

SOME INVESTIGATIONS IN LOW ENERGY
SUPERSYMMETRY



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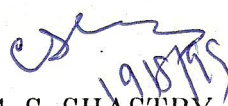
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Publications

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2. P. N. Pandita and P. Francis Paulraj, *Upper Bound on the Mass of the Lightest Higgs Boson in Supersymmetric Left-Right Model*, submitted to Phys. Rev. D.
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Chapter 1

Introduction

1.1 Standard Model

The Standard Model [1] is enormously successful in describing all the current experimental data [2]. However, despite its tremendous successes,¹ SM is not satisfactory, and many present and future experiments will look at the issues raised by the Standard Model. Perhaps the most important among these is: what breaks electroweak symmetry? Related to the nature of breakdown of electroweak symmetry is the question: is there supersymmetry? Nevertheless, the Standard Model remains the basis on which our quest for new physics must be built, so we start with a brief review of its basic features and examine whether its successes offer any hint of the direction in which to search for new physics.

The Standard Model (SM) of electroweak interactions is a gauge theory based² on the gauge group $SU(2)_L \times U(1)_Y$. The electroweak Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{\text{symm}} + \mathcal{L}_{\text{Higgs}}, \quad (1.1)$$

¹For the first time, clear evidence for new physics beyond the Standard Model may be emerging from non-accelerator neutrino experiments [3].

²Here we shall not discuss the strong interactions which are described by an unbroken $SU(3)_C$ gauge theory.

where $\mathcal{L}_{\text{symm}}$ involves only gauge bosons and fermions [4], and can be written as

$$\mathcal{L}_{\text{symm}} = -\frac{1}{4} \sum_{A=1}^3 F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{\psi}_L i \gamma^\mu D_\mu \psi_L + \bar{\psi}_R i \gamma^\mu D_\mu \psi_R. \quad (1.2)$$

This is the Yang-Mills Lagrangian for the gauge group $SU(2)_L \times U(1)_Y$ with fermion matter fields. Here

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad F_{\mu\nu}^A = \partial_\mu W_\nu^A - \partial_\nu W_\mu^A - g \epsilon_{ABC} W_\mu^B W_\nu^C, \quad (1.3)$$

are the gauge antisymmetric tensors constructed out of the gauge field B_μ associated with $U(1)$, and W_μ^A corresponding to the three $SU(2)$ generators; ϵ_{ABC} are the gauge structure constants which, for $SU(2)$, coincide with totally antisymmetric Levi-Civita tensor. The normalization of the $SU(2)$ gauge coupling g is therefore specified by Eq.(1.3).

The fermion fields are described by their left-hand and right hand components:

$$\psi_{L,R} = \left[\frac{(1 \mp \gamma_5)}{2} \right] \psi, \quad \bar{\psi}_{L,R} = \bar{\psi} \left[\frac{(1 \pm \gamma_5)}{2} \right], \quad (1.4)$$

with γ_5 and other Dirac matrices defined as in Bjorken and Drell [5]. In SM the left and right handed fermions have different transformation properties under the gauge group. Thus, mass terms for fermions (of the form $\bar{\psi}_L \psi_R + h.c.$) are forbidden in the symmetric limit. In particular all ψ_R are singlets in the Standard Model. The spectrum of known fermions in the Standard Model is shown below:

$$q_L = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \quad \begin{pmatrix} c_i \\ s_i \end{pmatrix}_L, \quad \begin{pmatrix} t_i \\ b_i \end{pmatrix}_L \quad \sim \quad (3, 2, \frac{1}{6}), \quad (1.5)$$

$$u_R = u_{iR}, \quad c_{iR}, \quad t_{iR} \quad \sim \quad (3, 1, \frac{2}{3}), \quad (1.6)$$

$$d_R = d_{iR}, \quad s_{iR}, \quad b_{iR} \quad \sim \quad (3, 1, -\frac{1}{3}), \quad (1.7)$$

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad \sim \quad (1, 2, -\frac{1}{6}), \quad (1.8)$$

$$l_R = e_R, \quad \mu_R, \quad \tau_R \quad \sim \quad (1, 1, -1), \quad (1.9)$$

where $i = 1, 2, 3$ denotes the $SU(3)_C$ index, and numbers in the parentheses denote the transformation properties under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. The q_L are left-handed $SU(2)$ doublets of quarks, whereas u_R, d_R are right handed quark singlets. Similarly, l_L are lepton doublets and l_R are singlets.

In the absence of mass terms, there are only vector and axial vector interactions in the Lagrangian that have the property of not mixing ψ_L and ψ_R . Fermion masses will be introduced, together with W^\pm and Z mass, by the mechanism of spontaneous symmetry breaking. The covariant derivatives D_μ in Eq.(1.2) are given by (g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively)

$$D_\mu = \left(\partial_\mu + ig \sum_{A=1}^3 t^A W_\mu^A + ig' \frac{Y}{2} B_\mu \right), \quad (1.10)$$

where t^A and $\frac{Y}{2}$ are the $SU(2)_L$ and $U(1)_Y$ generators, respectively in the appropriate representation of the fermions. The $SU(2)$ generators satisfy the commutation relations

$$[t^A, t^B] = i\epsilon_{ABC} t^C, \quad (1.11)$$

with $tr(t^A t^B) = \frac{\delta^{AB}}{2}$ in the fundamental representation. The $SU(2)$ piece in Eq.(1.10) appears only for the left-handed fermions ψ_L , which are isospin doublets, while the right handed fermions ψ_R are isospin singlets, and hence couple only to the $U(1)$ gauge boson B_μ , via the hypercharge $\frac{Y}{2}$. The electric charge generator Q (in units of e , the positron charge) is given by

$$Q = t^3 + \frac{Y}{2}. \quad (1.12)$$

We note that the normalization of the $U(1)$ gauge coupling g' in Eq.(1.10) is now specified as a consequence of Eq.(1.12).

All fermion couplings to the gauge bosons can be derived directly from Eqs.(1.2) and (1.10). The charged current (CC) couplings are

$$\begin{aligned} g (t^1 W_\mu^1 + t^2 W_\mu^2) &= g \left\{ \left[\frac{(t^1 + it^2)}{\sqrt{2}} \right] \frac{(W_\mu^1 - iW_\mu^2)}{\sqrt{2}} + h.c. \right\} \\ &= g \left\{ \left[\frac{(t^+ W_\mu^-)}{\sqrt{2}} \right] + h.c. \right\}, \end{aligned} \quad (1.13)$$

where $t^\pm = t^1 \pm it^2$ and $W^\pm = \frac{(W^1 \pm iW^2)}{\sqrt{2}}$. We, thus, obtain the charged current vertex

$$V_{\bar{\psi}\psi W} = g\bar{\psi}\gamma_\mu \left[\left(\frac{t^+}{\sqrt{2}} \right) \frac{(1 - \gamma_5)}{2} \right] \psi W_\mu^- + h.c. \quad (1.14)$$

In the neutral current (NC) sector, the photon A_μ and Z_μ , the mediator of weak neutral currents (NC), are orthogonal and normalized linear combinations of B_μ and W_μ^3 :

$$\begin{aligned} A_\mu &= \cos \theta_W B_\mu + \sin \theta_W W_\mu^3, \\ Z_\mu &= -\sin \theta_W B_\mu + \cos \theta_W W_\mu^3, \end{aligned} \quad (1.15)$$

where θ_W is the weak mixing angle. The photon is characterized by equal couplings to left and right fermions with a strength equal to the electric charge. From Eq.(1.12) for the charge matrix Q , we immediately obtain

$$g \sin \theta_W = g' \cos \theta_W = e, \quad (1.16)$$

or equivalently

$$\tan \theta_W = \frac{g'}{g}. \quad (1.17)$$

Given Eq.(1.16), one can easily derive the Z couplings:

$$\Gamma_{\bar{\psi}\psi Z} = \frac{g}{2 \cos \theta_W} \bar{\psi} \gamma_\mu [t^3(1 - \gamma_5) - 2Q \sin^2 \theta_W] \psi Z^\mu, \quad (1.18)$$

where $\Gamma_{\bar{\psi}\psi Z}$ is the notation for the vertex. In the Standard Model $t^3 = \pm \frac{1}{2}$.

In order to derive the effective four-fermion interactions that are equivalent, at low energies, to the CC and NC couplings given in Eqs.(1.14) and (1.18), we note that large masses, as experimentally observed, are provided for W^\pm and Z by $\mathcal{L}_{\text{Higgs}}$ (see later). For the left-left CC couplings, when the momentum transfer squared can be neglected in comparison to m_W^2 in the propagator of Born diagrams with single W exchange, we can write, using Eq.(1.14),

$$\mathcal{L}_{eff}^{CC} \simeq \left(\frac{g^2}{8m_W^2} \right) [\bar{\psi} \gamma_\mu (1 - \gamma_5) t^+ \psi] [\bar{\psi} \gamma_\mu (1 - \gamma_5) t^- \psi]. \quad (1.19)$$

By specializing further to the case of doublet fields such as $\nu_e - e^-$ or $\nu_\mu - \mu^-$, we obtain the tree-level relation of g with the Fermi coupling constant G_F measured from μ decay

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}. \quad (1.20)$$

Using the fact that $g \sin \theta_W = e$, we can write this as

$$m_W = \left(\frac{\pi \alpha}{\sqrt{2} G_F} \right)^{\frac{1}{2}} \frac{1}{\sin \theta_W} \simeq \frac{37.2802 \text{ GeV}}{\sin \theta_W}. \quad (1.21)$$

In the same manner, we obtain from Eq.(1.18) in the Born approximation the effective four-fermion NC interaction given by

$$\begin{aligned} \mathcal{L}_{eff}^{NC} \simeq & \sqrt{2} G_F \rho_0 \bar{\psi} \gamma_\mu [t^3(1 - \gamma_5) - 2Q \sin^2 \theta_W] \psi \\ & \times \bar{\psi} \gamma^\mu [t^3(1 - \gamma_5) - 2Q \sin^2 \theta_W] \psi, \end{aligned} \quad (1.22)$$

where

$$\rho_0 = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W}. \quad (1.23)$$

All couplings given above are obtained at tree level and are modified in higher orders of perturbation theory. In particular the relations between m_W and $\sin \theta_W$ as given in Eq.(1.21), as well as the observed value of ρ ($\rho = \rho_0$ at tree level) in different neutral current processes, are altered by radiative corrections which can be calculated in perturbation theory [6].

We now come to the Higgs part of the electroweak Lagrangian in Eq.(1.1), and the phenomenon of the generation of mass for gauge bosons and fermions. The Higgs Lagrangian is specified by gauge invariance and renormalizability to be

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger (D_\mu \phi) - V(\phi^\dagger \phi) - \bar{\psi}_L \Gamma \psi_R \phi - \bar{\psi}_R \Gamma^\dagger \psi_L \phi, \quad (1.24)$$

where ϕ is the Higgs multiplet. In the Standard Model, where all ψ_L transform as doublets and all right handed fermions ψ_R transform as singlets, ϕ is a $SU(2)_L$ doublet. The quantities Γ (which include coupling constants) are matrices that make the Yukawa couplings invariant under the Lorentz and gauge groups. The gauge

invariant potential $V(\phi^\dagger\phi)$ contains at most quartic terms in ϕ so that the theory is renormalizable:

$$V(\phi^\dagger\phi) = -\frac{1}{2}\mu^2\phi^\dagger\phi + \frac{1}{4}\lambda(\phi^\dagger\phi)^2. \quad (1.25)$$

Spontaneous gauge symmetry breaking is induced if the minimum of V , which is the classical analogue of quantum mechanical vacuum state (both are states of minimum energy) is obtained for non-vanishing values of the scalar field ϕ . We denote the vacuum expectation value of ϕ , i.e. the position of minimum, by v :

$$\langle 0|\phi|0\rangle = v \neq 0. \quad (1.26)$$

The fermion mass matrix is obtained from the Yukawa couplings by replacing $\phi(x)$ by v :

$$M = \bar{\psi}_L \mathcal{M} \psi_R + \bar{\psi}_R \mathcal{M}^\dagger \psi_L, \quad (1.27)$$

with

$$\mathcal{M} = \Gamma.v. \quad (1.28)$$

We note that by a suitable change of basis we can always make the matrix \mathcal{M} Hermitian, γ_5 -free, and diagonal. Indeed, we can make separate unitary transformations on ψ_L and ψ_R according to

$$\psi'_L = U\psi_L, \quad \psi'_R = V\psi_R, \quad (1.29)$$

and consequently

$$\mathcal{M} \rightarrow \mathcal{M}' = U^\dagger \mathcal{M} V. \quad (1.30)$$

This transformation does not alter the general structure of the fermion couplings in $\mathcal{L}_{\text{symm}}$.

If only one Higgs doublet is present, the change of basis that makes \mathcal{M} diagonal will at the same time diagonalize also the fermion-Higgs Yukawa couplings. Thus, in this case, no flavour-changing neutral Higgs exchanges are present. However, when there are several Higgs doublets, there would be flavour-changing neutral current couplings induced by Higgs exchanges. On the other hand one Higgs doublet for each electric charge sector, i.e. one doublet coupled only to u-type quarks, one doublet to d-type quarks, one doublet to charged leptons, would not lead to flavour-changing

neutral currents, because the mass matrices of fermions with different charges are diagonalized separately. For several Higgs doublets in a given charge sector it is also possible to generate CP violation by complex phases in the Higgs couplings. In the presence of six quark flavours [7], this CP-violation mechanism is not necessary. In fact, at present, the simplest model with only one Higgs doublet seems adequate for describing all observed phenomenon.

We now turn to the gauge-boson masses and their couplings to the Higgs boson. These effects are induced by the $(D_\mu\phi)^\dagger(D_\mu\phi)$ term in $\mathcal{L}_{\text{Higgs}}$ in Eq.(1.24), where

$$D_\mu\phi = \left[\partial_\mu + ig \sum_{A=1}^3 t^A W_\mu^A + ig' \left(\frac{Y}{2} \right) B_\mu \right] \phi. \quad (1.31)$$

Here t^A and $\frac{Y}{2}$ are the $SU(2)_L$ and $U(1)_Y$ generators in the representation spanned by ϕ . Not only doublets but all non-singlet Higgs representations can contribute to gauge-boson masses. The condition that photon remain massless is equivalent to the condition that vacuum is electrically neutral:

$$Q|v\rangle = \left(t^3 + \frac{Y}{2} \right) |v\rangle = 0. \quad (1.32)$$

The charged W mass is given by the quadratic terms in the W field arising from $\mathcal{L}_{\text{Higgs}}$, when ϕ is replaced by v . We obtain

$$m_W^2 W_\mu^+ W^{-\mu} = g^2 \left| \left(\frac{t^+ v}{\sqrt{2}} \right) \right|^2 W_\mu^+ W^{-\mu}, \quad (1.33)$$

whereas for the Z mass we get (using the definition (1.15))

$$\frac{1}{2} m_Z^2 Z_\mu Z^\mu = \left| \left[g \cos \theta_W t^3 - g' \sin \theta_W \left(\frac{Y}{2} \right) \right] v \right|^2 Z^\mu Z_\mu, \quad (1.34)$$

where the factor $\frac{1}{2}$ on the left hand side is for the proper normalization for the definition of a neutral vector field. Using (1.32) and (1.17) we obtain for the mass of the Z boson

$$\frac{1}{2} m_Z^2 = (g \cos \theta_W + g' \sin \theta_W) |t^3 v|^2 = \left(\frac{g^2}{\cos^2 \theta_W} \right) |t^3 v|^2. \quad (1.35)$$

For a Higgs doublet,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad v = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{\frac{\mu^2}{\lambda}}, \quad (1.36)$$

we have

$$|t^+ v|^2 = v^2, \quad |t^3 v|^2 = \frac{1}{4}v^2, \quad (1.37)$$

so that

$$m_W^2 = \frac{1}{2}g^2v^2, \quad m_Z = \frac{1}{2}\frac{g^2v^2}{\cos^2\theta_W}. \quad (1.38)$$

From (1.20), it follows that

$$v = 2^{-\frac{3}{4}}G_F^{-\frac{1}{2}} = 174.1\text{GeV}. \quad (1.39)$$

For Higgs doublets we have

$$\rho_0 = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1. \quad (1.40)$$

This relation is typical of one or more Higgs doublets [8] and would be spoiled if there were Higgs triplets in the model. In general

$$\rho_0 = \frac{\sum_i ((t_i)^2 - (t_i^3)^2 + t_i) v_i^2}{\sum_i 2(t_i^3)^2 v_i^2}, \quad (1.41)$$

for several Higgs multiplets with vacuum expectation values (VEVs) v_i , weak isospin t_i , and z-component of isospin t_i^3 . These results are valid at the tree level and are modified by calculable electroweak radiative corrections.

In the minimal version of the SM only one Higgs doublet is present. Then the fermion-Higgs couplings are proportional to the fermion masses. In fact, from the Yukawa couplings $g_{\phi\bar{\psi}\psi}(\bar{\psi}_L\phi\psi_R + h.c.)$, the mass m_ψ of the fermion is obtained by replacing ϕ by v , so that $m_\psi = g_{\phi\bar{\psi}\psi}v$. In the minimal SM three out of four Hermitian fields in ϕ are removed from the physical spectrum by the Higgs mechanism and become the third polarization state of W^+ , W^- , and Z. The fourth neutral Higgs boson is physical and should be found. If more doublets are present, two more charged and two more neutral Higgs scalars should be found for each additional doublet.

The couplings of the physical Higgs h to the gauge bosons can be simply obtained from $\mathcal{L}_{\text{Higgs}}$ by the replacement

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ v + \left(\frac{h}{\sqrt{2}}\right) \end{pmatrix}, \quad (1.42)$$

with the result

$$\begin{aligned} \mathcal{L}[h, W, Z] &= g^2 \left(\frac{v}{\sqrt{2}}\right) W_\mu^+ W^{-\mu} h + \left(\frac{g^2}{4}\right) W_\mu^+ W^{-\mu} h^2 \\ &+ \left[\frac{g^2 v Z_\mu Z^\mu}{2\sqrt{2} \cos^2 \theta_W}\right] h + \left[\frac{g^2}{8 \cos^2 \theta_W}\right] Z_\mu Z^\mu h^2. \end{aligned} \quad (1.43)$$

Once the vacuum expectation value $\langle 0|\phi(x)|0\rangle = v$ of the neutral Higgs boson is fixed in the minimal SM, the mass of the remaining physical Higgs boson is given by

$$m_h^2 = \mu^2 = \lambda v^2, \quad (1.44)$$

which is a free parameter in the Standard Model. The present direct experimental limit [9] on m_h from LEP is $m_h > 89.7 \text{ GeV}/c^2$.

In the SM with only one Higgs doublet a lower limit on m_h can be derived from the requirement of vacuum stability [10, 11, 12]. The limit is a function of the mass of the top-quark (m_t) and of the energy scale where the model breaks down and new physics appears. If one requires that λ remain positive up to $\Lambda = 10^{15} - 10^{19} \text{ GeV}$, then the resulting bound on m_h in the SM with only one Higgs doublet can be written as [11]

$$m_h > 134 + 2.1[m_t - 173.8] - 4.5 \frac{\alpha_3(m_Z) - 0.119}{0.006}, \quad (1.45)$$

where $\alpha_3 = g_3^2/4\pi$, with g_3 the $SU(3)_C$ gauge coupling constant. We see that the discovery of a Higgs particle with $m_h \lesssim 100 \text{ GeV}$ (the discovery limit at LEP II) would imply that the SM breaks down at a scale Λ of the order of a few TeV. It can be shown that the lower limit is not much relaxed even if strict vacuum stability is replaced by some sufficiently long metastability.

On the other hand an upper limit on the Higgs mass in the SM is important for assessing the chances of success of the Large Hadron Collider (LHC) as an accelerator

designed to detect the Higgs boson. The upper limit [13] arises from the requirement that the Landau pole associated with the non-asymptotically free behaviour of the $\lambda\phi^4$ theory does not occur below the scale Λ . The initial value of λ at the weak scale increases with m_h and the derivative is positive at large m_h . Thus if m_h is too large the Landau pole occurs at too low an energy. The upper limit on m_h for $m_t \sim 175$ GeV is given by $m_h \lesssim 180$ GeV for $\Lambda \sim m_{GUT} - m_{Pl}$, and $m_h \lesssim 0.5 - 0.8$ TeV for $\Lambda = 1$ TeV [14]. Thus, for $m_t \sim 174$ GeV, only a small range of values for m_h is allowed, $130 < m_h < 200$ GeV, if the Standard Model holds up to $\Lambda \sim m_{GUT}$ or m_{Pl} .

1.2 Why Supersymmetry?

At present there is no confirmed experimental evidence against the Standard Model. Nevertheless, there are several questions raised by the Standard Model which point toward physics beyond it. Central among these is the question of the generation of masses: do particle masses really originate from the Higgs mechanism, and, if so, why are these masses so small compared to the Planck mass $m_{Pl} \simeq 10^{19}$ GeV? In other words why is $m_W \ll m_{Pl}$? This is known as the mass hierarchy problem [15]. The Planck mass is the only candidate we have for a fundamental mass scale in physics, where gravity is expected to become as strong as other particle interactions. The hierarchy problem can also be rephrased as: Why is $G_F \gg G_N$ (the Newton's constant)?, since $G_F \sim \frac{1}{m_W^2}$ and $G_N \sim \frac{1}{m_{Pl}^2}$. We could set $m_W \ll m_{Pl}$ by hand, and ignore the problem. However, there is the question of radiative corrections. The radiative corrections to the squared Higgs mass in the Standard Model can be written as

$$\delta m_h^2 = \mathcal{O} \left(\frac{g^2}{16\pi^2} \right) \int^\Lambda d^4 k \frac{1}{k^2} = \mathcal{O} \left(\frac{\alpha}{\pi} \right) \Lambda^2, \quad (1.46)$$

where the cut off Λ in integral represents the scale up to which the Standard Model remains valid, and beyond which new physics sets in. If $\Lambda \simeq m_{Pl}$ or the grand unification scale, the quantum correction Eq.(1.46) is much larger than the physical value of $m_h \sim 100$ GeV. This is not a problem for renormalization theory: there could be large bare contribution with the opposite sign, and one could fine tune its

value to many significant figures so that the physical value m_h^2 comes out to be of the right order of magnitude. However, this seems unnatural, and would have to be repeated order by order in perturbation theory. In contrast, the one-loop radiative corrections to a fundamental fermion mass m_f are proportional to m_f itself, and only logarithmically divergent:

$$\delta m_f = \mathcal{O}\left(\frac{g^2}{16\pi^2}\right) m_f \int^\Lambda d^4k \frac{1}{k^4} = \mathcal{O}\left(\frac{\alpha}{\pi}\right) m_f \ln \frac{\Lambda}{m_f}. \quad (1.47)$$

This correction is no larger than the physical value, for any $\Lambda \lesssim m_{Pl}$. The reason for this is that there is an underlying chiral symmetry which is reflected in the m_f factor in (1.47) that keeps the quantum corrections naturally (logarithmically) small. The hope is to find a corresponding symmetry principle to make a small Higgs boson mass natural: $\delta m_h^2 \lesssim m_h^2$.

Supersymmetry [16] is at present the only known symmetry which can make the small Higgs boson mass natural, exploiting the fact that boson and fermion loop diagrams have opposite signs (see Fig.(1.1)). If there are equal number of fermions and bosons, and if they have equal couplings as in a supersymmetric theory, the quadratic divergences (1.46) cancel:

$$\begin{aligned} \delta m_h^2 &= -\left(\frac{g_F^2}{16\pi^2}\right) (\Lambda^2 + m_F^2) + \left(\frac{g_B^2}{16\pi^2}\right) (\Lambda^2 + m_B^2) \\ &= \mathcal{O}\left(\frac{\alpha}{4\pi}\right) |m_B^2 - m_F^2|, \end{aligned} \quad (1.48)$$

where $g_F^2/4\pi = g_B^2/4\pi = \alpha$ is the common coupling of bosons and fermions. This is no larger than the physical value, $\delta m_h^2 \lesssim m_h^2$, and hence naturally small, if ³

$$|m_B^2 - m_F^2| \lesssim 1 \text{ TeV}^2. \quad (1.49)$$

This naturalness argument is the only available theoretical motivation for thinking that supersymmetry may manifest itself at an accessible energy scale.

We note that the above argument is qualitative, and it does not tell us whether the supersymmetric partners of the known particles appear at 500 GeV, 1 TeV or 2 TeV.

³There is a logarithmic multiplicative factor in the right hand side of Eq.(1.48) which is not explicitly shown here.

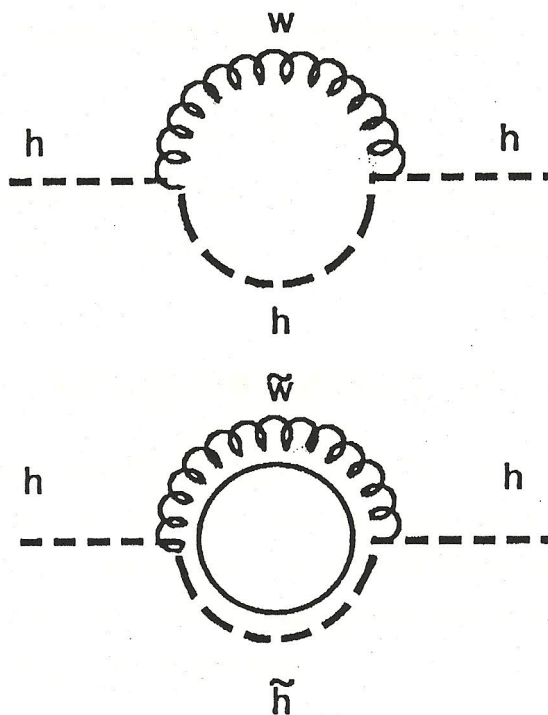


Figure 1.1: Contributions to the Higgs boson self-energy. Contributions from the individual graphs to the Higgs self-energy are separately quadratically divergent, but when both are included the divergence is removed. In models with broken supersymmetry a finite residual piece remains.

Supersymmetry is a symmetry that links bosons to fermions via spin- $\frac{1}{2}$ charges Q_α (where α is a spinorial index). It is the last possible symmetry of the particle scattering matrix [17]. All previously-known symmetries are generated by bosonic charges, which are, apart from the momentum operator P_μ associated with Lorentz invariance, scalar charges Q that relate different particles of the same spin J : $Q|J\rangle = |J'\rangle$, $Q \in U(1), SU(2), SU(3), \dots$. Indeed Coleman and Mandula [18] showed that it was impossible to mix such internal symmetries with Lorentz invariance using bosonic charges.

The possible algebra of spinorial charges connecting bosons to fermions can be easily explored [17]. Let $Q_\alpha^i, i = 1, 2, \dots, N$ be a set of spinorial charges. If they are to be symmetry generators, they must commute with the Hamiltonian

$$[Q_\alpha^i, H] = 0. \quad (1.50)$$

Hence, their anticommutator (which is bosonic) must also commute with H

$$[\{Q_\alpha^i, Q_\beta^j\}, H] = 0. \quad (1.51)$$

By the Coleman-Mandula theorem [18], this anticommutator must be a combination of the conserved Lorentz vector charge P_μ and some scalar charge Z^{ij} . The only possible form is in fact

$$\{Q_\alpha^i, Q_\beta^j\} = 2\delta^{ij}(\gamma^\mu C)_{\alpha\beta}P_\mu + Z^{ij}, \quad (1.52)$$

where we have used four-component spinors, C is the charge conjugation matrix and Z^{ij} is antisymmetric in the supersymmetry indices. Thus, this so called ‘‘central charge’’ vanishes for $N = 1$ supersymmetry which is of phenomenological interest.

The basic building blocks of $N = 1$ supersymmetric theories are supermultiplets containing the following helicity states [19]:

$$\text{chiral} : \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}, \quad \text{gauge} : \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}, \quad \text{graviton} : \begin{pmatrix} 2 \\ \frac{3}{2} \end{pmatrix}, \quad (1.53)$$

which are used to describe matter and Higgs bosons, gauge fields and gravity, respectively. We note that $N = 2$ supersymmetric theories will have left- and right-handed particles (helicity $\mp \frac{1}{2}$) in an identical representation of the gauge group, and hence cannot accommodate parity violation, and are not suitable for phenomenology. We shall denote the chiral supermultiplet in (1.53) by $\hat{\Phi}$. It contains a fermion (ψ), a scalar (ϕ), and the auxiliary field (F). Similarly, a vector or gauge supermultiplet will be denoted by \hat{W} , which contains the gauge boson (W), the gauge fermion (ω) and the auxiliary field (D), etc.

The simplest $N = 1$ supersymmetric theory contains a free fermion and a free boson [16, 20]:

$$\mathcal{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) + i\psi^\dagger \bar{\sigma} \cdot \partial \psi, \quad (1.54)$$

where we work with two-component spinors, and $\sigma_\mu = (1, \underline{\sigma})$, $\bar{\sigma}_\mu = (1, -\underline{\sigma})$ with $\underline{\sigma}$ being the Pauli matrices. The supersymmetry transformation laws are

$$\delta_\xi \phi = \sqrt{2}\xi^T C \psi, \quad \delta_\xi \psi = \sqrt{2}i\sigma \cdot \partial \phi C \xi^*, \quad (1.55)$$

where ξ is an infinitesimal spinor parameter, and C is the charge conjugation matrix: $C = -i\sigma^2 = C^*$, $C^{-1} = C^T = -C$. Under the transformation (1.55) the Lagrangian (1.54) changes by a total derivative, and hence the action $A = \int d^4x \mathcal{L}$ is invariant. We also see in (1.55) a reflection of the supersymmetry algebra (1.52): after two supersymmetry transformations, the fields (ϕ, ψ) are transformed by derivatives $(\partial\phi, \partial\psi)$, corresponding to the action of the momentum operator $P_\mu = i\partial_\mu$.

The example (1.54) can be extended to include the interactions:

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma} \cdot \partial \psi + F^\dagger F + \left(F \frac{\partial W}{\partial \phi} - \frac{1}{2} \psi^T C \psi \frac{\partial^2 W}{\partial \phi^2} + h.c. \right), \quad (1.56)$$

with supersymmetry transformation:

$$\begin{aligned} \delta_\xi \phi &= \sqrt{2} \xi^T C \psi, & \delta_\xi \psi &= \sqrt{2} i \sigma \cdot \partial \phi C \xi^*, \\ \delta_\xi F &= -\sqrt{2} i \xi^\dagger \bar{\sigma} \cdot \partial \psi. \end{aligned} \quad (1.57)$$

The field F is called an auxiliary field; it has no kinetic term, and may be eliminated by using the equation of motion

$$F^\dagger = -\frac{\partial W}{\partial \phi}. \quad (1.58)$$

Thus, all matter interactions are characterized by the analytic function $W(\hat{\Phi})$, which is called the superpotential. Renormalizability of the field theory requires the superpotential to be a cubic function of chiral superfields: for $W = \lambda \hat{\Phi}_1 \hat{\Phi}_2 \hat{\Phi}_3$, one obtains from (1.56) the following particle interactions:

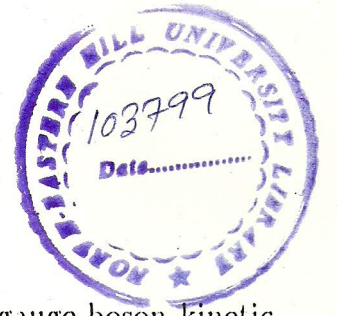
$$\lambda [(\psi_1^T C \psi_2) \phi_3 + (\psi_2^T C \psi_3) \phi_1 + (\psi_3^T C \psi_1) \phi_2] + |\lambda \phi_1 \phi_2|^2 + |\lambda \phi_2 \phi_3|^2 + |\lambda \phi_3 \phi_1|^2, \quad (1.59)$$

where the last three terms provide a quartic potential for the scalar fields ϕ_i and are called "F-terms". Furthermore, in addition to the gauge interactions of the chiral fermions and their bosonic partners, there are gaugino interactions

$$\sqrt{2} g [(\psi_i^T C (T^a)_j^i V_a) \phi^{j*} + h.c.], \quad (1.60)$$

where $(T^a)_j^i$ is the gauge representation matrix for the chiral fields. There is also another quartic potential term for the scalars:

$$V = \frac{g^2}{2} \sum_a |\phi^{i*} (T^a)_i^j \phi_j|^2, \quad (1.61)$$



which are called “D-terms”. We also note that the conventional gauge-boson kinetic term and the gauge interactions of fermions in the adjoint representation of the gauge group, such as the gauginos, are automatically supersymmetric.

1.3 The Supersymmetric Standard Model

In order to construct the minimal supersymmetric version of the Standard Model the first natural question is: can one construct it out of Standard Model particles alone? One can easily see that this is impossible, because the known bosons and fermions have different quantum numbers [21]. For example, gluons are in the octet ($\underline{8}$) representation of $SU(3)_C$, whereas quarks are in the triplet ($\underline{3}$) representation of $SU(3)_C$. Similarly, there are no weak isotriplet fermions as would be needed to partner the electroweak gauge bosons. The known leptons are isodoublets like the Higgs boson, but carry lepton number, so they cannot be the supersymmetric partners of the Higgs boson. For these reasons, new particles must be postulated [21] as supersymmetric partners of known particles (see Table 1.1). The spectrum of

Particle	Spin	Spartner	Spin
quark: q	$\frac{1}{2}$	squark	0
lepton: l	$\frac{1}{2}$	slepton	0
photon: γ	1	photino: $\tilde{\gamma}$	$\frac{1}{2}$
W	1	wino: \tilde{W}	$\frac{1}{2}$
Z	1	zino: \tilde{Z}	$\frac{1}{2}$
Higgs: H	0	higgsino: \tilde{h}	$\frac{1}{2}$

Table 1.1: Particles in the Standard Model and their supersymmetric partners

chiral superfields of the minimal supersymmetric standard model (MSSM) is shown in Table 1.2. We note that each superfield has a family index i ($i = 1, 2, 3$), representing the three known families of quarks and leptons, and their superpartners. This index is not explicitly shown.

Superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Particle Content
\hat{Q}	3	2	$\frac{1}{6}$	$(u_L, d_L), (\tilde{u}_L, \tilde{d}_L)$
\hat{U}^c	$\bar{3}$	1	$-\frac{2}{3}$	\bar{u}_R, \tilde{u}_R^*
\hat{D}^c	$\bar{3}$	1	$\frac{1}{3}$	\bar{d}_R, \tilde{d}_R^*
\hat{L}	1	2	$-\frac{1}{2}$	$(\nu_L, e_L), (\tilde{\nu}_L, \tilde{e}_L)$
\hat{E}^c	1	1	1	\bar{e}_R, \tilde{e}_R^*
\hat{H}_1	1	2	$-\frac{1}{2}$	(H_1, \tilde{h}_1)
\hat{H}_2	1	2	$\frac{1}{2}$	(H_2, \tilde{h}_2)

Table 1.2: Chiral Superfields of the MSSM

The minimal supersymmetric extension of the SM has the same gauge interactions as the Standard Model. In addition, there are couplings of the form (1.59) derived from the following superpotential:

$$W = h_U \hat{Q} \hat{U}^c \hat{H}_2 + h_D \hat{Q} \hat{D}^c \hat{H}_1 + h_L \hat{L} \hat{E}^c \hat{H}_1 + \mu \hat{H}_1 \hat{H}_2. \quad (1.62)$$

Here $\hat{Q}(\hat{L})$ denote isodoublets of supermultiplets containing $(u, d)_L$ $[(\nu, e)_L]$, \hat{D}^c [\hat{U}^c, \hat{E}^c] are singlets containing the left-handed conjugates d_L^c [u_L^c, e_L^c] of the right-handed d_R [u_R, e_R], and the superpotential couplings h_D [h_U, h_L] correspond to the Yukawa couplings of the SM that give masses to the d [u, l^-], respectively:

$$m_d = h_D \langle H_1 \rangle, \quad m_u = h_U \langle H_2 \rangle, \quad m_l = h_L \langle H_1 \rangle. \quad (1.63)$$

Each of these should be understood as a 3×3 matrix in generation space, which is to be diagonalized as in the Standard Model.

One feature of Table 1.2 needs an explanation. The SM contains a single $SU(2)_L$ doublet of Higgs bosons. In the supersymmetric (SUSY) extension of the SM, this scalar doublet acquires a SUSY partner which is an $SU(2)_L$ doublet of Majorana fermion fields, \tilde{h}_1 (the Higgsino), which contribute to the triangle $SU(2)_L$ and $U(1)_Y$ gauge anomalies. Since the fermions of the Standard Model have exactly the right quantum numbers to cancel the triangle anomalies among themselves [22], it follows

that the contribution from the fermionic partner of the Higgs doublet remains uncanceled. These contributions must be cancelled somehow if the SUSY theory is to be sensible. The simplest way is to add a second Higgs doublet with precisely the opposite $U(1)_Y$ quantum numbers from the first Higgs doublet. Then the contribution from the fermions of the second Higgs doublet will cancel the anomalies from the first doublet, leaving an anomaly free theory. We see from Table 1.2 that the fermions satisfy the conditions for anomaly cancellation:

$$\text{Tr} (Y^3) = \text{Tr} (T_{3L}^2 Y) = 0. \quad (1.64)$$

We further note that two Higgs doublets are also needed to give masses to both the up- and down- quarks and leptons in a supersymmetric theory, since one cannot use the complex conjugate of a Higgs superfield in the superpotential, as this would violate supersymmetry. Note also that the ratio of Higgs vacuum expectation values

$$\tan \beta \equiv \frac{\langle H_2 \rangle}{\langle H_1 \rangle}, \quad (1.65)$$

is undetermined and should be treated as a free parameter.

In addition to the chiral superfields, the MSSM will contain the massless vector superfields corresponding to the gauge bosons of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ and their Majorana fermion partners. These are shown in Table 1.3, where \hat{G}^a contains the gluons (g^a), and their supersymmetric partners, the gluinos (\tilde{g}^a); \hat{W}^i contains

Superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Particle Content
\hat{G}^a	8	1	0	g, \tilde{g}
\hat{W}^i	1	3	0	$W_i, \tilde{\omega}_i$
\hat{B}	1	1	0	B, \tilde{b}

Table 1.3: Vector Superfields of the MSSM

$SU(2)_L$ gauge bosons (W^i), and their fermionic partners, winos ($\tilde{\omega}^i$); and \hat{B} contains the $U(1)$ gauge field, B, and its fermionic partner, \tilde{b} (bino).

In addition to the Standard-Model-like superpotential terms (1.62), the following superpotential couplings are also allowed [23] by gauge invariance, supersymmetry and renormalizability:

$$W' = \lambda \hat{L} \hat{L} \hat{E}^c + \lambda' \hat{L} \hat{Q} \hat{D}^c + \lambda'' \hat{U}^c \hat{D}^c \hat{D}^c + \mu_i \hat{L}^i \hat{H}_2. \quad (1.66)$$

In general, λ , λ' , and λ'' could all be matrices which could mix the interactions of the 3 generations. Each of these violate conservation of either lepton number L or baryon number B . These couplings can mediate proton decay at tree level through the exchange of the scalar partner of the down quark. If SUSY partners of the SM particles have masses in the TeV region, then these interactions are severely restricted by experimental measurements [24]. The usual strategy is to require that all of these undesirable lepton and baryon number violating terms be forbidden by a symmetry [25], called R-parity (R_p). It is defined as a multiplicative quantum number such that all particles of the SM have $R_p = +1$, while their SUSY partners have $R_p = -1$. R-parity can also be defined as

$$R_p \equiv (-1)^{3(B-L)+2S}, \quad (1.67)$$

for a particle of spin S . Such a symmetry forbids the lepton and baryon number violating terms of the superpotential (1.66). The assumption of R_p conservation has profound experimental consequences which go beyond the details of a specific model. Conservation of R-parity implies that (i) SUSY partners can only be pair produced from SM particles; (ii) models with R_p conservation will have a lightest SUSY particle (LSP) which is stable, and (iii) the LSP will interact very weakly with ordinary matter, and a generic signal for R-parity conserving SUSY theories is missing transverse energy from the non-observed LSP.

Since nature is not supersymmetric, we must have a mechanism for supersymmetry breaking. This mechanism is not well understood at present. It is typically assumed that the SUSY breaking occurs at a high scale, say m_{Pl} , and results from some complete theory encompassing gravity. At the moment the usual approach is to assume that the MSSM, which is a theory at the electroweak scale, is an effective low energy theory [26]. The supersymmetry breaking is implemented by including

explicit “soft” mass terms for the scalar members of the chiral multiplets and for the gaugino members of the vector supermultiplets in the Lagrangian. These interactions are termed soft because they do not re-introduce the quadratic divergences which motivated the introduction of supersymmetry in the first place. The dimension of soft operators in the Lagrangian must be 3 or less, which means that the possible soft operators are mass terms, bilinear mixing terms (“B” terms), and trilinear scalar mixing terms (“A” terms). The origin of these supersymmetry breaking terms is left unspecified. The complete set of soft SUSY breaking terms (which respect R parity and the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry) is given by the Lagrangian [27]

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} = & m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - B\mu \epsilon_{ij}(H_1^i H_2^j + h.c.) \\
& + \tilde{M}_Q^2(\tilde{u}_L^* \tilde{u}_L + \tilde{d}_L^* \tilde{d}_L) + \tilde{M}_u^2 \tilde{u}_R^* \tilde{u}_R + \tilde{M}_d^2 \tilde{d}_R^* \tilde{d}_R \\
& + \tilde{M}_L^2(\tilde{e}_L^* \tilde{e}_L + \tilde{\nu}_L^* \tilde{\nu}_L) + \tilde{M}_e^2 \tilde{e}_R^* \tilde{e}_R \\
& + \frac{1}{2}[M_3 \tilde{g} \tilde{g} + M_2 \tilde{\omega}_i \tilde{\omega}_i + M_1 \tilde{b} \tilde{b}] \\
& + \frac{g}{\sqrt{2} M_W} \epsilon_{ij} \left[\frac{M_d}{\cos \beta} A_d H_1^i \tilde{Q}^j \tilde{d}_R^* + \frac{M_u}{\sin \beta} A_u H_2^j \tilde{Q}^i \tilde{u}_R^* \right. \\
& \left. + \frac{M_e}{\cos \beta} A_e H_1^i \tilde{L}^j \tilde{e}_R^* + h.c. \right]. \tag{1.68}
\end{aligned}$$

We note that terms of the type $m_\psi \psi^T C \psi$ which would give mass to the fermions in the chiral supermultiplets, and non-analytic trilinear scalar couplings are not allowed in (1.68).

One usually makes the hypothesis that the soft supersymmetry-breaking parameters $m_i, \tilde{M}_i, M_a, A_\lambda$, and B originate at some high GUT or gravity scale, perhaps from some supergravity or superstring mechanism. The physical values of these soft supersymmetry-breaking parameters are then subject to logarithmic renormalizations that may be calculated and resummed using the renormalization-group techniques [28]. It is usually assumed that the soft supersymmetry-breaking parameters are universal at the GUT or supergravity scale: $m_i^2 \equiv M_i^2 \equiv m_0^2$, $M_a \equiv m_{1/2}$, $A_\lambda \equiv A$, although such a universality is not very well motivated, since in particular, general supergravity models give no theoretical hint why they should be universal. If one assumes universality, the parameters $\mu, \tan \beta, m_0, m_{1/2}, A$ suffice to characterize MSSM phenomenology.

Assuming universality, typical renormalization group calculations [29] imply that the scalar masses are generally renormalized to larger values as the scale is reduced, but this is not necessarily the case if there are large Yukawa interactions such as those of the top quark. Such effects of the top quark Yukawa couplings must certainly be taken into account. Also the effects of bottom quark and τ lepton Yukawa couplings may be important if $\tan\beta$ is large. The potential significance of these Yukawa interactions is that they tend to drive $m_h^2 = (m_0^2 + \mu^2)$ to smaller values at smaller renormalization scales μ [30] via the renormalization group evolution

$$\mu \frac{\partial m_h^2}{\partial(\ln \mu)} = \frac{1}{(4\pi)^2} (3h_t^2(m_h^2 + m_q^2 + m_t^2) + \dots), \quad (1.69)$$

where $m_{\bar{q}}$ is a squark mass.

It is then possible to generate electroweak symmetry breaking dynamically, even if $m_h^2 > 0$ at the input scale along with the other scalar mass-squared parameters [30]. The appropriate renormalization scale for discussing the effective Higgs potential of the MSSM is $Q \lesssim 1 \text{ TeV}$, and the electroweak gauge symmetry will be broken if either or both of $m_{h_{1,2}}^2(Q) < 0$, as in the SM Higgs potential (1.25). Here $m_{h_1}^2 = m_1^2 + \mu^2$ and $m_{h_2}^2 = m_2^2 + \mu^2$, respectively. This is certainly possible for $m_t \sim 175 \text{ GeV}$ as observed.

The superpotential (1.62) of the MSSM contains a bilinear term $\mu \hat{H}_1 \hat{H}_2$ coupling the two Higgs superfields, with μ having the dimensions of mass. We note that this term is allowed by supersymmetry. With the bilinear term in the superpotential one would also expect a bilinear supersymmetry-breaking term $B\mu H_1 H_2$ in the scalar potential (see Eq.(1.68)), where B is expected to be of the order of supersymmetry breaking scale ($\lesssim 1 \text{ TeV}$). With a term of this form the mass of the only CP-odd Higgs boson in MSSM is given by

$$m_A^2 = \frac{2B\mu}{\sin 2\beta}. \quad (1.70)$$

There would seem to be two natural options for the parameter μ : either $\mu = 0$ due to some symmetry, or it acquires an extremely large mass. For $\mu = 0$, there is a Peccei-Quinn symmetry [31] in the potential of the MSSM, leading to a massless axion ($m_A = 0$), which is ruled out by experiment. The second option of having a

very large μ effectively obliterates the weak scale: the Higgs doublets are removed to the high mass scale, or (worse) electroweak symmetry is broken at high scale. Thus, a non-zero $\mu = \mathcal{O}(m_W)$ is required in order to break $SU(2)_L \times U(1)_Y$ successfully without producing an unacceptable axion. Softly broken supersymmetry ensures that the radiative corrections to μ are now under control so that $\mu = \mathcal{O}(m_W) \ll m_{Pl}$ is technically natural, thus solving the easy part of the hierarchy problem. However, it does not provide any dynamical reason why μ should be so small in the first place. Thus, it is a serious problem, why μ is of the order of electroweak scale. It endangers the whole idea of low energy supersymmetry. The simplest mechanism that provides a dynamical source for a term of the form $\mu \hat{H}_1 \hat{H}_2$ is the inclusion of an additional [32] singlet Higgs field \hat{N} . Then, if the superpotential contains a trilinear term $W \ni \lambda \hat{H}_1 \hat{H}_2 \hat{N}$ and if the scalar component of \hat{N} develops a vacuum expectation value $\langle N \rangle \equiv x$, a bilinear $\mu \hat{H}_1 \hat{H}_2$ mixing term with $\mu \equiv \lambda x$ is generated. In the presence of soft supersymmetry breaking, one would expect $x \lesssim \mathcal{O}(1 \text{ TeV})$, and hence $\mu \lesssim \mathcal{O}(1 \text{ TeV}) \ll m_{Pl}$, thereby solving the μ problem. We shall study [33, 34] some aspects of such an extension of MSSM, called the non-minimal supersymmetric standard model (NMSSM), in Chapter 2. In particular, we shall study the renormalization group evolution and infra-red fixed points of the Yukawa couplings of NMSSM, and carry out a detailed study of the of the stability of these fixed points.

The minimal supersymmetric standard model contains two Higgs doublet superfields \hat{H}_1 and \hat{H}_2 with opposite hypercharges so as to generate masses for up- and down-quarks (and leptons), and to cancel triangle gauge anomalies. After spontaneous symmetry breaking induced by the neutral components of H_1 and H_2 obtaining vacuum expectation values, the MSSM contains two neutral CP-even (h, H), one neutral CP-odd (A), and two charged Higgs bosons [35]. Because of underlying gauge invariance and supersymmetry, the lightest CP-even Higgs boson in MSSM has tree level upper bound of m_Z on its mass. Although radiative corrections to this result are appreciable, these are under control because of underlying softly broken supersymmetry [36]. This results in an upper bound of $m_h \lesssim 125 \text{ GeV}$ on the radiatively corrected lightest Higgs boson mass in MSSM.

Although there are two distinct scales, the electroweak breaking scale and the

supersymmetry (SUSY) breaking scale, in the MSSM, the (tree level) upper bound on the mass of the lightest Higgs boson is independent of the SUSY breaking scale (radiative correction induce only a logarithmic dependence on the SUSY breaking scale). The existence of such an upper bound on the mass of the lightest Higgs boson in MSSM has been investigated in situations where the underlying supersymmetric model respects baryon (B) and lepton (L) number conservation. However, as noted earlier, gauge invariance, supersymmetry, and renormalizability allow B and L violating terms (1.66) in the superpotential of MSSM. In MSSM these terms are eliminated by invoking the discrete R-parity symmetry (1.67). However, the assumption of R_p conservation appears to be *ad hoc*, since it is not required for the internal consistency of the MSSM. It is, therefore, more appealing to have a supersymmetric theory where R-parity is related to a gauge symmetry, and its conservation is automatic because of the invariance of the underlying theory under this gauge symmetry. Indeed, R_p conservation follows automatically in certain theories with gauged $(B - L)$, as is suggested by the appearance of $(B - L)$ in (1.67). It has been noted by several authors [37, 38] that if the gauge symmetry of MSSM is extended to left-right symmetry, $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the theory becomes automatically R-parity conserving. Such a left-right symmetry solves the problem of explicit B and L violation of MSSM, and has received [39] much attention recently. Such a naturally R-parity conserving theory necessarily involves the extension of the Standard Model gauge group, and since the extended gauge symmetry has to be broken, it involves “a new scale”, the scale of left-right symmetry breaking beyond the SUSY and $SU(2)_L \times U(1)_Y$ breaking scales of MSSM. It is, therefore, important to ask whether the upper bound on the lightest Higgs boson mass in such R-parity conserving theories depends on the scale of breakdown of the extended gauge group. It has been shown [40] that in the supersymmetric left-right model with minimal particle content the upper bound on the mass of the lightest neutral Higgs boson depends only on the gauge couplings and those vacuum expectation values (VEVs) which break $SU(2)_L \times U(1)_Y$. The upper bound does not depend on any other scales (vacuum expectation value) that exist in such models. In Chapter 3 we shall study [41] higher order radiative corrections to the lightest Higgs boson mass in the minimal version of the supersymmetric left-right

model in order to arrive at a precise value for the upper bound on the lightest Higgs boson mass in these models. Since the one-loop logarithmic correction proportional to m_t^4 is dominant, it is sufficient to consider two-loop leading and next-to-leading log contributions proportional to $m_t^4\alpha_3$ and $m_t^4\alpha_t$, where $\alpha_3 \equiv g_3^2/4\pi$, $\alpha_t \equiv h_t^2/4\pi$, respectively. We shall also compare our result with the corresponding result on the lightest Higgs boson mass in MSSM. In particular, we shall show that the upper bound on the lightest Higgs mass in this class of models lies considerably above the corresponding upper bound in the MSSM.

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