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Some anomalies in the treatment of ideal extremely relativistic Bose gas condensation

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We comment on some statements and results in the paper by Beckman, Karsch, and Miller. We also point out certain anomalies in treating the zero-mass case with a constraint.

In a recent paper,[1] Beckmann, Karsch, and Miller have analyzed the Bose-Einstein condensation of an ideal relativistic Bose gas in d dimensions. They solve the model in the usual way in the grand canonical ensemble by introducing a chemical potential μ corresponding to the constraint of the conserved number of bosons. By numerical evaluation of the equations for the density and the specific-heat anomaly, they are able to show the qualitative difference between the massive and massless relativistic systems. However, the results obtained in the paper are contradictory to the known results for photons (in the appropriate limit). Also, their treatment of the extreme relativistic and the massless cases is anomalous.

They mention the energy spectrum

$$\epsilon(p)=\sum_{i=1}^{d} c_i p_i^{\sigma} \ , \tag{1}$$

where c_i are constants [see their Eq. (1.1)]. In the paragraph following this equation, they claim that "another physically interesting case is that of a gas of massless particles where we have $\sigma=1$ together with all the coefficients equal to c, the velocity of light...." Later, in the same paragraph, they say that for $d \leq \sigma$ there exists no Bose-Einstein condensation (BEC), while $d > \sigma$ guarantees that it takes place. Firstly, Eq. (1) for $\sigma=1$ does not reduce to the spectrum

$$\epsilon(p)=c(p_1^2+p_2^2+\cdots+p_d^2)^{1/2} \ , \tag{2}$$

which is the true spectrum of massless particles. Instead, it reduces to

$$\epsilon(p)=c(p_1+p_2+\cdots+p_d) \ , \tag{3}$$

which is easily seen to represent a spectrum of d noninteracting massless particles all moving in one dimension! Fortunately, later in their calculation, they use the correct spectrum (2) for the massless case, so none of their calculations are affected by this statement. Secondly, their statement about the existence of BEC clearly does not apply to photons. For these particles, $d=3>1=\sigma$; their Table I will give a finite critical temperature T_c and Table II will give a jump in specific heat. None of these things happen for the usual photon gas.[2] All the thermodynamic properties of photons are analytic for all temperatures $0 \leq T < \infty$.

The reason why their result for the zero-mass bosons does not reduce to the photon result is as follows. Photons have zero mass, thus move with the speed of light. In QED, it is known[3] that these particles cannot support any internal quantum numbers that are conserved. Even their total number is not conserved. Therefore one should not introduce any constraint in studying the statistical mechanics of such particles. Hence, any calculation which artificially introduces a constraint on massless particles in analogy with the massive ones will not give photon results.

Finally, we comment briefly on their treatment of the ultrarelativistic case. It is curious that the results of the $m=0$ case and $m \neq 0$, but very small, are different [see, for example, their Eqs. (3.10) and (3.11)]. This would seem to suggest that the approach to the limit $m \rightarrow 0$ is nonuniform. The clue to this was recently found by Haber and Weldon,[4] who emphasized the following point. One starts at low temperatures with a given number of particles. Here the nonrelativistic form of the particle spectrum may be used. As the temperature increases, however, relativistic effects must be taken into account. These manifest themselves in two ways. Firstly, the full spectrum

$$\epsilon(p)=(|\vec{p}|^2+m^2)^{1/2} \tag{4}$$

must be used. Secondly, according to QED, the number of particles is no longer conserved when spontaneous pair production becomes important (at $k_B T \gtrsim mc^2$). At these temperatures (indeed, for logical consistency, at all temperatures) one must use the conservation law for the number of particles minus antiparticles.[5] If one does this, the photon results are recovered for $m=0$, and a correct description of the ultrarelativistic gas is obtained. (That this must be done has been known for the case of fermions for some time.[6]) Of course, the $m \rightarrow 0$ limit and the $m=0$ case both give the photon results now.

Lastly, we wish to say that the study of Bose gas with particle spectrum (4), but with a constraint on the number of particles alone, is perhaps worth studying in its own right as a statistical mechanical problem, but its relevance to an actual relativistic Bose gas is doubtful.

[1]R. Beckmann, F. Karsch, and D. E. Miller, Phys. Rev. A 25, 561 (1982); Phys. Rev. Lett. 43, 1277 (1979); D. E. Miller, R. Beckmann, and F. Karsch, in *Dynamics of Synergetic Systems,* edited by H. Haken (Springer, Berlin, 1980), pp. 39–46. See also S. de Groot, G. J. Hooyman, and C. A. ten Seldam, Proc. R. Soc. London, Ser. A 203, 266 (1950); A. Munster, Z. Phys. 144, 197 (1956); P. T. Landsberg, Proc. Cambridge Philos. Soc. 50, 65 (1954). For a detailed account of the problem and references, see P. T. Landsberg, in *Statistical Mechanics of Quarks and Hadrons,* edited by H. Satz (North-Holland, Amsterdam, 1981), pp. 355–382.

[2]See, for example, L. D. Landau and E. M. Lifshitz, *Statistical Physics,* 3rd ed., revised and enlarged by E. M. Lifshitz and L. P. Pitaevskii (Pergamon, New York, 1980), Part. 1, p. 183ff.

[3]K. M. Case and S. Gasiorowicz, Phys. Rev. 125, 1955 (1962); L. Durand, *ibid.* 128, 434 (1962).

[4]H. E. Haber and H. A. Weldon, Phys. Rev. Lett. 46, 1497 (1981); Phys. Rev. D 25, 502 (1982).

[5]J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, Reading, MA, 1967), p. 151; also, H. E. Haber and H. A. Weldon, Ref. 4.

[6]Landau and Lifshitz, Ref. 2, p. 315.