounds on the rare-kaon decay mode, $K_L\to\bar{\mu}e$, corresponding to $M_C\gtrsim3\times10^5$ GeV. Specifying the four sides of the quadrilateral in Fig. 2 in this fashion, the allowed solutions are shown by the shaded area.

The numerical values of M_C, ϵ, M_U, $\sin^2\theta_W$, and α_G^{-1} are shown in Table II for $M_C=10^5$–10^6 GeV and, in Table III for $M_C=10^7$–10^{11} GeV.

We find, ignoring the uncertainty in $\Lambda_{\overline{\rm MS}}$ and proton decay matrix elements,[24] that the modifications caused by the $d=5$ operator permit $10^5\lesssim M_C\lesssim10^{11}$ GeV. For every M_C, the parameter ϵ and the unification mass M_U are allowed over a wider range depending upon the 2σ or 1σ limit of $\sin^2\theta_W$. The solutions with smaller (larger) values of $\sin^2\theta_W$ are associated with larger (smaller) values of M_U and τ_p. Our solutions include the values of $M_C\sim10^5$–10^6 GeV, which predict rare-kaon decays to be observable for any value of $\sin^2\theta_W$ in the range of 0.22–0.24. The highest value of $M_U\simeq3\times10^{17}$ GeV is possible for $M_C=10^5$ GeV and $\sin^2\theta_W=0.22$. This has been shown by the point R in Fig. 1 which has been obtained by the projection of the corresponding point in Fig. 2.

As M_C increases, the unification mass, for a fixed value of $\sin^2\theta_W$, and the proton lifetime for the $p\to e^+\pi^0$ mode

TABLE I. One-loop solutions for SO(10) with single intermediate symmetry, $SU(2)_L\times U(1)_R\times SU(4)_C$, in the absence of gravity-induced corrections.

M_C (GeV)	M_U (GeV)	$\sin^2\theta_W$
10^5	9.2×10^{13}	0.273
10^7	1.2×10^{14}	0.260
10^9	1.5×10^{14}	0.247
10^{11}	2.0×10^{14}	0.233
10^{13}	2.57×10^{14}	0 220
10^{14}	2.95×10^{14}	0.214

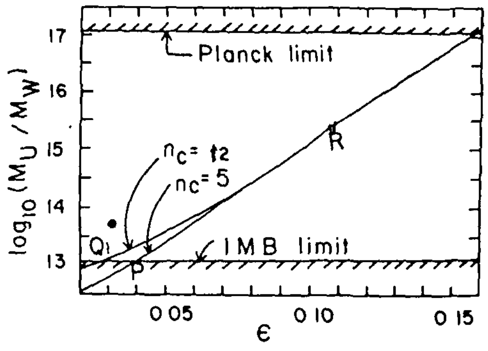

FIG. 1. Solutions of one-loop renormalization-group equations for M_U as a function of ϵ, and for $M_C=10^{n_c}$, $n_c=5$–12. The horizontal lines are the IMB (lower) and the Planck (upper) limits. The allowed upper limit, for $n_c=5$ shown as point R is obtained as the projection of the corresponding point in Fig. 2.

decreases. This has been shown in Fig. 3 for the 1σ and 2σ boundaries, and the central value of $\sin^2\theta_W=0.230$. For $M_C>10^8$ GeV, the allowed range of τ_p also decreases being restricted by the IMB limit form below. The IMB limit is found to be saturated nearly at $M_C\sim10^{10}$ (10^{11}) GeV if the value of $\sin^2\theta_W$ is allowed to be 0.225 (0.220). Including uncertainty in τ_p by a factor $10^{\pm2}$, arising out of uncertainties in the proton decay matrix element and the QCD parameter,[24] we find that the maximum allowed value of M_C can be increased by 1 order, (i.e., 10^{11}–10^{12} GeV) unless $\sin^2\theta_W$ is allowed to be significantly lower than 0.220.

Using Eq. (21) and the allowed range $M_C\sim10^5$–10^{12} GeV, we now calculate neutrino masses as shown in Fig. 4. They are found to vary over a wider range:

$$m_{\nu_e}\sim(2\times10^{-10}\text{–}2\times10^{-3})\ \text{eV}\,,$$
$$m_{\nu_\mu}\sim(10^{-5}\text{–}100)\ \text{eV}\,, \qquad (22)$$
$$m_{\nu_\tau}\sim(3\times10^{-2}\ \text{eV–}30\ \text{keV})\,,$$

where the lower (upper) limit corresponds to $M_C=10^{12}$ (10^5) GeV. The observable signatures of $SU(4)_C$ breaking by rare-kaon decay processes predict

$$m_{\nu_e}\sim(2\times10^{-4}\text{–}2\times10^{-3})\ \text{eV}\,, \qquad (23)$$
$$m_{\nu_\mu}\sim(11\text{–}110)\ \text{eV},\quad m_{\nu_\tau}\sim3\text{–}30\ \text{keV}\,.$$

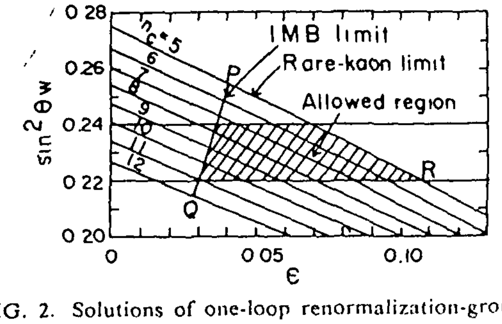

FIG. 2. Solutions of one-loop renormalization-group equations for $\sin^2\theta_W$ as a function of M_C and ϵ.

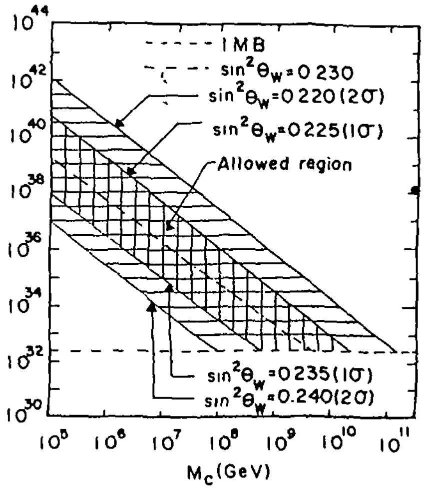

FIG. 3. Predictions on proton lifetime for the $p \to e^+\pi^0$ mode with $\Lambda_{\overline{MS}} = 160$ MeV, as a function of M_C and $\sin^2\theta_W$. The solid lines are for values of $\sin^2\theta_W$ corresponding to 1σ and 2σ limits. The dot-dashed line is for $\sin^2\theta_W = 0.230$. The IMB limit is shown by the dashed line.

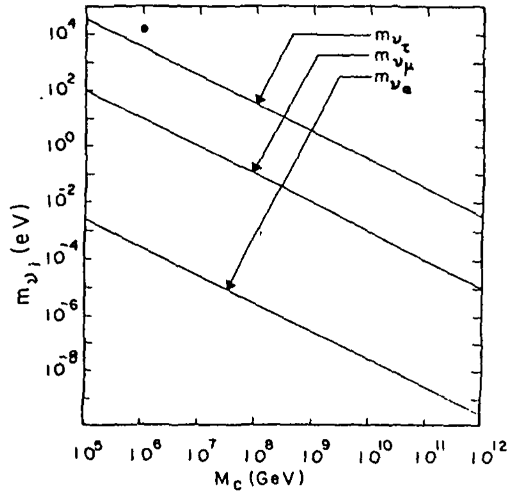

FIG. 4. Predictions on neutrino masses as a function of M_C.

Out of these, ν_μ and ν_τ masses are measurable by laboratory experiments. The masses are about 6–7 orders of magnitude larger than those obtained with single Pati-Salam intermediate symmetry,[6] but they are 3–4 orders of magnitude smaller than the models having low-mass $W_R^\pm$ and Z_R gauge bosons.[18,19,22]

To estimate, approximately, the order of magnitude of M_G that makes these gravity-induced corrections important, we use $\eta = (25\pi\alpha_G/2)^{1/2}\epsilon M_G/M_U$. Our estimation depends, crucially, on the assumption that $|\eta| \simeq 1$ as in the SW case.[6] Solutions having $\epsilon \simeq 0.03$–0.05 are found to be associated with lower values of the unification mass, $M_U \sim 10^{15}$ GeV. They require compactification scale nearly 2 orders smaller than $M_{\rm Pl}$. These solutions belong to the same class as noted[6] by SW in the context of SU(5) and SO(10) models. The other class of solutions found in this model are associated with $\epsilon \simeq 0.07$–0.10, and $M_U \sim 10^{16}$–10^{17} GeV. They require compactification scale $M_G \sim 10^{17}$–10^{18} GeV. In particular, the observable predictions for rare-kaon decay corresponding to $M_C \sim 10^5$–10^6 GeV are found to be also possible with $\sin^2\theta_W \simeq 0.22$–$0.23$, $\epsilon \simeq 0.1$, and $M_U \sim 10^{17}$ GeV, requiring $M_G \sim 10^{18}$ GeV. This scale is generally expected from the Kaluza-Klein-type compactification, where $M_G = M_{\rm Pl}/2\pi \simeq 1.6 \times 10^{18}$ GeV. If, on the other hand, η is allowed to be $|\eta| \simeq 0.1$ (10), our estimation would require M_G 1 order less (more) for every value of ϵ. For example, with $M_C \sim 10^5$ GeV, and $M_U \sim 10^{17}$ GeV, consistency of the solutions with $\epsilon \simeq 0.1$ requires $M_G \sim 10^{17}$ (10^{19}) GeV, if $|\eta| \simeq 0.1$ (10), instead of $|\eta| \simeq 1$.

IV. SUMMARY AND CONCLUSION

In theories exploiting Kaluza-Klein-type unification with gravity, nonrenormalizable terms involving higher-dimensional ($d>4$) operators, scaled by suitable powers of the compactification scale, are usually present. We have investigated the modifications caused by a $d=5$ operator on the SO(10) GUT with single G_{214} intermediate symmetry. In the absence of gravity-induced corrections, such a model is ruled out as it predicts τ_p significantly below the IMB limit. Including gravity-induced corrections, the SU(4)$_C$-breaking scale is found to be permitted over a wide range, $M_C \sim 10^5$–10^{12} GeV, leading to predictions on neutrino masses:

$$m_{\nu_e} \sim (10^{-10}\text{–}10^{-3})\ \text{eV}, \quad m_{\nu_\mu} \sim (10^{-5}\text{–}100)\ \text{eV},$$

$$m_{\nu_\tau} \sim 10^{-2}\ \text{eV} - 30\ \text{keV}.$$

TABLE II. Predictions for M_U, $\sin^2\theta_W$, and values of M_C, corresponding to observable rare-kaon decay, as a function of ϵ, in the presence of gravity-induced corrections, for the same chain as Table I. The proton lifetime is for the $p \to e^+\pi^0$ mode excluding uncertainties.

M_C (GeV)	ϵ	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_p (yr)
10^5	0.10	1.3×10^{17}	0.225	54.56	4.8×10^{40}
	0.09	5.9×10^{16}	0.230	53.93	2.0×10^{39}
	0.08	2.7×10^{16}	0.235	53.32	8.6×10^{37}
10^6	0.09	6.0×10^{16}	0.224	53.08	2.1×10^{39}
	0.08	2.8×10^{16}	0.229	52.47	9.6×10^{37}
	0.07	1.3×10^{16}	0.234	51.88	4.4×10^{36}
	0.06	6.3×10^{15}	0.239	51.30	2.4×10^{35}

TABLE III Same as Table II but for larger values of M_C

M_C (GeV)	ϵ	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_p (yr)
10^7	0 07	$1\,3\times10^{16}$	0 228	51 04	$4\,2\times10^{36}$
	0 06	$6\,7\times10^{15}$	0 233	50 48	$2\,9\times10^{35}$
	0 05	$3\,3\times10^{15}$	0 238	49 92	$1\,7\times10^{34}$
10^8	0 06	$7\,2\times10^{15}$	0 227	49 65	$3\,8\times10^{35}$
	0 05	$3\,6\times10^{15}$	0 232	49 10	$2\,3\times10^{34}$
	0 04	$1\,8\times10^{15}$	0 236	48 57	$1\,4\times10^{33}$
10^9	0 05	$3\,8\times10^{15}$	0 225	48 28	$2\,8\times10^{34}$
	0 04	$1\,9\times10^{15}$	0 230	47 75	$1\,7\times10^{33}$
10^{10}	0 04	$2\,1\times10^{15}$	0 223	46 94	$2\,4\times10^{33}$
10^{11}	0 03	$1\,2\times10^{15}$	0 221	45 63	$2\,0\times10^{32}$

Although m_{ν_e} is too small to be detected, m_{ν_μ} and m_{ν_τ} could be measured by laboratory experiments depending upon M_C

For the first time, we have obtained interesting SO(10) predictions, with single intermediate symmetry, for the observable $SU(4)_C$ breaking by rare-kaon decay modes at low energies with $M_C \sim 10^5$–10^6 GeV, and any value of $\sin^2\theta_W$ in the range 0 22–0 24 For such lower values of M_C, the predicted ν masses are 6–7 orders of magnitude larger than the SO(10) prediction with Pati-Salam intermediate symmetry,[6] but 3–4 orders smaller than models with low-mass right-handed gauge bosons [18 19 22] The proton lifetime is large depending upon the value of $\sin^2\theta_W$ For larger values of $M_C > 10^8$ GeV, the allowed range of τ_p decreases with increasing M_C For a fixed $\sin^2\theta_W$, τ_p decreases with M_C and the IMB limit is saturated when $M_C \sim 10^{11}$–10^{12} GeV The order of magnitude of the compactification scale, estimated in this model, is found to be in the range 10^{17}–10^{18} GeV, unless the parameter in the nonrenormalizable term has the value $|\eta| \simeq 10$, or larger

With the $SU(2)_L \times U(1)_R \times SU(4)_C$ gauge symmetry, existing at a scale $\mu \geq M_C = M_{Z_R} \geq 10^5$ GeV, and $M_{W_R} = M_U \sim 10^{15}$–$10^{17}$ GeV, there is negligible contribution to the $V+A$ structure of charged and neutral currents, in this model Similarly, the K_l-K_S mass difference and other CP-violating parameters have, essentially, the same prediction as the standard model At low energies, this model does not seem to predict any other detectable signatures, except rare kaon decays and neutrino masses

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SPONTANEOUS COMPACTIFICATION EFFECTS IN SO(10) WITH LOW-MASS $W_R^{\pm}$-GAUGE BOSONS WITHOUT OBSERVABLE PARITY RESTORATION

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In contrast to the conclusion for an SO(10) model including gravity induced corrections through a five-dimensional operator we note that low-mass right-handed gauge bosons are ruled out as the parity restoring gauge group $SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_C$ ($g_{2L}=g_{2R}$) is not allowed at low-mass scales Using the mechanism of decoupling parity and $SU(2)_R$ breakings the low-mass right-handed gauge bosons ($M_R\sim 100$ GeV–10 TeV) without observable parity restoration are found to be permitted when the five-dimensional operator is scaled by the compactification mass

Experiments being planned with ultrahigh-energy accelerators in the TeV ranges are expected to reveal new physics beyond the standard model that might be associated with new gauge symmetries. One of the promising gauge theories beyond $SU(3)_C\times SU(2)_L\times U(1)_Y$ ($\equiv G_{st}$) is the left–right model based upon the gauge symmetry $SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_C$ ($\equiv G_{2213}$) which has been the focus of considerable attention during the past years Ever since the proposal of left–right symmetry [1,2], many attempts have been made to obtain a low right-handed scale (M_R) associated with the spontaneous symmetry breaking (SSB) of G_{2213P} including the left–right discrete symmetry (P= parity when $g_{2L}=g_{2R}$), or excluding it ($g_{2L}\neq g_{2R}$) If the right-handed scale associated with the SSB of the G_{2213P} model (G_{2213} with $g_{2L}=g_{2R}$) is low, besides observing the $V+A$ structure of weak-charged and neutral currents by low-energy experiments and detecting the right-handed ($W_R^{\pm}$, Z_R) gauge bosons by the high-energy accelerators, it might also be possible to observe low-mass parity restoration In addition such a model provides a natural mechanism for neutrino masses and weak *CP* violation But when embedded in grand unified theories (GUTs) such models are known to be in serious conflict with cosmology, the well known problems being the presence of undesirable domain walls [3], and inadequate baryon number generation [4] In the conventional approach emphasizing the G_{2213P} model, P and $SU(2)_R$ are broken at the same scale When such a model is embedded in a GUT like SO(10) [5], the right-handed scale is found to be large ($M_R>10^{12}$ GeV) for $\sin^2\theta_w\simeq 0\ 230\pm 0\ 005$ This rules out verification of all possible signatures by low-energy experiments, or high-energy accelerators

When SO(10) breaks with two intermediate symmetries [6],

$$SO(10)\xrightarrow{M_U} G \xrightarrow{M_R^{+}} SU(2)_L\times U(1)_R\times U(1)_{B-L}\times SU(3)_C \xrightarrow{M_R^0} G_{st} \qquad (1)$$

a low-mass right-handed neutral gauge boson is known to be predicted with $M_R^0\sim 500$ GeV whether $G=G_{2213}$, or G_{2213P} In such cases the predicted value of the $W_R^{\pm}$-gauge boson mass is large, $M_R^{\pm}>10^{10}$ GeV [6] It has been noted very recently that, if the superheavy components of Higgs representations used in the SSB of the SO(10) model are permitted to have certain nondegenerate masses different from M_U by one order, the resulting uncertainty in the predicted value of $\sin^2\theta_w$ might permit low-mass right-handed $W_R^{\pm}$ gauge bosons in (1) when $G\equiv G_{2213}$ [7] How-

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ever we are interested in the model calculations exploiting the extended survival hypothesis under the natural assumptions that all superheavy masses are the same as M_U

It has been shown by Rizzo [8] that additional gravitational corrections due to a five-dimensional operator in the nonrenormalizable lagrangian,

$$\alpha_{NR} = \frac{C}{M_C} \mathrm{Tr}(F_{\mu\nu} \Phi_{(210)} F^{\mu\nu}) \quad (2)$$

with $M_C = 2\times 10^{19}$ GeV permits low-mass right-handed gauge bosons($M_R \sim 100$ GeV) in SO(10) with observable parity restoration In eq (2) $\Phi_{(210)}$ is the four-index antisymmetric Higgs-scalar representation **210** ⊂ SO(10),

$$F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu],$$

$$W_\mu = \tfrac{1}{4} \sum_{i,j=1}^{10} \sigma^{ij} W_\mu^{ij}, \quad C = -\tfrac{1}{8}\eta, \quad (3)$$

where $\frac{1}{2}\sigma^{ij}(W_\mu^{ij})$, with $i,j = 1, 2, \ldots, 10$, denote the 45 generators (gauge bosons) [9] of SO(10) [#1] and the constant C has been reparametrized The fact that the addition of such five-dimensional operators could drastically modify the GUT predictions was first proposed by Hill [11] and Shafi and Wetterich [12] As a distinguishing feature of the two suggestions note that the higher-dimensional operator was emphasized to be originating as an effect of quantum gravity in ref [11] while this was taken to be occurring naturally in some effective lagrangian as a result of compactification of extra dimensions with the compactification scale $M_C \sim 10^{17}$–10^{18} GeV [12] We follow the convention in which $i,j = 1, 2, \ldots, 6$ (7, 8, 9, 10) are the SO(6) (SO(4)) indices It is the purpose of the present paper to demonstrate that (i) low-mass right-handed gauge bosons in the G_{2213P} model as envisaged in ref [8] are ruled out, (ii) they are permitted in the SO(10) model with decoupled P- and $SU(2)_R$-breakings proposed recently by Chang, Mohapatra and one of us (M K P) [13], especially when eq (2) emerges as a result of compactification of extra dimensions with $M_C \lesssim M_{Pl}$ [12] When all superheavy-Higgs scalar masses are degenerate and the same as the unification mass, only through such a model the spontaneous compactification effects on SO(10) predict the possibility of observing $V+A$ structure of weak currents and the right-handed $W_R^\mp$ gauge bosons, but no observable parity restoration

Before the works of ref [13], a number of authors used the vacuum expectation value (VEV) of the neutral component (χ_0) of $\chi(1,1,15) \subset$ **45** ⊂ SO(10) to break

$$SO(10) \xrightarrow{\langle\chi_0\rangle \neq 0} G_{2213P},$$

where (1, 1, 15) denotes the transformation property of χ under $SU(2)_L \times SU(2)_R \times SU(4)_C$ ($\equiv G_{224}$) But is was found in ref [13] that, under the constraint of minimal finetuning, it is not possible to keep parity (P) unbroken below the GUT scale when the VEV of χ_0 is used It was noted that an element of SO(10), called $D = \sigma_{23}\sigma_{67}$, acts like P when all couplings in the SO(10)-invariant lagrangian are real The neutral components χ_0 and $\eta(1,1,1) \subset$ **210** ⊂ SO(10) are odd under D and, hence, under P, but the corresponding components of $\xi(1,1,15) \subset$ **210** and η (1, 1 1) ⊂ **54** are even Thus it is possible to descend down to G_{2213P} (G_{224P}) with P unbroken through the Higgs representation **210** (**54**) but not through **45** (**210**) Similarly when a VEV is assigned along directions odd under P, it is possible to break P at the GUT scale without breaking $SU(2)_R$ In such cases, P- and $SU(2)_R$-breakings are decoupled, and it is possible to descend down to G_{2213} (G_{224}) with P broken at the GUT scale through the Higgs representation **45** (**210**) In this paper while reexamining the gravity-induced effects in the SO(10) scenario considered by Rizzo [8],

$$\text{I} \quad SO(10) \xrightarrow[M_U]{210} G_{2213P} \xrightarrow[M_R]{126} G_{st}$$

$$\xrightarrow[M_W]{10} SU(3)_C \times U(1)_{em} \quad (4)$$

with $M_R \sim M_W$, we discuss yet another possibility of decoupling P- and $SU(2)_R$-breakings through **210** that predicts the existence of low-mass $W_R^\mp$ gauge bosons when compactification effects are included

When **210** acquires the VEV as

$$\langle \Phi_{(210)} \rangle = \frac{\varphi_0}{4\sqrt{6}} (-\Gamma_1\Gamma_2\Gamma_3\Gamma_4 + \Gamma_1\Gamma_2\Gamma_5\Gamma_6 + \Gamma_3\Gamma_4\Gamma_5\Gamma_6), \quad (5)$$

[#1] In this notation [9] every gauge boson appears in more than one element of the 16×16 matrix W_μ and the kinetic energy term is $-\frac{1}{8}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ Note the presence of $\frac{1}{8}$ instead of $\frac{1}{2}$ For details in SO(2N) models see ref [10]

$$\mathrm{SO}(10) \xrightarrow{M_U} \mathrm{G}_{2213P}$$

which has been used in the analysis of ref [8] by allowing G_{2213P} to survive down to the W_L-scale But since $\eta(1,1,1) \subset \mathbf{210}$ is odd under P, when VEV is assigned such that

$$\langle \Phi_{(210)} \rangle \sim \phi_0 (\Gamma_7 \Gamma_8 \Gamma_9 \Gamma_{10}) , \quad (6)$$

P breaks at the GUT scale Thus, while eq (5) gives the parity invariant vacuum with G_{2213P} gauge symmetry, addition of (5) and (6) with

$$\langle \phi_{210} \rangle = \frac{\phi_0}{8\sqrt{2}} (-\Gamma_1\Gamma_2\Gamma_3\Gamma_4 + \Gamma_1\Gamma_2\Gamma_5\Gamma_6 + \Gamma_3\Gamma_4\Gamma_5\Gamma_6 + \Gamma_7\Gamma_8\Gamma_9\Gamma_{10}) \quad (7)$$

yields the parity-violating vacuum having G_{2213} gauge symmetry In eqs (5)-(7) $-\langle \phi^{1234} \rangle = \langle \phi^{1256} \rangle = \langle \phi^{3456} \rangle = \langle \phi^{78910} \rangle$, where $\Phi_{(210)} = (1/4!)\Gamma_i\Gamma_j\Gamma_k\Gamma_l \times \varphi^{ijkl}$, φ^{ijkl} being antisymmetric in $ijkl$ For a symmetry breaking pattern of the type

$$\text{II} \quad \mathrm{SO}(10) \xrightarrow[M_U]{210} \mathrm{G}_{2213} \xrightarrow[M_R]{126} \mathrm{G}_{\mathrm{st}} \xrightarrow[M_W]{10} \mathrm{SU}(3)_C \times \mathrm{U}(1)_{\mathrm{em}} , \quad (8)$$

the renormalization group equations (RGEs) for the gauge coupling constants $\alpha_i(\mu) = g_i^2(\mu)/4\pi$ in the various ranges of the mass scales are

$$M_W \leqslant \mu \leqslant M_R \quad \frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_i(M_R)} + \frac{a_i}{2\pi} \ln \frac{M_R}{M_W} , \quad i = Y, 2L, 3C , \quad (9)$$

$$M_R \leqslant \mu \leqslant M_U \quad \frac{1}{\alpha_i(M_R)} = \frac{1}{\alpha_i(M_U)} + \frac{a_i'}{2\pi} \ln \frac{M_U}{M_R} , \quad i = \mathrm{BL}, 2L, 2R, 3C \quad (10)$$

Introduction of the five-dimensional operator in eq (2) is to modify the boundary conditions in the general form,

$$\alpha_{BL}(M_U)(1+\epsilon_{BL}) = \alpha_{2L}(M_U)(1+\epsilon_{2L}) = \alpha_{2R}(M_U)(1+\epsilon_{2R}) = \alpha_{3C}(M_U)(1+\epsilon_{3C}) = \alpha_G , \quad (11)$$

where $\alpha_G = g_0^2/4\pi$ g_0 being the bare coupling of the GUT lagrangian Using suitable combinations of the coupling constants, the matching relation $1/\alpha_Y(M_R) = \frac{3}{5}[\alpha_{2R}(M_R)]^{-1} + \frac{2}{5}[\alpha_{BL}(M_R)]^{-1}$, and the boundary conditions (11), the following equations for $\ln(M_U/M_W)$, $\sin^2\theta_w$, and the coupling constant α_G are obtained in a straightforward manner

$$\ln \frac{M_U}{M_W} = \frac{2\pi}{D} \Bigg(\frac{\frac{8}{3} + \epsilon_{2L} + \epsilon_{2R} + \frac{2}{3}\epsilon_{BL}}{\alpha_{3C}(M_W)} - \frac{1+\epsilon_{3C}}{\alpha(M_W)} \frac{a_{3C}}{a'_{3C}} + \frac{1}{2\pi} [(1+\epsilon_{3C})(a_{2L} + \frac{5}{3}a_Y - a'_{2L} - a'_{2R} - \frac{2}{3}a'_{BL}) + (\frac{8}{3} + \epsilon_{2L} + \epsilon_{2R} + \frac{2}{3}\epsilon_{BL})(a'_{3C} - a_{3C})] \ln \frac{M_R}{M_W} \Bigg) \quad (12)$$

$$\sin^2\theta_W = \frac{1}{D} \Bigg((1+\epsilon_{2L}) a'_{3C} - \frac{a'_{2L} a_{3C}}{a'_{3C}} (1+\epsilon_{3C}) + \frac{\alpha(M_W)}{\alpha_{3C}(M_W)} [a'_{2L}(\frac{5}{3} + \epsilon_{2R} + \frac{2}{3}\epsilon_{BL}) - (1+\epsilon_{2L})(a'_{2R} + \frac{2}{3}a'_{BL})] + \frac{\alpha(M_W)}{2\pi} \{ (\frac{5}{3} + \epsilon_{2R} + \frac{2}{3}\epsilon_{BL})(a'_{3C}a_{2L} - a'_{2L}a_{3C}) + (1+\epsilon_{2L})[a_{3C}(a'_{2R} + \frac{2}{3}a'_{BL}) - \frac{5}{3}a'_{3C}a_Y] + (1+\epsilon_{3C})[\frac{5}{3}a'_{2L}a_Y - a_{2L}(a'_{2R} + \frac{2}{3}a'_{BL})] \} \times \ln \frac{M_R}{M_W} \Bigg) , \quad (13)$$

$$\frac{1}{\alpha_G} = \frac{1}{D} \Bigg(\frac{a'_{3C}}{\alpha(M_W)} - \frac{a'_{2L} + a'_{2R} + \frac{2}{3}a'_{BL}}{\alpha_{3C}(M_W)} + \frac{1}{2\pi} [a_{3C}(a'_{2L} + a'_{2R} + \frac{2}{3}a'_{BL}) - a'_{3C}(a_{2L} + \frac{5}{3}a_Y)] \times \ln \frac{M_R}{M_W} \Bigg) , \quad (14)$$

where

$$D = a'_{3C}(\frac{8}{3} + \epsilon_{2L} + \epsilon_{2R} + \frac{2}{3}\epsilon_{BL}) - (1+\epsilon_{3C})(a'_{2L} + a'_{2R} + \frac{2}{3}a'_{BL}) \quad (15)$$

The case considered by Rizzo [8] corresponds to $M_R = M_W$ with $a_{2L} = a'_{2L} = a'_{2R} = \frac{4}{3}n_g + \frac{1}{6}(D+2T) - \frac{22}{3}$ $a_{3C} = a'_{3C} = \frac{4}{3}n_g - 11$ $a'_{BL} = \frac{4}{3}n_g + 3T$, where n_g is the fermion generation number and D (T) is the number of light Higgs doublets (triplets) In ref [8] the following values of the coefficients occurring in eqs (12)–(15) have been derived

$$\epsilon_{2L}=\epsilon_{2R}=\frac{15A}{\sqrt{2}}\,,\quad \epsilon_{BL}=\frac{A}{\sqrt{2}}\,,$$
$$\epsilon_{3C}=-\frac{20A}{\sqrt{2}}\,,\quad A=\frac{CM_U}{2(64\pi\alpha_G)^{1/2}M_{Pl}}\,, \tag{16}$$

such that all the gauge coupling constants at the boundary $\mu=M_U$ get modified according to eq (11) Noting that

$$-\Gamma_1\Gamma_2\Gamma_3\Gamma_4+\Gamma_1\Gamma_2\Gamma_5\Gamma_6+\Gamma_3\Gamma_4\Gamma_5\Gamma_6$$
$$=\mathrm{diag}(-1,-1,-1,3,-1\ -1,-1,3,-1,$$
$$-1,-1,3,-1,-1\ -1,3)\,, \tag{17}$$

we use the VEV given by (5) in eq (2) After a suitable rescaling of the gauge fields in the usual fashion we find that (2) gets absorbed in the kinetic energy terms for $SU(3)_C$, $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$ gauge fields resulting in

$$\epsilon_{2L}=\epsilon_{2R}=0\,,\quad \tfrac{1}{2}\epsilon_{BL}=-\epsilon_{3C}=\epsilon\,,$$
$$\epsilon=\frac{\eta\Phi_0}{4\sqrt{6}\,M_C}=\frac{\eta M_U}{2\sqrt{32\pi\alpha_G}\,M_C} \tag{18}$$

Thus we find that the modification to the boundary condition occurs only for the coupling constants $\alpha_{3C}(M_U)$, and $\alpha_{BL}(M_U)$, but not for $\alpha_{2L}(M_U)$ or $\alpha_{2R}(M_U)$ That eq (2) does not contribute to the modifications of $SU(2)_{L,R}$ kinetic energies can be further checked by using $i,j=7, 8, 9, 10$ in eq (3) and verifying that $\mathrm{Tr}(F^{(L,R)}_{\mu\nu}\langle\Phi_{(210)}\rangle F^{(L,R)\mu\nu})=0$ As we find the boundary condition (18) to be substantially different from that used in ref [8], we computed numerical solutions to eqs (12)–(15) under the condition (18), and with $M_R=M_W$, $M_C=2M_{Pl}=2\times10^{19}$ GeV, $\alpha_{3C}(M_W)\simeq0\,1$, and $\alpha^{-1}_{em}(M_W)=128$ as has been used in ref [8] In contrast to ref [8] we note that, in all cases of $D=T$, $\sin^2\theta_w$ given by eq (13) is independent of ϵ This happens due to the fact that the ϵ-dependence occurring in the first factor (D^{-1}) in eq (13) gets exactly cancelled by the same dependence in the numerator of the second factor For other combinations of $D\neq T$ the ϵ-dependence of $\sin^2\theta_w$ is weak Our numerical solutions for $D=T=1, 2$, $D=2$, $T=1$, and $D=1$, $T=2$ are shown in table 1 The lowest value of $\sin^2\theta_w$ is obtained for the case $D=1$, $T=2$ and is found to be 0 266 with $\epsilon=-0\,208$ for which $M_U=2M_{Pl}$ For all other values of $M_U<2M_{Pl}$ $\sin^2\theta_w>0\,266$ When we attempted to decrease $\sin^2\theta_w$ further with $\epsilon<-0\,208$, M_U exceeded $2M_{Pl}$ making the solutions unacceptable Thus, the possibility of low-mass right-handed gauge bosons with $M_R\sim M_W$ accompanied by observable low-mass parity restoration through G_{2213P} intermediate symmetry needs $\sin^2\theta_w\simeq0\,266$ which is far too large as compared to the present world average $\sin^2\theta_w\simeq0\,230\pm0\,005$

To find the lowest allowed value of M_R under the boundary conditions (11) and (18) we allowed $M_R\gg M_W$ in eqs (12)–(14) Some of our solutions with the same input parameters are shown in table 2 where the presence of $T>1$ has been taken between the scales M_R-M_U In the case $D=T=1$ ($D=1$, $T=2$) whenever we attempted to decrease $\sin^2\theta_w$ by decreasing $\epsilon<-0\,12$ ($\epsilon<-0\,2$), M_U exceeded $2M_{Pl}$ which ruled out the possibility of M_R below 10^9 GeV We find that $\sin^2\theta_w<0\,235$ constrains $M_R>10^9$ GeV Thus, the low-mass right-handed gauge bosons and observable parity restoration are ruled out in this model Even with such high values of $M_R<10^{12}$ GeV the model gives rise to stable domain walls [3] and negligible baryon asymmetry of the universe [4] unacceptable to the modern big-bang cosmology

In contrast to the conventional models the new SO(10) model [13] with separate P= and $SU(2)_R$-breaking scales does not suffer from the domain wall problem In such a model since the baryon generation in the early universe is related to the P-breaking scale (M_P) [14], it is possible to generate the observed baryon asymmetry of the universe with $M_R\ll M_P\sim M_U$ But the renormalization group constraints upto two-loop level have been found to permit $M_R>10^9$ GeV for $\sin^2\theta_w\ll0\,235$ with G_{2213} as the single intermediate symmetry To investigate the impact of the gravity induced corrections on this model we used the VEV given by eq (7) and computed the ϵ_i-parameters contributing to the modification of the boundary conditions,

$$\epsilon_{2R}=-\epsilon_{2L}=-\epsilon_{3C}=\tfrac{1}{2}\epsilon_{BL}=\epsilon\,,$$
$$\epsilon=\frac{\eta\Phi_0}{8\sqrt{2}\,M_C}=\frac{\eta M_U}{16M_C}\left(\frac{3}{2\pi\alpha_G}\right)^{1/2} \tag{19}$$

As explained earlier [13] within the constraint of

Table 1
Prediction of the SO(10) model with parity-restoring left–right gauge group at low-mass scales, $M_C = 2\times10^{19}$ GeV, and $M_R \sim 100$ GeV

Number of D and T	ϵ	M_U (GeV)	$\sin^2\theta_w$	α_G^{-1}	$C = -\frac{1}{8}\eta$
$D=T=1$	−0.05	2.6×10^{18}	0.274	49.0	0.14
	−0.08	2×10^{9}	0.274	49.8	0.03
$D=2, T=1$	−0.08	4.4×10^{18}	0.282	48.2	0.13
	−0.10	2×10^{19}	0.282	48.7	0.04
$D=1, T=2$	−0.18	4.4×10^{18}	0.266	44.14	0.31
	−0.208	2×10^{19}	0.266	44.50	0.08
$D=T=2$	−0.20	2.9×10^{18}	0.274	43.03	0.51
	−0.238	2×10^{19}	0.274	43.44	0.09

Table 2
Prediction of the SO(10) model with the parity-restoring gauge group as an intermediate symmetry and $M_C = 2\times10^{19}$ GeV

Number of D and T	ϵ	M_R (GeV)	M_U (GeV)	$\sin^2\theta_w$	α_G^{-1}	$C = -\frac{1}{8}\eta$
$D=T=1$	−0.12	10^{10}	4.6×10^{18}	0.233	46.5	0.19
	−0.12	10^{9}	7.9×10^{18}	0.238	47.1	0.11
$D=1, T=2$	−0.2	10^{9}	9.3×10^{18}	0.234	44.1	0.16
	−0.2	10^{8}	9.7×10^{18}	0.238	44.1	0.15

minimal finetuning of parameters, the left-handed triplet is made superheavy with its mass $\sim M_U$ and does not contribute to the RGEs of the coupling constants. At first, confining to the minimal number of Higgs particles needed for the SSB of the gauge symmetries, $D=T=1$ corresponds to including the following Higgs contributions in the two different mass ranges: $M_W \leqslant \mu \leqslant M_R$: $\phi(1, 2, \frac{1}{2})$, $M_R \leqslant \mu \leqslant M_U$: $\phi(2, 2, 0, 1) + \Delta_R(1, 3, 2, 1)$ where the transformation properties in the lower (higher) mass ranges are under G_{st} (G_{2213}). In the minimal case the coefficients occurring in eqs. (12)–(14) are

$$a_{2L} = -\frac{19}{6}, \quad a_Y = \frac{41}{10}, \quad a_{3C} = -7\,,$$
$$a'_{2L} = -3\,, \quad a'_{2R} = -\frac{7}{3}\,, \quad a'_{BL} = \frac{11}{2}\,, \quad a'_{3C} = -7 \qquad (20)$$

Modifying the boundary conditions as in eqs. (11) and (19) we solved eqs. (12)–(14) to obtain values of M_U, $\sin^2\theta_w$, and α_G for certain values of M_R as a function of ϵ. Some of our solutions are presented in table 3 where the entry in the last column is the parameter $C = -\frac{1}{8}\eta$ that has been computed using formula (18) and different values for the compactification scale (M_C). It is clear that low-mass right-handed gauge bosons with $M_R \sim 100$ GeV–10 TeV are permitted with $|C| = |\frac{1}{8}\eta| \simeq 0.2$–3 provided the compactification scale M_C is in the range of $\sim 10^{17}$–10^{18} GeV [12]. It may be noted that the compactification of the fifth dimension on a circle in the Kaluza–Klein model corresponds to $M_C = 10^{19}/2\pi$ GeV $= 1.6\times10^{18}$ GeV for which the value of the parameter C has also been calculated. It has been shown by Freund [15] that in Kaluza–Klein theories M_C could be easily made two orders of magnitude smaller than M_{Pl}. If we use $M_C = M_{Pl}$, the parameter C increases by a factor 10–100 making it unacceptably large for the minimal choice of Higgs representations ($D=T=1$). Thus, low-mass $W_R^{\pm}$-bosons are favoured in the SO(10) model which might be appearing as an effective gauge theory in four dimensions resulting from compactification of extra dimensions in some basic higher-dimensional theory [16].

We have carried out a similar analysis in the case $D=1$ and $T=2$ corresponding to the nonminimal

Table 3
Prediction of the SO(10) model with parity-violating left–right gauge group at lower mass scales ($M_R \sim 10^2$–10^4 GeV)

Number of D and T	ϵ	M_R (GeV)	M_U (GeV)	$\sin^2\theta_w$	α_G^{-1}	M_C (GeV)	$C=-\frac{1}{8}\eta$
$D=T=1$	0 05	10^4	$1\ 6\times10^{16}$	0 234	48 3	$1\ 6\times10^{18}$	−2 1
	0 06	10^4	10^{16}	0 232	48 1	10^{17}	−0 28
	0 08	10^3	$4\ 4\times10^{15}$	0 229	48 2	10^{17}	−0 76
	0 08	10^2	$8\ 2\times10^{15}$	0 233	49 0	10^{17}	−0 40
	0 07	10^2	$1\ 6\times10^{16}$	0 235	49 3	$1\ 6\times10^{18}$	−2 8
$D=1\ T=2$	0 01	10^4	$2\ \times10^{16}$	0 236	46 5	$1\ 6\times10^{18}$	−0 34
	0 02	10^3	$1\ 7\times10^{16}$	0 236	46 8	$1\ 6\times10^{16}$	−0 81
	0 04	10^2	$7\ 4\times10^{15}$	0 232	46 9	10^{17}	−0 23

choice of Higgs representations, the results are also reported in table 3 It is clear that low-mass $W_R^\pm$-bosons are favoured with $M_C\sim10^{17}$–10^{18} GeV when $|C|=0\ 2$–$0\ 8$ In this case, however, $M_C=M_{Pl}$ could be permitted provided C is allowed to be in the range 3–10

Although the renormalization group constraints including spontaneous compactification effects are found to permit a low scale like $M_R\sim100$–1000 GeV, there are several phenomenological constraints on the $W_R^\pm$-mass [17] The most stringent constraint from the K_L–K_S mass difference sets a lower bound [#2] $M_R\gtrsim2\ 5$ TeV in manifestly left–right symmetric models where the two gauge coupling constants, and the fermion mixing angles in the left- and the right-handed sectors are equal ($g_L=g_R$, $\theta_L=\theta_R$), but this lower bound can be decreased substantially in the asymmetric models ($g_L\neq g_R$, $\theta_L\neq\theta_R$) [18] One of the promising low-energy processes investigated extensively and expected to provide signatures of $V+A$ charged currents is the neutrinoless double β-decay With the Majorana neutrino mass $m_{\nu_e}\sim1$–2 eV available experimental data are consistent with a low W_R-mass ~3–4 TeV [19] If $M_R\sim10$–20 TeV the electric dipole moment of the neutron, a manifestation of CP and P violations, has been predicted to be close to the experimental limit, $d_n^e\lesssim10^{-26}$ $e$cm [17,20] The low M_R-scale also allows the neutral and charged Higgs scalar components of the right-handed triplet $\Delta_R(1,3,2,1)$ to have low masses The charged components can mediate neutrinoless double β-decay, muon decay, $\mu\to3e$, and muonium–antimuonium transitions Improved experimental measurements for these processes would set limits on the Higgs masses in the near future [17] One of the spectacular signatures of low-mass $W_R^\pm$-bosons at SSC energy would be through the decay modes

$$W_R^\pm\to e^+N_R\to e^\pm(\text{jets})\ ,$$

where N_R is the right-handed Majorana neutrino Detailed investigations have shown that the detection limit in this case is nearly $M_R\simeq8\ 6$ TeV [21]

It has been shown [22] that the see-saw mechanism for generating Majorana neutrino mass operates in a profound manner when the mechanism of decoupling P- and $SU(2)_R$-breakings is employed, as compared to the conventional method [17] With $M_R\sim1$ TeV the G_{2213} model predicts $m_{\nu_e}\sim$eV, $m_{\nu_\mu}\sim10$ keV, and $m_{\nu_\tau}\sim4$ MeV Such masses, if testified by laboratory measurements, would be in conflict with the cosmological bound according to which the sum of stable neutrino masses should not exceed ~40–100 eV Out of several mechanisms proposed to satisfy the cosmological bound with low M_R, the one that applies here is the decay of unstable heavier neutrinos to the stable light neutrinos like ν_e by the emission of a majoron [23] that arises as a result of spontaneous breaking of a global lepton number associated with the G_{2213} gauge symmetry The lightest neutrino mass $m_{\nu_e}\sim$eV compatible with the observable decay rate for the neutrinoless double β process, is far too large compared to the mass needed to solve the solar neutrino puzzle using the MSW [24] conjecture

[#2] For phenomenological constraints on the $W_R^\pm$- and Δ_R-boson masses see the recent review in ref [17] and references therein

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Models with natural seesaw mechanism for neutrino masses with identical parity- and $SU(2)_R$-breaking scales

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Chang and Mohapatra have observed that the implementation of the seesaw mechanism explaining small neutrino masses in left right symmetric or SO(10) models requires the parity (P) and $SU(2)_R$ breaking scales to be widely separated ($M_P >> M_R$) In this paper we show how the mechanism operates naturally even though the two scales are identical The gauge group immediately preceding the standard model emerges to be its minimal extension based upon $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ with a second neutral Z_R boson mass $M_R^0 \simeq 300$–10^5 GeV An embedding in the partial unification scheme leads to observable rare kaon decays In the two symmetry-breaking chains investigated in SO(10) with parity broken either at the unification scale ($M_P = M_U$), or at an intermediate scale ($M_P \gtrsim 10^{11}$ GeV), proton decay is predicted with lifetime near the observable limit, but, in the former case, rare kaon decays are also predicted near the observable limit when an intermediate gauge group $SU(2)_L \times U(1)_R \times SU(4)_C$ survives down to the scale $M_C \simeq 10^5$ GeV provided $\sin^2\theta_W \simeq 0.24$ The criterion for naturalness turns out to be wide separation between P and $U(1)_{B-L}$-breaking scales

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I INTRODUCTION

Out of several methods proposed to explain small neutrino masses, the seesaw mechanism[1,2] has been widely exploited in partially unified or grand unified theories (GUT's) of strong, weak, and electromagnetic interactions Recently Chang and Mohapatra[3,4] have made an important observation on the general validity of the mechanism[1,2] as a viable theory for neutrino masses They found that the implementation of the mechanism in the left-right-symmetric[5] (LRS) model or SO(10) GUT[6] needs a wide separation of parity- (P-) and $SU(2)_R$-breaking scales Starting with LRS models or GUT's such as SO(10), SU(16), or $SU(8)_L \times SU(8)_R$, it has been demonstrated earlier that a wide separation between P- and $SU(2)_R$ breaking scales can be realized in the presence of suitable Higgs representations,[7] or specific spontaneous symmetry-breaking patterns[8] In the case when the P- and $SU(2)_R$ breaking scales are identical, the present bound on neutrino masses does not permit the right-handed gauge bosons to be light ($M_{W_R} = M_{Z_R} = M_R = M_P > 10^8$–$10^{10}$ GeV), thus leaving no other testable signatures at lower energies In the latter situation the mechanism[1,2] has no role in explaining neutrino masses The main objective of this paper is to demonstrate that the seesaw mechanism is natural in certain models even if the two scales are identical This is achieved in spontaneous symmetry breaking (SSB) of a LRS gauge group or a GUT to the standard model in several steps such that $SU(2)_R \to U(1)_R$ in the first step In a subsequent step of SSB, $U(1)_R$ combines with the $U(1)_{B-L}$ present in the intermediate gauge group to form $U(1)_Y$ We provide two examples in SO(10) GUT where the proton lifetime is predicted to be within the observable limit of the second generation experiments In all cases the neutral current exhibits an admixture of $V + A$ structure corresponding to a low mass Z_R boson manifesting itself in the SSB of the minimally extended gauge group based upon

$$SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C \quad (\equiv G_{2113})$$

That a seesaw formula different from the usual lore exists in the G_{2113} model was first noted in Ref 9, but the left-handed triplet Δ_L carrying $B-L=2$ was taken to be light for the sake of convenience, which spoils the naturalness of the mechanism In all models leading to G_{2113} considered in this paper, the condition of minimal finetuning of parameters requires all the components of Δ_L to be much heavier than the $U(1)_{B-L}$ breaking scale which renders the mechanism to be natural

This paper is organized in the following manner In Sec II we review the work of Chang and Mohapatra illustrating the naturalness of the seesaw mechanism in gauge models with a wide separation between P- and $SU(2)_R$-breaking scales In Sec III we show how the naturalness criterion operates with identical P- and $SU(2)_R$-breaking scales using the LRS model and partial unification scheme In Secs IV and V we show how such models can be embedded in two different scenarios of SO(10) grand unification Section VI is devoted to summary and discussions

II NATURAL SEESAW MECHANISM WITH SEPARATE P- AND $SU(2)_R$-BREAKING SCALES

In this section we summarize the work of Chang and Mohapatra establishing the naturalness of the seesaw mechanism in left-right gauge models and GUT's with a wide separation between P- and $SU(2)_R$-breaking scales. For convenience we discuss the conventional mechanism in the context of LRS models[5] based upon the gauge group

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times P$$

$$(\equiv G_{2213P},\ g_{2L} = g_{2R}),$$

where the quarks (Q_L, Q_R) and leptons (ψ_L, ψ_R) of each generation, and Higgs scalars $(\phi, \Delta_L, \Delta_R)$, possess the following transformation properties under G_{2213}: $\psi_L(2,1,-1,1)$, $\psi_R(1,2,-1,1)$, $Q_L(2,1,\frac{1}{3},1)$, $Q_R(1,2,\frac{1}{3},1)$, $\phi(2,2,0,1)$, $\Delta_L(3,1,2,1)$, $\Delta_R(1,3,2,1)$. In order to derive the SSB, and implement the seesaw mechanism in the chain

$$G_{2213P} \xrightarrow[M_R]{\langle \Delta_R^0 \rangle = V_R} SU(2)_L \times U(1)_Y \times SU(3)_C (\equiv G_{s}) \xrightarrow[M_W]{\langle \phi^0 \rangle = k} U(1)_{em} \times SU(3)_C \ (\equiv G_{13}), \quad (1)$$

the scalars are assigned the following vacuum expectation values (VEV's)

$$\langle \Delta_L \rangle = \begin{vmatrix} 0 & 0 \\ V_L & 0 \end{vmatrix}, \quad \langle \Delta_R \rangle = \begin{vmatrix} 0 & 0 \\ V_R & 0 \end{vmatrix}, \quad \phi = \begin{vmatrix} k & 0 \\ 0 & k' \end{vmatrix} \quad (2)$$

leading to the neutrino mass term in the Lagrangian

$$\mathcal{L}_{mass} = (\nu^T N^T) \begin{vmatrix} m_{LL} & m_{LR} \\ m_{LR} & m_{RR} \end{vmatrix} \frac{\tau_2}{2} \begin{vmatrix} \nu \\ N \end{vmatrix}, \quad (3)$$

where $N = C(\bar{\nu}_R)^T$, $m_{LL} = h_3 V_L$, $m_{RR} = h_3 V_R$, $m_{LR} = h_1 k + h_2 k'$, and h_i's are Yukawa couplings. Under the condition $V_R \gg k \gg V_L, k'$, the generalization of Eq (2) to three generations leads to small (large) mass eigenvalues for the left- (right-)handed neutrinos $\nu_i(N_i)$

$$m_{\nu_i} \simeq \frac{m_i^{D^2}}{M_R}, \quad m_{N_i} = M_R, \quad i = 1,2,3, \quad (4)$$

M_R being the mass of $W_R^{\pm}$ and Z_R bosons where P and $SU(2)_R$ break simultaneously. The Dirac mass in (4) has been chosen to be the charged-lepton mass[2] ($m_1^D = m_e$, $m_2^D = m_\mu$, $m_3^D = m_\tau$) leading to $m_{\nu_e} \simeq 0.2$ eV, $m_{\nu_\mu} \simeq 10$ keV, and $m_{\nu_\tau} \simeq 4$ MeV. SO(10) breaks into G_{2213P} through the Higgs representation **210** at the unification scale (M_U), and the other scalars needed for (1) are contained in the representations **126** and **10**$\subset$SO(10). Also, SO(10) can break directly into G_s through the scalar representations **45**, and **126**$\subset$SO(10). Using the quark masses $m_1^D = m_u$, $m_2^D = m_c$, $m_3^D = m_t \simeq 80$ GeV and $M_U = M_R = 10^{15}$ GeV, the Gell-Mann–Ramond–Slansky[1]-type spectrum is $m_{\nu_e} \sim 10^{-11}$ eV, $m_{\nu_\mu} \sim 10^{-6}$ eV, and $m_{\nu_\tau} \simeq 10^{-2}$ eV. Such a feature of the mechanism as obtaining small ν_i masses simultaneously with small mixing angles was considered very natural until Chang and Mohapatra[3,4] observed that the presence of the terms

$$V_I = \sum_{i,j=1}^{2} \lambda_{ij} \mathrm{Tr}(\Delta_L^{\dagger} \phi_i \Delta_R \tau_2 \phi_j^{+} \tau_2) \quad (5)$$

in the Higgs potential, where $\phi_1 = \phi$ and $\phi_2 = \tau_2 \phi^* \tau_2$, leads to much larger induced values of $\langle \Delta_L^0 \rangle$ and the left-handed Majorana mass through Fig 1,

$$m_{LL}^{(I)} = \lambda h_3 \frac{k^2 V_R}{M_\Delta^2}, \quad (6)$$

even though one has $\langle \Delta_L^0 \rangle = 0$ to start with. Here λ is a function of scalar couplings and M_Δ is the mass of Δ_L. Thus, the seesaw mechanism[1,2] meant to explain small neutrino masses holds provided values given by (4) dominate over those in (6), which requires

$$\frac{\lambda V_R^2}{M_\Delta^2} \ll \left| \frac{h_1}{h_3} \right|^2 \quad (7)$$

For the maximum values of $M_\Delta = M_P = M_R$, obtained for $V_L = 0$ using extended survival hypothesis, the fine-tuning needed to satisfy (7), or $\lambda \ll (h_1/h_3)^2$, is arbitrary since the standard-model Yukawa coupling $h_1 \simeq 10^{-5}$, and there is no reason for h_3 to be small. Without arbitrary fine-tuning, (6) dominates over (4) and the bound on neutrino masses needs $M_\Delta = M_P = M_R \simeq 10^{10}-10^{11}$ GeV consistent with $m_{\nu_i} \simeq 1-10$ eV, $i = 1,2,3$. In such a situation the proposed mechanism[1,2] does not explain neutrino masses.

Introducing P-odd singlets of scalars in LRS, or using D-odd singlets already present in SO(10) (D = a discrete symmetry defined in Ref 7) in the representations **45** and **210**, P and $SU(2)_R$ breaking were decoupled earlier with the possibility $M_\Delta \simeq M_P \gg M_R$. Then m_{LL}^I in Eq (6) is made negligible compared to Eq (4) and the seesaw mechanism provides a natural explanation for neutrino masses even for low values of M_R. A number of symmetry-breaking patterns including two-loop effects have been worked out in SO(10) and found to be consistent with $M_P \gg M_R$. In these new SO(10) models the seesaw mechanism is natural.[7-10] In GUT's of higher rank such as $SU(8)_L \times SU(8)_R$ or SU(16) specific SSB pat-

FIG 1 Induced values of left-handed Majorana mass term that spoils the seesaw mechanism terns were found that generate asymmetry in the G_{2211} gauge group or partial unification scheme with $g_{2L} \neq g_{2R}$ (Ref 8) and wide separation between P and $SU(2)_R$ breakings. Similar arguments permit the seesaw mechanism to be natural in these GUTs.

III. NATURAL SEESAW MECHANISM IN MODELS WITH IDENTICAL P- AND $SU(2)_R$-BREAKING SCALES

In this section we demonstrate how the seesaw mechanism is natural in other gauge models with identical P- and $SU(2)_R$-breaking scales. In popular models LRS is associated with the gauge group G_{2213P}, or

$$SU(2)_L \times SU(2)_R \times SU(4)_C (\equiv G_{224P},\ g_{2L} = g_{2R}) .$$

GUT's such as SO(10), E_6, $SU(8)_L \times SU(8)_R$, or SU(16) contain these as subgroups. $B-L$ forms a diagonal generator of $SU(4)_C$. In the alternate class of models exhibiting a natural seesaw mechanism, although P and $SU(2)_R$ break at the same scale, $SU(2)_R \times U(1)_{B-L}$ or $SU(2)_R \times SU(4)_C$ breaks to $U(1)_Y$ in more than one steps. In the first step $U(1)_{B-L}$ or $SU(4)_C$ must remain unbroken but $SU(2)_R \rightarrow U(1)_R$ to generate wide separation between M_P and M_{B-L}, where M_{B-L} is the breaking scale of $U(1)_{B-L}$. This is achieved through the following chains in the two models

$$\text{(i)}\quad G_{2213P} \xrightarrow[\langle \chi^0 \rangle \neq 0]{M_P = M_R^+} G_{2113} \xrightarrow[\langle \Delta_R^0 \rangle \neq 0]{M_R^0} G_{s} ,$$

$$\text{(ii a)}\quad G_{224P} \xrightarrow[\langle \lambda^0 \rangle \neq 0]{M_P = M_R^+} G_{214} \xrightarrow[\langle \xi^0 \rangle \neq 0]{M_C} G_{2113} \xrightarrow[\langle \Delta_R^0 \rangle \neq 0]{M_R^0} G_{s} ,$$

$$\text{(ii b)}\quad G_{224P} \xrightarrow[\langle \sigma^0 \rangle \neq 0]{M_P = M_R^+ = M_C} G_{2113} \xrightarrow[\langle \Delta_R^0 \rangle \neq 0]{M_R^0} G_{s} ,$$

where $G_{214} \equiv SU(2)_L \times U(1)_R \times SU(4)_C$. In order to understand the naturalness of the seesaw mechanism it is necessary to know the order of the masses of Higgs scalars occurring as left handed and right-handed triplets which carry $B-L=2$. In case (i) the neutral component of the right handed Higgs scalar triplet with $B-L=0$ transforming as $X_f(1,3,0,1)$ under G_{2213P} gets a VEV at the scale $M_R^+ >> M_R^0$ to yield the minimally extended gauge group G_{2113}. In the second state the neutral component of the right handed Higgs scalar triplet carrying $B-L=2$ and transforming as Δ_R^0 $(1,-1,2,1)$ under G_{2113} gets a VEV to break $G_{2113} \rightarrow G_s$ at the scale $M_R^0 >> M_W$. In this case $W_R^\pm$ gauge-boson masses are of order M_R^+ whereas the right-handed neutral gauge-boson mass $\simeq M_R^0$. It may be noted that in the first step both P and $SU(2)_R$ break at the same scale ($\mu = M_P = M_R^+$) but $U(1)_R \times U(1)_{B-L}$ remain unbroken, which in turn break to form $U(1)_Y$ at the lower scale $M_R^0 = M_{B-L} << M_R^+ = M_P$. In (ii a) $G_{224P} \rightarrow G_{214}$ by the VEV of the neutral component of the Higgs scalar transforming as $\lambda(1,3,1)$ under G_{224}. In the second stage $G_{214} \rightarrow G_{2113}$ due to the VEV of the neutral omponent of the Higgs scalar transforming as $\xi(1,1,15)$ under G_{224} with $M_R^+ >> M_C$. In the case (ii b) the SSB $G_{224P} \rightarrow G_{2113}$ is achieved by the neutral component of the Higgs scalar transforming as $\sigma(1,3,15)$ under G_{224}. The symmetry breaking $G_{2113} \rightarrow G_s$ proceeds in the same manner as in case (i) through the VEV $\langle \Delta_R^0 \rangle \neq 0$. In the cases (i) and (ii) the final stage of SSB is achieved by the standard doublet of Higgs scalars. Thus compared to the seesaw mechanism envisaged in Sec. II new types of Higgs scalars are needed in certain cases to drive the SSB in the models. In the light Higgs-scalar sector there are two neutral particles: the standard Higgs scalar with mass $M_{\phi^0} \simeq M_W$ and the neutral component of the right-handed triplet with mass $M_{\Delta_R^0} \simeq M_R^0$. Since LRS is maintained at scales $\mu >> M_R^+$, the Higgs sector must be left-right symmetric for such values of μ. Using extended survival hypothesis, the charged components Δ_R^+, Δ_R^{++}, and Δ_R^{--} in the triplet $\Delta_R(1,3,2,1)$ under G_{2213} in case (i) acquire masses of order M_R^+. The left handed counterpart of Δ_R, i.e., $\Delta_L(3,1,2,1)$ under G_{2213} does not contribute to the SSB at any stage. Its role is to maintain LRS and, according to extended survival hypothesis, masses of all the components in Δ_L is of the order M_R^+. In the cases (ii a) and (ii b) the right-handed triplet is contained in the G_{224} representation $\Delta_R(1,3,10)$, whereas the left-handed triplet is contained in $\Delta_L(3,1,10)$. Only the neutral component of Δ_R^0 acquires a mass $\simeq M_R^0$ but all other components of Δ_R and Δ_L have masses of order M_R^+. In the case (ii a) all other components of $\xi(1,1,15)$ under G_{224} have masses $\simeq M_R^+$ except the neutral component which acquires a mass $M_\xi^0 \simeq M_C$. All the components of $X(1,3,1)$ under G_{224} have masses $\simeq M_R^+$. In the case (ii b) all the components of $\sigma(1,3,15)$ under G_{224} have masses $M_R^+ = M_P$.

Now using the seesaw mechanism and adding the induced mass term due to Fig. 1, we obtain, for the neutrino mass of ith generation,

$$m_{\nu_i} = \frac{\lambda h_3^{(i)}}{g^3} \frac{M_W^2 M_R^0}{M_R^{+2}} - \frac{(m_i^D)^2}{M_R^0}, \quad i = e, \mu, \tau , \tag{8}$$

where the first (second) term is the induced (seesaw mechanism) contribution and g is the appropriate gauge coupling. The dominance of the second term over the first, desired by the naturalness criterion, requires

$$m_i^D >> \frac{M_W M_R^0}{M_R^+} ,$$

or

$$R m_i^D >> M_W, \quad i = e, \mu, \tau , \tag{9}$$

where $R = M_R^+ / M_R^0$ and we have used $\lambda h_3^{(i)} / g^3 \simeq 1$, $i = e, \mu, \tau$. Inequality (9) is our new naturalness condition in order that the seesaw mechanism provides a meaningful theory for Majorana neutrino masses. If we use the charged lepton masses for m_i^D, then $R >> 10^5$ for the first generation. This automatically guarantees naturalness for the second and third generations since $m_\tau >> m_\mu >> m_e$. If $V + A$ structure of neutral currents

TABLE I Some predictions of the partial unification model $G_{2213P} \xrightarrow[M_R]{} G_{2113} \xrightarrow[M_I]{} G_{std}$ with $M_R^+ \simeq 1$ TeV is described in the text

M (GeV)	$M_R = M_I$ (GeV)	$\sin^2\theta_W$
10^5	5×10^{16}	0.230
	10^{17}	0.225
10^6	8×10^{16}	0.220
	2×10^{16}	0.230
	8×10^{15}	0.235

TABLE II Some predictions of the partial unification scheme $G_{2213P} \xrightarrow[M_P]{} G_{2113} \xrightarrow[M_R]{} G_{std}$

M_R (GeV)	$M_I = M_C = M_R$ (GeV)	$\sin^2\theta_W$
10^3	8×10^{13}	0.235
	1.4×10^{13}	0.230
10^5	10^{13}	0.235
	1.6×10^{13}	0.230

are desirable at low energies along with the detection of Z_R at the supercolliders, it is necessary that $M_R^0 \simeq 1$ TeV, which implies that the mechanism is natural if $M_R^+ = M_P \gg 10^8$ GeV. A very interesting feature of the new class of models specified in (i) and (ii) is the naturalness of the mechanism with the minimally extended gauge group G_{2113} at lower energies. Using renormalization group equations (RGEs) it is easy to satisfy the condition $M_R \gg M_R^0$ in the case (i) and the constraint arising out of $K_L - K_S$ mass difference as observed in Ref 9. In fact RGEs do not constrain $M_P = M_R^+$ as there are three unknown gauge coupling constants $g_{2L} = g_{2R}$, g_{BL}, and g_{3C} for $\mu \geq M_R^+ = M_P$. But in cases (ii a) and (ii b) there are two unknown gauge coupling constants, $g_{2L} = g_{2R}$ and g_{4C}, for $\mu \geq M_R^+ = M_P$ one of which can be eliminated using the fine structure constant matching at $\mu = M_W$. For the case (ii a) the relation between $\sin^2\theta_W$ and the mass scales can be expressed including one loop corrections as

$$\sin^2\theta_W = \frac{1}{2} - \frac{\alpha}{3\alpha_s} - \frac{8\alpha}{3\pi}\ln\frac{M_P}{M_C} - \frac{23\alpha}{6\pi}\ln\frac{M_C}{M_R^0} - \frac{11\alpha}{3\pi}\ln\frac{M_R^0}{M_W}\,, \tag{10}$$

where $\alpha \equiv \alpha(M_W) = e^2(M_W)/4\pi$ and $\alpha_S = g_3^2(M_W)/4\pi$. The corresponding equation for case (ii b) is obtained from Eq (10) by using $M_R^+ = M_P = M_C$. Solutions to the RGEs in cases (ii a) and (ii b) have been obtained with the QCD parameter $\Lambda_{\overline{MS}} \simeq 0.2$ GeV, where $\overline{MS}$ denotes the modified minimal subtraction scheme, $\sin^2\theta_W \simeq 0.22-0.24$ and for values of $M_R^0 \simeq (3\times10^3 - 10^5)$ GeV, some of which are presented in Tables I and II. For the case (ii a) we found 7×10^{13} GeV $\leq M_R^+ = M_P \leq 2\times10^{17}$ GeV for 10^{10} GeV $\gg M_C \gg 10^5$ GeV. In this case, in addition to predicting the low energy gauge group to be G_{2113} beyond the standard model the rare kaon decays are also predicted to be observable, corresponding to $M_C \simeq 10^5$ GeV. In the case (ii b) the solutions are consistent $M_I = M_C = M_R^+ \simeq 10^{12}$ GeV $- 5\times10^{11}$ GeV with a low mass Z_R boson. The parameter $R = M_R^+/M_R^0 \geq 10^7$ in both cases and is found to guarantee the naturalness condition. The neutrino mass spectrum for lower values of $M_R^0 \simeq 300$ GeV $-$ 1 TeV is of the type eV $-$ keV $-$ MeV for the three generations. In such cases m_{ν_μ} and m_{ν_τ} would violate the cosmological bound. One method of evading the cosmological bound is to make ν_μ and ν_τ unstable against Majoron emission as discussed in Sec VI. In the next section we examine embeddings of models (i) and (ii) in SO(10) grand unification.

IV IMPLEMENTATION IN SO(10) WITH G_{2213P} AS AN INTERMEDIATE SYMMETRY

In this section we show how the new seesaw mechanism operates in an SO(10) scenario where G_{2213P} and G_{2113} occur as the two intermediate symmetries. The well known problem in such a GUT scenario is the presence of undesirable domain walls[11] and inadequate baryon asymmetry[12] of the Universe if $M_P = M_R^+ \ll 10^{11}$ GeV. On the other hand if $M_P = M_R^+ \geq 10^{11}$ GeV the baryon asymmetry is compatible with the observed value and the domain walls created in the early Universe might have been removed by inflation. Our analyses in this paper demonstrate that the RGE's permit such solutions. We discuss the embeddings of these groups in SO(10) and find solutions to the unification mass (M_U) $\sin^2\theta_W$, and intermediate scales including renormalization effects on gauge coupling constants up to two loops and superheavy Higgs scalar effects[13-15]. The case (i) mentioned in Sec III can be embedded in SO(10) grand unification as follows

$$SO(10) \xrightarrow[M_U]{210} G_{2213P} \xrightarrow[M_P = M_R^+]{45} G_{2113} \xrightarrow[M_R^c = M_{B-L}]{126} G_{std} \xrightarrow[M_W]{10} G_{13}\,, \tag{11}$$

where the Higgs scalars mentioned in (i) are contained in the respective SO(10) representations $\chi \subset 45$, $\Delta_R \subset 126$, $\phi \subset 10$. In addition, the GUT symmetry breaks down to G_{2213P} when the neutral component of the Higgs scalar transforming as (1,1,0,15) under G_{2213P} and contained in 210 acquires VEV $\simeq M_U$. In order to make GUT predictions using the effective gauge theory approach,[13,15] the superheavy components in different Higgs representations needed for SSB in the model are noted below along with their masses and transformation properties under G_{2213} [15]

$$10 \supset M_{H_1}(1,1,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},3) + M_H(1,1,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{3}),$$

$$\begin{aligned}
126 \supset\ & M'_H(3,1,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{3}) + M'_{H_2}(3,1,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},6) + M'_{H_3}(1,3,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},3) \\
& + M_{H_4}(1,3,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},6) + M'_{H_5}(1,1,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},3) + M'_{H_6}(1,1,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{3}) \\
& + M_H(2,2,0,1) + M_{H_8}(2,2,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},3) + M'_{H_9}(2,2,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{3}) + M'_{H_{10}}(2,2,0,8),
\end{aligned}$$

$$45 \supset M_{S_1}(1,1,-\sqrt{\tfrac{3}{2}}\tfrac{2}{3},3) + M_S(1,1,\sqrt{\tfrac{3}{2}}\tfrac{2}{3},\bar{3}) + M_{S_3}(1,1,0,8) + M_{S_4}(2,2,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},3) + M_{S_5}(2,2,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{3}),$$

$$\begin{aligned}
210 \supset\ & M_{S_1}(3,1,-\sqrt{\tfrac{3}{2}}\tfrac{2}{3},3) + M'_{S_2}(3,1,\sqrt{\tfrac{3}{2}}\tfrac{2}{3},\bar{3}) + M'_{S_3}(3,1,0,8) \\
& + M_{S_4}(1,3,-\sqrt{\tfrac{3}{2}}\tfrac{2}{3},3) + M'_{S_5}(1,3,\sqrt{\tfrac{3}{2}}\tfrac{2}{3},3) + M'_{S_6}(1,3,0,8) + M_{S_7}(2,2,\sqrt{\tfrac{3}{2}},1) + M'_{S_8}(2,2,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},3) \\
& + M'_{S_9}(2,2,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},6) + M'_{S_{10}}(2,2,\sqrt{\tfrac{3}{2}},\bar{1}) + M'_{S_{11}}(2,2,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{3}) + M'_{S_{12}}(2,2,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{6})
\end{aligned} \tag{12}$$

If the component masses are taken to be arbitrarily nondegenerate the model loses its predictive power on the proton lifetime (τ_p) and $\sin\theta_W$. We examine their impact on GUT predictions by assuming the masses to be (a) degenerate, (b) nondegenerate but not arbitrary as they are constrained by a Coleman Weinberg-type condition in that a nondegeneracy factor up to 10 might be generated among different component masses in a single GUT representation [14]. In all cases τ_p ($p \to e^+\pi^0$) is predicted near the observable limit. In order to constrain masses under condition (b), we maximize τ_p using the RGE for $\ln(M_U/M_W)$, which leads to

$$\begin{aligned}
M'_{H_1} = M'_{H_7} = M'_{H_4} = M'_{H_9} = M_{S_4} = M'_{S_1} = M'_{S_2} = M'_{S_7} = M'_{S_8} = M'_{S_{10}} = M'_{S_{11}} = M_{S_5} = M^{(+)} \\
M_{H_1} = M_H = M'_H = M'_{H_5} = M'_{H_6} = M'_{H_{10}} = M'_{S_3} = M'_{S_9} = M'_{S_{12}} = M_{S_1} = M_{S_2} = M_{S_3} = M^{(-)}
\end{aligned} \tag{13}$$

Using a minimal number of Higgs scalars and three fermion generations we have computed the one and two loop coefficients in the equations for $\ln(M_U/M_W)$ and $\sin^2\theta_W$ given below

$$\begin{aligned}
\ln\frac{M_U}{M_W} = \frac{3\pi}{29}\left[\frac{1}{\alpha} - \frac{8}{3\alpha_s}\right] - \tfrac{11}{58}\ln\frac{M_R^+}{M_W} + \tfrac{1}{29}\ln\frac{M_R^0}{M_W} - \tfrac{1}{116}\big(-27\ln x_{2L}^U - 27\ln x_{2R}^U + \tfrac{24}{7}\ln x_{BL}^U - \tfrac{91}{7}\ln x_{3C}^U \\
+ \tfrac{1}{4}\ln x_{BL}^+ + 2\ln x_{1R}^+ + \tfrac{10}{19}\ln x_{2L}^+ - \tfrac{96}{7}\ln x_{3C}^+ + \tfrac{46}{41}\ln x_y^0 + \tfrac{10}{19}\ln x_{2L}^0 - \tfrac{96}{7}\ln x_{3C}^0\big) + \tfrac{1}{7}\left[21\ln\frac{M^{(+)}}{M_U} - 28\ln\frac{M^{(-)}}{M_U}\right],
\end{aligned} \tag{14}$$

$$\begin{aligned}
\sin^2\theta_W = \tfrac{15}{74} + \frac{9\alpha}{29\alpha_s} - \frac{92\alpha}{87\pi}\ln\frac{M_R^+}{M_W} + \frac{5\alpha}{58\pi}\ln\frac{M_R^0}{m_W} \\
- \frac{\alpha}{29\pi}\big(\tfrac{1013}{16}\ln x_{2L}^U - \tfrac{897}{16}\ln x_{2R}^U + \tfrac{103}{28}\ln x_{BL}^U + \tfrac{191}{14}\ln x_{3C}^U + \tfrac{91}{14}\ln x_{BL}^+ \\
+ \tfrac{193}{16}\ln x_{1R}^+ + \tfrac{307}{76}\ln x_{2L}^+ + \tfrac{191}{14}\ln x_{3C}^+ + \tfrac{403}{164}\ln x_y^0 + \tfrac{307}{76}\ln x_{2L}^0 + \tfrac{131}{14}\ln x_{3C}^0\big) \\
- \frac{\alpha}{174\pi}\left[33\ln\frac{M^{(+)}}{M_U} - 44\ln\frac{M^{(-)}}{M_U}\right],
\end{aligned} \tag{15}$$

where

$$x_i^U = \frac{\alpha_i(M_U)}{\alpha_i(M_R^+)}, \quad x_i^+ = \frac{\alpha_i(M_R^+)}{\alpha_i(M_R^0)}, \quad x_i^0 = \frac{\alpha_i(M_R^0)}{\alpha_i(M_W)}, \quad \text{and } \alpha_i(\mu) = \frac{g_i^2(\mu)}{4\pi}$$

Using an iterative convergence procedure that ensures fine structure constant matching[15] at $\mu = M_W$ we have computed M_U, τ_p, and $\sin^2\theta_W$ as a function of M_R^+ for the degenerate and nondegenerate cases as shown in Figs 2 and 3, respectively, while keeping Z_R light ($M_R^0 \simeq 1$ TeV), where $\eta^{-1} = \ln M^{(\pm)}/M_U$. Some interesting solutions are summarized in Table III

At the one loop level, neglecting superheavy scalar effects, the model predicts $M_U \simeq 10^{15}$ GeV and $\sin^2\theta_W \simeq 0.225$ for $\Lambda_{\overline{MS}} \simeq 250$ MeV and $M_P = M_R^+ \simeq 10^{11}$ GeV. This is consistent with $(\tau_p)_{max} \simeq 10^{33}$ yr, where we have included an uncertainty factor of 10^{-3} in τ_p arising

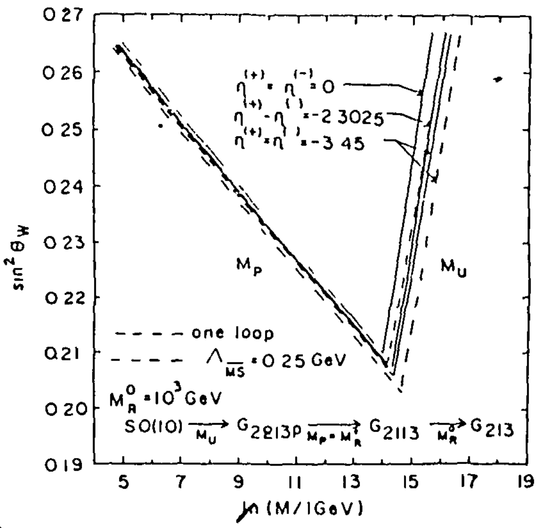


FIG 2 Predictions of the symmetry breaking pattern SO(10) → G_{2213P} → G_{213} as described in the text with $M_{Z_R} \simeq 1$ TeV with and without degenerate superheavy Higgs scalar contributions The dot dashed curve is for $\Lambda_{\overline{MS}} \simeq 0\ 250$ GeV, others are for $\Lambda_{\overline{MS}}=0\ 160$ GeV

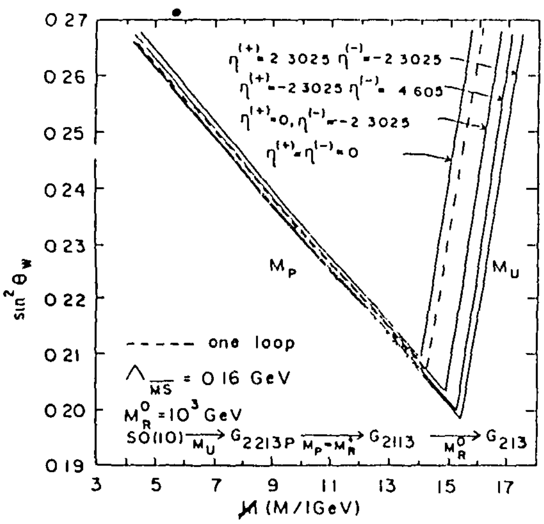


FIG 3 Same as Fig 2 but for nondegenerate superheavy scalar masses under a Coleman Weinberg type constraint and for $\Lambda_{\overline{MS}}=0\ 160$ GeV

out of uncertainties in the estimation of the proton decay matrix elements, branching ratios, and $\Lambda_{\overline{MS}}$ [16,17] Including contributions up to two loops and no superheavy-Higgs scalar effects, τ_p decreases by two orders corresponding to the curve $\eta^{(+)}=\eta^{(-)}=0$ in Fig 2 for which $\Lambda_{\overline{MS}}=160$ MeV Including the superheavy Higgs scalars lighter than M_U by a factor 10 (50) increases the two loop computation of τ_p by 2 (3) orders for $\Lambda_{\overline{MS}}=160$ MeV, and the decrease in $\sin^2\theta_W$ is only 0 0015 Allowing the possibility of $\Lambda_{\overline{MS}} \simeq 250$ MeV and the superheavy scalars lighter by a factor 50 from M_U, we find $(\tau_p)_{max}=10^{34}-10^{36}$ yr, with $M_P=M_R^+ \geq 10^{11}$ GeV and $\sin^2\theta_W=0\ 220-0\ 227$ as shown in Fig 2 and Table III Increasing M_R^0 from 1 TeV to 100 TeV does not have a significant impact on the GUT predictions In the case of nondegenerate superheavy components restricting $M_P=M_R^+ > 10^{11}$ GeV and $\sin^2\theta_W \simeq 0\ 22-0\ 23$, τ_p is found to increase over the one loop predictions by nearly 2 orders if $M^{(+)}=M_U$ and $M^{(-)}=M_U/10$ In this case $\tau_p \simeq 10^{33\pm3}-10^{34\pm3}$ yr, with $M_R^+=M_P=10^{11}-10^{12}$ GeV and $\sin^2\theta_W \simeq 0\ 22-0\ 225$ For larger values of nondegeneracy factor, τ_p could be larger as shown in Fig 3 The allowed values of the low mass of the Z_R boson (300 GeV – 1 TeV) are consistent with the eV keV MeV type of mass spectrum for the neutrinos of the three generations when we choose $m_1^D=m_e$, $m_2^D=m_\mu$, and $m_3^D=m_\tau$, as a

TABLE III Some predictions of the model SO(10) $\xrightarrow{M_U}$ G_{2213P} $\xrightarrow{M_P}$ G_{2113} on $\sin^2\theta_W$ and τ_p with $M_R^0=1$ TeV, $\Lambda_{\overline{MS}}=0\ 16$ GeV and different values of the parity violating scale (M_P) including superheavy Higgs scalar effects

$\eta^{(+)}$	$\eta^{(-)}$	$M_R^+=M_P$ (GeV)	M_U (GeV)	$\sin\theta_W$	α_G^{-1}	τ_p (yr)
−2 30	−2 30	8×10^8	2×10^{15}	0 240	36 1	$1\ 2\times10^{32\pm}$
		$5\ 6\times10^9$	$1\ 4\times10^{15}$	0 235	35 7	$3\times10^{31\pm3}$
		$2\ 5\times10^{11}$	7×10^{14}	0 225	32 0	$1\ 5\times10^{30\pm3}$
−3 45	−3 45	$4\ 5\times10^9$	2×10^{15}	0 235	32 4	$10^{32\pm3}$
		2×10^{11}	$1\ 2\times10^{15}$	0 226	31 6	$10^{31\pm3}$
−2 3	−4 6	$1\ 6\times10^{11}$	$2\ 4\times10^{15}$	0 230	31 1	$2\times10^{32\pm3}$
		10^{12}	$1\ 8\times10^{15}$	0 225	30 8	$10^{32\pm3}$
		$6\ 3\times10^{12}$	$1\ 3\times10^{15}$	0 221	30 4	$1\ 4\times10^{31\pm}$

consequence of natural seesaw mechanism.

One difficulty in having masses of the order of keV and MeV for ν_μ and ν_τ is the violation of the cosmological bound. The difficulty is removed by making them unstable with respect to decay into ν_e by the emission of a Majoron, which is obtained by introducing an additional global U(1) (l = lepton number) symmetry in the theory and breaking it spontaneously at a scale $M >> M_R^0$. The RGE's also permit solutions with larger values of $M_R' = M_{Z_R} \sim 10^{5}-10^{6}$ GeV. When $M_P = M_R' \simeq 10^{11}-10^{12}$ GeV, $R \sim 10^{4}-10$ for such larger values of M_R^0, which satisfies the naturalness criterion. In this case $m_{\nu_e} \sim m_{\nu_\mu} \sim m_{\nu_\tau} \sim 1-10$ eV and there is no conflict with the cosmological bound. The weak-interaction phenomenology with such large Z_R mass is indistinguishable from the standard-model predictions.

V. IMPLEMENTATION IN SO(10) WITH G_{214} AS AN INTERMEDIATE SYMMETRY

In this section we show how the seesaw mechanism can be implemented naturally with G_{214} as one of the two intermediate symmetries occurring in an SO(10) scenario. Compared to the case discussed in Sec. IV, this has the novel feature that both $SU(2)_R$ and P break at the GUT scale that is experimentally constrained as $M_I \lesssim 10^{15}$ GeV. The question of domain wall problem does not arise in this case; in addition, the proton decay rate could be close to the observable limit if RGE's permit $M_I \simeq 10^{15}$ GeV. For case (iib) we have found the unification mass too low to be allowed by proton-lifetime measurements unless additional fine tuning is permitted. On the other hand, case (iia) is promising in the context of the GUT scenario.

$$\mathrm{SO}(10) \xrightarrow[M_I]{54-45_1} G_{214} \xrightarrow[M_C]{45_-} G_{2113} \xrightarrow[M_R^0]{126} G_{213} \tag{16}$$

The Higgs scalars mentioned in Sec. III for case (iia) are contained in various SO(10) representations: $\sigma(1,3,1) \subset 45_1$, $\Sigma(1,1,15) \subset 45$, $\Delta_R(1,3,\overline{10}) \subset 126$, $\phi(2,2,1) \subset 10$, where the transformation properties mentioned are under G_{224}. Note that both 54 and 45_1 are needed for the SSB at $\mu \sim M_U$. The masses of superheavy components of different Higgs representations needed for SSB in the case (16) are noted below with their transformation properties under G_{214}:

$$\begin{aligned}
&10 \supset M_{H_1}(2,-\tfrac{1}{2},1) + M_H(1,0,6)\,,\\
&126 \supset M'_{H_1}(1,0,6) + M'_H(3,0,10) + M'_{H_3}(1,0,10) + M'_{H_4}(1,-1,10) + M_{H_5}(2,\tfrac{1}{2},15) + M'_{H_6}(2,-\tfrac{1}{2},15)\,,\\
&45_1 \supset M'_{S_1}(3,0,1) + M_S(1,0,15)\,,\\
&45 \supset M_{S_1}(3,0,1) + M_S(1,1,1) + M_{S_3}(1,0,1) + M_{S_4}(1,-1,1) + M_{S_5}(2,\tfrac{1}{2},6) + M_{S_6}(2,-\tfrac{1}{2},6)\,,\\
&54 \supset M''_{S_1}(3,1,1) + M''_{S_2}(3,0,1) + M''_{S_3}(3,-1,1) + M''_{S_4}(1,0,20) + M'_{S_5}(2,\tfrac{1}{2},6) + M'_{S_6}(2,-\tfrac{1}{2},6)
\end{aligned} \tag{17}$$

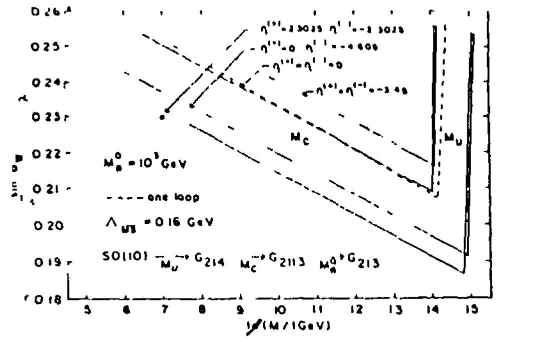
Maximization of τ_p leads to the following constraint on the superheavy-component masses:

$$\begin{aligned}
&M_{H_1} = M_H = M'_{H_4} = M_{S_1} = M_{S_2} = M_{S_4} = M_{S_5} = M_{S_6} = M'_{S_1} = M''_{S_1} = M'_{S_2} = M''_{S_3} = M''_{S_4} = M''_{S_6} = M^{(+)}\,,\\
&M_H = M'_H = M'_{H_3} = M'_{H_5} = M'_{H_6} = M'_{S_2} = M''_{S_4} = M^{(-)}
\end{aligned} \tag{18}$$

Using three generations of fermions with masses $\mu < M_W$, minimal number of Higgs scalars at various stages of SSB, and the superheavy Higgs scalar effects near $\mu \simeq M_U$ we compute $\ln M_U/M_W$ and $\sin^2\theta_W$ up to two loops as

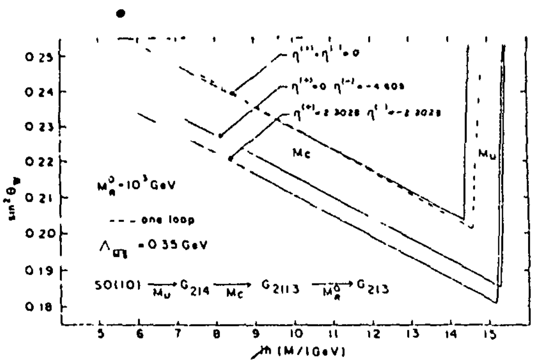
$$\begin{aligned}
\ln\frac{M_I}{M_W} = {} & \frac{6\pi}{67}\left[\frac{1}{\alpha} - \frac{8}{3\alpha_s}\right] - \frac{[illegible]}{67}\ln\frac{M_c}{M_W} + \frac{[illegible]}{67}\ln\frac{M_R^0}{M_W}\\
& - \frac{1}{[illegible]}\Big(\tfrac{10}{19}\ln x_{2L}^U + \tfrac{34}{13}\ln x_{1R}^U - \tfrac{[illegible]}{9}\ln x_{4c}^U + \tfrac{10}{19}\ln x_{2L}^C + 2\ln x_{1R}^C + \tfrac{1}{9}\ln x_{BL}^C - [illegible]\ln x_{3c}^C + \tfrac{46}{41}\ln x_Y^0 + \tfrac{10}{19}\ln x_{2L}^0 - \tfrac{[illegible]}{7}\ln x_{3c}^0\Big)\\
& - \frac{1}{67}\left[28\ln\frac{M^{(+)}}{M_U} - 29\ln\frac{M^{(-)}}{M_U}\right],
\end{aligned} \tag{19}$$

$$\begin{aligned}
\sin^2\theta_W = {} & \frac{[illegible]}{4} + \frac{61\alpha}{201\alpha_s} - \frac{437\alpha}{402\pi}\ln\frac{M_c}{M_W} + \frac{35\alpha}{402\pi}\ln\frac{M_R^C}{M_W}\\
& - \frac{\alpha}{67\pi}\Big(\tfrac{211}{76}\ln x_{2L}^U + \tfrac{4473}{124}\ln x_{1R}^U - \tfrac{[illegible]}{44}\ln x_{4c}^U + \tfrac{211}{76}\ln x_{2L}^C + \tfrac{[illegible]}{[illegible]}\ln x_{1R}^C\\
& + \tfrac{21}{6}\ln x_{BL}^C + \tfrac{433}{14}\ln x_{3c}^C + \tfrac{[illegible]}{[illegible]}\ln x_Y^0 + \tfrac{211}{76}\ln x_{2L}^0 + \tfrac{433}{14}\ln x_{3c}^0\Big) + \frac{\alpha}{804\pi}\left[-1633\ln\frac{M^{(+)}}{M_U} + 928\ln\frac{M^{(-)}}{M_U}\right],
\end{aligned} \tag{20}$$



log10

FIG 4 Predictions of the symmetry breaking pattern $SO(10) \rightarrow G_{214} \rightarrow G_{2113}$ as described in the text including superheavy Higgs scalar effects for $M_{Z_R} = 1$ TeV and $\Lambda_{\overline{MS}} = 0.16$ GeV



log10

FIG 5 Same as Fig 4 but for $\Lambda_{\overline{MS}} = 0.35$ GeV

where

$$x_i^L = \frac{\alpha_i(M_U)}{\alpha_i(M_C)}, \quad x_i^c = \frac{\alpha_i(M_c)}{\alpha_i(M_R^0)}, \quad \text{and} \quad x_i^0 = \frac{\alpha_i(M_R^0)}{\alpha_i(M_W)}.$$

Following the iterative convergence approach to solve two-loop renormalization group equations and using plausible values of superheavy component masses, our solutions for the intermediate scale and M_U for $M_R^0 \simeq 1$ TeV are shown in Figs 4 and 5 for $\Lambda_{\overline{MS}} \simeq 0.160$ GeV, and 0.350 GeV, respectively (Ref 17) where $\eta^{(\pm)} = \ln(M^{(\pm)}/M_U)$ Some of the interesting solutions are also presented in Table IV. At the one-loop level with $\Lambda_{\overline{MS}} = 0.350$ GeV the predicted value of τ_p is found to be very close to the observed experimental limit for $M_C = 10^{11}$ GeV and $\sin^2\theta_W \simeq 0.235$, but τ_p is found to be 1–2 orders less than the experimental limit for $\Lambda_{\overline{MS}} = 0.160$ GeV. When superheavy-Higgs-scalar effects are included in two-loop calculations, we find $\tau_p \simeq 10^{32.3} - 10^{34\pm3}$ yr, $\sin^2\theta_W \simeq 0.230$, $M_C \simeq 10^7$ GeV with $\Lambda_{\overline{MS}} \simeq 160$ MeV if the heavier (lighter) components differ by a factor 10 from the unification mass For larger values of $\Lambda_{\overline{MS}}$ or nondegeneracy factors, τ_p is found to increase further We find that this SO(10) model permits observable rare kaon decays corresponding to $M_c \simeq 10^5$ GeV provided $\sin^2\theta_W \simeq 0.24$ In all allowed solutions in this model $M_R^+ = M_U = M_P \simeq 10^{15}$ GeV With $M_R^0 \simeq 1$ TeV, $R \simeq 10^{12}$, and the naturalness criterion is easily satisfied As in Sec II, the low-mass Z_R boson yields the neutrino-mass spectrum as eV-keV-MeV for the three generations The violation of the cosmological bound by the ν_μ and ν_τ masses is avoided by making these neutrinos unstable against Majoron emission through the introduction of an additional global lepton-number symmetry $U(1)_l$ (Ref 18) But the RGE's also permit $M_R^0 \simeq 10^5 - 10^6$ GeV as the Z_R-boson mass for which $m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau} \simeq 1 - 10$ eV as a consequence of the natural seesaw mechanism with $R = 10^9 - 10^{10}$, and this is consistent with the cosmological bound with stable neutrinos In this case the predicted weak-interaction phenomenology at low energy cannot be distinguished from the standard model predictions

TABLE IV Some predictions of the model $SO(10) \xrightarrow{M_U} G_{214} \xrightarrow{M_C} G_{2113}$ for two values of $\Lambda_{\overline{MS}}$ on $\sin^2\theta_W$ and τ_p with $M_R^0 = 1$ TeV and different M_C including superheavy Higgs scalar-effects as described in the text

$\Lambda_{\overline{MS}}$ (GeV)	$\eta^{(+)}$	$\eta^{(-)}$	M_C (GeV)	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_p (yr)
	2 3	−2 3	4×10^5	10^{15}	0 240	48 3	$1.7\times10^{31\pm3}$
0 16			3×10^6	10^{15}	0 235	47 6	$1.4\times10^{31\pm3}$
	0	−4 6	3×10^6	1.3×10^{15}	0 240	42 5	$2.5\times10^{31\pm3}$
			1.6×10^8	1.1×10^{15}	0 230	40 8	$1.5\times10^{31\pm3}$
	2 3	−2 3	10^5	2.4×10^{15}	0 24	48 8	$4.3\times10^{32\pm3}$
0 35			5.6×10^6	2.2×10^{15}	0 230	47 4	$2.9\times10^{32\pm3}$
	0	−4 6	5.6×10^6	2.7×10^{15}	0 235	41 7	$5\times10^{32\pm3}$
			4×10^7	2.5×10^{15}	0 230	41 0	$3.8\times10^{32\pm3}$

VI. SUMMARY AND DISCUSSION

In this paper we have suggested the new possibility that the seesaw mechanism for neutrino masses could be natural in the context of the left-right-symmetric gauge group, partial unification scheme, and GUT's even if the scales of P and $SU(2)_R$ breakings are identical. In these models, the P-breaking scale is the same as the W_R gauge-boson mass ($M_P = M_R^+$) and the $U(1)_{R-L}$-breaking scale is the same as the Z_R-boson mass (M_R^0). The criterion which guarantees naturalness has been derived and is found to depend upon the largeness of the ratio $R = M_R^+/M_R^0 >> 10^5$. At the critical value of the ratio $R \simeq 10^5$, the induced and seesaw mechanism contributions are comparable, but for larger values of R the induced neutrino mass becomes smaller.

In the LRS model based upon the gauge group G_{2211P}, it is very easy to implement the mechanism as there is not much restriction on $M_P = M_R^+$. In the partial-unification scheme with one intermediate symmetry G_{2113}, the RGE permits $M_P = M_R^+ \simeq M_C = 10^{12} - 5\times 10^{13}$ GeV with $M_R^0 = M_{Z_R} = 300$ GeV-10^6 GeV [case (ii b)]. However, with two intermediate symmetries G_{214} and G_{2113} [case (ii a)] the solutions allow $M_P = M_R^+ \simeq 7\times 10^{13} - 10^{17}$ GeV for 10^{10} GeV $> M_C > 10^5$ GeV, predicting rare kaon decays to be observable by low-energy experiments besides a low-mass Z_R boson.

In the SO(10) model, implementation of the natural seesaw mechanism has been found to be possible with parity (P) surviving down to an intermediate scale $M_P = M_R^+ \simeq 10^{11}$ GeV or broken at the GUT scale $M_P = M_U = M_R^+ \gtrsim 10^{15}$ GeV. With G_{2211P} and G_{2113} intermediate symmetries, RGE's up to two loops, with superheavy-Higgs-scalar masses lighter than M_U by a factor of 10–50, are found to allow the intermediate P-breaking scale $M_P \simeq 10^{11} - 10^{12}$ GeV, observable proton decay by the second generation of experiments with $\tau_p \simeq 10^{33} - 10^{35}$ yr, and a low-mass Z_R boson ($M_R^0 \simeq 300 - 10^3$ GeV). In this case there is the possibility that the domain walls created in the early Universe might have been removed by inflation. In this context it is to be noted that the large P-violating scale can be associated with the breaking of Peccei-Quinn symmetry invoked to solve the strong CP problem and can be generated by the principle of geometric hierarchy from $M_{Pl} \simeq 10^{19}$ GeV and $M_{Z_R} \simeq 10^3$ GeV, or M_W. Further, it has been observed that while embedding a LRS gauge group as an intermediate symmetry in SO(10), the generation of an adequate baryon asymmetry of the Universe needs such a large P-violating scale.[12] In the other interesting SO(10) scenario with G_{214} and G_{2113} as the two intermediate symmetries, superheavy-Higgs-scalar masses differing by a factor 10 (lighter or heavier) from the unification mass allow $\tau_p \simeq 10^{35}$ yr with the possibility of observable rare kaon decays and a low-mass Z_R boson. In the two SO(10) models discussed here G_{2113} is allowed to be the gauge symmetry beyond the standard model with the permitted values of a Z_R-boson mass varying over a wider range 300–10^5 GeV.

The weak-interaction phenomenology at low energy does permit a low-mass Z_R boson ($M_R^0 \simeq 300$ GeV–1 TeV) in the G_{2113} model which yields fits to the neutral- and charged-current data similar to the standard-model predictions.[9,10] When such values of M_R^0 are used in the natural seesaw mechanism, the neutrino masses are of the order eV, keV, and MeV for the first-, second-, and third-generation neutrinos, respectively, out of which the latter two violate the cosmological bound. The cosmological bound can still be respected with low-Z_R masses by making ν_μ and ν_τ unstable with respect to the emission of a Majoron which is a massless scalar carrying 2 units of lepton number, and it is created when an additional global symmetry $U(1)_l$ (l = lepton number), attached to the models, breaks spontaneously.[18] With the other allowed possibility, $M_R^0 \simeq 10^5$ GeV, $m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau} \sim 1-10$ eV, there is no violation of the cosmological bound. The weak-interaction phenomenology at lower energies is then indistinguishable from the standard-model predictions within the available experimental accuracies. However, one novel feature in the partial-unification scheme and SO(10) model with G_{214} and G_{2213} intermediate symmetries is the prediction of observable rare kaon decays such as $K_L \to \mu \bar{e}$. The analyses carried out here in SO(10) can be easily implemented in other GUT's such as SO(2N) (N > 5), E_6, and SU(16) with similar predictions. However in $SU(8)_L \times SU(8)_R$ while all other low energy predictions are similar, it is possible to have a more stable proton since the gauge-boson-mediated interaction corresponding to the proton decay is absent.

Finally from the investigations carried out in this paper we conclude that scenarios different from those discussed by Chang and Mohapatra in Ref. 3 and worked out earlier[7,8] do exist in LRS models, partial unification schemes and GUT's in which the seesaw mechanism can provide a natural explanation for small Majorana neutrino masses even if the P- and $SU(2)_R$-breaking scales are identical.

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In Figs. 2-5, we meant $\log_{10}$(m/ 1GeV) but not ln. This may be corrected