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Gravity-induced large grand-unification mass in SU(5) with higher-dimensional operators

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Following the recent method due to Hill, Shafi, and Wetterich, we investigate the impact of higher-dimensional operators ($d \geq 5$) induced by gravity and compactification of extra dimensions, on the minimal SU(5) grand unified theory. Modifications caused by the $d=5$ operator alone seem to be ruled out, even if the compactification scale M_C is as low as 10^{17} GeV, as they require $\sin^2\theta_W \leq 0.203$ in conflict with the present world average. The addition of a six-dimensional operator is found to allow only high unification masses $M_U \sim (0.1-1)M_C$, with $M_C = 10^{17}-10^{19}$ GeV and $\sin^2\theta_W \simeq 0.22-0.24$. The grand-unification coupling constant is also found to be significantly smaller.

I. INTRODUCTION

Gauge theories of the Kaluza-Klein[1] type offer the exciting possibility of unification with gravity through the introduction of higher dimensions leading to the four-dimensional structure of the present Universe as a result of the compactification of extra dimensions.[2,3] Attempts have been made to generate effective four-dimensional gauge theories, such as the standard model and grand unified theories (GUT's), from the isometry group of the compactified manifold, to identify the observed fermions as the chiral representations of the effective gauge theories, and to compute the gauge couplings in terms of the characteristic length scales of extra spatial dimensions.[3-5] Although superstring theory[6] is expected to provide a realistic gauge unification of all basic interactions, a lot of interest still remains in conventional GUT's,[7-9] with and without gravity-induced effects. Although most of the GUT's with intermediate symmetries can satisfy the experimentally observed constraints on the proton lifetime (τ_p) for the $p \to e^+\pi^0$ mode and $\sin^2\theta_W$ (Ref. 10),

$$\tau_p \geq 3\times 10^{32} \text{ yr}, \quad \sin^2\theta_W = 0.230\pm 0.005 , \tag{1}$$

the minimal SU(5) model[7] and certain other GUT's with a grand desert are ruled out as they predict significantly lower values.[11-13] If the masses of other superheavy gauge bosons in SO(10) are nondegenerate and differ from the $X^{\pm 4/3}$ and $Y^{\pm 1/3}$ gauge-boson mass, the model can be made consistent with (1), even with a grand desert.[12] In a nonminimal SU(5) model,[14] however, τ_p and $\sin^2\theta_W$ can satisfy (1) by including one-loop contributions of nondegenerate superheavy scalars from additional representations such as **10**, **15**, **45**, and **50**. Since the grand unification occurs at a high scale ($M_U > 10^{15}$ GeV), it is natural to suppose that there could be significant modification to the GUT predictions by gravity-induced corrections. It is the purpose of this paper to compute such modifications to the minimal GUT predictions.

Recently, the impact of five-dimensional operators, scaled by the compactification scale (M_C), has been investigated to calculate the modifications caused by the minimal and other GUT predictions. Similar nonrenormalizable operators induced by gravity with dimensions $d \geq 5$ and scaled by powers $(M_C)^{-(d-4)}$ are subject only to the symmetries of low-energy theory and are known to occur, for example,[15] in the presence of gravitational instantons for $M_C \sim M_{\text{Pl}}$. The five-dimensional operators are seen to arise naturally as a result of compactification of extra dimensions in a Kaluza-Klein-type theory.[2] The impact of such an operator on the quark-to-lepton mass ratio m_d/m_e predicted by the minimal SU(5) model was examined by Ellis and Gaillard.[16]

In the case of a supersymmetric SU(5) GUT, significant modifications to τ_p and $\sin^2\theta_W$ have been noted[17] with $M_C \sim M_{\text{Pl}} = 10^{19}$ GeV. In the SO(10) GUT, with $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ as intermediate symmetry, the masses of the $W_R^{\pm}$ gauge bosons have been brought down to the order of the W scale leading to the possible observation of low-mass parity restoration in the future.[18] In the minimal SU(5) GUT, if the compactification scale is allowed to be about two orders lower than M_{Pl}, which is possible within certain Kaluza-Klein-type theories,[19] Shafi and Wetterich[20] have observed a very significant increase of τ_p so as to be compatible with the experimental limit for the $p \to e^+\pi^0$ mode.

In this paper we use the method due to Shafi and Wetterich,[20] and Hill,[17] to compute modifications of the minimal SU(5) predictions in the presence of $d \geq 5$ operators, especially in view of the recent measurements on $\sin^2\theta_W$. We note that the modifications of the $d=5$ operator alone, taken to be making the minimal GUT compatible with the experimental data on τ_p, are now ruled out as these solutions require $\sin^2\theta_W$ significantly below the accepted world average, even if $M_C = 10^{17}$ GeV. As our main result, we then examine the modifications caused by adding a $d=6$ operator in the Lagrangian. We find that the only permissible values of the unification mass should be of the order $(0.1-1)\,M_C$, where M_C could be anywhere between 10^{17} and 10^{19} GeV. Interestingly enough, the allowed values of the electroweak mixing angle can be made consistent with the currently available world average with $\sin^2\theta_W \simeq 0.22-0.24$ for every value of the unification mass. Another interesting aspect of the present analysis is that the bare-grand-unification coupling α_G turns out to be nearly 2 orders of magnitude smaller than the earlier results. We also obtain perturbative and positivity bounds on certain parameters and mention a new relation among them.

In Sec. II we obtain general formulas for the unification mass and the electroweak mixing angle including five-dimensional operators and particular forms of still higher-dimensional operators in the Lagrangian. In Sec. III we discuss earlier results with five-dimensional operators. In Sec. IV we report numerical analysis including five- and six-dimensional operators. Our conclusions are stated in Sec. V.

II. FORMULAS FOR GAUGE COUPLINGS, UNIFICATION MASS, AND $\sin^2\theta_W$

As has been emphasized earlier, gravitational effects could induce nonrenormalizable operators of dimension $d \geq 5$, scaled by powers of $M_C^{-(d-4)}$, into the normalizable Lagrangian, but only the impact of $d=5$ operators have been examined so far on the unification mass M_U and $\sin^2\theta_W$. Such operators are restricted only by the symmetries of the theory at lower energies. Denoting ϕ as the scalar in the adjoint representation $\mathbf{24} \subset \mathrm{SU}(5)$, the effect of the nonrenormalizable operators can be included in the following manner in the SU(5)-invariant Lagrangian:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\mathrm{NR}} , \tag{2a}$$

$$\mathcal{L}_{\mathrm{NR}} = \sum_{n=1,2,\ldots} \mathcal{L}_{\mathrm{NR}}^{(n)} , \tag{2b}$$

$$\mathcal{L}_{\mathrm{NR}}^{(n)} = -\frac{1}{2}\frac{\eta^{(n)}}{M_C^n}\mathrm{Tr}(F_{\mu\nu}\phi^n F^{\mu\nu}) , \tag{2c}$$

$$\mathcal{L}_0 = -\tfrac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) , \tag{2d}$$

$$F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] , \tag{2e}$$

$$(A_\mu)^a{}_b = A_\mu^i \left[\frac{\lambda_i}{2}\right]^a{}_b , \tag{2f}$$

$$\mathrm{Tr}(\lambda_i\lambda_j) = \tfrac{1}{2}\delta_{ij} . \tag{2g}$$

In Eq. (2), A^i is the ith component of the gauge field, λ_i is the corresponding generator, and $\eta^{(n)}$, $n=1,2,\ldots$, are the unknown parameters. In Refs. 17 and 20, the case with the five-dimensional operator corresponds to $\eta^{(1)} \neq 0$ and $\eta^{(n)}=0$ for $n \geq 2$. It may be noted that the expression (2c) for higher-dimensional operators given in Eq. (2c) is not the most general one, especially when $n \geq 2$. For example, with $n=2$, other gauge-invariant operators not covered by (2c) are $\mathrm{Tr}(\phi^2)\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ and $\mathrm{Tr}(F_{\mu\nu}\phi)\mathrm{Tr}(F^{\mu\nu}\phi)$ the latter being more troublesome for computations of the physical quantities of interest in this paper. We confine to the choice (2c) for the sake of convenience and obtaining modifications to M_U and $\sin^2\theta_W$ with a constraint on the parameters as shown in Eq. (14a) in Sec. IV. Using the vacuum expectation value of **24** as

$$\langle\phi\rangle = (\tfrac{1}{15})^{1/2}\phi_0 \mathrm{diag}(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2}) , \tag{3}$$

denoting g_3, g_2, and g_1 as the $\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$, and $\mathrm{U}(1)_Y$ coupling constants, respectively, and the gravity-induced changes as

$$\begin{aligned} g_3^2(M_U) &\to g_3^2(M_U)(1+\epsilon_C) , \\ g_2^2(M_u) &\to g_2^2(M_U)(1+\epsilon_L) , \\ g_1^2(M_U) &\to g_1^2(M_U)(1+\epsilon_Y) , \end{aligned} \tag{4}$$

we obtain, using Eqs. (2)–(4),

$$\begin{aligned} \epsilon_C &= \sum_{n=1,2,\ldots} \epsilon^{(n)} , \\ \epsilon_L &= -\tfrac{3}{2}\epsilon^{(1)} + \tfrac{9}{4}\epsilon^{(2)} - \tfrac{27}{8}\epsilon^{(3)} + \cdots , \\ \epsilon_Y &= -\tfrac{1}{2}\epsilon^{(1)} + \tfrac{7}{4}\epsilon^{(2)} - \tfrac{13}{8}\epsilon^{(3)} + \cdots , \end{aligned} \tag{5}$$

where the ellipsis in (5) includes the effect of operators $d>7$ and

$$\epsilon^{(n)} = \left[\frac{1}{\sqrt{15}}\frac{\phi_0}{M_C}\right]^n \eta^{(n)}, \quad n=1,2,\ldots . \tag{6}$$

Using $\alpha_G = g_0^2/4\pi$, where g_0 is the bare GUT coupling, and the relation

$$\phi_0 = [6/(5\pi\alpha_G)]^{1/2} M_U , \tag{7}$$

Eq. (6) can be rewritten as

$$\eta^{(n)} = \left[\left[\frac{25\pi\alpha_G}{2}\right]^{1/2}\frac{M_C}{M_U}\right]^n \epsilon^{(n)} . \tag{8}$$

Imposing the condition of equality of the three coupling constants at M_U,

$$g_3^2(1+\epsilon_C) = g_2^2(1+\epsilon_L) = g_1^2(1+\epsilon_Y) = g_0^2 , \tag{9}$$

the one-loop renormalization-group equations[21]

$$\frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_i(M_U)} + \frac{a_i}{2\pi}\ln\frac{M_U}{M_W}, \quad i=1,2,3 \tag{10}$$

are solved with $a_1 = \frac{41}{10}$, $a_2 = -\frac{19}{6}$, $a_3 = -7$ to yield

$$\ln\frac{M_U}{M_W}=\frac{1}{D}\left[1+\left[\epsilon_C-\frac{5\epsilon_Y+3\epsilon_L}{3}\frac{\alpha}{\alpha_s}\right]\left[1-\frac{8}{3}\frac{\alpha}{\alpha_s}\right]^{-1}\right]\times\ln\frac{M^{(5)}}{M_W}\ , \tag{11a}$$

$$\sin^2\theta_W=\left[\sin^2\theta_W^{(5)}-\tfrac{19}{134}\epsilon_C+\tfrac{1}{67}\left[21+\frac{41}{2}\frac{\alpha}{\alpha_s}\right]\epsilon_L+\frac{95}{402}\frac{\alpha}{\alpha_s}\epsilon_Y\right]\Big/D\ , \tag{11b}$$

$$\frac{1}{\alpha_G}\equiv\frac{4\pi}{g_0^2}=\frac{3}{67}\left[\frac{11}{3\alpha_s}+\frac{7}{\alpha}\right]\Big/D\ , \tag{11c}$$

$$D=1+\tfrac{1}{67}(11\epsilon_C+21\epsilon_L+35\epsilon_Y)\ , \tag{11d}$$

where $M^{(5)}$ and $\sin^2\theta^{(5)}$ denote the one-loop predictions of the minimal model, without gravity-induced effects, including only one set of **24+5** of Higgs fields and three generations of fermions:

$$\ln\frac{M^{(5)}}{M_W}=\frac{6}{67\alpha}\left[1-\frac{8}{3}\frac{\alpha}{\alpha_s}\right]\ ,$$

$$\sin^2\theta_W^{(5)}=\frac{23}{134}+\frac{109}{201}\frac{\alpha}{\alpha_s}\ . \tag{12}$$

In Eqs. (11) and (12), $\alpha^{-1}(M_W)=127.54$ and $\alpha_s=g_3^2(M_W)/4\pi=0.1088$ corresponding to $\Lambda_{\overline{\rm MS}}=160$ MeV, where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme. Solutions in the similar forms including only the five-dimensional operators have been obtained in Refs. 17 and 20. Here we note that the effects of all higher-dimensional operators are contained in the parameters ϵ_C, ϵ_L, and ϵ_Y as illustrated in Eq. (5). Thus, Eq. (11) through Eq. (5), can, in principle, account for all gravity-induced corrections due to higher-dimensional operators.

III. SOLUTIONS WITH FIVE-DIMENSIONAL OPERATORS

In this section we briefly review the earlier solutions obtained with $d=5$ operator noting that they are ruled out because of experimental constraints on τ_p and $\sin^2\theta_W$. Such a conclusion was already reached by Hill[17] with $M_C\sim M_{\rm Pl}=10^{19}$ GeV. We, therefore, discuss Shafi-Wetterich[20] solutions with $M_C\sim10^{17}$ GeV where SU(5) has been stated to survive the then existing experimental data. Using $\epsilon^{(2)}=\epsilon^{(3)}=\cdots=0$ and $\epsilon^{(1)}=\epsilon=\eta\phi^0/(\sqrt{15}M_C)$ in Eqs. (5) and (11) yields $\epsilon_C=\epsilon$, $\epsilon_L=-3\epsilon/2$, and $\epsilon_Y=-\epsilon/2$ and $D=1-38\epsilon/67$, as in Ref. 20, leading to

$$\frac{1}{\alpha_G}=\frac{11\alpha_s^{-1}+21\alpha^{-1}}{67-38\epsilon}\ , \tag{13a}$$

$$\ln\frac{M_U}{M_W}=\frac{6\pi}{67-38\epsilon}[\alpha^{-1}-\tfrac{8}{3}\alpha_s^{-1}+(\tfrac{7}{3}\alpha_s^{-1}+\alpha^{-1})\epsilon]\ , \tag{13b}$$

$$\sin^2\theta_W=\frac{1}{67-38\epsilon}\left[\frac{23}{2}+\frac{109}{3}\frac{\alpha}{\alpha_s}-\left[41+\frac{116}{3}\frac{\alpha}{\alpha_s}\right]\epsilon\right]\ . \tag{13c}$$

For different assumed values of the parameter ϵ, the gauge coupling constant α_G, unification mass M_U, and $\sin^2\theta_W$ are computed as has been done before. The basic parameter η occurring in the Lagrangian is calculated using the relation

$$\eta=\left[\frac{25\pi}{2}\frac{67-38\epsilon}{11\alpha_S^{-1}+21\alpha^{-1}}\right]^{1/2}\frac{M_C}{M_U}\epsilon\ . \tag{13d}$$

It is worth mentioning the new additional fact that the ϵ parameter can be bounded from above and below, in this case, using the positivity and perturbative constraints on α_G. From Eq. (13a), the positivity of α_G suggests that $\epsilon<\frac{67}{38}=1.76$, whereas the perturbative constraint ($\alpha_G<1$) yields, with $\alpha_s=0.1088$ and $\alpha^{-1}=127.54$, $\epsilon>-70$, i.e.,

$$-70<\epsilon<1.76\ .$$

The lower bound is dominated by α^{-1} and does not vary significantly in the allowed range of $\alpha_s(M_W)$ corresponding to $\Lambda_{\overline{\rm MS}}=0.160\pm0.100$ GeV.

Numerical solutions for the unification mass, τ_p, $\sin^2\theta_W,\alpha_G^{-1}$, and η for different values of the ϵ parameter are presented in Table I. For calculating η, the value of the compactification scale has been taken to be $M_C=10^{17}$ GeV as before.[20] The uncertainty by a factor $10^{\pm2}$ in τ_p arises due to the uncertainty in $\Lambda_{\overline{\rm MS}}$ and the matrix elements for $p\to e^+\pi^0$. In order that τ_p agrees with the experimental limit, $\tau_p\geq3\times10^{32}$ yr, it is clear that $\epsilon>0.015$ which needs $\sin^2\theta_W<0.203$, even though M_C is allowed to be as low as 10^{17} GeV. Such lower values of $\sin^2\theta_W$ in the modified solutions, needed for the stability of the proton, are clearly in contradiction with the recent world average $\sin^2\theta_W=0.23\pm0.005$.

TABLE I. Modifications of the minimal GUT predictions with $d=5$ operator. The parameter η has been calculated with $M_C=10^{17}$ GeV.

ϵ	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_p (yr)	η
0.005	4.32×10^{14}	0.208	41.60	$3.58\times10^{30\pm2}$	1.10
0.010	5.80×10^{10}	0.205	41.72	$1.16\times10^{31\pm2}$	1.66
0.015	7.78×10^{14}	0.203	41.85	$3.77\times10^{31\pm2}$	1.86
0.020	1.05×10^{15}	0.199	41.95	$1.25\times10^{32\pm2}$	1.84

IV. NEW SOLUTIONS WITH FIVE- AND SIX-DIMENSIONAL OPERATORS

As we mentioned in Sec. III, modifications with $d=5$ operator in the minimal GUT seem to be ruled out as they require $\sin^2\theta_W<0.203$, in order to yield $\tau_p \geq 3\times 10^{32}$ yr. But, following the similar philosophy as in Refs. 17 and 20, we investigate whether inclusion of still higher-dimensional operators could predict τ_p and $\sin^2\theta_W$ consistent with the available experimental data. As the next economic choice we include both $d=5$ and $d=6$ operators in $\mathcal{L}_{NR}$ and find that the most natural and plausible solutions which correspond to logical values of the parameters in the Lagrangian, yield $M_U\sim(0.1-1)M_C \sim 10^{16}-10^{19}$ GeV, and $\sin^2\theta_W=0.22-0.24$, for each value of M_U. In this case the relations between ϵ parameters are

$$\epsilon_Y=\tfrac{2}{5}\epsilon_C+\tfrac{3}{5}\epsilon_L \ , \quad (14a)$$

$$\epsilon^{(1)}=\tfrac{9}{15}\epsilon_C-\tfrac{4}{15}\epsilon_L \ , \quad (14b)$$

$$\epsilon^{(2)}=\tfrac{2}{5}\epsilon_C+\tfrac{4}{15}\epsilon_L \ . \quad (14c)$$

Note that the relation (14a) is also valid in the $d=5$ case.

The basic parameters of the Lagrangian are the η parameters, rather than the ϵ parameters. Except the positivity and perturbative constraints on ϵ, as already discussed in this paper, there seems to be no theoretical constraint on the η parameters. But, in order that the modified Lagrangian makes some sense, the following general criteria on the parameters need to be satisfied, and we treat the solutions as acceptable when either criteria (i) and (ii) or (i) and (iii) are satisfied: (i) The magnitude of $\eta^{(n)}$, $n=1,2,\ldots$ is not very large; (ii) although the individual values of the η parameters may differ, one possibility is that they are of the same order. (iii) If the gravity-induced corrections might be acting as the terms in a perturbation series, for reasons hitherto unknown to us, the other possibility might be that $|\eta_2|<|\eta_1|$.

Now we discuss briefly how far criterion (i) has been satisfied by earlier solutions with $d=5$ operator where there is only one parameter, $\eta(1)\equiv\eta$. Shafi and Wetterich[20] have obtained modifications with $\eta\sim 1$. Although Hill[17] has investigated over the range of parameters $\eta=-20-20$, i.e., with maximum allowed $\eta\sim 10$, no plausible solutions have been obtained for the nonsupersymmetric minimal GUT; but, in the supersymmetric SU(5), significant and acceptable modifications have been observed for $\eta=-2-2$. In the SO(10) grand unification, with Pati-Salam gauge group as the intermediate symmetry, solutions with $\eta\sim 1$ have been found[21] to yield τ_p and $\sin^2\theta_W$ consistent with Eq. (1); while, with $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}(g_{2L}=g_{2R})$ as the proposed and possible low-energy symmetry ($M_{W_R}\sim M_{W_L}$), acceptable solutions have been obtained with $\eta\simeq -1$, with $\sin^2\theta_W\simeq 0.22$ and $\tau_p \gtrsim 10^{40}$ yr. It is worth mentioning how the first criterion controls the value of the compactification scale in relation to the unification mass. With only $d=5$ operator, Shafi and Wetterich have found that $M_C=10^{17}$ GeV is necessary in order to obtain $\eta\sim 1$. On the other hand if $M_C\sim 10^{18}-10^{19}$ GeV, $\eta\sim 10-100$, for $M_U\sim 10^{15}$ GeV with the same values of ϵ. Thus, largeness of η, besides the smallness of $\sin^2\theta_W$, prevents having $M_C\sim M_{\rm Pl}$, with only the $d=5$ operator for the minimal GUT. In the present case, however, we will find that criteria (i)–(iii) can clearly rule out solutions with $M_U\sim 10^{15}$ GeV, but allow only those with high unification masses which depend on the compactification scale.

Using Eqs. (11a)–(11d) and (14a) we first compute values of ϵ_C and ϵ_L such that $M_U \gtrsim 10^{15}$ GeV and $\sin^2\theta_W\simeq 0.22-0.24$, and the corresponding value of the GUT coupling, α_G. Using Eqs. (14b) and (14c) we then compute the numerical values of $\epsilon^{(1)}$ and $\epsilon^{(2)}$. Some of these solutions are presented in Table II for different values of the unification mass. In the second step, to test whether all such type of solutions are acceptable, we compute the values of the basic parameters $\eta^{(1)}$ and $\eta^{(2)}$, using Eq. (8), with the knowledge of $\epsilon^{(1)}$, $\epsilon^{(2)}$, α_G, and M_U from Table II, and several reasonable values of M_C existing in the literature. Such computations are shown in Tables III and IV. On the basis of criteria (i)–(iii) we find that all the numerical solutions for M_U and $\sin^2\theta_W$, can be classified into the following categories.

(A) $M_U \ll M_C$. Here the inequality is used to mean values of M_U less than M_C by 2 or more orders. Since $\epsilon^{(1)}$ and $\epsilon^{(2)}$ are of the same order, it is clear from Eq. (8) that if $M_U\ll M_C$, $|\eta^{(2)}|\gg|\eta^{(1)}|$. For example, with $M_U=10^{15}$ GeV, we obtain $(\eta^{(1)},\eta^{(2)})=(-4.05,\ -10^2)$, $(-40.5,\ -10^4)$, and $(-405.6,\ -10^6)$, for $M_C=10^{17}$, 10^{18}, and 10^{19} GeV, respectively. Further, the combination $(M_U,M_C)=(10^{16},\ 10^{18})$ GeV, $(10^{17},\ 10^{19})$ GeV, and $(10^{16},\ 10^{19})$ GeV, correspond to the parameter values $(\eta_1,\eta_2)=(-3.07,\ -57.56)$, $(-2.50,\ -38)$, and $(-30.7,$

TABLE II. Parameters ϵ_C, ϵ_L, ϵ_Y, $\epsilon^{(1)}$, and $\epsilon^{(2)}$ computed using one-loop renormalization-group equations and corrections due to $d=5$ and 6 operators. Relations among ϵ parameters are given in Eq. (14).

ϵ_L	ϵ_C	ϵ_Y	$\epsilon^{(1)}$	$\epsilon^{(2)}$	M_U (GeV)	$\sin^2\theta_W$	α_G
−0.9841	−0.9833	−0.9838	−0.3276	−0.6558	10^{15}	0.2305	3.905×10^{-4}
−0.9913	−0.9899	−0.9907	−0.3296	−0.6603	10^{16}	0.232	2.22×10^{-4}
−0.9945	−0.9930	−0.9939	−0.3306	−0.6624	10^{17}	0.239	1.461×10^{-4}
−0.9957	−0.9940	−0.9950	−0.3309	−0.6631	10^{18}	0.238	1.188×10^{-4}
−0.9964	−0.9945	−0.9956	−0.331	−0.6635	10^{19}	0.232	1.039×10^{-4}

TABLE III. Values of the parameters $\eta^{(1)}$ and $\eta^{(2)}$ and the ratio $|\eta^{(2)}/\eta^{(1)}|$ for the class (A) solutions.

M_U (GeV)	M_C (GeV)	$\sin^2\theta_W$	$\eta^{(1)}$	$\eta^{(2)}$	$\lvert\eta^{(2)}/\eta^{(1)}\rvert$
10^{15}	10^{17}		-4.056	-1.005×10^{2}	24.778
	10^{18}	0.2305	-40.568	-1.005×10^{4}	2.477×10^{2}
	10^{19}		-405.68	-1.005×10^{6}	2.477×10^{3}
10^{16}	10^{18}	0.232	-3.077	-57.564	18.707
	10^{19}		-30.774	-5.756×10^{3}	1.870×10^{2}
10^{17}	10^{19}	0.239	-2.504	-38.00	15.1757

-5.7×10^3), respectively. Thus, if M_U is lower than M_C by more than 1 order, besides the magnitudes of the parameters being large, their ratio $|\eta^{(2)}/\eta^{(1)}|$ becomes far too large for the Lagrangian to make any sense. The values ($\eta^{(1)}$, $\eta^{(2)}$) and the ratio $|\eta^{(2)}/\eta^{(1)}|$ are further magnified if $M_C\sim M_{\rm Pl}$ and $M_U\sim10^{15}-10^{16}$ GeV. The reason for such large values of the ratio is due to the fact that $|\eta^{(2)}/\eta^{(1)}|\propto M_C/M_U$. Such large values of the ratio clearly violate criteria (ii) and (iii) when M_U is several orders less than M_C. Because of these highly undesirable values of the parameters and their ratio, we conclude that if the addition of a six-dimensional operator is going to make any sense, in the presence of a five-dimensional operator, the solutions with M_U several orders smaller than M_C are not acceptable. For the most general expectation of the compactification scale,[17] $M_C\sim M_{\rm Pl}$, the lower unification masses, $M_U\sim10^{15}-10^{17}$ GeV, are clearly ruled out.

(B) $M_U\sim(0.1-1)M_C$. In contrast with the class (A) solutions, it is evident from Table III that there are other solutions which satisfy either $|\eta_2/\eta_1|\sim1$, or $|\eta_2/\eta_1|\sim0.1$. When $M_U\sim0.1M_C$, criteria (i) and (ii) seem to be satisfied. For example, $M_U\sim0.1M_C\sim10^{16}$, 10^{17}, and 10^{18} GeV correspond to the parameter; values $(\eta_1,\eta_2)\simeq(-0.307,\ -0.575)$, $(-0.25\ \ -0.38)$, and $(-0.226,\ -0.309)$ and the ratio $|\eta_2/\eta_1|\simeq1.87$, 1.51, and 1.36, respectively. Thus, combining criteria (i) and (ii) yields allowed values of high unification masses $M_U\sim0.1M_C$. But, as mentioned earlier, criterion (iii), an alternative to (ii), could also be possible, if the nonrenormalizable terms act like perturbation on the normalizable Lagrangian. The gravity-induced effects on SU(5) do permit such solutions satisfying criteria (i) and (iii). For example, high unification masses $M_U=M_C=10^{17}$, 10^{18}, and 10^{19} GeV correspond to $(\eta^{(1)},\eta^{(2)})\simeq(-0.025,\ -3.8\times10^{-3})$, $(-0.022,\ -3.09\times10^{-3})$, and $(-0.0211,\ -2.707\times10^{-3})$, and the ratio $|\eta^{(2)}/\eta^{(1)}|\simeq0.152$, 0.132, and 0.128, respectively. For every allowed value of $M_U\sim0.1M_C$, satisfying criteria (i) and (ii), or $M_U\sim M_C$, satisfying (i) and (iii), the value of $\sin^2\theta_W$ is found to be in the range 0.22–0.24. Some allowed solutions belonging to class (B) and satisfying criteria (i) and (ii) or (i) and (iii) are presented in Table IV. With the possible values of the compactification scale, $M_C=10^{17}-10^{19}$ GeV, the high values of the unification mass are found to cover rather a wider range, $M_U\sim(0.1-1)M_C\simeq10^{16}-10^{19}$ GeV. Another interesting new feature of the present solutions is the smallness of the bare GUT coupling constant, $\alpha_G\sim10^{-4}$, as compared to all earlier results existing in the literature. Such a small numerical value of α_G is understood by noting that

$$\alpha_G=\frac{67+25\epsilon_C+42\epsilon_L}{3\left[\dfrac{11}{3\alpha_s}+\dfrac{7}{\alpha}\right]}\,, \tag{15}$$

where the numerator tends to be small as $\epsilon_C\simeq\epsilon_L\to-1$. The small value of α_G decreases the proton decay rate resulting in a very significant increase in τ_p. Thus, according to the present observations, the enhancement in τ_p occurs due to two sources: largeness of M_U and smallness of α_G. Introducing a factor of $10^{\pm2}$ uncertainty, the minimum value of τ_p corresponding to the lowest allowed $M_U\sim10^{16}$ GeV turns out to be $\tau_p\sim10^{38}$ yr, where a factor of 10^4 enhancement due to smallness of α_G has been included. If, on the other hand, we confine to the most general expectation, $M_C\sim M_{\rm Pl}=10^{19}$ GeV, the GUT does not seem to have unification significantly below $M_U\sim10^{18}$ GeV.

Before closing this section it might be necessary to

TABLE IV. Same as Table III, but for class (B) solutions satisfying criteria (i) and (ii) or (i) and (iii) as described in the text.

M_C (GeV)	M_U (GeV)	$\sin^2\theta_W$	α_G	$\eta^{(1)}$	$\eta^{(2)}$	$\lvert\eta^{(2)}/\eta^{(1)}\rvert$
10^{17}	10^{16}	0.232	2.22×10^{-4}	-0.3077	-0.5756	1.870
	10^{17}	0.239	1.461×10^{-4}	-0.0250	-0.0038	0.1520
10^{18}	10^{17}	0.239	1.461×10^{-4}	-0.2504	-0.3800	1.5175
	10^{18}	0.238	1.188×10^{-4}	-0.0226	-0.003	0.1327
10^{19}	10^{18}	0.238	1.188×10^{-4}	-0.226	-0.3094	1.369
	10^{19}	0.232	1.039×10^{-4}	-0.0211	-0.0027	0.1279

clarify certain points regarding the self-consistency of the treatment of compactification effects through an expansion in higher-dimensional operators. As evident from Table II, $\epsilon^{(2)}/\epsilon^{(1)}\sim 2$, where $\epsilon^{(n)}$ is related to $\eta^{(n)}$ by Eq. (8). This might give the impression that the expansions for ϵ_C, ϵ_L, and ϵ_Y expressed in Eq. (5) are not converging. But we have taken only the first two out of a large number of terms in the series in Eq. (5) to show that they fully account for the available data on $\sin^2\theta_W$, and large values of M_U. This implies that, so far as the available values of $\sin^2\theta_W$ and allowed values of τ_p are concerned, $\epsilon^{(n)}\sim 0$ for $n>2$, thus guaranteeing convergence of the series and the self-consistency of the method for $M_U\sim(0.1-1)M_C$. Another way of looking into the convergence of expansions is the following. Since the first two terms are capable of explaining the available experimental values of $\sin^2\theta_W$, for M_U in the range $10^{16}-10^{19}$ GeV, it is certainly true that at least the same values of $\sin^2\theta_W$ and M_U are possible by taking larger number of terms such that $|\epsilon^{(n+1)}|\ll|\epsilon^{(n)}|$, for $n\gg 2$; this guarantees convergence of the series and self-consistency of the method adopted.

V. SUMMARY, CONCLUSION, AND DISCUSSION

The minimal SU(5) model predicts a proton lifetime about 2–3 orders less than the observed experimental lower limit and $\sin^2\theta_W\simeq 0.21$. Including gravity-induced effects through $d=5$ operators, scaled by the compactification mass, although Hill[17] obtained quite lower values of $\sin^2\theta_W$, and, hence, ruled out any modification with $M_C\sim M_{\rm Pl}$, Shafi and Wetterich[20] found that the minimal GUT could be saved by such compactification effects if $M_C=10^{17}$ GeV. But, as we have noted here, these solutions can be consistent with experiments provided $\sin^2\theta_W<0.203$ which seem to disagree with the present world average: $\sin^2\theta_W=0.230\pm 0.005$.

To investigate further, whether the SU(5) predictions can be improved by spontaneous compactification effects, we have investigated the combined role of both five- and six-dimensional operators, which is allowed, in principle, at least, and in the same spirit. Out of, at least, three different possible forms for the six-dimensional operator, we have chosen only one for the sake of simplicity and convenience, and also for obtaining modifications to GUT predictions within the constraint expressed by Eq. (14a). Although our computation in Table II indicates $\sin^2\theta_W\simeq 0.23-0.24$, we have checked that with slight change of ϵ_C and ϵ_L the allowed range is $\sin^2\theta_W\simeq 0.22-0.24$. It seems, at first sight, as if all numerical solutions with $M_U=10^{15}-10^{19}$ GeV and $\sin^2\theta_W=0.22-0.24$, as shown in Table II, are allowed. But when we compute the basic parameters $\eta^{(1)}$ and $\eta^{(2)}$ and their ratio $|\eta^{(2)}/\eta^{(1)}|$, where $\eta^{(1)}(\eta^{(2)})$ occurs as the coefficient of the five- (six-) dimensional operators, we find that $|\eta^{(2)}|\gg|\eta^{(1)}|$ for those solutions for which $M_U/M_C\lesssim 10^{-2}$, with $M_C=10^{17}-10^{19}$ GeV. As the Lagrangian does not make sense with such parameters, the corresponding solutions with low unification masses $M_U\simeq 10^{15}$ GeV are ruled out. This criterion, ruling out $M_U\simeq 10^{15}$ GeV is found to be strongly valid if the compactification occurs at the most generally acceptable scale, $M_C=M_{\rm Pl}$.

The present analysis reveals that the gravity-induced corrections with $d=5$ and 6 operators permit high unification mass, $M_U\sim(0.1-1)M_C\sim 10^{16}-10^{19}$ GeV, if $M_C=10^{17}-10^{19}$ GeV, with $\sin^2\theta_W\simeq 0.22-0.24$ for every M_U. The values of the η parameters are never found to be large for such solutions belonging to class (*B*), and the ratio $|\eta_2/\eta_1|\sim 1$ for those solutions with $M_U\sim 0.1M_C$, but $|\eta_2/\eta_1|\sim 0.1$ for others with $M_U\sim M_C$. Although such parameters with $|\eta_2/\eta_1|\sim 1$ are generally expected in unified gauge theories, the other values with $|\eta_2/\eta_1|\sim 0.1$ suggest that the successive terms containing higher-dimensional operators might be acting as perturbation upon the renormalizable Lagrangian. Using the most general value, $M_C\simeq M_{\rm Pl}$, we find that solutions with $M_U\simeq 10^{15}-10^{17}$ GeV are ruled out and the gravity-induced effects permit only $M_U\sim 10^{18}-10^{19}$ GeV. For the first time we find here a grand unified theory with the GUT coupling constant as small as $\alpha_G\sim 10^{-4}$. The enhancement of the proton lifetime occurs due to two factors: largeness of M_U and smallness of α_G. Thus, if the addition of five- and six-dimensional operators to the GUT Lagrangian is going to make sense, the predictions of minimal SU(5) with an unstable proton and $\sin^2\theta_W<0.215$ are modified to be consistent with an extremely stable proton ($\tau_p>10^{38}$ yr) and $\sin^2\theta_W\simeq 0.22-0.24$.

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