

Comment on "Supersymmetric Phase Transition"

Recently Pandita and Singh¹ have proposed a prototype Lagrangian which is invariant under supersymmetric transformation and leads to a phase transition similar to the Bose-Einstein condensation.² I show here on broad thermodynamic grounds that the assumptions made by Pandita and Singh in their model would always give the Bose-Einstein condensation irrespective of the details of the supersymmetric interaction or the Lagrangian density functional. The supersymmetric phase transition would occur in systems whose space-time dimensionality exceeds two.^{2,3} Furthermore, in four and higher dimensions, the nature of the phase transition would remain unaffected even if some of the simplifying assumptions in the model were relaxed.

Consider a set of energy levels E_i , $i=1,2,\dots,N$, which may be simultaneously occupied by bosons and fermions. I assume that particles occupying different energy levels do not interact with each other, but that bosons and fermions in the same energy state may interact with each other. The thermodynamic internal energy may be written phenomenologically as

$$U = \sum_i (E_i - \mu) [\langle n_i \rangle_1 + \langle n_i \rangle_\infty + g \langle n_i^a \rangle \langle n_i^b \rangle_\infty],$$

where μ denotes a chemical-potential-like adjustable parameter,⁴ n_i is the number of particles occupying the energy level E_i , angular brackets denote thermal averages, and the subscripts 1 and ∞ denote thermal averages with respect to Fermi and Bose-Einstein distributions, respectively. The parameter g denotes the coupling strength between boson and fermion fields, and a and b are positive constants. Bosons may all condense into a single energy level; therefore b must be chosen equal to unity to keep the internal energy an extensive quantity scaling linearly with the total number of particles N . For any positive a ,

$$\langle n_i^a \rangle_1 = [\exp \beta (E_i - \mu) + 1]^{-1},$$

and

$$\langle n_i \rangle_\infty = [\exp \beta (E_i - \mu) - 1]^{-1},$$

where $\beta = (k_B T)^{-1}$. Therefore the product $\langle n_i^a \rangle_1 \times \langle n_i \rangle_\infty$, for any value of a , has a Bose-Einstein distribution corresponding to one-half of the temperature of the noninteracting system. These phenomenological considerations show that inter-

acting bosons and fermions in the same multiplet obey Bose-Einstein statistics. The parameter μ may be fixed by the general requirement that the internal energy should scale linearly with N . This would always result in a Bose-Einstein condensation. For the usual parametric dependence of E_i on i , the Bose-Einstein condensation occurs only when the dimensionality of the system (space-time dimensionality in the present case rather than the spatial dimensionality as mentioned in the first line on p. 1551 of Ref. 1) exceeds two.

In the above discussion I have not considered any interaction between different energy levels except for the effective interaction which arises from the constraint which fixes μ . Some reflection along the lines of the renormalization-group theory of critical phenomena shows that the upper critical dimensionality for Bose-Einstein condensation is four. Therefore in four and higher space-time dimensions the character of the phase transition would remain unchanged even when interactions between particles occupying different energy levels are considered.

In conclusion, the broad thermodynamic considerations given above support the point that a supersymmetric system at $T=0$ continues in the same unbroken-symmetry state up to a finite critical temperature above which the symmetry is spontaneously broken.

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¹P. N. Pandita and S. Singh, Phys. Rev. Lett. **50**, 1550 (1983).

²See L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, Oxford, 1980), Pt. 2. Bose-Einstein condensation is closely related to the phase transition in the spherical model of Berlin and Kac on which much greater literature exists. For an account of the properties of the spherical model, see G. S. Joyce, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic, New York, 1972), Vol. 2.

³See M. B. Paranjape, T. Taormina, and L. C. R. Wijewardhana, Phys. Rev. Lett. **50**, 1350 (1983).

⁴The use of a chemical potential in a microscopic theory is questionable as pointed out in Ref. 3 because of the noncommutativity of supercharges. However, the use of an adjustable parameter in the free energy does not suffer from this defect.