

**SHOCK WAVES IN A MIXTURE  
OF A GAS AND SMALL SOLID  
PARTICLES**



**ABSTRACT**

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**DEPARTMENT OF MATHEMATICS**

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# ABSTRACT

The study of shock waves in a mixture of a gas and small solid particles is of great importance due to its applications to nozzle flow; lunar ash flow; bomb blast; coal-mine blast; underground, volcanic and cosmic explosions; metallized propellant rocket; supersonic flight in polluted air; collision of coma with a planet and many other engineering problems (see [1-4]). In this dissertation we have attempted to study some of the recent works done in the area of propagation of shock waves in a mixture of a gas and small solid particles. The dissertation consists of four chapters.

The first chapter is introductory. It gives, in brief, an idea about Two Phase Flow, Fundamental Equations of Mixture of a Gas and Small Solid Particles, Shock Waves, Jump Conditions across a Shock Wave, Radiative Transfer Equations, Energy Equation in the Presence of Thermal Conduction and The Concept of Self-Similarity, as used in the dissertation.

In the chapter 2, we have studied following two types of problem:

- (i) Propagation of strong shock waves in a mixture of a gas and small solid particles, and
- (ii) Propagation of shock waves in a dusty gas with exponentially varying density.

In the first problem, we have studied similarity solutions of a strong shock wave propagation in a mixture of a gas and small solid particles. Similarity solution exists only when the shock is very strong and the surrounding

medium is of a constant density and at rest and with negligible counterpressure. The non-dimensional fundamental equations are derived and studied. The results depend on three non-dimensional parameters; i.e. (i) the ratio of specific heats of the gas  $\gamma$ , (ii) the mass concentration of the solid particles  $k_p$  in the mixture and (iii) the ratio of the density of the solid particles to that of initial density of the gas  $G_1$ . Numerical solutions for various values of  $\gamma$ ,  $k_p$  and  $G_1$  are presented and discussed.

In the second problem, we have studied non similarity solutions of a shock wave propagation in a dusty gas with exponentially varying density. The variation of flow-variables with distance, in the flow-field behind a shock wave are obtained at different times. The equilibrium flow conditions are assumed to be maintained. Effects of an increase in the mass concentration of the solid particles  $k_p$  on the flow variables and on the distance between the inner contact surface and the shock front are studied.

In the chapter 3, we have studied following two types of piston problem :

- (i) A self-similar solution of a shock propagation in a dusty gas, and
- (ii) Similarity solutions for the flow behind an exponential shock in a dusty gas.

In the first problem, analytical solutions are obtained for the unsteady, one-dimensional self-similar flow field between a strong shock and a moving piston in a dusty gas. The piston is assumed to be moving with time according to the power law. The dusty gas is assumed to consist of a mixture of a small solid particles and a perfect gas, in which solid particles are continuously distributed. It is assumed that the equilibrium flow-condition

is maintained and variable energy input is continuously supplied by the piston. The variation of flow variables with distance for a decelerated piston, a constant velocity piston and an accelerated piston are studied.

In the second problem, we have studied similarity solutions for the flow of a dusty gas behind a strong exponential shock driven out by a piston moving with time according to an exponential law. The medium is assumed to be a mixture of small solid particles and a perfect gas. In order to get some essential features of the shock propagation, small solid particles are considered as pseudo-fluid and it is assumed that the equilibrium flow condition is maintained in the flow-field, and that the viscous-stress and heat conduction of the mixture are negligible. Solutions are obtained, in both cases, when the flow between the shock and the piston is isothermal or adiabatic. Effects of a change in the mass concentration of the solid particles in the mixture  $k_p$  and the ratio of the density of solid particles to that of initial density of the gas  $G_1$  on the flow variables in the region between the shock and the piston are also studied.

In the final chapter, we have again studied two types of problem :

- (i) Propagation of shock waves in a dusty gas with radiation heat flux and exponentially varying density, and
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The equilibrium flow conditions are assumed to be maintained and the radiation is considered to be of a diffusion type for an optically thick grey gas model. The shock wave moves with variable velocity and the total energy of the wave is non-constant. Non-similar solutions are obtained and the effects of variation of the radiation parameter and time are studied. The effects of an increase in (i) the mass concentration of solid particles in the mixture and (ii) the ratio of the density of solid particles to the initial density of gas on the flow variables in the region behind the shock are also studied.

In the second problem, propagation of spherical shock waves in a dusty gas with heat conduction and radiation heat flux, in which density varies exponentially, is investigated. The dusty gas is assumed to be a mixture of small solid particles and a perfect gas. The equilibrium flow conditions are assumed to be maintained, and the heat conduction is expressed in terms of Fourier's law and the radiation is considered to be of the diffusion type for an optically thick grey gas model. The thermal conductivity  $K$  and the absorption coefficient  $\alpha_R$  are assumed to vary with temperature and density. The shock wave moves with variable velocity and the total energy of the wave is non-constant. Non-similar solutions are obtained and the effects of variation of the heat transfer parameters and time are studied. The effects of an increase in (i) the mass concentration of solid particles in the mixture and (ii) the ratio of the density of solid particles to the initial density of the gas on the flow variables in the region behind the shock are also studied at given times.

# Bibliography

- [1] Pai, S.I., Menon, S. and Fan, Z.Q.: *Similarity solutions of a strong shock wave propagation in a mixture of a gas and dusty particles*. Int. J.Eng, Sci. 18(12), 1365-1373 (1980).
- [2] Higashino, F. and Suzuki, T. : *The effects of particles on blast wave in a dusty gas*. Z. Naturf. a 35, 1330-1336 (1980).
- [3] Miura, H. and Glass, I. I. : *On the passage of a shock wave through a dusty gas layer*. Proc. R. Soc. Lond. A 385, 85-105 (1983).
- [4] Gretler, W. and Regenfelder, R. : *Strong shock waves generated by a piston moving in a dust-laden gas under isothermal condition*. Eur. J. Mech. B 24, 205- 218 (2005)
- [5] O'Keefe, J.A. and Adams,E.W.: *Tektite structure and lunar ash flows*. Jour .Geophy. Res. 70(16), 3819 - 3829(1965).
- [6] Pai,S.I.: *Two phase flows*.Vieweg. Tracts in Pure Appl.Phys.Vol 3.Vieweg Vevlog Braunschweg (1977).

- [7] Burgers, J.M.: *Flow Equations for Composite Gases*. Academic Press, New York (1969).
- [8] Naidu, G.N., Venkatanandan, K. and Ranga Rao, M.P. : *Approximate analytical solution for self similar flows of a dusty gas with variable energy*. Int. J. Eng. Sci. 23, 39-49 (1985).
- [9] Vishwakarma, J.P.: *Propagation of shock waves in a dusty gas with exponentially varying density*. Eur. Phys J. B. 16, 369-372 (2000).
- [10] Steiner, H. and Hirschler, T.: *A self similar solution of a shock propagation in a dusty gas*. Eur. J. Mech. B/ fluids 21, 371-380 (2002).
- [11] Vishwakarma, J.P. and Pandey, S.N.: *Propagation of strong spherical shock waves in a dusty gas*. Physica scripta 68, 259-263 (2003).
- [12] Vishwakarma, J.P. and Nath, G.: *Similarity solutions for unsteady flow behind an exponential shock in a dusty gas*. 74, 493-498 (2006).
- [13] Singh, K.K. and Vishwakarma, J.P.: *Propagation of spherical shock waves in a dusty gas with radiation heat-flux*. J. Theo. Appl. Mech. 45(4), 801-817 (2007).
- [14] Vishwakarma, J.P., Nath, G. and Singh, K.K.: *Propagation of shock waves in a dusty gas with heat conduction, radiation heat-flux and exponentially varying density*. Phys. Scr. 78, 11 pp (2008).
- [15] Alan Jeffrey : *Magnetohydrodynamics*. Oliver and Boyd Ltd., London (1966).

- [16] Helliwell J.B.: *Phys. Fluids* 9,1869 (1966).
- [17] Landau,L.D. and Lifshitz, E.M.: *A Course of Theoretical Physics*. Vol.5, Statistical Physics, Pergamon Press,Oxford (1958).
- [18] Sedov, L.I.: *Similarity and dimensional methods in mechanics*. Academic Press, New York (1959).
- [19] Korobeinikov, V.P.: *Problems in the theory of point explosion in Gases*. Proceedings of the Steklov Institute of Mathematics, No. 119, American Mathematical Society (1976).
- [20] Ray, G.D. and Bhowmick, J.B.: *Propagation of cylindrical and spherical explosion waves in an exponential medium*. Def. Sci.J. 9-14 (1974).
- [21] Verma, B.G. and Vishwakarma, J.P.: *Propagation of magnetogasdynamic plane shock waves in an exponential medium*.Nuovo Cim. 32, 267-272(1976).
- [22] Miura, H. and Glass, I. I. : *Development of the flow induced by a piston moving impulsively in a dusty gas*. Proc. Roy. Soc. Lond. A 397, 295-305 (1985).
- [23] Marble, F. E.: *Dynamics of dusty gases*. Rev. Fluid Mech. 2, 397 - 446(1970).
- [24] Moelwyn-Hughes : *Physical Chemistry*. Pergamon Press, London (1961).

- [25] Ranga Rao, M. P. and Ramanna, B.V.: *Unsteady flow of a gas behind an exponential shock*. J. Math. Phys Sci. 10, 465 - 476 (1976).
- [26] Higashino, F.: *Characteristic method applied to blast waves in a dusty gas*. Z. Naturforsch 38a, 399- 406 (1983).
- [27] Lambach, D.D. and Probst, R. F.: *Self - similar strong shocks with radiation in a decreasing exponential atmosphere*. Phys. Fluids 13, 1178 - 1183 (1970).
- [28] Sachdev, P.L. and Ashraf, S.: *Converging spherical and cylindrical shocks with zero temperature gradient in the rear flow - field*. J . Appl. Math. Phys. (ZAMP) 22, 1095 - 1102 (1971).
- [29] Zhuravskaya, T.A. and Levin, V.A. : *The propagation of converging and diverging shock waves under intense heat exchange conditions*. J. Appl. Math.Mech. 60, 745 - 752 (1996).
- [30] Elliott,L.A.: *Similarity methods in radiation hydrodynamics*. Proc. R. Soc. Lond. A 258, 287-301 (1960).
- [31] Wang, K. C.: *Approximate solution of a plane radiating piston problem*. Phys. fluids , 1922-19280 (1966).
- [32] Helliwell, J. B.: *Self-similar piston problem with radiative heat transfer*. J. Fluid Mech. 37, 497 -512 (1969),
- [33] Ghoneim, A. F.,Kamel,M. M, Berger, S. A. and Oppenheim, A. K.: *Effects of internal heat transfer on the structure of self-similar blast waves*. J. Fluid Mech. 117, 473-491 (1982).

- [34] Singh, J. B. and Srivastava, S. K.: *Propagation of spherical shock waves in an exponential medium with radiation heat flux*. *Astrophys. Space Sci.* 88, 277- 282 (1982).
- [35] Zel'dovich, Ya. B. and Raizer, Yu.P.: *Physics of Shock Waves and High Temperature Hydrodynamic Phenomena Vol. II*, Academic Press ,New York (1967).
- [36] Bhowmick, J. B.: *An exact analytical solution in radiating gas dynamics*, *Astrophys Space Sci.* 74, 481-485(1981).
- [37] Laumbach, D. D. and Probst, R. F.: A point explosion in a cold exponential atmosphere Part II Radiating flow . *J. Fluid Mech.* 40, 833-858 (1970).
- [38] Verma, B.G. and Vishwakarma, J.P.: *Axially symmetric explosion in magneto-gasdynamics*. *Astrophysics.Space Sci.* 69, 177-188 (1980).
- [39] Abdel- Raouf, A.M. and Gretler, W.: *Quasi-similar solution for blast wave with internal heat transfer effects*,. *Fluid Dyn.Res.* 8, 273-285 (1991).
- [40] Pomaraming, G.C.: *The Equation of Radiation Hydrodynamics* (Int. Ser. Monographs in Natural Philosophy Vol. 54) (Oxford: Pergamon) (1973).

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**To**

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**JANUARY, 2010**

# CERTIFICATE

I certify that the dissertation entitled "SHOCK WAVES IN A MIXTURE OF A GAS AND SMALL SOLID PARTICLES" submitted by Ms. Sarujinee Gogoi in partial fulfilment of the requirement of the degree of Master of Philosophy in Mathematics is the outcome of a study undertaken by the candidate.

I certify that the sources from which ideas have been borrowed have been duly referred to.

The material in this dissertation has not been presented for the award of a degree in any university before.

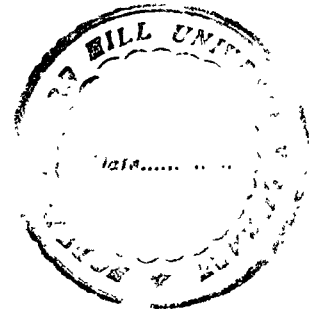
This dissertation may be placed before the examiners for evaluation and necessary formalities. I certify that this dissertation is worthy of consideration by the examiners.

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## DECLARATION

I, Sarujinee Gogoi, hereby declare that the subject matter in this dissertation is the record of work done by me, that the contents of this dissertation did not form basis of the award of any previous degree to me or to the best of my knowledge to anybody else, and that the dissertation has not been submitted by me for any research degree in any other university/institute.

This dissertation is being submitted to the North-Eastern Hill University for the degree of Master of Philosophy in Mathematics.

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Sarujinee Gogoi

# PREFACE

In this dissertation we have attempted to study some of the recent works done in the area of propagation of shock waves in a mixture of a gas and small solid particles. The dissertation has been divided into four chapters and each chapter has been subdivided into a number of sections.

The first chapter is introductory. It gives, in brief, an idea about Two Phase Flow, Fundamental Equations of Mixture of a Gas and Small Solid Particles, Shock Waves, Jump Conditions across a Shock Wave, Radiative Transfer Equations, Energy Equation in the Presence of Thermal Conduction and The Concept of Self-Similarity, as used in the dissertation.

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In the final chapter, we have again studied two types of problem :

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In the first problem, propagation of spherical shock waves in a dusty gas with radiation heat flux and exponentially varying density is investigated. The equilibrium flow conditions are assumed to be maintained and the radiation is considered to be of a diffusion type for an optically thick grey gas model. The shock wave moves with variable velocity and the total energy of the wave is non-constant. Non-similar solutions are obtained and the effects of variation of the radiation parameter and time are studied. The effects of an increase in (i) the mass concentration of solid particles in the mixture and

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# Chapter 1

## **BASIC CONCEPTS AND FUNDAMENTAL EQUATIONS**

In this chapter we give some basic concepts about two-phase flow, shock waves and self-similarity. We also derive fundamental equations and jump conditions across a shock wave in a mixture of a gas and small solid particles which will be used in forth coming chapters.

### **1.1 TWO-PHASE FLOW**

By two-phase flow we mean a special flow problem in which we have to consider the mechanics of two phases of matter simultaneously. In classical fluid mechanics we treat the flow problems of a homogeneous fluid which is

in one state only i.e either in liquid, gas or plasma state. In such problems, the solid bodies in the flow field are usually assumed to be rigid bodies so that the solids may be considered as given boundary conditions of the fluid flow problems. However, in many engineering problems as well as fluid flow in nature, we have to treat the flow problems of a mixture of substances in different states and the solid body may not be considered as rigid bodies of given shape. Such a system may be called, in general , the multiphase system and the corresponding flow may be called multiphase flow. The most common types of multiphase flow consist of two phases of some substances only.

Since the properties of a substance in different state are greatly different, the two-phase flows should be classified according to the states in the flow field. Each class should be treated independently from the other. Hence we may classify the two phase flows as follows:

- (i) liquid - gas flow;
- (ii) liquid - solid flow;
- (iii) Gas - solid flow;
- (iv) liquid - plasma flow which also includes the mixture of electrically conducting liquid and gas;
- (v) plasma - solid flow which also includes the mixture of electrically charged solid particles and a gas;
- (vi) gas - plasma flow which also includes the mixture of different gases.

In the present study, we are concerned only with the two phase flow of mixture of a gas and small solid particles. This is a well known problem of internal ballistics, i.e, the flow of gas behind a projectile being expelled by

gun powder. Recently this problem has been extensively studied because it is connected with lunar ash flows [1]. In order to have a general view of the whole flow problem of a mixture of gas and solid particles, we may divide the flow into several stages in which the flow behaves differently in different stages as shown in the following example.

For simplicity, let us consider well packed gun powder. In the first stage, when the gun powder begins to burn, there is a very small amount of flow of gas in the gun powder. The gun powder is practically undisturbed. The flow is similar to the case of gas flowing through a porous medium. This stage is usually called the fixed bed stage. As the combustion continues, the amount of gas flow increases. As the gas flow reaches a critical value, called the flow for fluidization, at which the character of the solid powder changes abruptly to a pseudofluid, waves can be set up in the gun powder. This pseudofluid behaves as a fluid so that it tends to form a level surface. The flow field at this stage may be called the dense phase of the fluidized bed. If the flow rate is increased, the flow is seen to become irregular; bubbles of the gas rise through the packed gun powder and burst. This process is known as slugging. Further increase of the flow rate will cause a disturbed and irregular regime in which the flow becomes so rapid that it will push the bullet and the gas will carry some gun powder with it. Beyond this point, the region is called the dilute phase in which the solid matter occupies less than 5% of the total volume and mixes with the gas in the flow field. This dilute phase is simply called the two-phase flow of a mixture of a gas and small solid particles in a narrow sense.

Many two phase flow problems of gas-solid mixture consider this dilute

phase. For the dilute phase, we may assume that the size of the solid particles is small enough so that we may assume that the average properties of these particles may be obtained. Furthermore, since the number of the solid particles is large, we may consider the solid particles as a pseudofluid if the velocity of the gas is sufficiently high. Thus, we consider the mixture of gas and solid particles as a mixture of two fluids.

There are many engineering problems, in which this dilute phase of solid - gas flow is a good approximation of the actual condition.

## 1.2 FUNDAMENTAL EQUATIONS OF THE MIXTURE OF A GAS AND SMALL SOLID PARTICLES

Flowing Pai [2] and Pai et al [3], in this section we derive the fundamental equations and some simple thermodynamic relations of a mixture of a gas and small solid particles.

We consider only the case of particles being a pseudo-fluid. The solid particles are spheres of identical mass  $m_p$ , radius  $r_p$  and specific heat  $C_s$ . We may consider the mixture as a mixture of two fluids: one is the real fluid, gas and the other is the pseudo-fluid of solid particles. For each species  $r$  in the mixture, we would like to know its velocity vector  $\vec{q}_r$ , its temperature  $T_r$ , its pressure  $p_r$  and its density  $\rho_r$ . In this case, we have two definitions for

density: the species density and the partial density. Since these definitions are important in derivation of the fundamental equations, we are going to write down these definitions for the two species in the mixture as follows:

We consider an element of the mixture of a gas  $g$  and solid particles  $p$  with total mass  $M = M_g + M_p$  and with total volume  $V = V_g + V_p$  where subscript  $g$  refers to the value for the gas and subscript  $p$  refers to that of the solid particles.

It is convenient to introduce the number density of the solid particles  $n_p$  which is the number of solid particles per unit volume at a point in the flow field. The volume occupied by the solid particles  $V_p$  is

$$V_p = n_p \cdot V \cdot \bar{\tau}_p , \quad (1.2.1)$$

where  $\bar{\tau}_p = \frac{4}{3}\pi r_p^3$  is the volume of a solid particle in the mixture. Without subscript, we refer to the value of the mixture as a whole.

The mass of the solid particles in the volume  $V$  of the mixture is

$$M_p = m_p n_p V . \quad (1.2.2)$$

The species density of the solid particles is defined as

$$\rho_{sp} = \frac{M_p}{V_p} = \frac{m_p n_p V}{n_p V \bar{\tau}_p} = \frac{m_p}{\bar{\tau}_p} . \quad (1.2.3)$$

Hence, the species density of solid particles is a constant for a given problem.

The partial density of pseudo-fluid of solid particles is defined as

$$\bar{\rho}_p = \frac{M_p}{V} = m_p n_p . \quad (1.2.4)$$

Also,

$$\bar{\rho}_p = \frac{M_p}{V} = \frac{V_p}{V} \cdot \frac{M_p}{V_p} = Z \rho_{sp} , \quad (1.2.5)$$

where  $Z = \frac{V_p}{V} = n_p \bar{\tau}_p$  is called the volume fraction of solid particles in the mixture. Hence

$$\bar{\rho}_p = \frac{M_p}{V} = m_p n_p = Z \rho_{sp} = \rho_{sp} n_p \bar{\tau}_p . \quad (1.2.6)$$

The partial density of the pseudo-fluid of solid particles  $\bar{\rho}_p$  is one of the fundamental variables in our analysis and it is proportional to  $Z$  or  $n_p$ .

Similarly, we have also the species density of the gas and the partial density of the gas too. The species density of the gas is defined as

$$\rho_g = \frac{M_g}{V_g} \quad (1.2.7)$$

and the partial density of gas is defined as

$$\bar{\rho}_g = \frac{M_g}{V} = \frac{M_g}{V_g} \cdot \frac{V_g}{V} = \rho_g \left( \frac{V - V_p}{V} \right) = \rho_g \left( 1 - \frac{V_p}{V} \right) = (1 - Z) \rho_g . \quad (1.2.8)$$

Only when  $Z \ll 1$ , the partial density of the gas is approximately equal to the species density.

Now we are going to derive the fundamental equations and some simple thermodynamic relations of the mixture of a gas and a pseudo-fluid of solid particles based on the two-fluid theory.

### (i) Equation of State and Thermodynamics

For each species in the mixture of a gas and a pseudo-fluid of solid particles, we have one equation of state. For the gas, we may use the perfect gas law which is

$$p_g = R^* \bar{\rho}_g T_g = R^* (1 - Z) \rho_g T_g = (1 - Z) p , \quad (1.2.9)$$

where  $p_g$  is the partial pressure of the gas in the mixture and  $T_g$  is the partial temperature of the gas and  $R^*$  is the gas constant .The total pressure of the mixture is  $p$  which is obtained also from the perfect gas law:

$$p = R^* \rho_g T_g . \quad (1.2.10)$$

Since the total pressure of the mixture is the sum of the partial pressure of the gas  $p_g$  and the partial pressure of the pseudofluid of solid particles  $p_p$ , we have  $p = p_g + p_p$ . With the help of the equations (1.2.9) and (1.2.10) we find that the partial pressure of the solid particles must be

$$p_p = Zp . \quad (1.2.11)$$

The equation of state for the pseudo-fluid of solid particles is simply

$$\rho_{sp} = constant . \quad (1.2.12)$$

We consider the thermodynamic equilibrium condition such that  $T_p = T_g = T$ . The density of the mixture as a whole is given by

$$\rho = \bar{\rho}_p + \bar{\rho}_g = Z\rho_{sp} + (1 - Z)\rho_g . \quad (1.2.13)$$

The mass concentration of pseudo-fluid of solid particles is defined as

$$k_p = \frac{M_p}{M} = \frac{M_p/V}{M/V} = \frac{\bar{\rho}_p}{\rho} = \frac{Z\rho_{sp}}{\rho} . \quad (1.2.14)$$

In equilibrium flow  $k_p$  is a constant in the whole flow field. Therefore from (1.2.14)

$$\frac{Z}{\rho} = constant , \quad (1.2.15)$$

in the whole flow field, for equilibrium flow.

Also from equations (1.2.13) and (1.2.14), we get

$$Z = \frac{k_p}{k_p + (1 - k_p)G}, \quad (1.2.16)$$

where  $G = \frac{\rho_{sp}}{\rho_g}$ .

Since the temperature of the solid particles  $T_p$  does not associate with the random translational kinetic energy of the particles and the temperature of the gas does not relate with its translational kinetic energy, it is not profitable to define a temperature of the mixture as a whole as in the usual treatment of a mixture of two gases[4]. Thus we retain the two temperatures  $T_p$  and  $T_g$ . In general, we shall write  $T_g = T$  for simplicity and  $T_p$  may be equal to or different from  $T$ . In the thermodynamic equilibrium condition, however, we have  $T_p = T_g = T$ . From equation (1.2.10), (1.2.13) and (1.2.14), we find the following relation between the pressure and the density of the mixture as a whole

$$p = \frac{1 - k_p}{1 - Z} \rho R^* T = \frac{\rho R_m T}{1 - Z}, \quad (1.2.17)$$

where

$$R_m = (1 - k_p)R^* \quad (1.2.18)$$

and  $R_m$  may be considered as an effective gas constant of the mixture and  $\rho$  is given by equation (1.2.13).

The internal energy of the mixture per unit mass  $U_m$  is related to the internal energies of the two species by the following relation;

$$\rho U_m = \bar{\rho}_p U_{mp} + \bar{\rho}_g U_{mg}$$

or

$$\rho U_m = Z \rho_{sp} C_{sp} T_p + (1 - Z) \rho_g C_v T \quad (1.2.19)$$

or

$$U_m = k_p C_{sp} T_p + (1 - k_p) C_v T , \quad (1.2.20)$$

where  $C_{sp} = C_s + C_{vp}$  ,  $C_s$  being the specific heat of solid particles due to internal degree of freedom of solid particles and  $C_{vp}$ , the effective specific heat at constant volume of pseudo-fluid of solid particles. Here we assume that  $C_{sp}$  and  $C_v$  are constant for simplicity. For thermodynamic equilibrium condition, we have the specific heat of the mixture at constant volume  $C_{vm}$  as follows:

$$C_{vm} = k_p C_{sp} + (1 - k_p) C_v , \quad (1.2.21)$$

where  $C_v$  is the specific heat of the gas at constant volume.

The enthalpy of the mixture per unit mass is

$$H_m = U_m + \frac{p}{\rho} = k_p (C_{sp} T + \frac{p}{\rho_{sp}}) + (1 - k_p) C_p T , \quad (1.2.22)$$

where  $C_p$  is the specific heat of the gas at constant pressure. For thermodynamic equilibrium condition, the specific heat of the mixture at constant pressure is then:

$$C_{pm} = k_p C_{sp} + (1 - k_p) C_p . \quad (1.2.23)$$

The ratio of specific heats of the mixture is

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \gamma \cdot \frac{\left(1 + \frac{\delta\beta'}{\gamma}\right)}{(1 + \delta\beta')} , \quad (1.2.24)$$

where

$$\gamma = \frac{C_p}{C_v}, \beta' = \frac{C_{sp}}{C_v}, \text{ and } \delta = \frac{k_p}{1 - k_p}. \quad (1.2.25)$$

Subtracting equation (1.2.21) from (1.2.23) and using  $C_p - C_v = R^*$ , we obtain

$$C_{pm} - C_{vm} = (1 - k_p)R^*. \quad (1.2.26)$$

From equations (1.2.24) and (1.2.26), we get

$$C_{vm} = \frac{(1 - k_p)R^*}{(\Gamma - 1)}$$

and

$$C_{pm} = \frac{(1 - k_p)R^*T}{(\Gamma - 1)}.$$

Therefore,

$$U_m = C_{vm}T = \frac{1 - k_p}{\Gamma - 1}R^*T$$

or, using equation (1.2.17)

$$U_m = \frac{(1 - Z)p}{(\Gamma - 1)\rho}. \quad (1.2.27)$$

If we consider the mixture as a homogeneous medium, the first law of thermodynamics for the mixture gives

$$dQ = dU_m - \frac{p}{\rho^2}d\rho, \quad (1.2.28)$$

where  $dQ$  is the heat addition to the mixture.

For isentropic change of state of gas-particle mixture, we have  $dQ = 0$ , equation (1.2.28) gives

$$\frac{1}{\Gamma - 1} \frac{dT}{T} = \frac{1}{1 - Z} \frac{d\rho}{\rho}. \quad (1.2.29)$$

Since  $Z = \frac{k_p \rho}{\rho_{sp}}$ , for constant  $k_p$  and  $T_p = T$  case, from the integration of equation (1.2.29), we have

$$T \left( \frac{\rho}{1 - Z} \right)^{-(\Gamma-1)} = \text{constant}. \quad (1.2.30)$$

Similarly, from equation (1.2.17) for a given  $k_p$  and  $T_p = T$ , we have

$$\frac{dp}{p} = \frac{dT}{T} + \frac{1}{1 - Z} \frac{d\rho}{\rho}. \quad (1.2.31)$$

From equations (1.2.29) and (1.2.31) we have,

$$\frac{dp}{p} = \frac{\Gamma}{1 - Z} \frac{d\rho}{\rho} \quad (1.2.32)$$

or

$$p \left( \frac{\rho}{1 - Z} \right)^{-\Gamma} = \text{constant}. \quad (1.2.33)$$

We may calculate the so-called equilibrium speed of sound of the mixture 'a' from equation (1.2.33) as follows:

$$a^2 = \frac{dp}{d\rho} = \frac{\Gamma(1 - k_p)R^*T}{(1 - Z)^2} = \frac{\Gamma R_m T}{(1 - Z)^2}$$

or

$$a^2 = \frac{\Gamma p}{\rho(1 - Z)}. \quad (1.2.34)$$

### (ii) Equation of Continuity

For each species in the mixture, we have one equation of continuity which gives the conservation of mass of that species. If we combine equations of continuity of both the species, we may obtain an equation of continuity for the mixture as a whole.

The equation of continuity for one-dimensional unsteady flow of the mixture can be written as [2,3]

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{\nu \rho u}{r} = 0, \quad (1.2.35)$$

where  $\nu = 0, 1$  and  $2$  correspond to the plane, cylindrical and spherical symmetry, respectively,  $\rho$  is the density of the mixture and  $u$  the flow velocity of the mixture.

### (iii) Equation of Motion

For each species, the conservation of momentum gives the corresponding equation of motion for that species. Combining the equations of motion of both the species, we may obtain an equation of motion for the mixture as a whole.

The equation of motion for one-dimensional unsteady flow of the mixture can be written as [2, 3]

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0, \quad (1.2.36)$$

where  $p$  is the total pressure of the mixture. Here, it is assumed that the viscous stress and heat conduction of the mixture are negligible.

#### (iv) Equation of Energy

For each species, the conservation of energy gives the corresponding equation of energy for that species. Combining the equations of energy of both the species, we may obtain an equation of energy of the mixture as a whole.

The equation of energy for one-dimensional unsteady flow of the mixture can be written as [2, 3]

$$\frac{\partial U_m}{\partial t} + u \frac{\partial U_m}{\partial r} - \frac{p}{\rho^2} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0, \quad (1.2.37)$$

where  $U_m$  is the internal energy of the mixture per unit mass.

### 1.3 SHOCK WAVES

In this section, we give, in brief, an idea about shock waves.

Shock waves are the most important distinctive features of supersonic flow of a fluid, across which the medium undergoes sudden and often considerable changes in velocity, pressure, density and temperature. The occurrence of shock waves is commonly associated with the supersonic flight, explosion of atomic bomb on the earth surface, thermonuclear explosion and electric discharges. The occurrence of shock waves in a mixture of a gas and small solid particles is encountered in several branches of engineering and science (see, for example 2,3,5,6,7,8,9,10,11)

Shock waves are the most conspicuous phenomena occurring in non-linear wave propagation. Even without being caused by initial discontinuities, they may appear and be propagated. The underlying mathematical fact is that, unlike linear partial differential equations, non linear equations often do not

admit solutions which can be continuously extended whenever the differential equations themselves remain regular.

Shock waves appear when elements in fluid approach one another with a velocity larger than the local sound speed. It is true, of course, that a shock wave is not a discontinuity in the strict sense. It has a finite thickness across which the physical properties change continuously. The study of 'structure' of a shock wave involves the study of pressure, temperature, density, velocity etc., in the small but finite thickness of shock wave. The thickness of shock wave is of the order of few molecular mean free path.

Relative to the shock wave, the flow on the upstream side must be supersonic, on the downstream side the flow relative to the wave may be either supersonic or subsonic, depending on the inclination of the incident stream from the normal to the wave. If the incident stream is parallel to the normal to the wave, the flow behind the wave is always subsonic relative to the wave.

## 1.4 JUMP CONDITIONS ACROSS A SHOCK WAVES IN A MIXTURE OF A GAS AND SMALL SOLID PARTICLES

In this section we first derive jump conditions across a shock front in general form and then from these we derive jump conditions in particular form which will be used in forthcoming chapters.

The jump conditions to be satisfied by jumps in the terms  $H$ ,  $\vec{F}$ , and  $G$

of the general conservation law

$$\frac{\partial H}{\partial t} + \text{div} \vec{F} = G \quad (1.4.1)$$

across general discontinuity surface is given by (Jeffrey[12]),

$$[\tilde{\lambda}H - \vec{F} \cdot \hat{n}] + [\vec{g} \cdot \hat{n}] = 0 , \quad (1.4.2)$$

where  $H$  and  $G$  are scalar functions,  $\vec{F}$  is a vector function,  $\tilde{\lambda} = \vec{q} \cdot \hat{n}$  is the normal speed of the discontinuity surface assumed to be moving with the arbitrary velocity  $\vec{q}$ ,  $\hat{n}$  being the outward drawn unit vector normal to the discontinuity surface,  $\vec{g}$  is some vector function such that  $G = \text{div} \vec{g}$  and the symbol  $[X]$  denotes the jump  $X_2 - X_1$  in the quantity  $X$  across the discontinuity surface.

Now we apply the jump equation (1.4.2) to conservation equations to obtain the jump conditions in general form.

#### (i) Jump Condition for Continuity Equation

The continuity equation expressing the conservation of mass of the fluid is

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \vec{u}) = 0 , \quad (1.4.3)$$

where  $\vec{u}$  is the fluid velocity vector.

Now applying equation (1.4.2) to equation (1.4.3), we find the following jump condition:

$$[\tilde{\lambda}\rho - \rho \vec{u} \cdot \hat{n}] = 0 . \quad (1.4.4)$$

It is useful to display this result in a different form by expressing it in terms of

$$\tilde{u}_n = \vec{u} \cdot \hat{n} - \tilde{\lambda} , \quad (1.4.5)$$



the normal fluid velocity component relative to the normal velocity  $\tilde{\lambda}$  of the discontinuity surface. Alternatively expressed, the jump condition (1.4.4) becomes,

$$[\rho \tilde{u}_n] = 0 . \quad (1.4.6)$$

### (ii) Jump Condition for Momentum Equation

The momentum equation is

$$\rho \frac{D\vec{u}}{Dt} = -grad p. \quad (1.4.7)$$

Using equation (1.4.3), equation (1.4.7) becomes

$$\frac{\partial}{\partial t}(\rho \vec{u}) + \vec{u} div(\rho \vec{u}) + (\rho \vec{u} \cdot \nabla) \vec{u} = -grad p \quad (1.4.8)$$

Writing above equation in component form, we have

$$\frac{\partial}{\partial t}(\rho u_x) + u_x div(\rho \vec{u}) + (\rho \vec{u} \cdot \nabla) u_x = -\frac{\partial p}{\partial x} ,$$

which can be put in the form

$$\frac{\partial}{\partial t}(\rho u_x) + div(\rho u_x \vec{u}) = -\frac{\partial p}{\partial x} . \quad (1.4.9)$$

Similarly,

$$\frac{\partial}{\partial t}(\rho u_y) + div(\rho u_y \vec{u}) = -\frac{\partial p}{\partial y} , \quad (1.4.10)$$

$$\frac{\partial}{\partial t}(\rho u_z) + div(\rho u_z \vec{u}) = -\frac{\partial p}{\partial z} . \quad (1.4.11)$$

Writing the jump conditions for above three equations and combining them, we get

$$[\tilde{\lambda} \rho \vec{u} - \rho \vec{u}(\vec{u} \cdot \hat{n})] = [p] \hat{n} \quad (1.4.12)$$

Expressing it in the form  $\tilde{u}_n = \vec{u} \cdot \hat{n} - \tilde{\lambda}$ , we get

$$[\rho \vec{u} \tilde{u}_n + p \hat{n}] = 0 \quad (1.4.13)$$

### (iii) Jump Condition for Energy Equation

The energy equation is

$$\frac{\partial E}{\partial t} + \text{div}(E\vec{u}) = -\text{div}(p\vec{u}) . \quad (1.4.14)$$

where  $E = \frac{1}{2}\rho\vec{u}^2 + \rho U_m$

Jump condition for above equation across the discontinuity surface is

$$[\tilde{\lambda}E - E\vec{u} \cdot \hat{n}] - [p\vec{u} \cdot \hat{n}] = 0 . \quad (1.4.15)$$

Expressing it in the form  $\tilde{u}_n = \vec{u} \cdot \hat{n} - \tilde{\lambda}$ , we get

$$[E\tilde{u}_n + p(\vec{u} \cdot \hat{n})] = 0 . \quad (1.4.16)$$

The jump condition (1.4.6) expresses the simple fact that mass flow through the discontinuity surface is constant. We shall denote the mass flow through the surface by  $m$ . A discontinuity surface will only be called a shock when the mass flow  $m$  through the surface is non zero. Hence fluid particles must cross a shock front.

The jump conditions (1.4.6), (1.4.13) and (1.4.16) have been derived quite general for an element of an arbitrary shock front moving with local normal velocity  $\tilde{\lambda} = \vec{q} \cdot \hat{n}$ .

To simplify our jump conditions let us assume that the normal  $\hat{n}$  is in the direction of the fluid flow and velocity of the shock (discontinuity surface) is

also in the same direction so that  $\vec{u} \cdot \hat{n} = u_n = |\vec{u}| = u$ ,  $\tilde{u}_n = u_n - \tilde{\lambda} = u - \tilde{\lambda} = \tilde{u}$  and  $\tilde{\lambda} = U$ , the shock velocity in the direction of the fluid flow.

Now we consider that a shock wave is propagated into a mixture of a gas and small solid particles of density  $\rho_1$ , at rest ( $u_1 = 0$ ) and with counter pressure  $p_1$ , where subscript '1' refers to the values immediately in front of the shock. Following pai et.all [3], we assume that the viscous stress and heat conduction of the mixture are negligible, and the equilibrium flow condition is maintained in the flow field so that the velocity of the gas  $u_g$  and that of the pseudo-fluid of solid particles  $u_p$  are equal and they are also equal to the velocity of the mixture  $u$ . The temperature of the gas  $T_g$ , that of the solid particles  $T_p$ , and that of the mixture  $T$  are also equal.

Under the above assumptions, the jump condition for continuity equation i.e(1.4.6) reduces into the following form;

$$[\rho\tilde{u}] = 0 , \quad (1.4.17)$$

which gives

$$\rho_2(U - u_2) = \rho_1 U = m(\text{say}) , \quad (1.4.18)$$

where subscript '2' refers to the values immediately behind the shock front.

The jump condition for momentum equation i.e (1.4.13) reduces into the following form:

$$[\rho u\tilde{u} + p] = 0 , \quad (1.4.19)$$

which by using  $\tilde{u} = u - \tilde{\lambda}$ , and equation (1.4.17) gives

$$p_2 + \rho_2(U - u_2)^2 = p_1 + \rho_1 U^2 . \quad (1.4.20)$$

The jump condition for energy equation i.e (1.4.16) reduces into the following form:

$$[E\tilde{u} + pu] = 0 , \quad (1.4.21)$$

which by using  $\tilde{u} = u - \tilde{\lambda}$ ,  $E = \frac{1}{2}\rho u^2 + \rho U_m$ , and equations (1.4.18) and (1.4.20) gives

$$U_{m_2} + \frac{p_2}{\rho_2} + \frac{1}{2}(U - u_2)^2 = U_{m_1} + \frac{p_1}{\rho_1} + \frac{1}{2}U^2 . \quad (1.4.22)$$

Also from equation (1.2.15) ,we have

$$\frac{Z_2}{\rho_2} = \frac{Z_1}{\rho_1} .$$

Thus we have the following jump conditions:

$$\begin{aligned} \rho_2(U - u_2) &= \rho_1 U = m(\text{say}), \\ p_2 + \rho_2(U - u_2)^2 &= p_1 + \rho_1 U^2 , \\ U_{m_2} + \frac{p_2}{\rho_2} + \frac{1}{2}(U - u_2)^2 &= U_{m_1} + \frac{p_1}{\rho_1} + \frac{1}{2}U^2 , \\ \frac{Z_2}{\rho_2} &= \frac{Z_1}{\rho_1} . \end{aligned} \quad (1.4.23)$$

## 1.5 RADIATIVE TRANSFER EQUATION

The problems of radiative energy transfer in fluids have received increasing attention in recent years as a consequence of the increasing speeds of bodies through the atmosphere and the very high temperatures attained by gases in motion. Effects of radiation are significance in the fields of nuclear power and space research, for instance.

Radiation plays an important role in many hydrodynamic processes relevant to strong shock waves and explosions. Its role is not confined to luminescence of the heated body. It can take part in energy transfer and heat exchange, causing energy losses, thus affecting the hydrodynamic movement of the matter.

On the flow-field of a gas, there are three radiative effects expressed in terms of the radiation pressure, radiation energy density and radiation flux.

**(i) Radiation pressure ( $p_R$ ):**

The only component of radiation stress which differs from zero is the radiation pressure  $p_R$  which may be written as

$$p_R = \frac{1}{3} a_1 T^4, \quad (1.5.1)$$

where  $T$  is the temperature of the gas in  $^{\circ}K$  and  $\frac{a_1 c}{4}$  is known as the Stefan-Boltzmann constant,  $c$  being the velocity of light.

**(ii) Radiation energy ( $E_R$ ):**

The radiation energy per unit mass of the fluid is given by

$$E_R = \frac{a_1 T^4}{\rho}, \quad (1.5.2)$$

where  $\rho$  is the density of the fluid.

**(iii) Radiation flux ( $q_i$ ):**

The net amount of energy passing through per unit area per unit time is called the flux through the surface and for optically thick medium it is given by expression (Helliwell[13]),

$$q_i = -D_R \frac{\partial}{\partial x_i} (\rho E_R), \quad (1.5.3)$$

where  $q_i$  is the total radiative flux passing across a co-ordinate plane  $x_i = \text{constant}$  at a point,  $D_R = \frac{cL_R}{3}$  is the Rosseland diffusion coefficient of radiation,

$$L_R = \frac{1}{K_1\rho}, \quad (1.5.4)$$

$L_R$  being the Rosseland mean free path of radiation and  $K_1$ , which depends upon temperature  $T$  and density  $\rho$ , is the opacity or Rosseland mean absorption coefficient.

## 1.6 ENERGY EQUATION IN THE PRESENCE OF THERMAL CONDUCTION

In an ideal fluid the law of conservation of energy is expressed by (Landau and Lifshitz [14]),

$$\frac{\partial}{\partial t} \left( \frac{1}{2} \rho v^2 + \rho U_m \right) = -\text{div} \left[ \rho \vec{v} \left( \frac{1}{2} v^2 + w \right) \right]. \quad (1.6.1)$$

The expression on the left is the rate of change of the energy in unit volume of the fluid, while that on the right is the divergence of the energy flux density,  $w$  is the heat function and  $U_m$  the internal energy per unit mass. If the temperature of the fluid is not constant throughout its volume, there will be, besides the energy transfer due to motion of the fluid, a transfer of heat by what is called thermal conduction. This signifies the direct molecular transfer of energy from points where the temperature is high to those where it is low.

We denote by  $\vec{q}_c$  the heat flux density due to thermal conduction. The flux  $\vec{q}_c$  is related to the variation of temperature through the fluid by the following relation:

$$\vec{q}_c = -K \text{grad}T. \quad (1.6.2)$$

The constant  $K$  is called the thermal conductivity of the fluid which is always positive. The coefficient  $K$  is in general a function of temperature and pressure.

Thus the total energy flux in a fluid when there is thermal conduction is  $\rho\vec{v}(\frac{1}{2}v^2 + w) - K \text{grad}T$ . Accordingly, the general law of conservation of energy is given by the equation

$$\frac{\partial}{\partial t}(\frac{1}{2}\rho v^2 + \rho U_m) = -\text{div}[\rho\vec{v}(\frac{1}{2}v^2 + w) - K \text{grad}T]. \quad (1.6.3)$$

It is convenient, however, to put it in the following form by transforming it with the aid of equations of motion (Landau and Lifshitz[14])

$$\rho T(\frac{\partial S}{\partial t} + \vec{v} \cdot \text{grad}S) = \text{div}(K \text{grad}T), \quad (1.6.4)$$

where  $S$  is the entropy per unit mass of the fluid. Note that if we neglected the conductivity of the fluid, the energy equation (1.6.4) reduces to the energy equation of an ideal fluid.

## 1.7 THE CONCEPT OF SELF-SIMILARITY

In this section we give a brief idea about self-similarity.

The fluid motion is said to be one-dimensional when all its properties depend on only one geometric coordinate and on the time. The motion in which the distributions of the flow variables remain similar to themselves with time and vary only as a result of change in scale is called self-similar. Self-similar motion is of great importance in gas dynamics. In this case the flow variables do not depend on the co-ordinates and time separately, but depend only on particular combinations of them. The methods of dimensional analysis can be used to find exact solutions of certain problems of one-dimensional unsteady motion of compressible fluid. The spherical, cylindrical and plane waves produce one-dimensional motion.

The basic physical variables in the Eulerian approach are the velocity  $u$ , the density  $\rho$ , and the pressure  $p$ . The characteristic parameters are the linear coordinate  $r$ , the time  $t$  and the constants which enter into the equations, the boundary and the initial conditions of the problems. Since the dimensions of the quantities  $p$  and  $\rho$  contain the mass, at least one constant  $a$ , the dimensions of which also contain the mass, must be a characteristic parameter. Hence, as in [15], without loss of generality, we can assume that

$$\dim a = ML^K T^s . \quad (1.7.1)$$

We can then write for the velocity, density and pressure as

$$u = \frac{r}{t} V, \quad \rho = \frac{a}{r^{K+3} t^s} G, \quad p = \frac{a}{r^{K+1} t^{s+2}} P , \quad (1.7.2)$$

where  $V, G$  and  $P$  are arbitrary quantities and therefore, can depend only on non-dimensional combinations of  $r$  and  $t$  and other parameters of the problem. In general, they are functions of two non-dimensional variables. But, if

an additional characteristic parameter  $b$  can be introduced with dimensions independent of those of  $a$ , the number of independent variable which can be formed by combining  $a$  and  $b$  is reduced to one. Since the dimensions of the constant  $a$  contain the mass, we can choose the constant  $b$  in a manner such that its dimensions do not contain the mass, i.e.

$$\dim b = L^m T^n. \quad (1.7.3)$$

The single non-dimensional independent variable in this case will be  $\frac{r^m t^n}{b}$  which can be replaced for  $m \neq 0$  by the variable

$$\eta = \frac{r}{b^{1/m} t^{\delta'}}. \quad (1.7.4)$$

where  $\delta' = -\frac{n}{m}$ .

If  $m = 0$ ,  $V, G$  and  $P$  depend only on time  $t$  and the velocity  $u$  is proportional to  $r$ .

This argument shows that, when the characteristic parameters include two constants with independent dimensions in addition to  $r$  and  $t$ , the partial differential equations satisfied by the velocity, density and pressure in the one-dimensional unsteady motion of a compressible fluid can be replaced by ordinary differential equations for  $V, G$  and  $P$ . The solution of these ordinary differential equations can sometimes be obtained exactly in closed form and, in other cases, approximated by using numerical integration.

# Bibliography

- [1] O'Keefe, J.A. and Adams,E.W.: *Tektite structure and lunar ash flows.*  
Jour .Geophy. Res. 70(16), 3819 - 3829(1965).
- [2] Pai,S.I.: *Two phase flows.*Vieweg. Tracts in Pure Appl.Phys.Vol  
3.Vieweg Vevlog Braunschweg (1977).
- [3] Pai, S.I., Menon, S. and Fan, Z.Q.: *Similarity solutions of a strong shock  
wave propagation in a mixture of a gas and dusty particles.* Int. J.Eng,  
Sci. 18(12), 1365-1373 (1980).
- [4] Burgers,J.M.: *Flow Equations for Composite Gases.* Academic  
Press,New York (1969).
- [5] Naidu, G.N., Venkatanandan,K. and Ranga Rao, M.P. : *Approximate  
analytical solution for self similar flows of a dusty gas with variable  
energy.* Int. J.Eng. Sci. 23, 39-49 (1985).
- [6] Vishwakarma, J.P.: *Propagation of shock waves in a dusty gas with  
exponentially varying density.* Eur.Phys J.B. 16, 369-372 (2000).

- [7] Steiner, H. and Hirschler, T.: *A self similar solution of a shock propagation in a dusty gas*. Eur.J.Mech. B/ fluids 21,371-380 (2002).
- [8] Vishwakarma, J.P. and Pandey, S.N.: *Propagation of strong spherical shock waves in a dusty gas*. Physica scripta 68, 259-263 (2003).
- [9] Vishwakarma, J.P. and Nath, G.: *Similarity solutions for unsteady flow behind an exponential shock in a dusty gas*. Phys. Scr. 74,493-498 (2006).
- [10] Singh, K.K. and Vishwakarma, J.P.: *Propagation of spherical shock waves in a dusty gas with radiation heat-flux*. J.Theo. Appl. Mech. 45(4), 801-817 ( 2007).
- [11] Vishwakarma, J.P., Nath, G. and Singh, K.K.: *Propagation of shock waves in a dusty gas with heat conduction, radiation heat-flux and exponentially varying density*. Phys.Scr.78,11 pp (2008).
- [12] Alan Jeffrey : *Magnetohydrodynamics*. Oliver and Body Ltd., London (1966).
- [13] Helliwell J.B.: *Phys. Fluids* 9,1869 (1966).
- [14] Landau, L.D. and Lifshitz, E.M.: *A Course of Theoretical Physics*. Vol.5, Statistical Physics, Pergamon Press, Oxford (1958).
- [15] Sedov, L.I.: *Similarity and dimensional methods in mechanics*. Academic Press, New York (1959).

## Chapter 2

# PROPAGATION OF SHOCK WAVES IN A MIXTURE OF A GAS AND SMALL SOLID PARTICLES

When a gaseous and dusty coma collides with a planet, the resultant density of coma may be increasing or decreasing or may be constant. Due to collision of such astrophysical bodies, shock waves are formed and these shock waves may be strong or weak (Pai et al[1]). Pai et al[1] generalized the well known solution of a point explosion in gas (Sedov [2], Korobeinikov[3]), to the case of two phase flow of a mixture of perfect gas and small solid particles, and brought out the essential effects due to presence of dusty particles on such a strong shock wave.

The propagation of plane shock wave in a medium where density increases exponentially was studied by Ray and Bhowmick [4], verma and vishwakarma [5] and many others. Vishwakarma [6] generalized the solution of Ray and Bhowmick [4] in a gas to the case of two-phase flow of a mixture of gas and small solid particles in which density obeys the exponential law. He found the significant effects of the presence of dust particles in the medium on the variation of density and pressure.

The solutions obtained by Pai et al [1] are similar whereas those obtained by Vishwakarma [6] are non-similar one.

In this chapter, we study propagation of shock waves in a mixture of a gas and small solid particles with constant and exponentially varying density using similarity and non-similarity methods respectively.

## 2.1 SIMILARITY SOLUTIONS OF A STRONG SHOCK WAVE PROPAGATION IN A MIXTURE OF A GAS AND SMALL SOLID PARTICLES

This section is devoted to study similarity solutions of a strong shock wave propagation in a mixture of a gas and small solid particles with constant density. We follow Pai et al[1] here.

### 2.1.1 INTRODUCTION

We consider one-dimensional unsteady flow of a mixture of a gas and small solid particles. Since we are interested in the shock wave propagation, we may assume that the viscous stress and heat conduction of the mixture are negligible. Furthermore, in order to get some essential features of the shock wave propagation, we assume that the equilibrium flow condition is maintained in the flow field (Pai et al[1]).

### 2.1.2 FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

The fundamental equations for one-dimensional unsteady flow of a mixture of a gas and small solid particles can be written as

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{\nu \rho u}{r} = 0, \quad (2.1.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0, \quad (2.1.2)$$

$$\frac{\partial U_m}{\partial t} + u \frac{\partial U_m}{\partial r} - \frac{p}{\rho^2} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0, \quad (2.1.3)$$

where  $\nu = 0, 1, \text{ or } 2$  corresponds to plane, cylindrical or spherical symmetry,

$\rho$  is the density of mixture,

$u$  the flow velocity,

$p$  the pressure mixture,

$U_m$  the internal energy per unit mass of the mixture,

$r$  the distance, and

$t$  the time.

The equation of state of the mixture of gas and small solid particles can be written as (Pai et al[1], Vishwakarma [6])

$$p = \frac{(1 - k_p)}{(1 - Z)} \rho R^* T, \quad (2.1.4)$$

where  $R^*$  is the gas constant,  $T$  the temperature,  $k_p$  the mass concentration of solid particles and  $Z$  the volume fraction of solid particles in the mixture.

The relation between  $k_p$  and  $Z$  is given by

$$k_p = \frac{Z \rho_{sp}}{\rho}, \quad (2.1.5)$$

where  $\rho_{sp}$  is the species density of solid particles. In equilibrium flow,  $k_p$  is a constant in the whole flow field.

The internal energy per unit mass of the mixture may be written as follows

$$U_m = [k_p C_{sp} + (1 - k_p) C_v] T = C_{vm} T, \quad (2.1.6)$$

where  $C_{sp}$  is specific heat of the solid particles,  $C_v$  is the specific heat of the gas at constant volume process and  $C_{vm}$  is the specific heat of the mixture at constant volume process. The specific heat of the mixture at constant pressure process is

$$C_{pm} = k_p C_{sp} + (1 - k_p) C_p , \quad (2.1.7)$$

where  $C_p$  is the specific heat of the gas at constant pressure process.

The ratio of the specific heats of the mixture is given by (Pai et al [1], Vishwakarma[6])

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \frac{\gamma \left(1 + \frac{\delta \beta'}{\gamma}\right)}{1 + \delta \beta'} , \quad (2.1.8)$$

where

$$\gamma = \frac{C_p}{C_v}, \quad \delta = \frac{k_p}{1 - k_p}, \quad \text{and} \quad \beta' = \frac{C_{sp}}{C_v}.$$

The internal energy is therefore, given by

$$U_m = \frac{p(1 - Z)}{\rho(\Gamma - 1)}. \quad (2.1.9)$$

We consider that a strong shock wave is propagated into a medium of density  $\rho_1$ , at rest ( $u_1 = 0$ ) and with negligibly small counter pressure  $p_1 \cong 0$ , where subscript '1' refers to the values immediately in front of the shock. The boundary conditions at the strong shock with  $u_1 = 0$  are as follows:

$$\begin{aligned} \rho_2(U - u_2) &= \rho_1 U , \\ p_2 + \rho_2(U - u_2)^2 &= \rho_1 U^2 , \\ U_{m2} + \frac{p_2}{\rho_2} + \frac{1}{2}(U - u_2)^2 &= \frac{1}{2}U^2 , \\ \frac{Z_2}{\rho_2} &= \frac{Z_1}{\rho_1} , \end{aligned} \quad (2.1.10)$$

where  $U$  is the velocity of the shock front which is a function of time  $t$ , and subscript '2' refers to the values immediately behind the shock front.

The initial volume fraction of the solid particles  $Z_1$  is, in general not a constant. But the volume occupied by the solid particles is very small because the density of the solid particles is much larger than that of the gas (Miura and Glass[7]), hence  $Z_1$  may be assumed as a small constant. The expression for  $Z_1$  is (Naidu et al[8])

$$Z_1 = \frac{k_p}{G_1(1 - k_p) + k_p}, \quad (2.1.11)$$

where  $G_1 = \frac{\rho_{sp}}{\rho_{g1}}$  is the ratio of the density of solid particles to the initial density of the gas.

### 2.1.3 SIMILARITY SOLUTIONS

Following Pai et al[1], in this section we shall find solutions for  $\nu = 0$  and  $\rho_1 = \text{constant}$ .

The boundary conditions (2.1.10) can also be put in the following form:

$$\begin{aligned} \rho_2(U - u_2) &= \rho_1 U, \\ p_2 &= \rho_2 u_2 (U - u_2), \\ \rho_2(U - u_2) \left[ \frac{1}{\Gamma - 1} \frac{p_2}{\rho_2} + \frac{1}{2} u_2^2 \right] - p_2 u_2 - \frac{Z_2(U - u_2)p_2}{\Gamma - 1} &= 0. \end{aligned} \quad (2.1.12)$$

To obtain the similarity solutions, we write the unknown variables in the following form

$$u = U i(\eta), \quad \rho = \rho_1 j(\eta), \quad p = \rho_1 U^2 k(\eta), \quad Z = Z_1 j(\eta), \quad (2.1.13)$$

where  $i$ ,  $j$  and  $k$  are functions of the non-dimensional variable(similarity variable)  $\eta = \frac{r}{r_2(t)}$  only and  $r_2(t)$  is the location of the shock front at time  $t$  so that

$$U(t) = \frac{dr_2}{dt} . \quad (2.1.14)$$

The variable  $\eta$  assumes the value 1 at the shock front.

By using equations (2.1.1) and (2.1.9), energy equation (2.1.3) can be put in the form

$$\rho \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right) \left[ \frac{p}{\rho} (1 - Z) \right] + (\Gamma - 1) p \frac{\partial u}{\partial r} = 0 . \quad (2.1.15)$$

Using similarity transformations (2.1.13), equations (2.1.1), (2.1.2) and(2.1.15) can be transformed into the following form

$$i'j + j'(i - \eta) = 0 , \quad (2.1.16)$$

$$i'j(i - \eta) + k' = \frac{1}{2}ij , \quad (2.1.17)$$

$$i'k\Gamma + k'(1 - jZ_1)(i - \eta) = k(1 - jZ_1) , \quad (2.1.18)$$

where the prime refers to the differentiation with respect to  $\eta$ .

Solving equations (2.1.16) to (2.1.18) for  $i'$ ,  $j'$  and  $k'$ , we obtain

$$i' = \frac{\frac{1}{2}ij[k\Gamma - j(i - \eta)^2(1 - jZ_1)] - jk[\frac{1}{2}i\Gamma - (i - \eta)(1 - jZ_1)]}{j(i - \eta)} , \quad (2.1.19)$$

$$j' = - \left[ \frac{\frac{1}{2}ij\{k\Gamma - j(i - \eta)^2(1 - jZ_1)\} - jk\{\frac{1}{2}i\Gamma - (i - \eta)(1 - jZ_1)\}}{(i - \eta)^2\{k\Gamma - j(i - \eta)^2(1 - jZ_1)\}} \right] , \quad (2.1.20)$$

$$k' = \frac{jk[\frac{1}{2}i\Gamma - (i - \eta)(1 - jZ_1)]}{k\Gamma - j(i - \eta)^2(1 - jZ_1)} . \quad (2.1.21)$$

Using (2.1.13), the shock conditions (2.1.12) transform into

$$i(1) = k(1) = \frac{2(1 - Z_1)}{\Gamma + 1}, \quad j(1) = \frac{\Gamma + 1}{\Gamma - 1 + 2Z_1}. \quad (2.1.22)$$

Because of the dependence of the boundary conditions (2.1.22) and the equations (2.1.19) to (2.2.21) on the initial volume fraction  $Z_1$ , the similar solution exist only when  $Z_1$  is a constant, i.e for constant initial density only.

Normalizing the variables  $u$ ,  $p$ ,  $\rho$  and  $T$  with their respective values at the shock, we obtain

$$\begin{aligned} \frac{u}{u_2} &= \frac{i(\eta)}{i(1)}, \quad \frac{p}{p_2} = \frac{k(\eta)}{k(1)}, \quad \frac{\rho}{\rho_2} = \frac{j(\eta)}{j(1)} \\ \frac{T}{T_2} &= \left( \frac{k(\eta)/j(\eta)}{k(1)/j(1)} \right) \left( \frac{1 - Z_1 j(\eta)}{1 - Z_1 j(1)} \right). \end{aligned} \quad (2.1.23)$$

From the similar solutions  $i(\eta)$ ,  $j(\eta)$  and  $k(\eta)$ , we can find the relation between the velocity of the shock front  $U(t)$  and the total energy release  $E_0$  of the flow field. The total energy release is

$$E_0 = 2 \int_0^{r_2} \rho \left( \frac{1 - Z p}{\gamma - 1 \rho} + \frac{1}{2} u^2 \right) dr. \quad (2.1.24)$$

The motion of the shock wave can be easily determined without solving the equations of motion of the medium. For this we can proceed as follows:

Following Sedov[2], we take the following quantities as fundamental constants:

$$\rho_1 \text{ and } \frac{E}{\rho_1},$$

where  $E$  is a constant having the same dimensions as the energy  $E_0$  liberated during explosion. The dimension of  $E$  for the plane case is

$$[E] = MT^{-2}.$$

Evidently, the constant  $E$  is directly proportional to  $E_0$ :

$$E_0 = \alpha E ,$$

where  $\alpha$  is a constant.

In this case the only dimensionless variable parameter  $\eta$  is given by

$$\eta = \frac{r}{\left(\frac{E}{\rho_1}\right)^{\frac{1}{3}} t^{\frac{2}{3}}} .$$

Let us assume that at the shock front,  $\eta$  assumes the value  $\eta^*$  (constant).

Therefore

$$\eta^* = \frac{r_2}{\left(\frac{E}{\rho_1}\right)^{\frac{1}{3}} t^{\frac{2}{3}}} .$$

The constant  $\eta^*$  can be set equal to any non-zero number. For the sake of simplicity, we shall set  $\eta^* = 1$ . Therefore for the plane waves, the motion of the shock wave is given by

$$r_2 = \left(\frac{E}{\rho_1}\right)^{1/3} t^{2/3}, \quad U = \frac{2}{3} \left(\frac{E}{\rho_1}\right)^{1/3} t^{-1/3} = \frac{2}{3} \sqrt{\frac{E}{\rho_1}} \cdot \frac{1}{\sqrt{r_2}}$$

or

$$r_2 = \left(\frac{E_0}{\rho_1 \alpha}\right)^{1/3} t^{2/3}, \quad U = \frac{2}{3} \left(\frac{E_0}{\rho_1 \alpha}\right)^{1/3} t^{-1/3} = \frac{2}{3} \sqrt{\frac{E_0}{\rho_1 \alpha}} \cdot \frac{1}{\sqrt{r_2}} .$$

Hence we may write

$$U(t) = \frac{dr_2}{dt} = \frac{2}{3} \frac{r_2}{t} = \frac{2}{3} \left(\frac{E_0}{\rho_1 \alpha}\right) r^{-1/2} = \frac{2}{3} \left(\frac{E_0}{\rho_1 \alpha}\right)^{1/3} t^{-1/3} . \quad (2.1.25)$$

To find  $\alpha$ , we substitute equations (2.1.13) and (2.1.25) into equation (2.1.24), we obtain

$$\alpha = \frac{8}{9} \int_0^1 \left[ \frac{1 - Z_1 j}{\Gamma - 1} k + \frac{j i^2}{2} \right] d\eta . \quad (2.1.26)$$

From the similar solution, we obtain  $\alpha$  from equation (2.1.26) and then substituting  $\alpha$  of equation (2.1.26) into equation (2.1.25), we find the location of the shock front  $r_2(t)$  and the velocity of the shock front  $U(t)$  as a function of  $E_0$  and  $t$ .

#### 2.1.4 RESULTS AND DISCUSSION

The similar solutions of equations (2.1.19) to (2.1.21) with the boundary conditions (2.1.22) depend on the non-dimensional parameters  $\Gamma$  and  $Z_1$  or  $\gamma$ ,  $k_p$  and  $G_1$ . In other words, the similar solutions depend on

- (i) the properties of the gas which is characterized by the ratio of the specific heats  $\gamma = \frac{C_p}{C_v}$ ,
- (ii) the mass concentration of solid particles in the mixture  $k_p$ , and
- (iii) the density ratio between the solid particles and the gas  $G_1$ .

Distribution of the flow variables are obtained from equations (2.1.19) to (2.1.21) by numerical integration. Runge-Kutta method of fourth order is used for numerical integration, starting from the shock front ( $\eta = 1$ ) and continuing upto centre of symmetry ( $\eta = 0$ ). For the purpose of numerical integration the following values of the non-dimensional parameters are used (see Pai et al[1])

$$\gamma = 1.3 , 1.4 , 2.0 , 3.0 , 6.0 ;$$

$$k_p = 0.1 , 0.2 , 0.3 , 0.4 , 0.6 , 0.7 ;$$

$$G_1 = 1.0 , 10.0 , 100.0 , 1000.0 .$$

In figs. 1-5, the effects of  $\gamma$  for a given  $k_p$  and  $G_1$  on the quantities  $\frac{u}{u_2}$ ,  $\frac{\rho}{\rho_2}$ ,  $\frac{p}{p_2}$ ,  $\frac{T}{T_2}$  and  $Z$  are shown. In fig.1, we find that for given  $k_p$  and  $G_1$ , the velocity  $\frac{u}{u_2}$  decreases as  $\gamma$  increases while in fig.2, we find that the density  $\frac{\rho}{\rho_2}$  increases with increase of  $\gamma$ . Fig.3 shows the variation of pressure  $\frac{p}{p_2}$  which is different from those of  $\frac{u}{u_2}$  and  $\frac{\rho}{\rho_2}$ . Near the shock front ( $\eta = 1$ ), the pressure increases with increase of  $\gamma$  while near the the centre ( $\eta = 0$ ), the pressure decreases as  $\gamma$  increases . In fig.4, we find that the temperature  $\frac{T}{T_2}$  increases as  $\gamma$  decreases. In general, the temperature increases as  $\eta$  decreases. Fig.5 shows the variation of the volume fraction of the solid particles  $Z$  in the flow field. Near the shock front ( $\eta = 1$ ),  $Z$  decreases as  $\gamma$  increases while near the centre ( $\eta = 0$ ),  $Z$  increases with increase of  $\gamma$ .

In figs. 6-10, the effects of  $k_p$  and  $G_1$  for a given value of  $\gamma$  on the quantities  $\frac{u}{u_2}$ ,  $\frac{\rho}{\rho_2}$ ,  $\frac{p}{p_2}$ ,  $\frac{T}{T_2}$  and  $Z$  are shown. For  $\gamma = 1.4$  and  $G_1 = 1.0$ , fig.6 shows that the velocity  $\frac{u}{u_2}$  increases with the increase of  $k_p$ . For  $k_p = 0.6$  and  $\gamma = 1.4$ , the value of  $\frac{u}{u_2}$  decreases as  $G_1$  increases. In fig.7, we find that for  $\gamma = 1.4$  and  $G_1 = 1.0$ , the density  $\frac{\rho}{\rho_2}$  increases with increase of  $k_p$  but for large  $G_1$  such that  $G_1 = 100$  or larger, the density  $\frac{\rho}{\rho_2}$  decreases with

increase of  $G_1$ . Fig.8 shows the variation of pressure  $\frac{p}{p_2}$ . For  $\gamma = 1.4$  and  $G_1 = 1.0$ ,  $\frac{p}{p_2}$  increases with increase of  $k_p$  but for  $\gamma = 1.4$  and  $G_1 = 100$ ,  $\frac{p}{p_2}$  decreases with increase of  $k_p$ . In fig.9, we find that for  $\gamma = 1.4$  and  $G_1 = 1.0$ , the temperature decreases with increase of  $k_p$  but for large  $G_1 = 100$ , the temperature increases with increase of  $k_p$ . In fig.10, we find that the values of  $Z$  increase with increase of  $k_p$  but decrease with the increase of  $G_1$ .

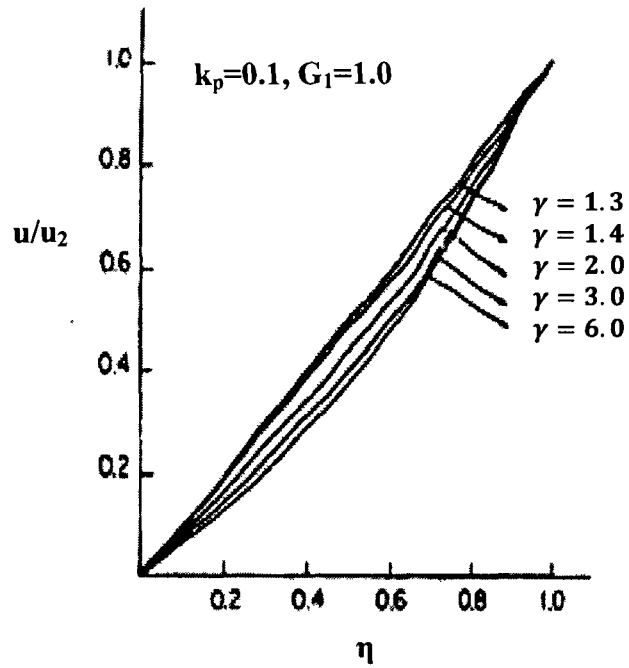


Fig.1 The variation of the velocity ( $u/u_2$ ) behind the shock with similarity parameter  $\eta$  at  $k_p=0.1$  and  $G_1=1.0$  for various  $\gamma$

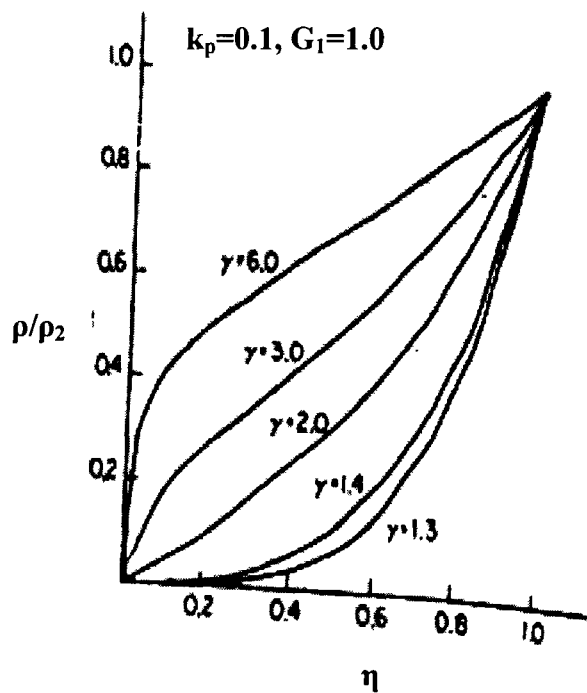


Fig.2 The variation of the density ( $\rho/\rho_2$ ) behind the shock with similarity parameter  $\eta$  at  $k_p=0.1$  and  $G_1=1.0$  for various  $\gamma$

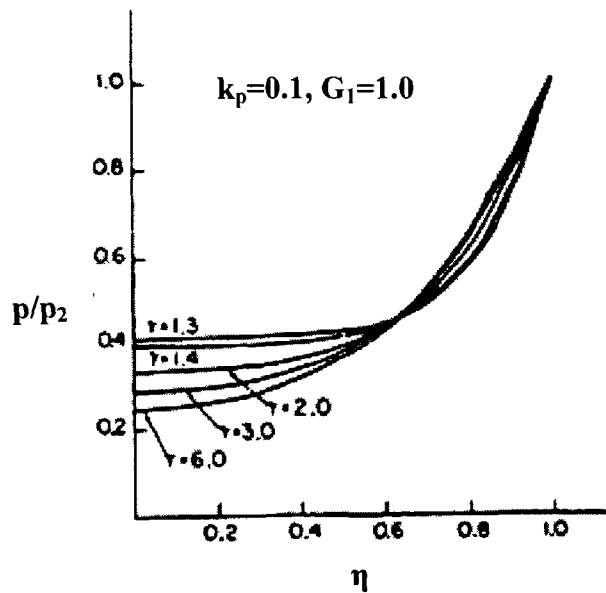


Fig.3 The variation of the pressure ( $p/p_2$ ) behind the shock with similarity parameter  $\eta$  at  $k_p=0.1$  and  $G_1=1.0$  for various  $\gamma$

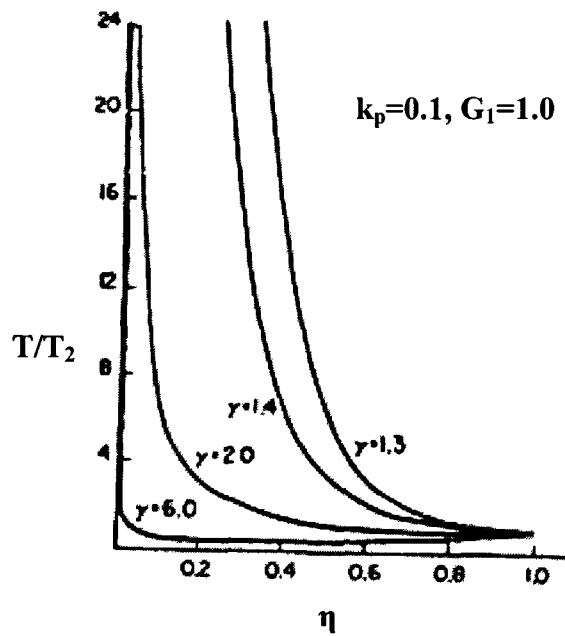


Fig.4 The variation of the temperature ( $T/T_2$ ) behind the shock with similarity parameter  $\eta$  at  $k_p=0.1$  and  $G_1=1.0$  for various  $\gamma$

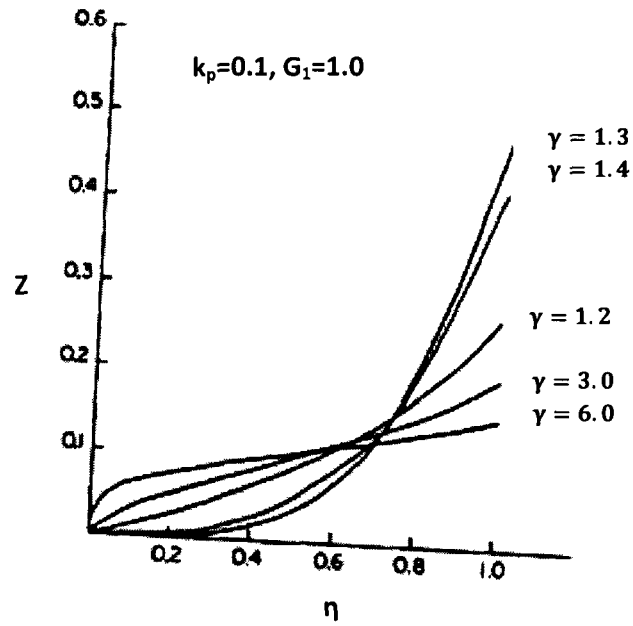


Fig.5 The variation of the volume fraction of the solid particles  $Z$  in the mixture behind the shock with similarity parameter  $\eta$  at  $k_p=0.1$  and  $G_1=1.0$  for various  $\gamma$

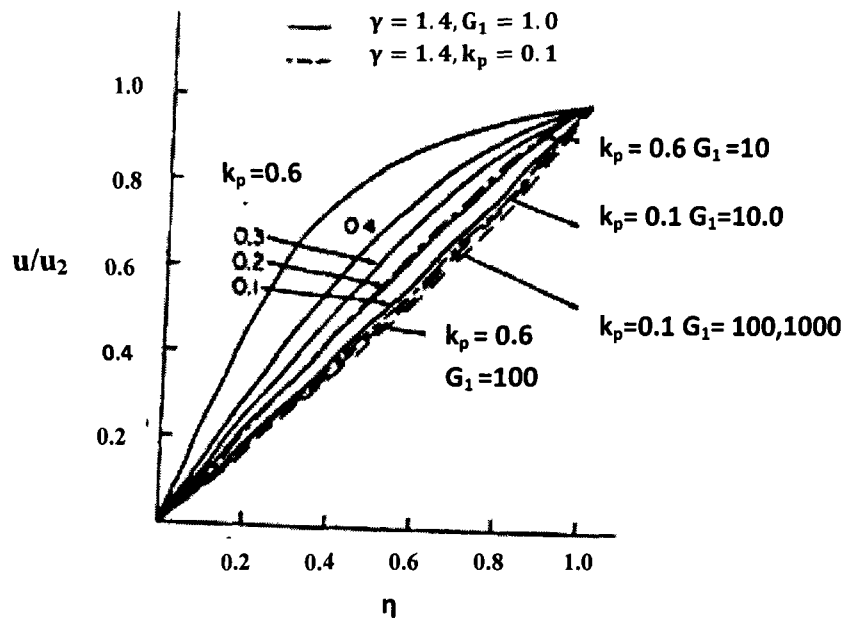


Fig.6 The variation of the velocity ( $u/u_2$ ) behind the shock with similarity parameter  $\eta$  at  $\gamma=1.4$  for various  $k_p$  and  $G_1$

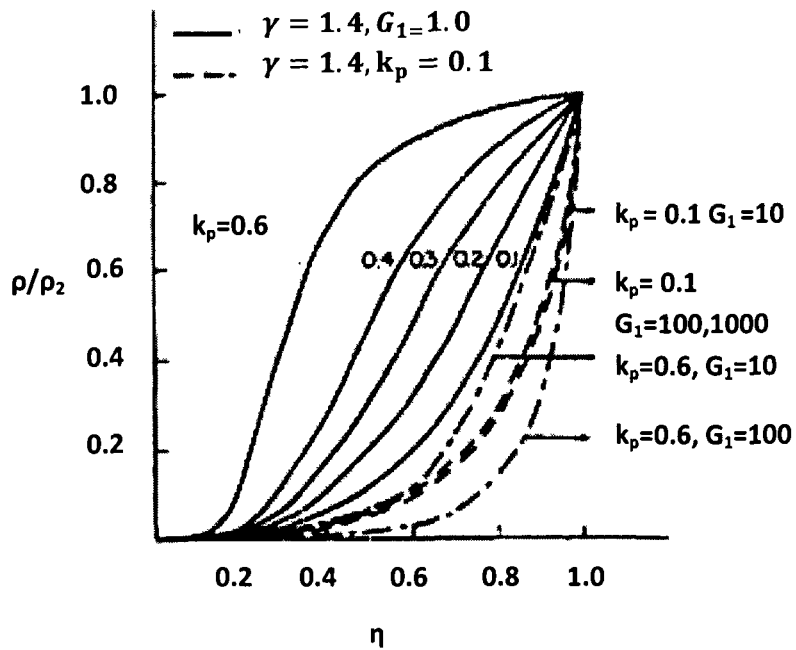


Fig.7 The variation of the density ( $\rho/\rho_2$ ) behind the shock with similarity parameter  $\eta$  at  $\gamma=1.4$  for various  $k_p$  and  $G_1$

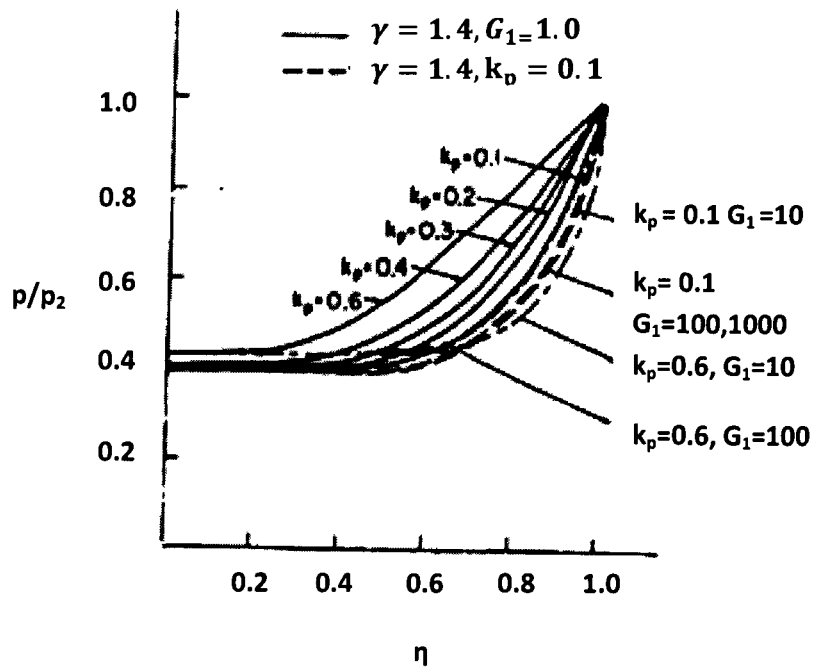


Fig.8 The variation of the pressure ( $p/p_2$ ) behind the shock with similarity parameter  $\eta$  at  $\gamma=1.4$  for various  $k_p$  and  $G_1$

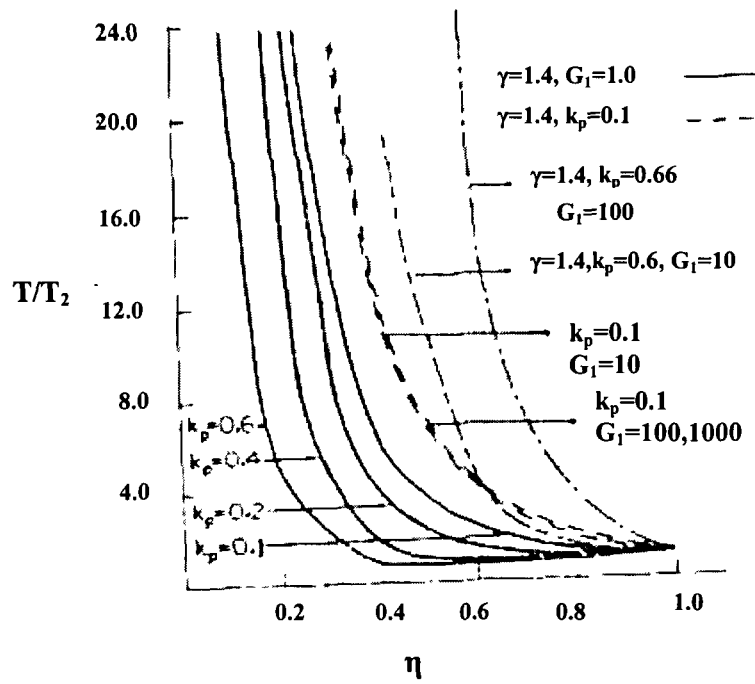


Fig.9 The variation of the temperature ( $T/T_2$ ) behind the shock with similarity parameter  $\eta$  at  $\gamma = 1.4$  for various  $k_p$  and  $G_1$

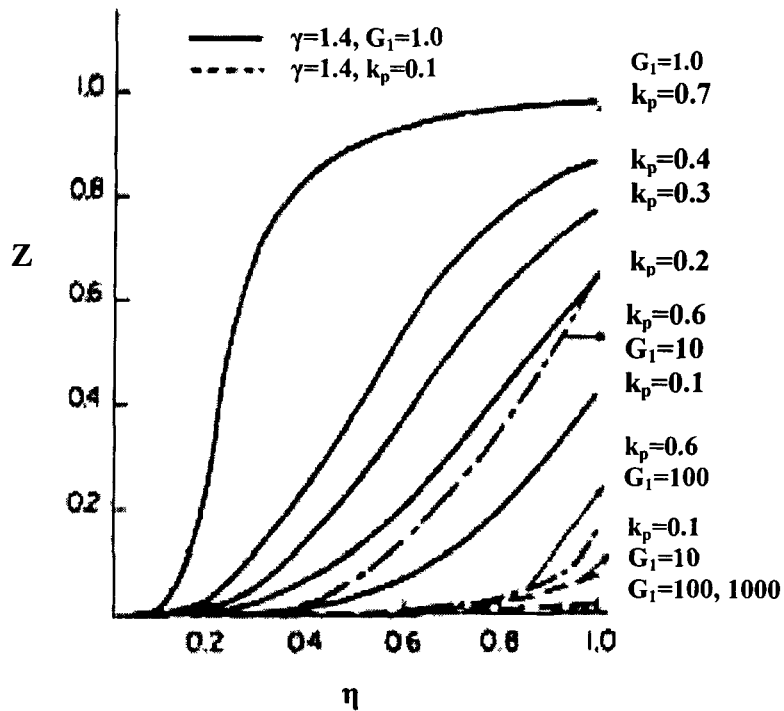


Fig.10 The variation of the volume fraction of the solid particles behind the shock with the similarity parameter  $\eta$  at  $\gamma = 1.4$  for various  $k_p$  and  $G_1$

## 2.2 PROPAGATION OF SHOCK WAVES IN A DUSTY GAS WITH EXPONENTIALLY VARYING DENSITY

This section is devoted to study non-similarity solutions of a shock wave propagation in a dusty gas with exponentially varying density. We follow Vishwakarma [6] here.

### 2.2.1 FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

Here we consider the same type of flow of a mixture of a gas and small solid particles as taken in last section 2.1, so fundamental equations and boundary conditions for this problem are also same as that in section 2.1.2.

Here we assume that the initial density of the medium (the mixture of a gas and small solid particles) obeys the exponential law,

$$\rho = \mu e^{\alpha_1 r} , \quad (2.2.1)$$

where  $\alpha_1$  and  $\mu$  are positive constants.

From jump conditions (2.1.10), we get

$$\begin{aligned} u_2 &= (1 - \beta)U , \\ \rho_2 &= \frac{\rho_1}{\beta} , \\ p_2 &= (1 - \beta)\rho_1 U^2 , \end{aligned}$$

$$Z_2 = \frac{Z_1}{\beta}, \quad (2.2.2)$$

where  $U = \frac{dR}{dt}$  is the shock velocity, and  $R$  the distance of the shock front from the plane, the line or the point of symmetry. also the quantity  $\beta$  is given by

$$\beta = \frac{\Gamma + 2Z_1 - 1}{\Gamma + 1}. \quad (2.2.3)$$

Let the solution of equations (2.1.1) to (2.1.3) be of the form

$$\begin{aligned} u &= t^{-1}V(\xi), \\ \rho &= t^{\Omega}D(\xi), \\ p &= t^{\Omega-2}P(\xi), \end{aligned} \quad (2.2.4)$$

where

$$\xi = te^{\lambda r}, \quad \lambda \neq 0 \quad (2.2.5)$$

and  $\Omega$  and  $\lambda$  are constants to be determined subsequently. We choose the shock surface to be given by

$$\xi_0 = \text{constant}, \quad (2.2.6)$$

so that its velocity is given by

$$U = -\frac{1}{\lambda t}. \quad (2.2.7)$$

From equation (2.2.7), it is clear that  $\lambda < 0$ . The solutions of the equations (2.1.1) to (2.1.3) in the form (2.2.4) are compatible with the shock conditions only if,

$$\Omega = 2 \text{ and } \lambda = -\frac{\alpha_1}{2}. \quad (2.2.8)$$

As a consequence of equations (2.2.1), (2.2.5), (2.2.7) and (2.2.8), the effective shock Mach number  $M_e$  ahead of the shock is given by

$$M_e^2 = \frac{U^2}{a_1^2} = \frac{U_2}{\left[ \frac{\Gamma p_1}{\rho_1(1-Z_1)} \right]} = \frac{4(1-Z_1)\mu}{\Gamma p_1 \alpha_1^2 \xi_0^2} = \text{constant} .$$

Since  $M_e$  comes out to be a constant and  $p_1$  can be taken to be of order zero for a very strong shock, we conclude that the shock retains its great strength even for a large time. Hence our solutions obtained in the next section are applicable for any time  $t > \tau$  till  $Z_1$  remains constant,  $\tau$  being the duration of initial impulse.

Also from equations (2.2.7) and (2.2.8), we obtain

$$R = \frac{2}{\alpha_1} \log \frac{t}{\tau} . \quad (2.2.9)$$

## 2.2.2 SOLUTION OF THE EQUATIONS

From equations (2.2.4), (2.2.7), (2.2.8), we obtain

$$\frac{\partial \rho}{\partial t} = \alpha \rho U - u \frac{\partial \rho}{\partial r} , \quad (2.2.10)$$

$$\frac{\partial p}{\partial t} = -U \frac{\partial p}{\partial r} , \quad (2.2.11)$$

$$\frac{\partial u}{\partial t} = uU\lambda - U \frac{\partial u}{\partial r} . \quad (2.2.12)$$

Let us suppose that

$$r' = \frac{r}{R}, \quad u' = \frac{u}{U}, \quad p' = \frac{p}{p_2}, \quad \rho' = \frac{\rho}{\rho_2} . \quad (2.2.13)$$

Using equations (2.2.10) to (2.2.13) in the fundamental equations (2.1.1) to (2.1.3), we obtain

$$\frac{du'}{dr'} = -\frac{u'R\alpha_1}{2(1-u')} + \frac{(1-\beta)\beta}{\rho'(1-u')} \frac{dp'}{dr'} , \quad (2.2.14)$$

$$\frac{1}{\rho'} \frac{d\rho'}{dr'} = \frac{\alpha_1 R}{1-u'} + \frac{1}{1-u'} \frac{du'}{dr'} + \frac{\nu u'}{r'(1-u')}, \quad (2.2.15)$$

$$\frac{1}{p'} \frac{dp'}{dr'} = \frac{\Gamma}{(1-Z)\rho'} \frac{d\rho'}{dr'} - \frac{\Gamma\alpha_1 R}{(1-Z)(1-u')}. \quad (2.2.16)$$

Solving equations (2.2.14) to (2.2.16) for  $\frac{dp'}{dr'}$ ,  $\frac{du'}{dr'}$  and  $\frac{d\rho'}{dr'}$ , we obtain

$$\frac{dp'}{dr'} = \frac{\rho' p' \Gamma \beta}{2r'} \left[ \frac{2\nu u'(1-u') - \alpha_1 R u' r'}{\rho'(\beta - Z_1 \rho')(1-u')^2 - \Gamma \beta^2 p'(1-\beta)} \right], \quad (2.2.17)$$

$$\frac{du'}{dr'} = \frac{p' \Gamma \beta^2 (1-\beta)}{2r'(1-u')} \left[ \frac{2\nu u'(1-u') - \alpha_1 R u' r'}{\rho'(\beta - Z_1 \rho')(1-u')^2 - \Gamma \beta^2 p'(1-\beta)} \right] - \frac{u' \alpha_1 R}{2(1-u')}, \quad (2.2.18)$$

$$\begin{aligned} \frac{d\rho'}{dr'} = & \frac{\alpha_1 R \rho'}{1-u'} + \frac{\rho' p' \Gamma \beta^2 (1-\beta)}{2r'(1-u')^2} \left[ \frac{2\nu u'(1-u') - \alpha_1 R u' r'}{\rho'(\beta - Z_1 \rho')(1-u')^2 - \Gamma \beta^2 p'(1-\beta)} \right] \\ & - \frac{u' \alpha_1 R \rho'}{2(1-u')^2} + \frac{\nu u' \rho'}{(1-u')r'}. \end{aligned} \quad (2.2.19)$$

In terms of dimensionless variables  $r'$ ,  $p'$ ,  $\rho'$  and  $u'$  the shock conditions take the form

$$r' = 1, \quad p' = 1, \quad \rho' = 1, \quad u' = 1 - \beta. \quad (2.2.20)$$

Equations (2.2.17) to (2.2.19) along with the boundary conditions (2.2.20) give the solution of our problem. The solution thus obtained is a non-similar one since the motion behind the shock can be determined only when a definite value for time is prescribed.

### 2.2.3 RESULTS AND DISCUSSION

Distribution of the flow variables  $u' = \frac{u}{U}$ ,  $p' = \frac{p}{p_2}$  and  $\rho' = \frac{\rho}{\rho_2}$  are obtained from equations (2.2.17) to (2.2.19) for spherical shock ( $\nu = 2$ ) at given instants by numerical integration. Runge-Kutta method is used for

numerical integration, starting from the shock front ( $r' = 1$ ) and proceeding inwards. For the purpose of numerical integration the following values of the non-dimensional parameters are used (see Pai et al[1], Miura and Glass[7], Vishwakarma[6])

$$\begin{aligned}\gamma &= 1.4 ; \\ k_p &= 0, 0.1, 0.4 ; \\ G_1 &= 100 ; \\ \beta' &= 1 ; \\ \frac{t}{\tau} &= 2, 4 .\end{aligned}$$

Fig.1 shows that in the initial stages of the motion ( $\frac{t}{\tau} = 2$ ), the velocity  $u'$  increases as we move from shock front to the inner contact surface, but at latter stages ( $\frac{t}{\tau} = 4$ ), it decreases after attaining a maximum. Also, it shows that for small values of  $k_p$ , the values of  $u'$  tend to be those for the corresponding perfect gas.

Fig.2 shows that the pressure  $p'$  decreases as we move from shock front to the inner contact surface. This decrease of pressure is faster for dusty gas in comparison to that for the corresponding perfect gas.

Fig.3 shows that the density  $\rho'$  increases as we move from shock front to the inner contact surface which is in contrast with the corresponding case of perfect gas (Ray and Bhowmick[4]), where it decreases.

Figs.1 to 3 show that the effects of an increase in mass concentration of solid particles  $k_p$  are:

(i) to increase the velocity  $u'$

(ii) to increase the slopes of the pressure and density profiles in the region behind the shock front, and

(iii) to decrease the distance between the inner contact surface and the shock front. This results due to the fact that shock speed is reduced when shock moves in a dusty gas with comparatively higher  $k_p$ .

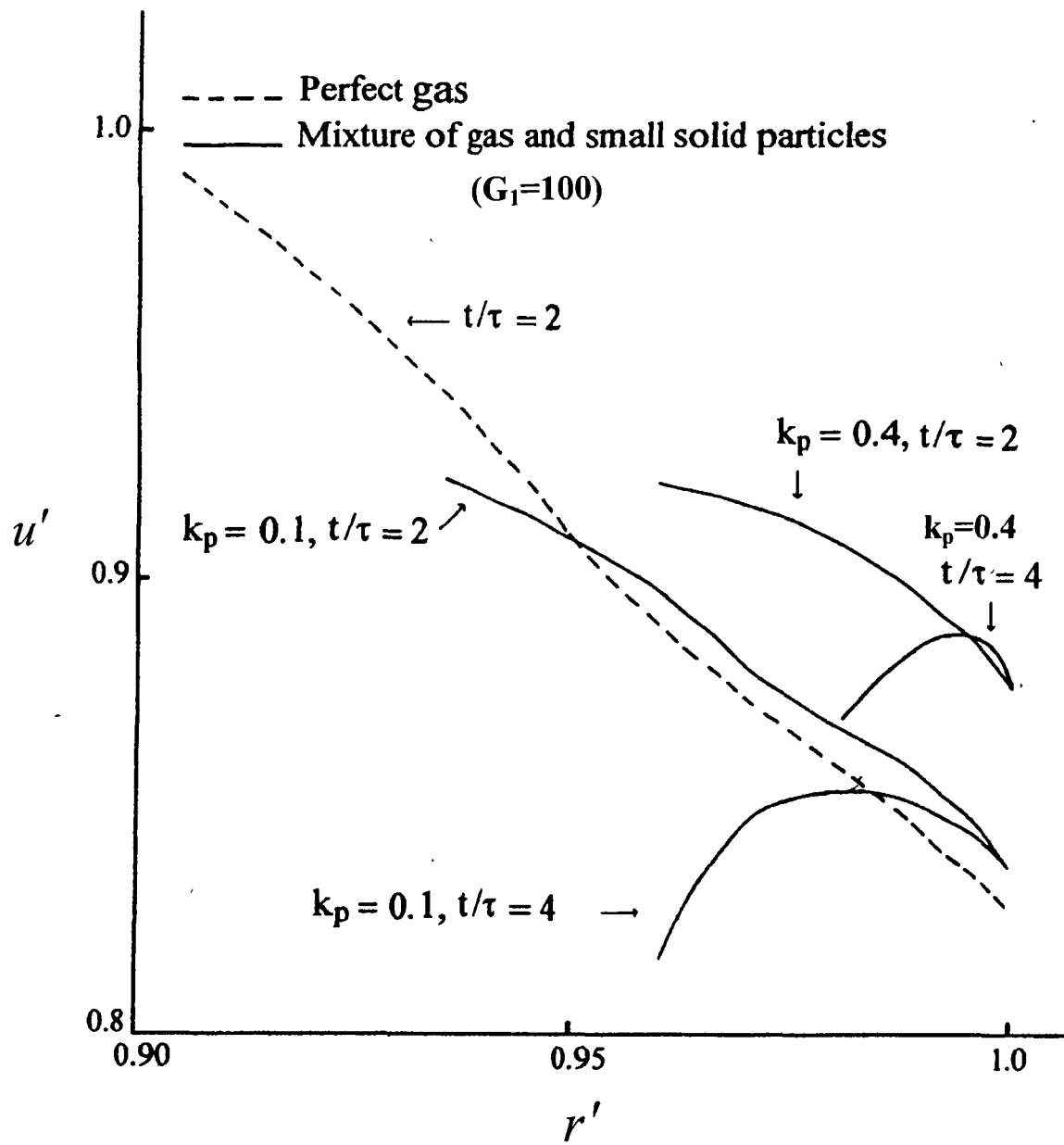


Fig.1 Variation of reduced flow velocity  $u'$  in the region behind the shock front

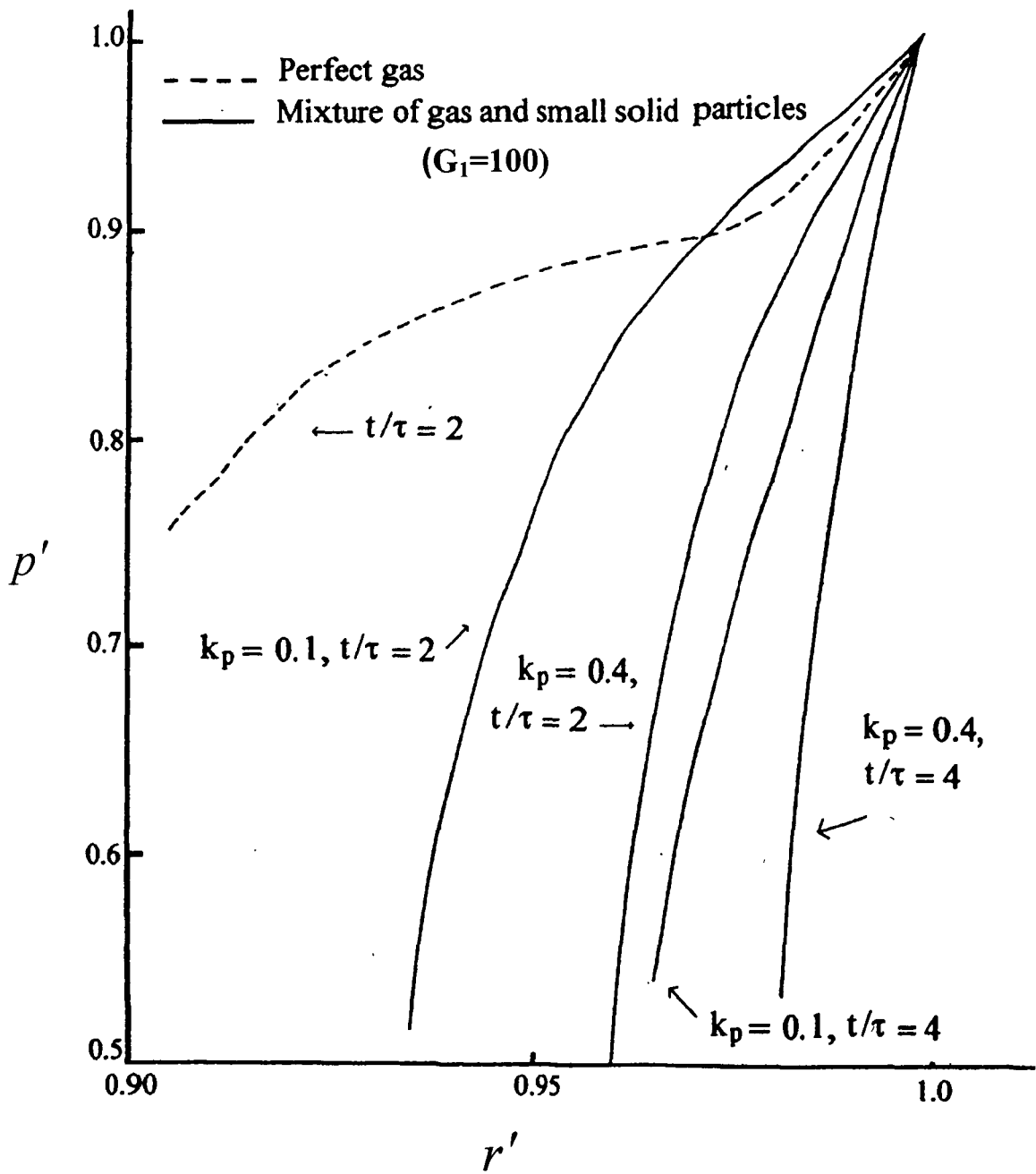


Fig.2 Variation of reduced pressure  $p'$  in the region behind the shock front

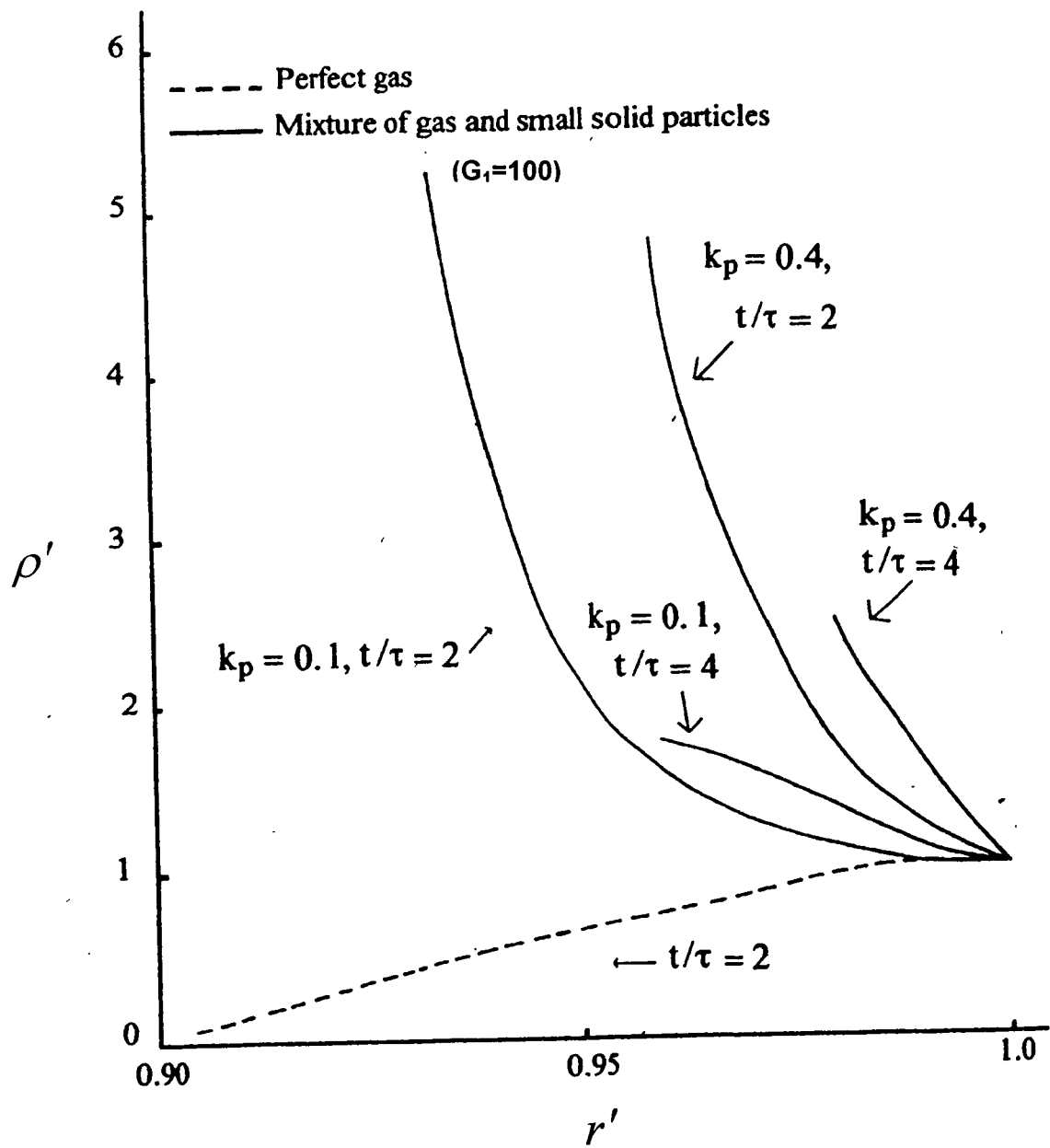


Fig.3 Variation of reduced density  $\rho'$  in the region behind the shock front

# Bibliography

- [1] Pai,S.I., Menon, S. and Fan, Z. Q.: *Similarity solutions of a strong shock wave propagation in mixture of a gas and dusty particles.* Int.J.Engg.Sci.18(12), 1365-1373(1980).
- [2] Sedov, L.I.: *Similarity and Dimensional Methods in Mechanics.* Chap iv, Academic Press, New york (1959).
- [3] Korobeinikov, V.P.: *Problems in the theory of point explosion in Gases.* Proceedings of the Steklov Institute of Mathematics, No. 119, American Mathematical Society (1976).
- [4] Ray, G.D. and Bhowmick, J.B.: *Propagation of cylindrical and spherical explosion waves in an exponential medium.* Def. Sci.J. 9-14 (1974).
- [5] Verma, B.G. and Vishwakarma, J.P.: *Propagation of magnetogasdynamic plane shock waves in an exponential medium.*Nuovo Cim. 32, 267-272(1976).
- [6] Vishwakarma, J.P.:*Propagation of shock waves in a dusty gas with exponentially varying density.* Eur. Phys. J.B 16, 369-372 (2000).

- [7] Miura, H. and Glass, I. I. : *Development of the flow induced by a piston moving impulsively in a dusty gas*. Proc. Roy. SOC. Lond. A 397, 295-305 (1985).
- [8] Naidu, G.N., Venkatanandan,K. and Ranga Rao, M.P.: *Approximate analytical solutions for self-similar flows of a dusty gas with variable energy*. Int.J. Eng.Sci. 23, 39-49 (1985).

## Chapter 3

# SELF-SIMILAR PISTON PROBLEMS IN A DUSTY GAS

In the last chapter we considered the shock waves produced on account of sudden explosion in a medium and studied the propagation of shock waves in a dusty gas with constant and exponentially varying density. In this chapter we consider the shock waves produced by a plane, cylindrically symmetric or spherically symmetric piston and study self-similar motion of a dusty gas displaced by a piston moving according to (i) power law and (ii) exponential law.

## 3.1 A SELF- SIMILAR SOLUTION OF A SHOCK PROPAGATION IN A DUSTY GAS

This section is devoted to study a self-similar flow of a dusty gas behind a shock driven out by a piston moving with time according to power law. The dusty gas is assumed to consists of a mixture of small solid particles and a perfect gas, in which solid particles are continuously distributed. It is assumed that the equilibrium flow-condition is maintained and variable energy input is continuously supplied by the piston. We follow Steiner and Hirschler[1] here.

### 3.1.1 FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

The conservation equations for an unsteady, plane, cylindrically or spherically symmetric flow field between a shock and a piston moving behind it in a mixture of a perfect gas and small solid particles may be written as(Pai[2])

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{\nu \rho u}{r} = 0, \quad (3.1.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0, \quad (3.1.2)$$

$$\frac{\partial U_m}{\partial t} + u \frac{\partial U_m}{\partial r} - \frac{p}{\rho^2} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0, \quad (3.1.3)$$

where  $\nu = 0, 1$  or  $2$  corresponds to plane, cylindrical or spherical symmetry,

$\rho$  is the density of mixture,

$u$  the flow velocity,

$p$  the pressure of mixture,

$U_m$  the internal energy per unit mass of the mixture,

$r$  the distance, and

$t$  the time.

The equation of state of the mixture of perfect gas and small solid particles can be written as (Pai [2], Pai et al [3], Vishwakarma [4]),

$$p = \frac{(1 - k_p)}{(1 - Z)} \rho R^* T, \quad (3.1.4)$$

where  $R^*$  is the gas constant,  $T$  the temperature,  $Z = \frac{V_{sp}}{V}$  the volume fraction and  $k_p = \frac{m_{sp}}{m}$  the mass fraction (concentration) of the solid particles in the mixture, where  $m_{sp}$  and  $V_{sp}$  are the total mass and the volumetric extension of the solid particles and  $V$  and  $m$  are the total volume and total mass of the mixture.

The relation between  $k_p$  and  $Z$  is given by

$$k_p = \frac{Z \rho_{sp}}{\rho}, \quad (3.1.5)$$

where  $\rho_{sp}$  is the species density of solid particles. In equilibrium flow,  $k_p$  is constant in whole flow field. Therefore from equation (3.1.5)

$$\frac{Z}{\rho} = \text{constant}, \quad (3.1.6)$$

as  $\rho_{sp}$  is constant.

Also we have the relation (Pai[2])

$$Z = \frac{k_p}{G(1 - k_p) + k_p}, \quad (3.1.7)$$

where  $G = \frac{\rho_{sp}}{\rho_g}$  is the ratio of the species density of solid particles to the species density of the gas.

The internal energy per unit mass of the mixture may be written as

$$U_m = [k_p C_{sp} + (1 - k_p) C_v] T = C_{vm} T, \quad (3.1.8)$$

where  $C_{sp}$  is the specific heat of the solid particles,  $C_v$  the specific heat of the gas at constant volume and  $C_{vm}$  the specific heat of the mixture at constant volume.

The specific heat of the mixture at constant pressure is

$$C_{pm} = k_p C_{sp} + (1 - k_p) C_p, \quad (3.1.9)$$

where  $C_p$  is the specific heat of the gas at constant pressure.

The ratio of the specific heats of the mixture is given by (pai[2], Pai et al [3], Marble[5])

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \frac{\gamma \left(1 + \frac{\delta \beta'}{\gamma}\right)}{1 + \delta \beta'}, \quad (3.1.10)$$

where  $\gamma = \frac{C_p}{C_v}$ ,  $\delta = \frac{k_p}{1 - k_p}$  and  $\beta' = \frac{C_{sp}}{C_v}$ .

Now,

$$C_{pm} - C_{vm} = (1 - k_p)(C_p - C_v) = (1 - k_p) R^*. \quad (3.1.11)$$

The internal energy per unit mass of the mixture is, therefore, given by

$$U_m = \frac{p(1-Z)}{\rho(\Gamma-1)}. \quad (3.1.12)$$

From the first law of thermodynamics and the equation of state (3.1.4), we may calculate the speed of sound in the mixture, as

$$a = \left( \frac{dp}{d\rho} \right)_S^{\frac{1}{2}} = \left[ \frac{\Gamma p}{\rho(1-Z)} \right]^{\frac{1}{2}}, \quad (3.1.13)$$

where  $\left( \frac{dp}{d\rho} \right)_S$  denotes the derivative of  $p$  with respect to  $\rho$  at constant entropy  $S$ .

From equation (3.1.7) we have

$$Z_1 = \frac{V_{sp}}{V_1} = \frac{k_p}{G_1(1-k_p) + k_p}, \quad (3.1.14)$$

where  $G_1 = \frac{\rho_{sp}}{\rho_{g1}}$  is the ratio of the density of the solid particles to the initial density of the gas, and subscript '1' refers to the values in the initial state.

The compressibility (adiabatic) of the mixture may be calculated as (Moelwyn-Hughes[6])

$$\zeta = -\rho \left( \frac{d(\frac{1}{\rho})}{dp} \right)_S = \frac{1}{\rho a^2} = \frac{1-Z}{\Gamma p}. \quad (3.1.15)$$

The volume fraction of solid particles in the mixture lowers the compressibility of the mixture, while the mass of the solid particles increases the total mass, and thereby may add to the inertia of the mixture. This can be demonstrated in two limiting cases of the mixture at the initial state. For

$G_1 = 1$ , it follows from (3.1.14),(3.1.4) and(3.1.15) that

$$Z_1 = k_p, \quad \rho_1 = \frac{p_1}{R^*T_1} \quad \text{and} \quad \zeta_1 = \frac{1 - k_p}{\Gamma p_1} . \quad (3.1.16)$$

Equation(3.1.16) shows that an increase in the volume fraction of solid particles (which is equal to  $k_p$  for  $G_1 = 1$ ) lowers the compressibility of the mixture in the initial state. In other limiting case, i.e, for  $G_1 \rightarrow \infty$ , the density of the solid particles  $\rho_{sp} \rightarrow \infty$  and the volumetric extension of the solid particles  $V_{sp}$  tends to zero. Therefore  $Z_1$  tends to zero according to equation (3.1.14). Hence it follows from equations (3.1.4) and(3.1.15) that

$$\rho_1 = \frac{p_1}{R^*T_1} \left( \frac{1}{1 - k_p} \right), \quad \zeta_1 = \frac{1}{\Gamma p_1} . \quad (3.1.17)$$

In this case the compressibility is not affected by the presence of solid particles. The solid particles contribute only to increase the mass and inertia of the mixture of solid particles and a perfect gas.

We consider that a strong shock wave, driven by a piston is propagated into a medium (mixture of perfect gas and small solid particles) of constant density  $\rho_1$  at rest ( $u_1 = 0$ ) and with negligibly small counter pressure  $p_1 \cong 0$ . The boundary conditions at the strong shock are as follows,

$$\begin{aligned} \rho_2(U_s - u_2) &= \rho_1 U_s , \\ p_2 + \rho_2(U_s - u_2)^2 &= \rho_1 U_s^2 , \\ U_{m2} + \frac{p_2}{\rho_2} + \frac{1}{2}(U_s - u_2)^2 &= \frac{1}{2}U_s^2 , \\ \frac{Z_2}{\rho_2} &= \frac{Z_1}{\rho_1} , \end{aligned} \quad (3.1.18)$$

where subscript ‘2’ refers to values immediately behind the shock,  $U_s$  is the velocity of the shock front which is a function of time and  $Z_1$  is given by equation (3.1.14).

From shock conditions (3.1.18), we have

$$\begin{aligned} u_2 &= (1 - \beta)U_s, \quad \rho_2 = \frac{\rho_1}{\beta}, \\ p_2 &= (1 - \beta)\rho_1 U_s^2, \quad Z_2 = \frac{Z_1}{\beta}, \end{aligned} \quad (3.1.19)$$

where  $\beta$  is given by the relation

$$\beta = \frac{\Gamma - 1 + 2Z_1}{(\Gamma + 1)}. \quad (3.1.20)$$

### 3.1.2 SELF-SIMILARITY TRANSFORMATION

The inner boundary of the flow-field behind the shock is assumed to be an expanding piston. In the frame work of self-similarity (Sedov[7]) the velocity  $U_p = \frac{dr_p}{dt}$  of the piston is assumed to obey a power law which results in

$$U_p = \frac{dr_p}{dt} = U_o \left( \frac{t}{t_0} \right)^n, \quad (3.1.21)$$

where  $r_p$  is the radius of the piston and  $t_0$  denotes the time at a reference state. Equation (3.1.21) encloses three cases of the formation of a strong shock, where the counter-pressure ahead of the shock can be neglected: for  $n = 0$  the piston instantaneously starts to move at  $t = 0$  with constant velocity  $U_0$ . As a result a strong shock is produced from the beginning, if the piston velocity  $U_0$  is sufficiently high. For  $n > 0$  the piston is continuously accelerated, and a shock is produced which arrives at a strong shock limit

at large times. For  $-1 < n < 0$  the piston velocity suddenly rises at  $t = 0$  from zero to infinite velocity leading to the formation of a strong shock in the initial phase. In the cases of cylindrical and spherical symmetries ( $\nu > 0$ ) we can not take the exponent  $n \leq -1$  for physical reasons. In regard to the shock boundary condition self-similarity requires that the velocity of the shock  $U_s = \frac{dr_s}{dt}$  is proportional to the velocity of the piston, that is

$$U_s = \frac{dr_s}{dt} = CU_0 \left( \frac{t}{t_0} \right)^n, \quad (3.1.22)$$

where  $r_s$  is the shock radius and  $C$  is a constant.

Using (3.1.22) the time and space co-ordinate can be transformed into a dimensionless self-similarity variable  $\eta$  as

$$\eta = \frac{r}{r_s} = \left[ \frac{(n+1)t_0^n}{U_0 C} \right] \left( \frac{r}{t^{n+1}} \right). \quad (3.1.23)$$

The variable  $\eta$  assumes the value '1' at the shock front and  $\eta_p = \frac{r_p}{r_s}$  at the piston. To obtain the similarity solutions, we write the unknown variables in the following form (Steiner and Hirschler [1]),

$$\begin{aligned} u &= \frac{r}{t} V(\eta), \quad \rho = \rho_1 D(\eta), \\ p &= \rho_1 \frac{r^2}{t^2} P(\eta), \quad Z = Z_1 D(\eta), \end{aligned} \quad (3.1.24)$$

where  $V$ ,  $D$  and  $P$  are functions of  $\eta$  only.

With the help of equations (3.1.24), equations(3.1.1) to (3.1.3) can be transformed and simplified to

$$[V - (n+1)] \frac{dD}{d\eta} + D \frac{dV}{d\eta} + (\nu+1) \frac{DV}{\eta} = 0, \quad (3.1.25)$$

$$[V - (n + 1)] \frac{DV}{D\eta} + \frac{1}{D} \frac{dP}{d\eta} + \frac{V(V - 1)}{\eta} + \frac{2P}{D\eta} = 0, \quad (3.1.26)$$

$$\frac{dP}{d\eta} - \frac{\Gamma P}{D(1 - Z_1 D)} \frac{dD}{d\eta} + \frac{2P(V - 1)}{\eta[V - (n + 1)]} = 0. \quad (3.1.27)$$

Solving equations (3.1.25) to (3.1.27) for  $\frac{dV}{d\eta}$ ,  $\frac{dP}{d\eta}$  and  $\frac{dD}{d\eta}$ , we get

$$\frac{dV}{d\eta} = \frac{-[V - (n + 1)] dD}{D} \frac{dD}{d\eta} - (\nu + 1) \frac{V}{\eta}, \quad (3.1.28)$$

$$\frac{dP}{d\eta} = [V - (n + 1)]^2 \frac{dD}{d\eta} + \frac{DV}{\eta} [\nu\{V - (n + 1)\} - n] - \frac{2P}{\eta}, \quad (3.1.29)$$

$$\frac{dD}{d\eta} = \frac{2P - DV[\nu\{V - (n + 1)\} - n] - \frac{2P(V - 1)}{V - (n + 1)}}{\eta[N + \{V - (n + 1)\}^2]}, \quad (3.1.30)$$

where

$$N = \frac{-\Gamma P}{D(1 - Z_1 D)}. \quad (3.1.31)$$

Using equations (3.1.24), the shock conditions (3.1.19) transformed into

$$V(1) = (1 - \beta)(n + 1), \quad D(1) = \frac{1}{\beta},$$

$$P(1) = (1 - \beta)(n + 1)^2. \quad (3.1.32)$$

The piston's path coincides at  $\eta_p = \frac{r_p}{r_s}$  with a particle path. Using equations (3.1.21) and (3.1.24) the relation

$$V(\eta_p) = n + 1 \quad (3.1.33)$$

can be derived. In addition to the shock conditions (3.1.32) the kinematic condition (3.1.33) at piston surface must be satisfied.

The ordinary differential equations (3.1.28) to (3.1.30) with boundary conditions (3.1.32) can now be numerically integrated to obtain the solution for the flow behind the shock surface.

The individual flow variables  $u, p$  and  $\rho$  can be related to their corresponding values immediately behind the shock, which are denoted by the subscript '2', as follows

$$\frac{u}{u_2} = \frac{V(\eta)}{V(1)} \eta, \quad \frac{\rho}{\rho_2} = \frac{D(\eta)}{D(1)}, \quad \frac{p}{p_2} = \frac{P(\eta)}{P(1)} \eta^2. \quad (3.1.34)$$

### 3.1.3 RESULTS AND DISCUSSION

Distribution of the flow variables between the shock surface ( $\eta = 1$ ) and the piston ( $\eta = \eta_p$ ) are obtained from equations (3.1.28), (3.1.29) and (3.1.30) using Runge - Kutta method. For the purpose of numerical calculations, values of the constant parameters are taken to be (Miura and Glass[8], Pai et al[3], Vishwakarma[4] )  $\nu = 2$ ;  $\gamma = 1.4$ ;  $\beta' = 1$ ;  $k_p = 0.2$ ;  $G_1 = 10$ ;  $n = -0.2, 0, 2$ . The value  $\nu = 2$  corresponds to spherical shocks. Also,  $n = -0.2$  corresponds to a decelerated piston,  $n = 0$  to the constant velocity piston and  $n = 2$  to an accelerated piston.

Figs. 1 - 3 show the variation of the flow variables  $\frac{u}{u_2}$ ,  $\frac{\rho}{\rho_2}$  and  $\frac{p}{p_2}$  for the cases  $n = -0.2$ ,  $n = 0$  and  $n = 2$  for  $G_1 = 10$  and  $k_p = 0.2$ . As can be seen from equation (3.1.30) for non-dimensional density  $D$ , there is

a singularity at the piston, where  $V = n + 1$ , because the equation for  $D$  becomes singular there. In the constant piston velocity case associated with  $n = 0$  this singularity is removable and a finite solution for  $D(= \frac{\rho}{\rho_2} D(1))$  is obtained as shown in fig.2. In the cases of accelerated ( $n = 2$ ) and decelerated ( $n = -0.2$ ) piston, the singularity is non-removable, and the derivative of the density tends to positive and negative infinity, respectively, as shown in figs. 3 and 1. This singularity can be physically interpreted as follows (Steiner and Hirschler[1]): the path of the accelerated piston converges with the path of the particle immediately ahead condensing the gas to infinity whereas the path of the decelerated piston diverges from the path of the particle immediately ahead rarefying the gas.

Figs. 1 to 3 also show that the pressure and the velocity radially decrease due to spherical geometry, except for the decelerated piston, where the pressure radially increases (fig.1). In the latter the decrease in pressure due to geometry is not compensated by the compression of the dust.

It is also found that the effects of an increase in the value of  $n$  are to increase the flow variables  $\frac{u}{u_2}$ ,  $\frac{\rho}{\rho_2}$  and  $\frac{p}{p_2}$  and to decrease the distance of the piston from the shock front.

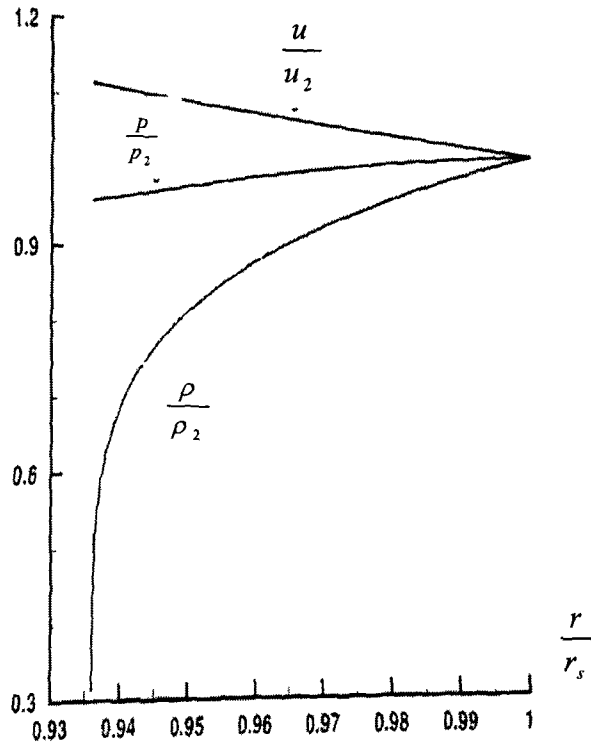


Fig.1 The radial profiles of the velocity  $u$ , the pressure  $p$  and the density  $\rho$  for an decelerated piston ( $n = -0.2$ ). All quantities are related to their corresponding values  $u_2$ ,  $p_2$ ,  $\rho_2$  and  $r_s$  at the shock

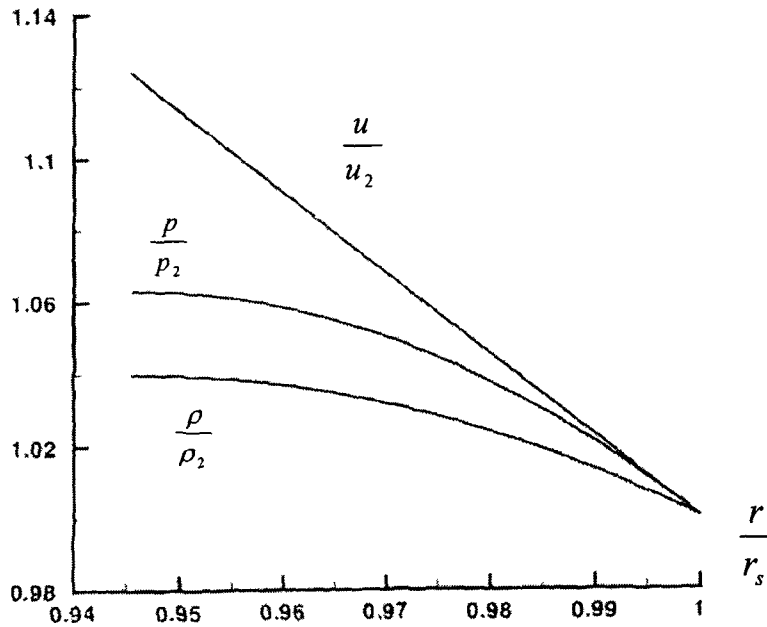


Fig.2 The radial profiles of the velocity  $u$ , the pressure  $p$  and density  $\rho$  for an constant velocity piston ( $n = 0$ ). All quantities are related to their corresponding values  $u_2$ ,  $p_2$ ,  $\rho_2$  and  $r_s$  at the shock

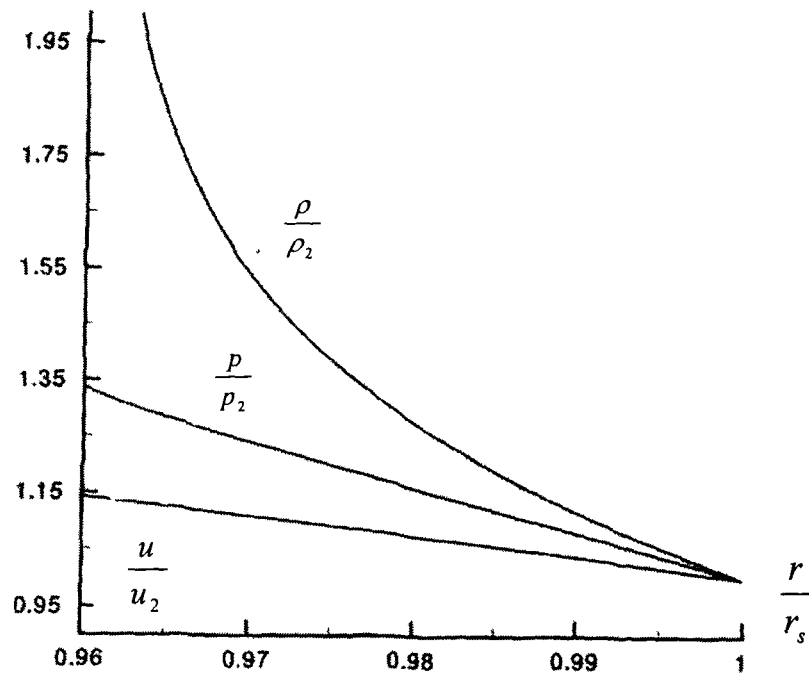


Fig.3 The radial profiles of the velocity  $u$ , the pressure  $p$  and the density  $\rho$  for an accelerated piston ( $n = 2$ ). All quantities are related to their corresponding values  $u_2$ ,  $p_2$ ,  $\rho_2$  and  $r_s$  at the shock.

## 3.2 SIMILARITY SOLUTIONS FOR THE FLOW BEHIND AN EXPONENTIAL SHOCK IN A DUSTY GAS

In this section, we present similarity solutions for the flow of a dusty gas behind a strong exponential shock driven out by a piston moving with time according to an exponential law. We follow Vishwakarma and Nath[9] here.

### 3.2.1 INTRODUCTION

Ranga Rao and Ramana [10] obtained approximate analytical solutions for the problem of unsteady self-similar motion of a perfect gas displaced by a piston moving according to an exponential law. Higashino[11] investigated the propagation of shock waves in a dusty gas by characteristic method by applying the motion of a piston (plane, cylindrical or spherical) obeying an exponential law. In the present study, we investigate the one-dimensional unsteady self-similar flow of dusty gas behind a strong shock driven out by a cylindrical or spherical piston moving with time according to an exponential law, namely

$$r_p = A \exp(\lambda t), \quad \lambda > 0, \quad (3.2.1)$$

where  $r_p$  is the radius of the piston,  $A$  and  $\lambda$  are dimensional constants and  $t$  is the time.

Since we are concerned with self-similar motions, we may assume that

$$R = B \exp(\lambda t), \quad (3.2.2)$$

where  $R$  is the shock radius, and  $B$  a dimensional constant which is to be determined.

In order to get some features of the shock propagation, small solid particles are considered as a pseudo-fluid, and it is assumed that the equilibrium flow condition is maintained in the flow-field, and that the viscous stress and heat conduction of the mixture are negligible (Pai et al[3]).

We study both the cases, when the flow between the shock and the piston is adiabatic or isothermal. The assumption of isothermal flow is physically realistic, when radiation heat transfer effects are implicitly present. As the shock propagates, the temperature behind it increases and becomes very large so that there is intense transfer of energy by radiation. This causes the temperature gradient to approach zero, i.e. the dependent temperature tends to become uniform behind the shock front and flow becomes isothermal (Korobeinikov[12], Laumbach and Probst[13], Sachdev and Ashraf[14]). With this assumption we obtain, in sections 3.2.2 and 3.2.3, the similarity solutions. In section 3.2.4, we present the solutions for the flow taken to be adiabatic.

A comparative study between the solutions of isothermal and adiabatic flows will be made in section 3.2.5.

### **3.2.2 FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS-ISOTHERMAL FLOW**

The fundamental equations for one-dimensional, unsteady and isothermal flow of a mixture of a gas and small solid particles can be written as (Pai et

al[3], Zhuravskaya and Levin[15], Sachdev and Ashraf[14]),

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0, \quad (3.2.3)$$

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{\nu \rho u}{r} = 0, \quad (3.2.4)$$

$$\frac{\partial T}{\partial r} = 0, \quad (3.2.5)$$

where  $\nu = 1$  or  $2$  corresponds to the cylindrical or spherical symmetry,  $\rho$ ,  $u$ ,  $p$ , and  $T$  are the density, the flow velocity, the pressure and the temperature of the mixture, and  $r$  and  $t$  are the distance and time.

The equation of state of the mixture of perfect gas and small solid particles can be written as (Pai[2], pai et al[3], Vishwakarma[4]),

$$p = \frac{(1 - k_p)}{(1 - Z)} \rho R^* T, \quad (3.2.6)$$

where  $R^*$  is the gas constant,  $k_p$  the mass concentration of solid particles and  $Z$  the volume fraction of solid particles in the mixture.

The relation between  $k_p$  and  $Z$  is given by

$$k_p = \frac{Z \rho_{sp}}{\rho},$$

where  $\rho_{sp}$  is the species density of solid particles.

Like section 3.1.1, the internal energy per unit mass of the mixture is

$$U_m = \frac{p(1 - Z)}{\rho(\Gamma - 1)}, \quad (3.2.7)$$

wher  $\Gamma$ , the ratio of the specific heats of the mixture, is given by

$$\Gamma = \gamma \frac{(1 + \delta\beta'/\gamma)}{(1 + \delta\beta')}, \quad (3.2.8)$$

where  $\gamma = \frac{C_p}{C_v}$ ,  $\delta = \frac{k_p}{1 - k_p}$  and  $\beta' = \frac{C_{sp}}{C_v}$ .

Also, we have the relation

$$Z = \frac{k_p}{(1 - k_p)G + k_p}, \quad (3.2.9)$$

where  $G = \frac{\rho_{sp}}{\rho_g}$  is the ratio of the species density of solid particles to the species density of the gas.

The laws of conservation of mass, momentum and energy across the shock front propagation with velocity  $U \left( = \frac{dR}{dt} \right)$  into the mixture of perfect gas and small solid particles at rest ( $u_1 = 0$ ) give the following shock conditions

$$\rho_2(U - u_2) = \rho_1 U = m_s \text{ (say) },$$

$$p_2 - p_1 = m_s u_2,$$

$$U_{m_2} + \frac{p_2}{\rho_2} + \frac{1}{2}(U - u_2)^2 - \frac{F_2}{m_s} = U_{m_1} + \frac{p_1}{\rho_1} + \frac{1}{2}U^2 - \frac{F_1}{m_s},$$

$$\frac{Z_2}{\rho_2} = \frac{Z_1}{\rho_1}, \quad (3.2.10)$$

where the suffices '1' and '2' refer to the values just ahead and just behind the shock front, respectively, and  $F$  is the radiation heat flux. If the shock is strong,

$$p_1 = U_{m_1} = 0. \quad (3.2.11)$$

Then from the shock conditions (3.2.11), we get

$$u_2 = (1 - \beta)U, \quad \rho_2 = \frac{\rho_1}{\beta},$$

$$p_2 = (1 - \beta)\rho_1 U^2, \quad Z_2 = \frac{Z_1}{\beta}, \quad (3.2.12)$$

where  $\beta$  ( $0 < \beta < 1$ ) is given by the relation

$$\frac{\beta - Z_1}{(\Gamma - 1)} - \frac{1}{2}(1 - \beta) = \frac{F_2 - F_1}{p_2 U}. \quad (3.2.13)$$

As the shock is strong,  $F_2 - F_1$  is negligible in comparison with the product of  $p_2$  and  $U$  (Laumbach and Probstein [13]). Therefore, equation (3.2.13) reduces to

$$\beta = \frac{\Gamma - 1 + 2Z_1}{(\Gamma + 1)}. \quad (3.2.14)$$

The expression for the initial volume fraction of the solid particles  $Z_1$  is given by, from equation (3.2.9)

$$Z_1 = \frac{k_p}{(1 - k_p)G_1 + k_p}, \quad (3.2.15)$$

where  $G_1 = \frac{\rho_{sp}}{\rho_{g1}}$ , is the ratio of the species density of solid particles to the initial species density of the gas in the mixture.

Equation (3.2.6) together with equation (3.2.5) give

$$\frac{p}{p_2} = \frac{(1 - Z_2)\rho}{(1 - Z)\rho_2}. \quad (3.2.16)$$

### 3.2.3 SIMILARITY SOLUTIONS

For the problem under consideration we take the similarity transformation in the following form

$$\begin{aligned} u &= UV(\eta), \quad \rho = \rho_1 D(\eta), \\ p &= \rho_1 U^2 P(\eta), \quad Z = Z_1 D(\eta), \end{aligned} \quad (3.2.17)$$

where  $V, D$  and  $P$  are functions of the non-dimensional variable (similarity variable)  $\eta = \frac{r}{R}$  only. The variable  $\eta$  assumes the value '1' at the shock front

and  $\eta_p$  on the piston. Equations (3.2.1), (3.2.2) and the relation  $\eta_p = \frac{r_p}{R}$  yield a relation between A and B in the form

$$A = B\eta_p . \quad (3.2.18)$$

Equation (3.2.16) with the aid of equations (3.2.17) and (3.2.12) yield a relation between  $P$  and  $D$  in form

$$P(\eta) = \frac{(1 - \beta)(\beta - Z_1)D(\eta)}{1 - Z_1D(\eta)} . \quad (3.2.19)$$

By use of equations (3.2.17), equations (3.2.3), (3.2.4) and (3.2.19) can be transformed to

$$(V - \eta)\frac{dV}{d\eta} + \frac{1}{D}\frac{dP}{d\eta} + V = 0 , \quad (3.2.20)$$

$$(V - \eta)\frac{dD}{d\eta} + D\frac{dV}{d\eta} + \frac{\nu DV}{\eta} = 0 , \quad (3.2.21)$$

$$\frac{dP}{d\eta} = \frac{(1 - \beta)(\beta - Z_1)}{(1 - Z_1D)^2} \frac{dD}{d\eta} . \quad (3.2.22)$$

Solving equations (3.2.20), (3.2.21) and (3.2.22) for  $\frac{dD}{d\eta}$  and  $\frac{dV}{d\eta}$ , we obtain

$$\frac{dD}{d\eta} = \frac{[(V - \eta)\nu - \eta]DV(1 - Z_1D)^2}{\eta[(1 - \beta)(\beta - Z_1) - (V - \eta)^2(1 - Z_1D)^2]} , \quad (3.2.23)$$

$$\frac{dV}{d\eta} = \frac{[V(V - \eta)(1 - Z_1D)^2 - (1 - \beta)(\beta - Z_1)\frac{\nu V}{\eta}]}{[(1 - \beta)(\beta - Z_1) - (V - \eta)^2(1 - Z_1D)^2]} . \quad (3.2.24)$$

The shock conditions (3.2.12) with the help of equations (3.2.17) take the form

$$V(1) = (1 - \beta), \quad D(1) = \frac{1}{\beta}, \quad P(1) = (1 - \beta). \quad (3.2.25)$$

In addition to the shock conditions (3.2.25), the condition to be satisfied at the piston surface is that velocity of the fluid is equal to the velocity of the piston itself.

Now the velocity of the fluid at the piston surface is

$$u_p = mRV(\eta_p)$$

and the velocity of the piston is

$$\frac{dr_p}{dt} = m r_p.$$

Therefore,

$$u_p = \frac{dr_p}{dt}$$

gives

$$V(\eta_p) = \eta_p. \quad (3.2.26)$$

Now, equations (3.2.23) and (3.2.24) can be numerically integrated to obtain the solution of the problem.

### 3.2.4 ADIABATIC FLOW

In this section, we present the similarity solution for the adiabatic flow behind a strong shock driven out by a cylindrical or spherical piston moving

according to the exponential law (3.2.1). The strong shock conditions, which serve as the boundary conditions for the problem, are given by (Pai et al[3])

$$\begin{aligned}
 u_2 &= \frac{2(1 - Z_1)U}{(\Gamma + 1)}, \\
 \rho_2 &= \frac{(\Gamma + 1)}{(\Gamma - 1 + 2Z_1)}\rho_1, \\
 p_2 &= \frac{2(1 - Z_1)}{(\Gamma + 1)}\rho_1 U^2, \\
 Z_2 &= \frac{(\Gamma + 1)}{(\Gamma - 1 + 2Z_1)}Z_1, \tag{3.2.27}
 \end{aligned}$$

which are the same as given by equations (3.2.12) and (3.2.14) in the case of isothermal flow. For adiabatic flow, equation (3.2.5) is replaced by

$$\frac{\partial U_m}{\partial t} + u \frac{\partial U_m}{\partial r} - \frac{p}{\rho^2} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0. \tag{3.2.28}$$

By use of equations (3.2.17), equations (3.2.3), (3.2.4) and (3.2.28) can be transformed to

$$(V - \eta) \frac{dV}{d\eta} + \frac{1}{D} \frac{dP}{d\eta} + V = 0, \tag{3.2.29}$$

$$(V - \eta) \frac{dD}{d\eta} + D \frac{dV}{d\eta} + \frac{\nu DV}{\eta} = 0, \tag{3.2.30}$$

$$(V - \eta) \frac{dP}{d\eta} - \frac{\Gamma P(V - \eta)}{D(1 - Z_1 D)} \frac{dD}{d\eta} + 2P = 0. \tag{3.2.31}$$

Solving equations (3.2.29) , (3.2.30), and (3.2.31) for  $\frac{dV}{d\eta}$ ,  $\frac{dD}{d\eta}$  and  $\frac{dP}{d\eta}$ , we obtain

$$\frac{dV}{d\eta} = \frac{\nu\Gamma PV + 2P(1 - Z_1D)\eta - (V - \eta)VD(1 - Z_1D)\eta}{\eta[(V - \eta)^2D(1 - Z_1D) - \Gamma P]}, \quad (3.2.32)$$

$$\frac{dD}{d\eta} = \frac{D[\nu\Gamma PV + 2P(1 - Z_1D)\eta - \eta(V - \eta)VD(1 - Z_1D)]}{\eta[(V - \eta)^2D(1 - Z_1D) - \Gamma P](\eta - V)} - \frac{\nu DV}{\eta(V - \eta)}, \quad (3.2.33)$$

$$\frac{dP}{d\eta} = \frac{[(V - \eta)D\{\nu\Gamma PV + 2\eta P(1 - Z_1D) - (V - \eta)VD(1 - Z_1D)\eta\}]}{\eta[(V - \eta)^2D(1 - Z_1D) - \Gamma P]}. \quad (3.2.34)$$

The shock conditions (3.2.27) with the aid of equations (3.2.17) take the form

$$V(1) = \frac{2(1 - Z_1)}{\Gamma + 1}, \quad D(1) = \frac{\Gamma + 1}{\Gamma - 1 + 2Z_1}, \quad P(1) = \frac{2(1 - Z_1)}{\Gamma + 1}. \quad (3.2.35)$$

In addition to the shock conditions (3.2.35), the kinematic condition (3.2.26) at the piston surface must be satisfied.

The ordinary differential equations (3.2.32) to (3.2.34) can now be numerically integrated to obtain the solutions for the adiabatic flow behind the shock surface .

### 3.2.5 RESULTS AND DISCUSSION

Distribution of the flow variables between the shock surface ( $\eta = 1$ ) and the piston ( $\eta = \eta_p$ ) are obtained from equations (3.2.19), (3.2.23) and(3.2.24)

for isothermal flow, and from equations (3.2.32) to (3.2.34) for adiabatic flow. For the purpose of numerical calculations, the values of the constant parameters are taken to be (Pai et al[3], Miura and Glass[8], Vishwakarma[4])  $\nu = 2$ ;  $\gamma = 1.4$ ;  $\beta' = 1$ ;  $k_p = 0, 0.2, 0.4$ ; and  $G_1 = 1, 10, 100$ . The value  $\nu = 2$  corresponds to spherical shocks and the value  $k_p = 0$  to the dust free case.

Figs. 1 to 3 show the variation of the non-dimensional flow variables  $V, D$  and  $P$  in isothermal and adiabatic cases.

It is found that the effects of an increase in the value of  $G_1$  are

- (i) to increase the flow variables  $V, D$  and  $P$  in the flow field behind the shock (see figs. 1 to 3);
- (ii) to decrease the distance between the shock front and the piston; and
- (iii) to decrease  $\beta$  (i.e. to increase the shock strength) (see table 1).

Effects of an increase in the value of  $k_p$  are as follows:

- (i) to decrease the flow - variables  $V, D$  and  $P$ , when  $G_1 = 1, 10$  and to increase them when  $G_1 = 100$ ,
- (ii) to increase the distance of piston from the shock front when  $G_1 = 1$ . At higher values of  $G_1$ , the effect is small and of opposite nature (see table 1); and
- (iii) to increase  $\beta$  (i.e. to decrease the shock strength) when  $G_1 = 1$ , and to increase it, when  $G_1 \geq 50$ . the strength of the shock is almost unaltered by an increase in  $k_p$ , when  $G_1 = 10$ .

The above effects are found in both the cases, whether the flow is isothermal or adiabatic.

In the adiabatic case, there is an unbounded density distribution near the piston in some cases whereas in isothermal case, the density is finite at the piston for all values of  $k_p$  and  $G_1$ . Thus the assumption of zero temperature gradient brings a profound change in the density distribution as compared to that of the adiabatic flow (except the case when  $G_1 = 1$ ); whereas the pressure and velocity distributions are little affected.

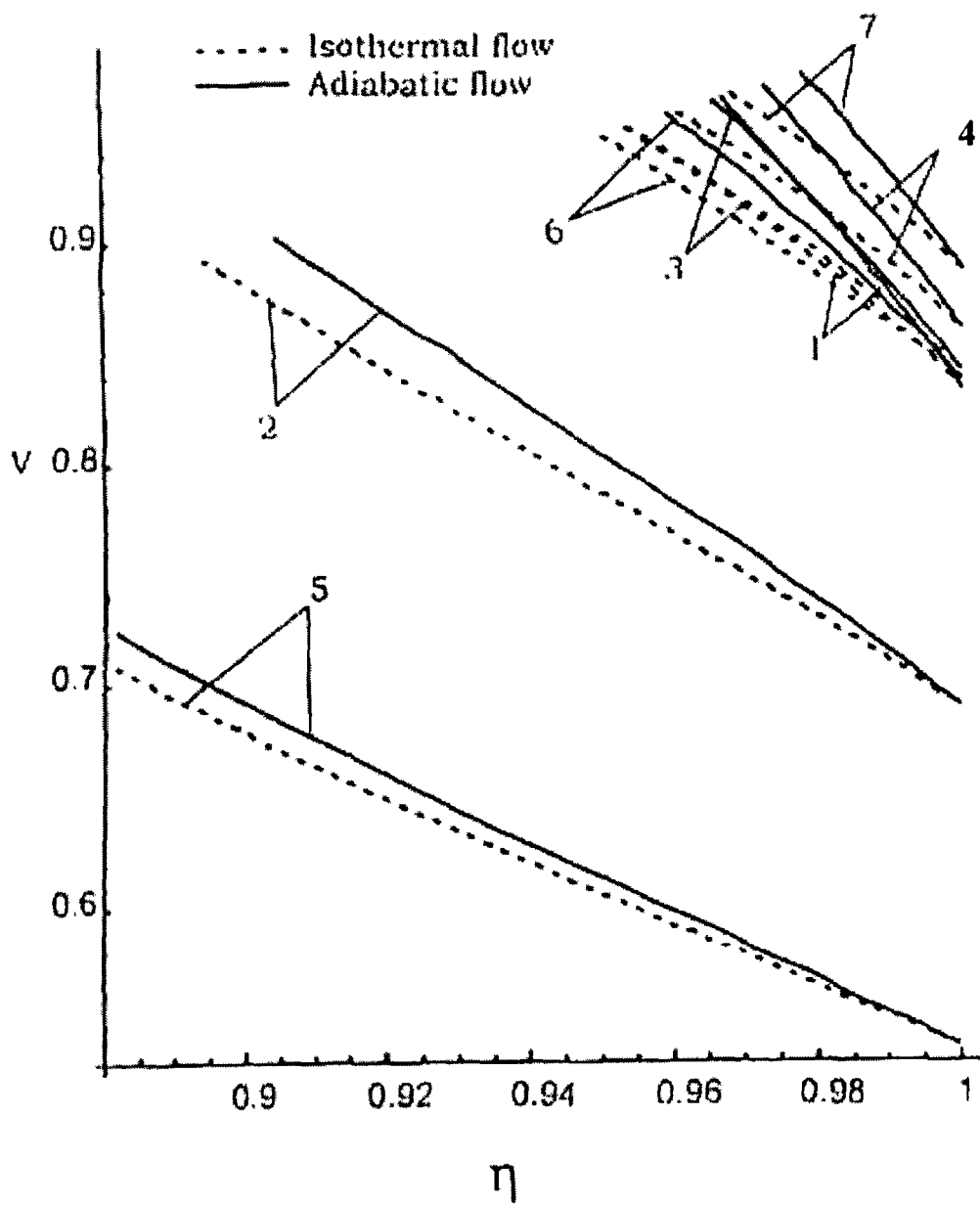


Fig. 1. Velocity distribution in the region behind the shock front  
 (1)  $k_p=0$  (perfect gas); (2)  $k_p=0.2, G_1=1$ ; (3)  $k_p=0.2, G_1=10$ ; (4)  $k_p=0.2, G_1=100$ ;  
 (5)  $k_p=0.4, G_1=1$ ; (6)  $k_p=0.4, G_1=10$ ; (7)  $k_p=0.4, G_1=100$

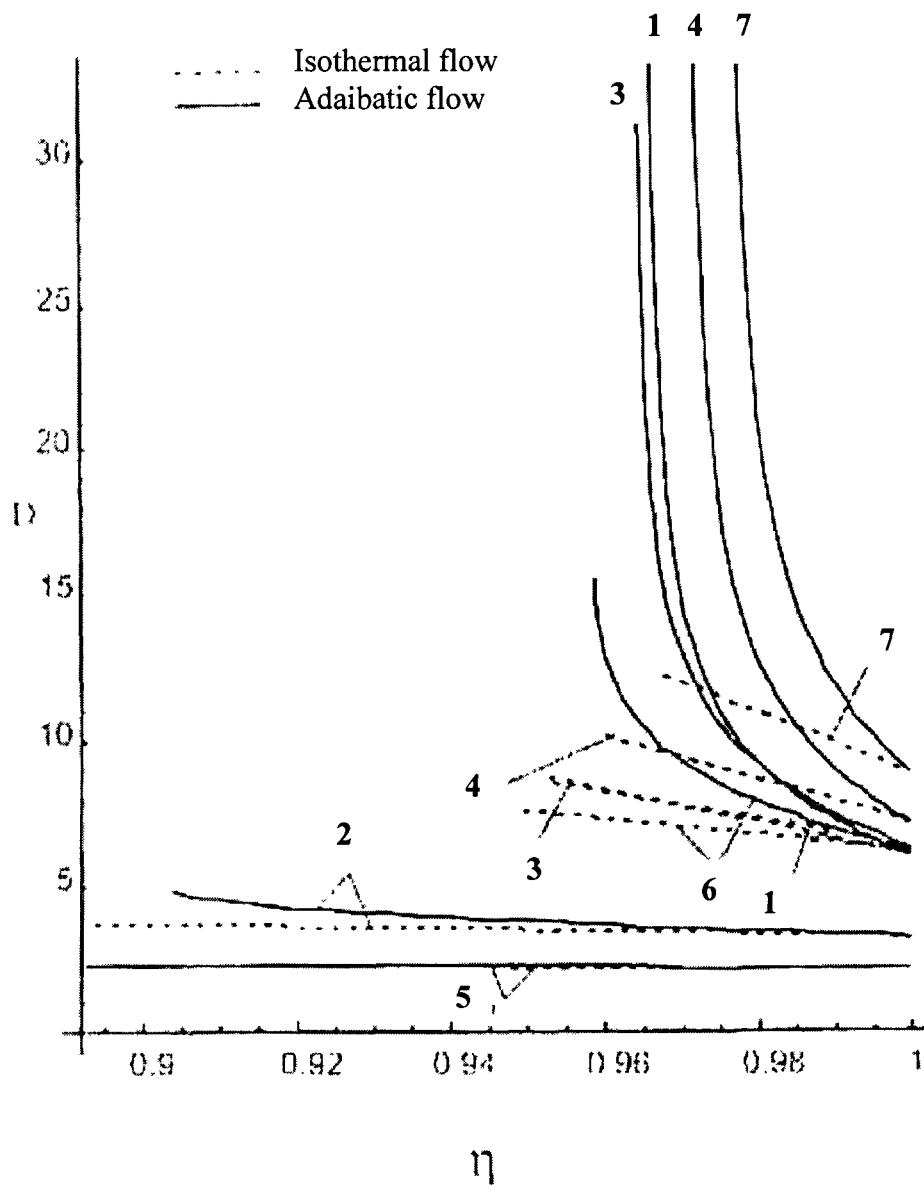


Fig. 2. Density distribution in the region behind the shock front  
 (1)  $k_p=0$  (perfect gas); (2)  $k_p=0.2$ ,  $G_1=1$ ; (3)  $k_p=0.2$ ,  $G_1=10$ ; (4)  $k_p=0.2$ ,  $G_1=100$ ;  
 (5)  $k_p=0.4$ ,  $G_1=1$ ; (6)  $k_p=0.4$ ,  $G_1=10$ ; (7)  $k_p=0.4$ ,  $G_1=100$

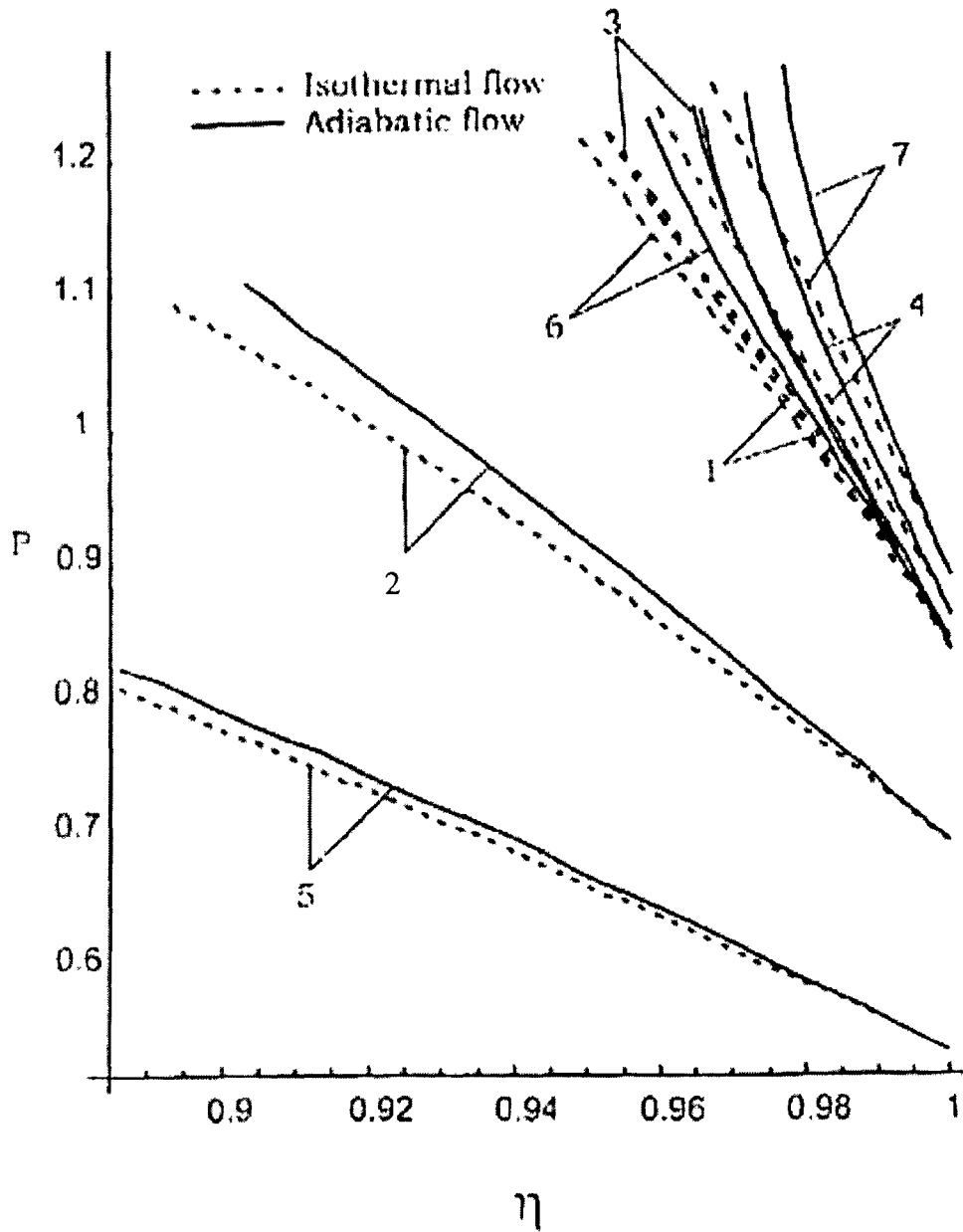


Fig. 3. Pressure distribution in the region behind the shock front  
 (1)  $k_p = 0$  (perfect gas); (2)  $k_p = 0.2$ ,  $G_1 = 1$ ; (3)  $k_p = 0.2$ ,  $G_1 = 10$ ; (4)  $k_p = 0.2$ ,  $G_1 = 100$ ;  
 (5)  $k_p = 0.4$ ,  $G_1 = 1$ ; (6)  $k_p = 0.4$ ,  $G_1 = 10$ ; (7)  $k_p = 0.4$ ,  $G_1 = 100$

# Bibliography

- [1] Steiner, H. and Hirschler, T.: *A self - similar solution of a shock propagation in a dusty gas*. Eur. J. Mech.B 21, 371 - 380 (2002).
- [2] Pai, S.I. : *Two phase flows*. Vieweg tracts in pure and applied physics, Vol.3, Vieweg, Brauns chweig, Chap v (1977)
- [3] Pai, S.I., Menon, S. and Fan, Z.Q.: *Similarity solution of a strong shock wave propagation in a mixture of a gas and dusty particle*. Int. J.Eng.Sci.18,1365 - 1373 (1980).
- [4] Vishwakarma, J.P.: *Propagation of shock waves in a dusty gas with exponentially varying density*. Eur. Phys. J . B 16, 369 - 372 (2000).
- [5] Marble, F. E.: *Dynamics of dusty gases*. Rev. Fluid Mech. 2, 397 - 446(1970).
- [6] Moelwyn-Hughes : *Physical Chemistry*. Pergamon Press, London (1961).
- [7] Sedov, L.I.: *Similarity and Dimensional Methods in Mechanics*. Academic Press, New York, Chap iv (1959).

- [8] Miura, H. and Glass, I. I. : *Development of the flow induced by a piston moving impulsively in a dusty gas*. Proc. Roy. Soc. Lond. A 397, 295-305 (1985).
- [9] Vishwakarma, J. P. and Nath, G.: *Similarity solutions for unsteady flow behind an exponential shock in a dusty gas*. Phys. Scr. 74, 493 - 498 (2006).
- [10] Ranga Rao, M. P. and Ramanna, B.V.: *Unsteady flow of a gas behind an exponential shock*. J. Math. Phys Sci. 10, 465 - 476 (1976).
- [11] Higashino, F.: *Characteristic method applied to blast waves in a dusty gas*. Z. Naturforsch 38a, 399- 406 (1983).
- [12] Korobeinikov, V. P. : *Problems in the theory of point explosion in gases*, In: Proceedings of the Steklov institute of Mathematics , No - 119. American Mathematical Society, Providence, R I (1976).
- [13] Lambach, D.D. and Probstein, R. F.: *Self - similar strong shocks with radiation in a decreasing exponential atmosphere*. Phys. Fluids 13, 1178 - 1183 (1970).
- [14] Sachdev, P.L. and Ashraf, S.: *Converging spherical and cylindrical shocks with zero temperature gradient in the rear flow - field*. J . Appl. Math. Phys. (ZAMP) 22, 1095 - 1102 (1971).
- [15] Zhuravskaya, T.A. and Levin, V.A. : *The propagation of converging and diverging shock waves under intense heat exchange conditions*. J. Appl. Math.Mech. 60, 745 - 752 (1996).

## Chapter 4

# PROPAGATION OF SHOCK WAVES IN A DUSTY GAS WITH INTERNAL HEAT TRANSFER EFFECTS

The influence of radiation on a shock wave and on the flow field behind the shock front has always been of great interest, in the field of nuclear power and space research. Consequently explosion and piston problems with radiation heat transfer effects have been studied (Elliot[1], Wang[2], Helliwell[3], Ghoniem et al[4]). In this chapter we shall study the following two types of problem:

(i) Propagation of shock waves in a dusty gas with radiation heat flux and exponentially varying density; and

(ii) Propagation of shock waves in a dusty gas with heat conduction and radiation heat flux.

## 4.1 PROPAGATION OF SHOCK WAVES IN A DUSTY GAS WITH RADIATION HEAT FLUX AND EXPONENTIALLY VARY- ING DENSITY

This section is devoted to study the propagation of spherical shock waves in a dusty gas with radiation heat flux and exponentially varying density. We follow Singh and Vishwakarma[5] here.

### 4.1.1 INTRODUCTION

Since the phenomena associated with heat transfer in a radiating fluid are extremely complex whether the fluid is at rest or in motion, steady or unsteady , so we study the problem under certain assumptions. We assumed that the gas is grey and opaque, and the shock is isothermal . Radiation pressure and radiation energy are considered to be very small in comparison to material pressure and energy, respectively, and therefore only the radiation flux is taken into account. In order to get some essential features of shock propagation, small solid particles are considered as a pseudo- fluid, and it is assumed that the equilibrium flow condition is maintained in the flow field, and that the viscous stress and the heat condition of the mixture are negligible.

### 4.1.2 FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

The fundamental equations for one-dimensional , spherically symmetric and unsteady flow of a mixture of a gas and small solid particles taking radiation flux into account, can be written as (Vishwakarma [6], Singh and Srivastava[7])

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{2\rho u}{r} = 0 , \quad (4.1.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0 , \quad (4.1.2)$$

$$\frac{\partial U_m}{\partial t} + u \frac{\partial U_m}{\partial r} - \frac{p}{\rho^2} \left( \frac{\partial \rho}{\partial r} + u \frac{\partial \rho}{\partial r} \right) + \frac{1}{\rho r^2} \frac{\partial}{\partial r} (q_R r^2) = 0 , \quad (4.1.3)$$

where  $\rho$  is the density of the mixture,  $u$  the flow velocity in the radial direction,  $p$  the pressure of the mixture,  $U_m$  the internal energy per unit mass of the mixture,  $q_R$  the radiation heat flux,  $r$  the distance, and  $t$  the time.

Assuming local thermodynamic equilibrium , and taking Rosseland's diffusion approximation, we have

$$q_R = -\frac{CL_R}{3} \frac{\partial}{\partial r} (aT^4) , \quad (4.1.4)$$

where  $\frac{ac}{4}$  is the Stefan - Boltzmann constant ;  $c$  the velocity of light ; and  $L_R$  the mean free path of radiation , which is a function of density  $\rho$  and absolute temperature  $T$ .

Following Wang [2], we have

$$L_R = L_{R_0} \rho^{\alpha^*} T^{\beta^*} , \quad (4.1.5)$$

where  $\alpha^*$  and  $\beta^*$  are constants.

The equation of state of a mixture of gas and small solid particles can be written as (Pai[8])

$$p = \left( \frac{1 - k_p}{1 - Z} \right) \rho R^* T, \quad (4.1.6)$$

where  $R^*$  is the constant,  $Z$  the volume fraction of solid particles in the mixture and  $k_p$  the mass concentration of the solid particles in the mixture.

The relation between  $k_p$  and  $Z$  is given by

$$k_p = \frac{Z \rho_{sp}}{\rho}, \quad (4.1.7)$$

where  $\rho_{sp}$  is the species density of the solid particles. In the equilibrium flow,  $k_p$  is a constant in the whole flow field.

The ratio of the specific heats of the mixture is given by (Marble[9], Pai[8])

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \frac{\gamma(1 + \frac{\delta\beta'}{\gamma})}{1 + \delta\beta'}, \quad (4.1.8)$$

where

$C_{pm}$  is the specific heat of the mixture at constant pressure,

$C_{vm}$  is the specific heat of the mixture at constant volume,

$$\gamma = \frac{C_p}{C_v}, \quad \delta = \frac{k_p}{1 - k_p}, \quad \beta' = \frac{C_{sp}}{C_v},$$

$C_p$  is the specific heat of the gas at constant pressure,

$C_v$  is the specific heat of the gas at constant volume, and

$C_{sp}$  is the specific heat of solid particles.

The internal energy per unit mass of the mixture is given by (Pai [8])

$$U_m = \frac{p(1-Z)}{\rho(\Gamma-1)}. \quad (4.1.9)$$

We consider that a spherical shock wave is propagated into the medium, at rest, with small constant counter pressure. Also, the initial density of the medium is assumed to obey exponential law

$$\rho = \mu e^{\alpha_1 r}, \quad (4.1.10)$$

where  $\alpha_1$  and  $\mu$  are suitable constants.

The shock is assumed to be isothermal (formation of the isothermal shock is a result of the mathematical approximation in which the flux is taken to be proportional to the temperature gradient. This excludes the possibility of a temperature jump, (see for example Zel'dovich and Raizer[10] , Bhowmick [11], Singh and Srivastava [7]) and, hence, the conditions across it are

$$\rho_2(U - u_2) = \rho_1 U = m_s \text{ (say)},$$

$$p_2 + \rho_2(U - u_2)^2 = p_1 + \rho_1 U^2,$$

$$U_{m_2} + \frac{p_2}{\rho_2} + \frac{1}{2}(U - u_2)^2 - \frac{q_{R_2}}{m_s} = U_{m_1} + \frac{p_1}{\rho_1} + \frac{1}{2}U^2,$$

$$\frac{Z_2}{\rho_2} = \frac{Z_1}{\rho_1},$$

$$T_2 = T_1, \quad (4.1.11)$$

where  $U = \frac{dR}{dt}$  denotes the velocity of the shock at  $r = R(t)$ , indices '1' and '2' refer to the values just ahead and just behind the shock surface, and  $q_{R_1} = 0$  (Laumbach and Probst[12]). From shock conditions (4.1.11), we get

$$\begin{aligned} u_2 &= (1 - \beta)U, \quad \rho_2 = \frac{\rho_1}{\beta}, \\ p_2 &= (1 - Z_1)\rho_1 U^2, \quad Z_2 = \frac{Z_1}{\beta}, \end{aligned} \quad (4.1.12)$$

$$q_{R_2} = (1 - \beta) \left[ \frac{(1 + \Gamma)\beta + (1 - \Gamma) - 2Z_1}{2(\Gamma - 1)} - \frac{1 - Z_1}{(\Gamma - 1)M_e^2} \right] \rho_1 U^2,$$

where  $\beta$  is given by

$$\beta = Z_1 + \frac{1 - Z_1}{\Gamma M_e^2} \quad (4.1.13)$$

and

$$M_e^2 = \frac{U^2}{a_1^2}, \quad a_1^2 = \frac{\Gamma p_1}{\rho_1(1 - Z_1)},$$

$M_e$  being the shock - Mach number referred to the speed of sound  $a_1$  in the dusty gas.

The initial volume fraction of the solid particles  $Z_1$  is, in general not a constant. But the volume occupied by the solid particles is very small because the density of the solid particles is much larger than that of the gas (Miura and Glass [13]), hence  $Z_1$  may be assumed as a small constant. The expression for  $Z_1$  is (Naidu et al[14])

$$Z_1 = \frac{k_p}{G_1(1 - k_p) + k_p}, \quad (4.1.14)$$

where  $G_1 = \frac{\rho_{sp}}{\rho_{g1}}$  is the ratio of the density of solid particles to the initial density of the gas. Values of  $Z_1$  for some typical values of  $k_p$  and  $G_1$  are given in table 1.

Table 1. Values of  $Z_1$  for some typical values of  $k_p$  and  $G_1$

$k_p$	$G_1$	$Z_1$
0.2	10	0.02439
	50	0.00498
	100	0.00249
0.4	10	0.06250
	50	0.01316
	100	0.00662

Let the solution of the equations (4.1.1) to (4.1.4) be of the form (Vishwakarma [6], Singh and Srivastava [7], Bhowmick [11], Verma and Vishwakarma [15])

$$u = t^{-1}V(\xi), \quad \rho = t^\Omega D(\xi),$$

$$p = t^{\Omega-2}P(\xi), \quad q_R = t^{\Omega-3}Q(\xi), \quad (4.1.15)$$

where

$$\xi = te^{\lambda r}, \quad \lambda \neq 0 \quad (4.1.16)$$

and constants  $\Omega$  and  $\lambda$  are to be determined subsequently . We choose the shock surface to be given by

$$\xi_0 = \text{constant}, \quad (4.1.17)$$

so that its velocity is given by

$$U = -\frac{1}{\lambda t} \quad (4.1.18)$$

which represents the outgoing shock surface, if  $\lambda < 0$ .

The solution of the equations (4.1.1) to (4.1.4) in the form (4.1.15) are compatible with the shock conditions if

$$\Omega = 2, \quad \lambda = -\frac{\alpha_1}{2}, \quad \alpha^* = 1, \quad \beta^* = -\frac{5}{2}. \quad (4.1.19)$$

Since necessarily  $\lambda < 0$ , relation (4.1.19) shows that  $\alpha_1 > 0$ , which means shock expands outwardly in an exponentially increasing medium.

The strength of the shock, under these conditions remains constant, for

$$M_e^2 = \frac{U^2}{a_1^2} = \frac{U^2}{\frac{\Gamma p_1}{\rho_1(1-Z_1)}} = \frac{4(1-Z_1)\mu}{\Gamma p_1 \alpha_1^2 \xi_0^2} = \text{constant}.$$

Also from equations (4.1.18) and (4.1.19), we obtain

$$R = \frac{2}{\alpha_1} \log \frac{t}{\tau}, \quad (4.1.20)$$

where  $\tau$  is the duration of the initial impulse.

### 4.1.3 SOLUTION OF THE EQUATIONS

The flow variables in the flow-field behind the shock front will be obtained by solving the equations (4.1.1) to (4.1.4). From equations (4.1.15), (4.1.18) and (4.1.19), we obtain

$$\frac{\partial u}{\partial t} = u\lambda U - U \frac{\partial u}{\partial r}, \quad (4.1.21)$$

$$\frac{\partial \rho}{\partial t} = U\rho\alpha_1 - U \frac{\partial \rho}{\partial r}, \quad (4.1.22)$$

$$\frac{\partial p}{\partial r} = -U \frac{\partial p}{\partial r}. \quad (4.1.23)$$

Using equations (4.1.21) to (4.1.23) and the transformations

$$r' = \frac{r}{R}, \quad u' = \frac{u}{U}, \quad p' = \frac{p}{p_2},$$

$$\rho' = \frac{\rho}{\rho_2}, \quad q'_R = \frac{q_R}{q_{R_2}} \quad (4.1.24)$$

in the fundamental equations (4.1.1) to (4.1.4), we obtain

$$\frac{d\rho'}{dr'} = \frac{\rho'}{1-u'} \left[ 2 \log \frac{t}{\tau} + \frac{du'}{dr'} + \frac{2u'}{r'} \right], \quad (4.1.25)$$

$$\frac{dp'}{dr'} = \frac{\rho'}{(1-Z_1)\beta} \left[ (1-u') \frac{du'}{dr'} + u' \log \frac{t}{\tau} \right], \quad (4.1.26)$$

$$\frac{dq'_R}{dr'} = \frac{(1 - Z_1)}{(1 - \beta) \left[ \frac{(1 + \Gamma)\beta + (1 - \Gamma) - 2Z_1}{2} - \frac{1 - Z_1}{M_e^2} \right]} \times \left[ (1 - u') \left( 1 - \frac{Z_1 \rho'}{\beta} \right) \frac{dp'}{dr'} + \Gamma \frac{p'}{\rho'} \left\{ 2 \rho' \log \frac{t}{\tau} - (1 - u') \frac{d\rho'}{dr'} \right\} \right] - \frac{2q'_R}{r'}, \quad (4.1.27)$$

$$q'_R = -\frac{NL}{\log \frac{t}{\tau}} \sqrt{\frac{p'(\beta - Z_1 \rho')}{\rho'}} \left[ (\beta - Z_1 \rho') \frac{dp'}{dr'} - \left\{ Z_1 + \frac{1}{\rho'} (\beta - Z_1 \rho') \right\} p' \frac{d\rho'}{dr'} \right], \quad (4.1.28)$$

where

$$N = \frac{4acL_{R_0}\alpha_1}{3\sqrt{R^{*3}}}, \quad (4.1.29)$$

is a non-dimensional radiation parameter and

$$L = \frac{(\Gamma - 1)\sqrt{(1 - Z_1)^3}}{2(1 - \beta) \left[ \frac{(1 + \Gamma)\beta + (1 - \Gamma) - 2Z_1}{2} - \frac{1 - Z_1}{M_e^2} \right] \beta \sqrt{(1 - K_p)^3}}. \quad (4.1.30)$$

Solving equations (4.1.25) to (4.1.28) for  $\frac{d\rho'}{dr'}$ ,  $\frac{dp'}{dr'}$ ,  $\frac{dq'_R}{dr'}$  and  $\frac{du'}{dr'}$ , we obtain

$$\frac{d\rho'}{dr'} = \frac{\rho'}{1 - u'} \left[ 2 \log \frac{t}{\tau} + \frac{du'}{dr'} + \frac{2u'}{r'} \right], \quad (4.1.31)$$

$$\frac{dp'}{dr'} = \frac{\rho'}{(1 - Z_1)\beta} \left[ (1 - u') \frac{du'}{dr'} + u' \log \frac{t}{\tau} \right], \quad (4.1.32)$$

$$\frac{dq'_R}{dr'} = \frac{(1 - Z_1)}{(1 - \beta) \left[ \frac{(1 + \Gamma)\beta + (1 - \Gamma) - 2Z_1}{2} - \frac{1 - Z_1}{M_e^2} \right]} \times$$

$$\left\{ \left[ \frac{(\beta - Z_1\rho')(1 - u')^2\rho'}{(1 - Z_1)\beta^2} - \Gamma p' \right] \frac{du'}{dr'} + \frac{(1 - u')(\beta - Z_1\rho')\rho'u' \log \frac{t}{\tau}}{(1 - Z_1)\beta^2} - \frac{2\Gamma p'u'}{r'} \right\} - \frac{2q'_R}{r'},$$

(4.1.33)

$$\frac{du'}{dr'} = \frac{1}{p'\beta^2(1 - Z_1) - (\beta - Z_1\rho')\rho'(1 - u')^2} \left[ \frac{q'_R(1 - Z_1)\beta(1 - u')\sqrt{\rho'} \log \frac{t}{\tau}}{NL\sqrt{p'}\sqrt{(\beta - Z_1\rho')}} + \right.$$

$$\left. (1 - u')(\beta - Z_1\rho')\rho'u' \log \frac{t}{\tau} - 2p'\beta^2(1 - Z_1) \log \frac{t}{\tau} - \frac{2p'u'\beta^2(1 - Z_1)}{r'} \right].$$

(4.1.34)

The adiabatic compressibility of the mixture of the gas and small solid particles may be calculated as (c.f. Moelwyn - Hughes[16])

$$C = \frac{1}{\rho} \left( \frac{\partial \rho}{\partial p} \right)_S = \frac{1 - Z}{\Gamma p},$$

(4.1.35)

where  $\left( \frac{\partial \rho}{\partial p} \right)_S$  denotes the derivative of  $\rho$  with respect to  $p$  at a constant entropy  $S$ . The non - dimensional compressibility  $c' = \frac{c}{c_2}$  can be expressed as

$$c' = \frac{\left( 1 - \frac{Z_1}{\beta} \rho' \right)}{p' \left( 1 - \frac{Z_1}{\beta} \right)}.$$

(4.1.36)

Also, the total energy of the flow- field behind the shock front is given by

$$E = 4\pi \int_0^R \rho(U_m + \frac{1}{2}u^2)r^2 dr. \quad (4.1.37)$$

Using equations (4.1.24) and (4.1.19), equation (4.1.37) becomes

$$E = \frac{16\pi\mu R^3}{\beta\alpha_1^2\xi_0^2} \int_0^1 \left[ \frac{p'(1-Z_1)(\beta-Z_1\rho')}{\Gamma-1} + \frac{1}{2}\rho'u'^2 \right] r'^2 dr'. \quad (4.1.38)$$

Hence the total energy of the shock wave is non-constant and varies with  $R^3$ .

In terms of dimensionless variables  $r', p', \rho', u'$  and  $q'_R$ , the shock conditions take the form

$$r' = 1, \quad p' = 1, \quad \rho' = 1, \quad u' = 1 - \beta, \quad q'_R = 1. \quad (4.1.39)$$

Equations (4.1.31) to (4.1.34) along with the boundary conditions (4.1.39) give the solution of our problem. The solution so obtained is a non - similar one, since the motion behind the shock can be determined only when a definite value for time is prescribed.

#### 4.1.4 RESULTS AND DISCUSSION

The distribution of the flow variables behind the shock front is obtained by numerical integration of equations (4.1.31) to (4.3.34) with boundary conditions (4. 1. 39). For the purpose of numerical integration, the values of the constant parameters are taken to be (Pai et al[17], Miura and Glass[13], Vishwakarma [6], Singh and Srivastava [7])  $\gamma = 1.4$ ;  $k_p = 0, 0.2, 0.4$ ;  $G_1 = 10, 50$ ;  $\beta' = 1$ ;  $M_e^2 = 20$ ;  $N = 0.6, 0.8, 10$ ; and  $\frac{t}{\tau} = 2, 4$ . Starting from the

shock front, the numerical integration is carried out until the singularity of the solution

$$p'\beta^2(1 - Z_1) - (\beta - Z_1\rho')\rho'(1 - u')^2 = 0$$

is reached. This marks the inner boundary of the disturbance and at this surface, the value of  $i'(=i_p)$  remains constant.

Figs. 1 to 8 show the variation of the reduced flow variables  $\rho'$ ,  $p'$ ,  $q'_R$  and  $u'$  with reduced distance  $r'$  at various values of the parameters  $\frac{t}{\tau}$ ,  $N$ ;  $k_p$ ,  $G_1$  and fig.9 shows the variation of the non - dimensional compressibility  $c'$  with reduced distance  $r'$  at various values of  $k_p$ ,  $G_1$ . Table 2,3 and 4 display the density ratio  $\frac{1}{\beta}$  across the shock and the piston of the inner boundary surface  $r'_p$  (say) for various values of constant parameters.

Table 2. Density ratio  $\frac{\rho_2}{\rho_1} = \frac{1}{\beta}$  across the shock front for different values of  $k_p$  and  $G_1$

$k_p$	$G_1$	$\frac{\rho_2}{\rho_1} = \frac{1}{\beta}$
0		27.99998
0.2	10	16.30122
	50	23.43556
	100	24.82961
0.4	10	9.96989
	50	18.88506
	100	21.42447

Table 3. Position of the inner boundary surface for different values of the radiation parameter  $N$  and time  $\frac{t}{\tau}$  with  $k_p = 0.2$  and  $G_1 = 50$

$\frac{t}{\tau}$	$N$	Position of the inner boundary surface( $r'_p$ )
2	0.6	0.95844
	0.8	0.95828
	10	0.95769
4	0.6	0.97232
	0.8	0.97212
	10	0.97127

Table 4. Position of the inner boundary surface for different values of  $k_p$  and  $G_1$  with  $N = 10$  and  $\frac{t}{\tau} = 2$

$k_p$	$G_1$	Position of the inner boundary surface( $r'_p$ )
0		0.96241
0.2	10	0.94595
	50	0.95769
	100	0.95923
0.4	10	0.92257
	50	0.95101
	100	0.95494

The effects of an increase in the value of the radiation parameter  $N$  are (see figs. 1 to 4):

- (i) to decrease the density  $\rho'$  and to increase the pressure  $p'$  at any point in the flow- field behind the shock. The decrease of density and the increase of pressure become significant near the inner boundary surface,
- (ii) to increase the radiation heat flux  $q'_R$  and the velocity  $u'$  near the inner boundary surface , and
- (iii) to increase, slightly, the distance of the inner boundary surface from the shock surface (see table 3).

The effects of an increase in the time  $(\frac{t}{\tau})$  are (see figs. 1 to 4):

- (i) to decrease the density  $\rho'$  and the pressure  $p'$ ,
- (ii) to increase the radiation flux  $q'_R$  and the velocity  $u'$ , and
- (iii) to decrease the distance of the inner boundary surface from the shock surface (see table 3).

Figs. 5 to 8 show that for given values of  $N$  and  $G_1$  , the effects of an increase in the mass concentration of the solid particles  $k_p$  at a given instant are

- (i) to increase the density  $\rho'$ , the pressure  $p'$ , the radiation heat flux  $q'_R$  and to decrease the following velocity  $u'$ , and

(ii) to increase the distance between the inner contact surface and the shock front (see table 4). This means that an increase in the mass concentration of the solid particles has an effect to decrease the shock strength.

Also figs. 5 to 8 show that for given values of  $N$  and  $k_p$ , the effects of an increase in the ratio of the density of the solid particles to the initial density of the gas  $G_1$  at a given instant are

- (i) to decrease the density  $\rho'$ , the pressure  $p'$ , the radiation heat flux  $q'_R$  and to increase the flow velocity  $u'$ , and
- (ii) to decrease the distance between the inner contact surface and the shock front (see table 4). This means that an increase in the ratio of the density of the solid particles to the initial density of gas has an effect to increase the shock strength.

The effects of an increase in  $k_p$  or  $G_1$  on the shock strength may be explained with the help of the compressibility of the mixture as follows :

Fig.9 shows that the compressibility decreases as the value of  $k_p$  increases, whereas it increases as the value of  $G_1$  increases. The decrease in the compressibility causes weaker compression of the gas behind the shock and, hence, a decrease in the shock strength. The increase in the compressibility causes stronger compression of the gas behind the shock and, hence, an increase in the shock strength.



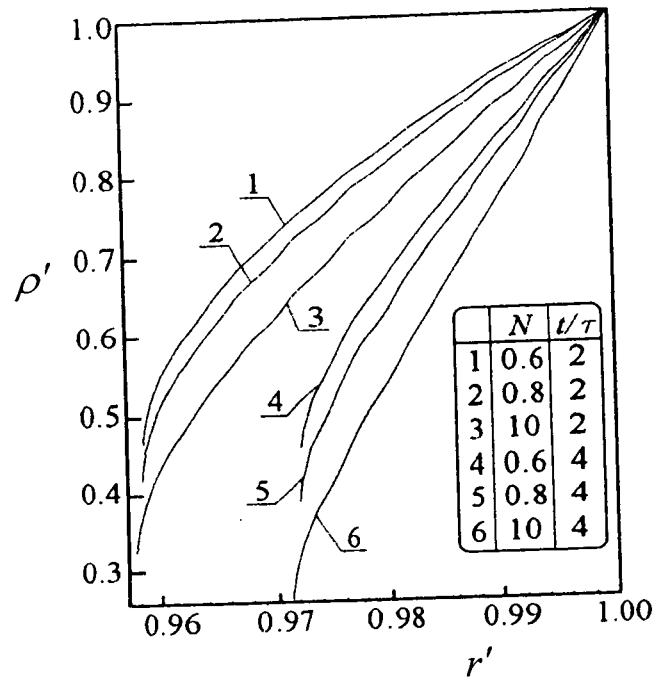


Fig.1 Variation of reduced density  $\rho'$  in the region behind the shock front for  $k_p=0.2$  and  $G_1=50$

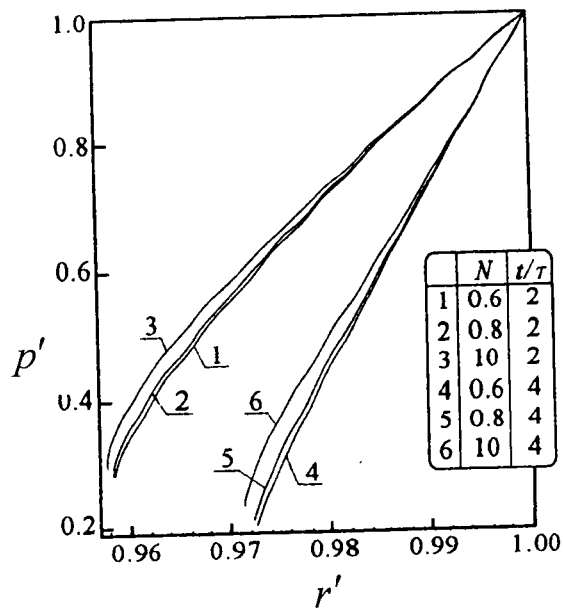


Fig.2 Variation of reduced pressure  $p'$  in the region behind the shock front for  $k_p=0.2$  and  $G_1=50$

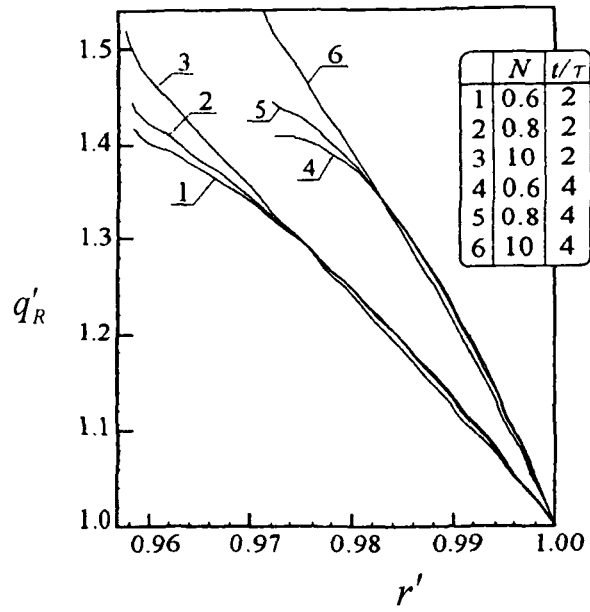


Fig.3 Variation of reduced radiation heat flux  $q'_R$  in the region behind the shock front for  $k_p=0.2$  and  $G_1=50$

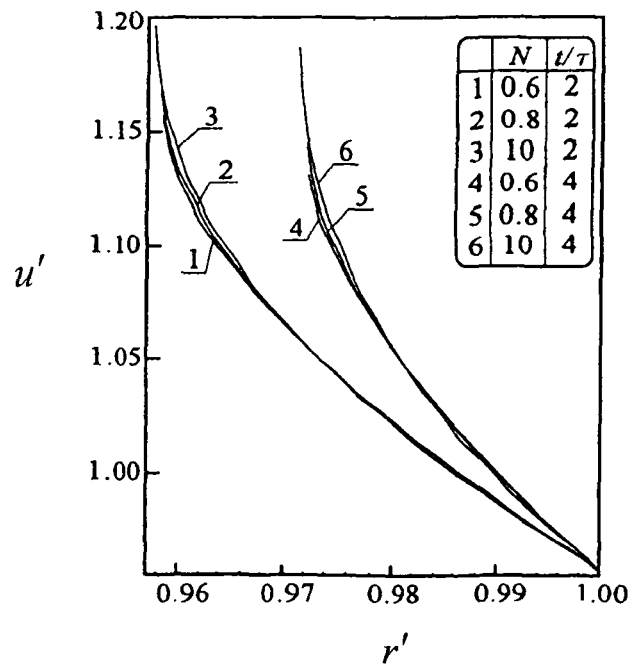


Fig.4 Variation of reduced flow velocity  $u'$  in the region behind the shock front for  $k_p=0.2$  and  $G_1=50$

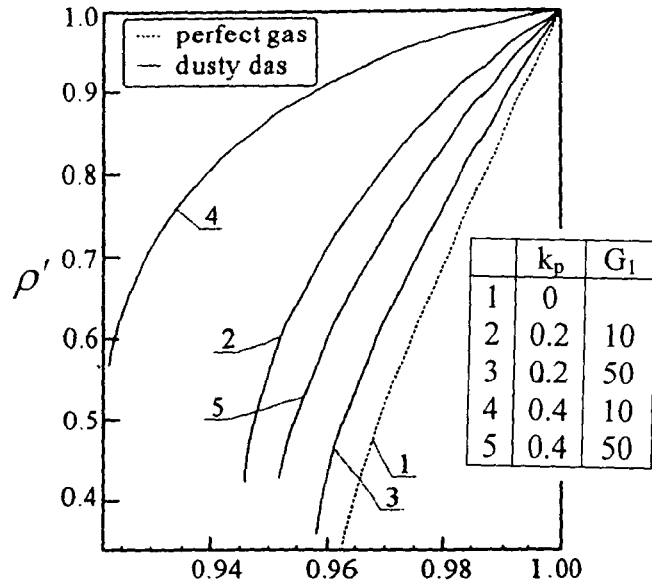


Fig.5 Variation of reduced density  $\rho'$  in the  $r'$  region behind the shock front for  $N=10$  and  $t/\tau=2$

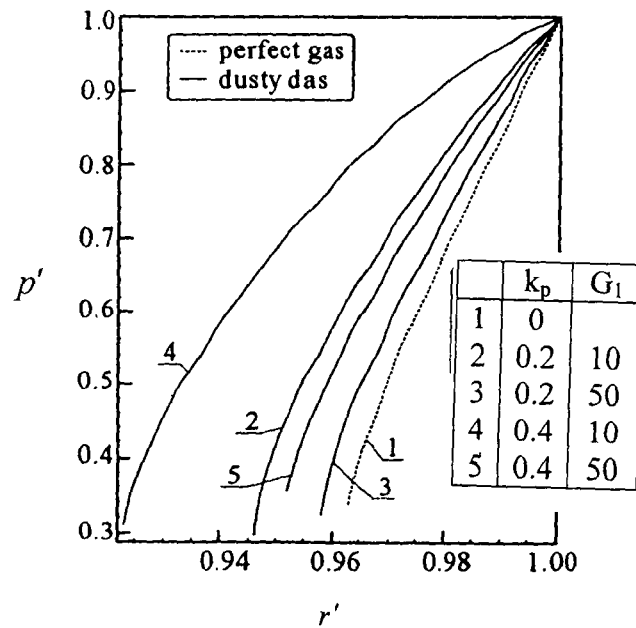


Fig.6 Variation of reduced pressure  $p'$  in the region behind the shock front for  $N=10$  and  $t/\tau=2$

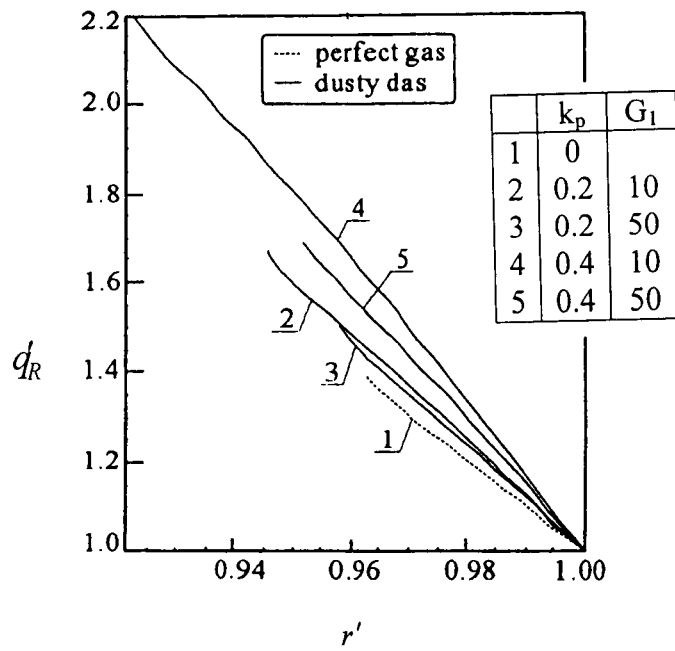


Fig.7 Variation of reduced radiation heat flux  $q'_R$  in the region behind the shock front for  $N=10$  and  $t/\tau=2$

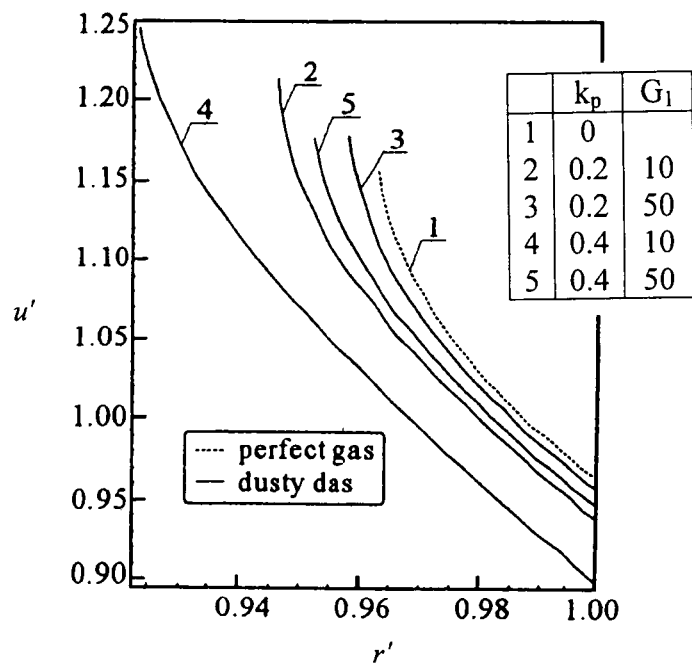


Fig.8 Variation of reduced flow velocity  $u'$  in the region behind the shock front for  $N=10$  and  $t/\tau=2$

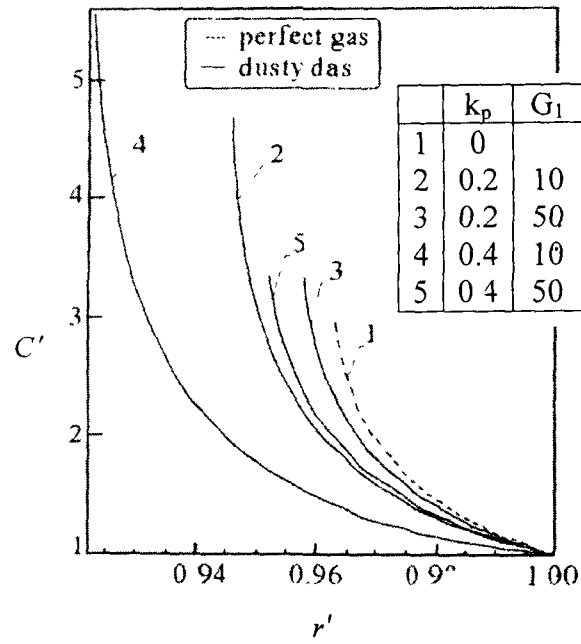


Fig.9 Variation of non-dimensional compressibility  $C'$  in the region behind the shock front for  $N=10$  and  $t/\tau=2$

## 4.2 PROPAGATION OF SHOCK WAVES IN A DUSTY GAS WITH HEAT CONDUCTION AND RADIATION HEAT FLUX

This section is devoted to study the propagation of spherical shock waves in a dusty gas with heat conduction and radiation heat flux, in which density varies exponentially. We follow Vishwakarma et al[18] here.

### 4.2.1 INTRODUCTION

In the present work, we study a non-self -similar solution for the shock propagation in a dusty gas with heat conduction and radiation heat flux, in which density varies exponentially. In order to get some essential features of shock propagation, the solid particles are considered as a pseudo - fluid and it is assumed that the equilibrium flow condition is maintained in the flow-field, and that the viscous stress of the mixture is negligible. The heat transfer fluxes are expressed in terms of Fourier's law for heat - conduction and a diffusion radiation mode for an optically thick grey dusty gas, which is typical of large - scale explosions. The thermal conductivity and absorption coefficient of the gas are assumed to be proportional to appropriate powers of temperature and density (Ghoniem et al[4]). Also it is assumed that the dusty gas is opaque, and the shock is isothermal. The radiation pressure and radiation energy are neglected. The gas ahead of the shock is assumed to be at rest.

## 4.2.2 FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

The fundamental equations for one-dimensional, spherically symmetric and unsteady flow of a mixture of gas and small solid particles with heat conduction and radiation heat flux may, in Eulerian coordinates, be expressed as (Abdel - Raouf and Gretler[19], Ghoniem et al[4], Vishwakarma [6])

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{2\rho u}{r} = 0, \quad (4.2.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0, \quad (4.2.2)$$

$$\frac{\partial U_m}{\partial t} + u \frac{\partial U_m}{\partial r} - \frac{p}{\rho^2} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) + \frac{1}{\rho r^2} \frac{\partial}{\partial r} (qr^2) = 0, \quad (4.2.3)$$

where  $r$  and  $t$  are independent space and time coordinates,  $\rho$  is the density of the mixture,  $p$  the pressure,  $u$  the flow velocity,  $U_m$  the internal energy per unit mass of the mixture and  $q$  the heat flux.

The total heat flux  $q$ , which appears in the energy equation may be decomposed as

$$q = q_C + q_R, \quad (4.2.4)$$

where  $q_C$  is the conduction heat flux, and  $q_R$  the radiation heat flux.

According to Fourier's law of heat conduction

$$q_C = -K \frac{\partial T}{\partial r}, \quad (4.2.5)$$

where  $K$  is the coefficient of thermal conductivity of the dusty gas and  $T$  the absolute temperature.

Assuming local thermodynamic equilibrium and using the radiative diffusion model for an optically thick grey gas (Pomraning [20]), the term  $q_R$ , which represents radiative heat flux, may be obtained from the differential approximation of the radiation - transport equation in the diffusion limit as

$$q_R = -\frac{4}{3} \left( \frac{\sigma}{\alpha_R} \right) \frac{\partial T^4}{\partial r}, \quad (4.2.6)$$

where  $\sigma$  is the Stefan - Boltzmann constant and  $\alpha_R$  is the Rosseland mean absorption coefficient.

The thermal conductivity  $K$  and absorption coefficient  $\alpha_R$  are assumed to vary with temperature and density. These can be written in the form of power laws, namely (Ghoniem et al[4])

$$K = K_0 \left( \frac{T}{T_0} \right)^{\beta_C} \left( \frac{\rho}{\rho_0} \right)^{\delta_C},$$

$$\alpha_R = \alpha_{R_0} \left( \frac{T}{T_0} \right)^{\beta_R} \left( \frac{\rho}{\rho_0} \right)^{\delta_R}, \quad (4.2.7)$$

where subscript '0' denote a reference state. The exponents in the above equations should be compatible with the shock conditions of the problem and the form of the required solution.

The above system of equations should be supplemented with an equation of state. The medium is assumed as a mixture of a perfect gas and small solid particles. Then the equation of state of the mixture can be written as (Pai[8])

$$p = \left( \frac{1 - k_p}{1 - Z} \right) \rho R^* T, \quad (4.2.8)$$

where  $R^*$  is the gas constant,  $k_p$  the mass concentration of solid particles and  $Z$  the volume fraction of solid particles in the mixture.

The relation between  $k_p$  and  $Z$  is given by

$$k_p = \frac{Z \rho_{sp}}{\rho}, \quad (4.2.9)$$

where  $\rho_{sp}$  is the species density of solid particles. In the equilibrium flow,  $k_p$  is a constant in the whole flow- field. Therefore,

$$\frac{Z}{\rho} = constant \quad (4.2.10)$$

in the whole flow field. Also we have the relation (Pai[8])

$$Z = \frac{k_p}{G(1 - k_p) + k_p}, \quad (4.2.11)$$

where  $G = \frac{\rho_{sp}}{\rho_g}$  is the ratio of the density of the solid particles to the species density of the gas.

The ratio of the specific heats of the mixture is given by (Pai [8], Marble[9])

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \frac{\gamma(1 + \frac{\delta\beta'}{\gamma})}{1 + \delta\beta'}, \quad (4.2.12)$$

where

$C_{pm}$  is the specific heat of the mixture at constant pressure,

$C_{vm}$  is the specific heat of the mixture at constant volume,

$$\gamma = \frac{C_p}{C_v}, \quad \delta = \frac{k_p}{1 - k_p}, \quad \beta' = \frac{C_{sp}}{C_v},$$

$C_p$  - the specific heat of the gas at constant pressure,

$C_v$  - the specific heat of the gas at constant volume, and

$C_{sp}$  - the specific heat of solid particles.

The internal energy per unit mass of the mixture is given by (Pai [8])

$$U_m = \frac{p(1 - Z)}{\rho(\Gamma - 1)}. \quad (4.2.13)$$

According to equation of state the isothermal speed of sound is

$$a_{iso} = \left( \frac{\partial p}{\partial \rho} \right)_T^{\frac{1}{2}} = \sqrt{\frac{p}{\rho(1 - Z)}}. \quad (4.2.14)$$

The disturbance is headed by an isothermal shock and, hence, the conditions across it are

$$\rho_2(U - u_2) = \rho_1 U,$$

$$p_2 + \rho_2(U - u_2)^2 = p_1 + \rho_1 U^2,$$

$$U_{m_2} + \frac{p_2}{\rho_2} + \frac{1}{2}(U - u_2)^2 - \frac{q_2}{\rho_1 U} = U_{m_1} + \frac{p_1}{\rho_1} + \frac{1}{2}U^2,$$

$$\frac{Z_2}{\rho_2} = \frac{Z_1}{\rho_1},$$

$$T_2 = T_1, \quad (4.2.15)$$

where  $U = \frac{dR}{dt}$  denotes the velocity of the shock at  $r = R(t)$ , indices '1' and '2' refer to the values just ahead and just behind the shock surface, and  $q_1 = 0$  (Laumbach and Probststein [12]).

From the shock conditions (4.2.15), we get

$$u_2 = (1 - \beta)U ,$$

$$\rho_2 = \frac{\rho_1}{\beta} ,$$

$$p_2 = (1 - Z_1)\rho_1 U^2 , \quad (4.2.16)$$

$$Z_2 = \frac{Z_1}{\beta} ,$$

$$q_2 = (1 - \beta) \left[ \frac{(1 + \Gamma)\beta + (1 - \Gamma) - 2Z_1}{2(\Gamma - 1)} - \frac{1 - Z_1}{(\Gamma - 1)M_e^2} \right] \rho_1 U^3 .$$

The quantity  $\beta$  is given by the equation

$$\beta = Z_1 + \frac{1 - Z_1}{\Gamma M_e^2} , \quad (4.2.17)$$

where

$$Z_1 = \frac{k_p}{G_1(1 - k_p) + k_p} , \quad (4.2.18)$$

$G_1$  is the ratio of the density of the solid particles to the initial density of the gas and  $M_e$  is the shock- Mach number referred to the speed of sound  $a_1 = \left[ \frac{\Gamma p_1}{\rho_1(1 - Z_1)} \right]^{\frac{1}{2}}$  in the dusty gas i.e.

$$M_e^2 = \frac{U^2}{a_1^2} = \frac{U^2 \rho_1 (1 - Z_1)}{\Gamma p_1} . \quad (4.2.19)$$

Also, we have

$$M_e^2 = \frac{\gamma(1 - Z_1)M^2}{\Gamma} \quad (4.2.20)$$

where  $M$  is shock - Mach number referred to the speed of sound in dust - free perfect gas  $\left(\frac{\gamma p_1}{\rho_1}\right)^{\frac{1}{2}}$  and is defined by

$$M = \left(\frac{\gamma p_1}{\rho_1 U^2}\right)^{\frac{1}{2}} . \quad (4.2.21)$$

The initial density of the medium is assumed to obey an exponential law, namely

$$\rho_1 = \mu e^{\alpha_1 R} , \quad (4.2.22)$$

where  $\mu$  and  $\alpha_1$  are suitable constants.

Let the solution of equations (4.2.1) to (4.2.4) be of the form (Bhowmick [11], Verma and Vishwakarma [15], Singh and Srivastava [7], Vishwakarma [6])

$$u = t^{-1}V(\xi), \quad \rho = t^{\Omega}D(\xi) ,$$

$$p = t^{\Omega-2}P(\xi), \quad q = t^{\Omega-3}Q(\xi) , \quad (4.2.23)$$

where

$$\xi = te^{\lambda r}, \quad \lambda \neq 0 \quad (4.2.24)$$

and the constants  $\Omega$  and  $\lambda$  are to be determined subsequently . We choose the shock surface to be given by

$$\xi_0 = \text{constant} \quad (4.2.25)$$

so that its velocity is given by

$$U = -\frac{1}{\lambda t}, \quad (4.2.26)$$

which represents an out going shock surface if  $\lambda < 0$ .

The solutions of equation (4.2.1) to (4.2.1) in the form (4.2.23) are compatible with the shock conditions and equation (4.2.5) and (4.2.6), if 0.9966

$$\Omega = 2, \quad \lambda = -\frac{\alpha_1}{2}, \quad \beta_C = -\frac{1}{2} + \delta_C, \quad \beta_R = \frac{7}{2} + \delta_R. \quad (4.2.27)$$

Also, from equations (4.2.26) and (4.2.27) , we obtain

$$R = \frac{2}{\alpha_1} \log \frac{t}{t_0}, \quad (4.2.28)$$

where  $t_0$  is the duration of the almost instantaneous explosion.

### 4.2.3 SOLUTION OF THE EQUATIONS

The flow variables in the flow - field behind the shock front will be obtained by solving the equations (4.2.1) to (4.2.6). From equations (4.2.23), (4.2.26) and (4.2.27), we obtain

$$\frac{\partial u}{\partial t} = \lambda u U - U \frac{\partial u}{\partial r}, \quad (4.2.29)$$

$$\frac{\partial \rho}{\partial t} = -2\rho\lambda U - U \frac{\partial \rho}{\partial r}, \quad (4.2.30)$$

$$\frac{\partial p}{\partial r} = -U \frac{\partial p}{\partial r}. \quad (4.2.31)$$

Using equations (4.2.29) to (4.2.31) and the transformations

$$r' = \frac{r}{R}, \quad u' = \frac{u}{U}, \quad \rho' = \frac{\rho}{\rho_2}, \quad p' = \frac{p}{p_2}, \quad q' = \frac{q}{q_2} \quad (4.2.32)$$

in the fundamental equations (4.2.1) to (4.2.3), we obtain

$$\frac{d\rho'}{dr'} = \frac{\rho'}{(1-u')} \left[ \frac{du'}{dr'} + 2 \log \frac{t}{t_0} + \frac{2u'}{r'} \right], \quad (4.2.33)$$

$$\frac{dp'}{dr'} = \frac{\rho'}{(1-Z_1)\beta} \left[ (1-u') \frac{du'}{dr'} + u' \log \frac{t}{t_0} \right], \quad (4.2.34)$$

$$\begin{aligned} \frac{dq'}{dr'} = & \frac{(1-Z_1)}{(1-\beta) \left[ \frac{(1+\Gamma)\beta + (1-\Gamma) - 2Z_1}{2} - \frac{1-Z_1}{M_e^2} \right]} \times \\ & \left[ (1-u') \left( 1 - \frac{Z_1 \rho'}{\beta} \right) \frac{dp'}{dr'} + \Gamma \frac{\rho'}{\rho'} \left\{ 2\rho' \log \frac{t}{t_0} - (1-u') \frac{d\rho'}{dr'} \right\} \right] - \frac{2q'}{r'}. \end{aligned} \quad (4.2.35)$$

Using equations (4.2.33) and (4.2.34), equation (4.2.35) becomes

$$\frac{dq'}{dr'} = \frac{(1 - Z_1)}{(1 - \beta) \left[ \frac{(1 + \Gamma)\beta + (1 - \Gamma) - 2Z_1}{2} - \frac{1 - Z_1}{M_e^2} \right]} \left[ \left\{ \frac{(1 - u')^2(\beta - Z_1\rho')\rho'}{(1 - Z_1)\beta^2} - \Gamma p' \right\} \frac{du'}{dr'} + \frac{(1 - u')(\beta - Z_1\rho')\rho'u'}{(1 - Z_1)\beta^2} \log \frac{t}{t_0} - \frac{2\Gamma p'u'}{r'} \right] - \frac{2q'}{r'}. \quad (4.2.36)$$

By using equations (4.2.5) to (4.2.7) in (4.2.4), we get

$$q = \frac{-16\sigma}{3\alpha_{R_0}} \left[ T^{\frac{7}{2} + \delta_R} \rho_0^{\delta_R} T^{-\frac{1}{2} - \delta_R} \rho^{-\delta_R} \right] \frac{\partial T}{\partial r} - \frac{K_0}{T_0^{-\frac{1}{2} + \delta_C} \rho_0^{\delta_C}} \left[ T^{-\frac{1}{2} + \delta_C} \rho^{\delta_C} \right] \frac{\partial T}{\partial r}. \quad (4.2.37)$$

From equation (4.2.8) we get

$$\frac{\partial T}{\partial r} = \frac{1}{(1 - k_p)R^*} \left[ \frac{(1 - Z) \partial p}{\rho} - \frac{p \partial \rho}{\rho^2} \right]. \quad (4.2.38)$$

Using equations (4.2.38), (4.2.16), (4.2.19), (4.2.28) and (4.2.32) in (4.2.37), we get

$$q' = -LX \left( \frac{\rho'}{p'} \right)^{\frac{1}{2}} \left[ \frac{(\beta - Z_1\rho') dp'}{\beta\rho' dr'} - \frac{p' d\rho'}{\rho'^2 dr'} \right], \quad (4.2.39)$$

where

$$L = \frac{\Gamma^{\frac{3}{2}} (1 - Z_1)^{\frac{3}{2}}}{2(1 - \beta) \left( \log \frac{t}{t_0} \right) \left[ \frac{(1 + \Gamma)\beta + (1 - \Gamma) - 2Z_1}{2(\Gamma - 1)} - \frac{1 - Z_1}{(\Gamma - 1)M_e^2} \right]},$$

$$X = \left[ \frac{\Gamma_R p_0^{1+\delta_R} (\Gamma p_1 M_e^2)^{-1-\delta_R} (\beta - Z_1 \rho')^{-\left(\frac{1}{2}+\delta_R\right)} p'^{-\delta_R}}{\beta^{-(1+\delta_R)}} + \frac{\Gamma_C p_0^{1-\delta_C} (\Gamma p_1 M_e^2)^{-1+\delta_C} (\beta - Z_1 \rho')^{-\frac{1}{2}+\delta_C} p'^{\delta_C}}{\beta^{-(1-\delta_C)}} \right]$$

$\Gamma_C$  and  $\Gamma_R$  are the conductive and radiative non-dimensional heat transfer parameters, respectively. The parameters  $\Gamma_C$  and  $\Gamma_R$  depend on the thermal conductivity  $K$  and the mean free path of radiation  $\frac{1}{\alpha_R}$ , respectively, and they are defined by

$$\Gamma_C = \frac{K_0 T_0 \alpha_1}{\rho_0 a_0^3 (1 - Z_0)^{2+\delta_C}} \quad \text{and} \quad \Gamma_R = \frac{16 \sigma T_0^4 \alpha_1}{3 \alpha_{R_0} \rho_0 a_0^3 (1 - Z_0)^{2-\delta_R}}, \quad (4.2.40)$$

where  $a_0 = \sqrt{\frac{\Gamma p_0}{\rho_0 (1 - Z_0)}}$  is the speed of sound in the reference state.

From equations (4.2.33), (4.2.34) and (4.2.39), we get

$$\frac{du'}{dr'} = \frac{\frac{g'}{LX} (p' \rho')^{\frac{1}{2}} (1 - u') + \frac{(\beta - Z_1 \rho') \rho' u' (1 - u')}{(1 - Z_1) \beta^2} \left( \log \frac{t}{t_0} \right) - 2p' \left( \log \frac{t}{t_0} \right) - \frac{2p' u'}{r'}}{p' - \frac{(\beta - Z_1 \rho') (1 - u')^2 \rho'}{(1 - Z_1) \beta^2}}. \quad (4.2.41)$$

By using equations (4.2.32) and (4.2.16) in (4.2.14), we get

$$\frac{a_{iso}}{U} = \sqrt{\frac{(1 - Z_1) \beta^2 p'}{(\beta - Z_1 \rho') \rho'}}, \quad (4.2.42)$$

where  $T' = \frac{T}{T_2}$ .

The adiabatic compressibility of the mixture of the gas and small solid particles may be calculated as (Moelwyn - Hughes[16])

$$C = \frac{1}{\rho} \left( \frac{\partial \rho}{\partial p} \right)_S = \frac{1 - Z}{\Gamma p}, \quad (4.2.43)$$

where  $\left(\frac{\partial \rho}{\partial p}\right)_S$  denotes the derivative of  $\rho$  with respect to  $p$  at constant entropy  $S$ .

The non-dimensional compressibility  $C' = \frac{C}{C_2}$  can be expressed as

$$C' = \frac{1 - \left(\frac{Z_1}{\beta}\right)\rho'}{p' \left(1 - \left(\frac{Z_1}{\beta}\right)\right)}. \quad (4.2.44)$$

Also, the total energy of the disturbance is given by

$$E = 4\pi \int_{\bar{r}}^R \rho \left( U_m + \frac{1}{2} u^2 \right) r^2 dr, \quad (4.2.45)$$

where  $\bar{r}$  is the position of the inner boundary of the disturbance. Using equations (4.2.13), (4.2.32) and (4.2.16), equation (4.2.45) becomes

$$E = \frac{16\pi\mu R^3}{\alpha_1^2 \xi_0^2 \beta} \int_{\bar{r}'}^1 \left[ \frac{1}{2} \rho' u'^2 + \frac{p'(1 - Z_1)(\beta - Z_1 \rho')}{(\Gamma - 1)} \right] r'^2 dr'. \quad (4.2.46)$$

Hence the total energy of the shock wave is not constant and varies as  $R^3$ . This increase of total energy may be achieved by the pressure exerted on the fluid by the inner expanding surface (a contact surface or a piston).

In terms of dimensionless variables  $r', u', \rho', p'$ , and  $q'$ , the shock conditions (4.2.16) take the form

$$r' = 1, \quad u' = (1 - \beta), \quad \rho' = 1, \quad p' = 1, \quad q' = 1. \quad (4.2.47)$$

Equations (4.2.33), (4.2.34), (4.2.36) and (4.2.41) along with the boundary conditions (4.2.47) give the solution of our problem. The solution so obtained is a non-similar one, since the motion behind the shock can be determined only when a definite value for time is prescribed.

#### 4.2.4 RESULTS AND DISCUSSION

The distribution of the flow variables behind the shock front is obtained by numerical integration of equations (4.2.33),(4.2.34),(4.2.36) and (4.2.41) with the boundary conditions (4.2.47) by the Runge - Kutta method of the fourth order. For the purpose of numerical integration, the values of the constant parameters are taken to be (Ghoniem et al[4], Vishwakarma[6])  $\gamma = 1.4, k_p = 0, 0.2, 0.4; G_1 = 10, 50, 100; \beta' = 1; M^2 = 25; \delta_C = 1; \delta_R = 2; \Gamma_C = 0.05, 0.1, 0.8, \infty; \Gamma_R = 10, 50, 100, 200, 1000;$  and  $\frac{t}{t_0} = 2, 5$ . The value  $k_p = 0$  corresponds to the dust free case. Starting from the shock front ,the the numerical integration is carried out until the singularity of the solution

$$p' \beta^2 (1 - Z_1) - (\beta - Z_1 \rho') \rho' (1 - u')^2 = 0 \quad (4.2.48)$$

is reached. This marks the inner boundary of the disturbance and at this surface  $r'$  remains constant.

Figs. 1 - 3 show the variation of the reduced flow variables  $\rho', p', q', u', \frac{a_{iso}}{U}$  and the non- dimensional compressibility  $C'$  with reduced distance  $r'$  at various values of the parameters  $k_p, G_1; \Gamma_C, \frac{t}{t_0}; \Gamma_R, \frac{t}{t_0}$  and tables 1 - 3 show the position of the inner boundary surface in different cases.

Table 1. Position of the inner boundary surface for different values of  $k_p$  and  $G_1$  with  $\Gamma_C = 0.05$ ,  $\Gamma_R = 10$  and  $\frac{t}{t_0} = 2$

$k_p$	$G_1$	Position of the inner boundary surface $\bar{r}'$
0	0	0.9863
0.2	10	0.9692
	50	0.9872
	100	0.9881
0.4	10	0.9371
	50	0.9804
	100	0.9966

Table 2. Position of the inner boundary surface for different values of  $\frac{t}{t_0}$  and  $\Gamma_C$  with  $k_p = 0.2$ ,  $G_1 = 10$  and  $\Gamma_R = 10$ .

$\frac{t}{t_0}$	$\Gamma_C$	Position of the inner boundary surface $\bar{r}^i$
2	0.05	0.9672
	0.1	0.9631
	0.8	0.9529
	$\infty$	0.9515
5	0.05	0.9840
	0.1	0.9821
	0.8	0.9726
	$\infty$	0.9710

Table 3. Position of the inner boundary surface for different values of  $\frac{t}{t_0}$  and  $\Gamma_R$  with  $k_p = 0.2$ ,  $G_1 = 10$  and  $\Gamma_C = 0.1$

$\frac{t}{t_0}$	$\Gamma_R$	Position of the inner boundary surface $\bar{r}'$
2	50	0.9618
	100	0.9608
	200	0.9595
	1000	0.9556
5	50	0.9808
	100	0.9798
	200	0.9785
	1000	0.9749

Figs. 1(a) - (f) show that for given values of  $\Gamma_C$ ,  $\Gamma_R$  and  $G_1$ , the effects of an increase in the mass concentration of the solid particles  $k_p$  at given instant are

- (i) to increase the pressure  $p'$ , the isothermal sound speed  $\frac{a_{iso}}{U}$  and the total heat flux  $q'$  at any point in the flow field behind shock,
- (ii) to decrease the tendency of the density  $\rho'$  attaining a peak point (a maximum) in the flow -field behind the shock,
- (iii) to decrease the velocity  $u'$  and the non-dimensional compressibility  $C'$  in general. The decrease in the compressibility causes weaker compression of the gas behind the shock and, hence, a decrease in the shock strength, and
- (iv) to increase the distance between the inner boundary surface and the shock front (see table 1). This means an increase in the mass concentration of the solid particles has an effect of decreasing the shock strength, which is same as concluded in (iii) above.

Also , figs. 1(a)- (f) show that for given values of  $\Gamma_C$ ,  $\Gamma_R$ , and  $k_p$ , the effects of an increase in the ratio of the density of the solid particles to the initial density of the gas  $G_1$  at a given instant are

- (i) to decrease the pressure  $p'$ , the total heat flux  $q'$  and isothermal sound speed  $\frac{a_{iso}}{U}$ ,
- (ii) to increase the velocity  $u'$  , the density  $\rho'$  and the non-dimensional compressibility  $C'$ . The increase in the compressibility causes stronger compression of the gas behind the shocks and, hence, an increase in the shock strength, and

- (iii) to decrease the distance between the inner boundary surface and the shock front (see table 2). This means that an increase in the ratio of the density of the solid particles to the initial density of the gas has an effect of increasing the shock strength which is same as indicated in (ii) above.

The effects of an increase in the time  $(\frac{t}{t_0})$  are (see figs. 2 and 3)

- (i) to increase the density  $\rho'$  for lower values of  $\Gamma_C$  and  $\Gamma_R$  and to decrease it for higher values of  $\Gamma_C$  or  $\Gamma_R$ ,
- (ii) to decrease the pressure  $p'$  and the total heat flux  $q'$ ,
- (iii) to decrease the isothermal sound speed  $\frac{a_{iso}}{U}$  for higher values of  $\Gamma_C$  or  $\Gamma_R$ , and to displace the point of maximum isothermal sound speed nearer to the shock front for lower values of  $\Gamma_C$  and  $\Gamma_R$ . (see figs. 2(e) and 3(e)),
- (iv) to increase the fluid velocity  $u'$  and the non-dimensional compressibility  $C'$ , and
- (v) to decrease the distance between the inner boundary surface and the shock front (see tables 2 and 3).

Effects of an increase in the value of conductive heat transfer parameter  $\Gamma_C$  are

- (i) to decrease the density  $\rho'$ , the total heat flux  $q'$  and the fluid velocity  $u'$ ,
- (ii) to increase the pressure  $p'$  and the non-dimensional compressibility  $C'$ , and
- (iii) to increase the distance between the inner boundary surface and the shock front (see table 2).

Effects of an increase in the value of radiative heat transfer parameter  $\Gamma_R$  are very similar to those of an increase in  $\Gamma_C$ .

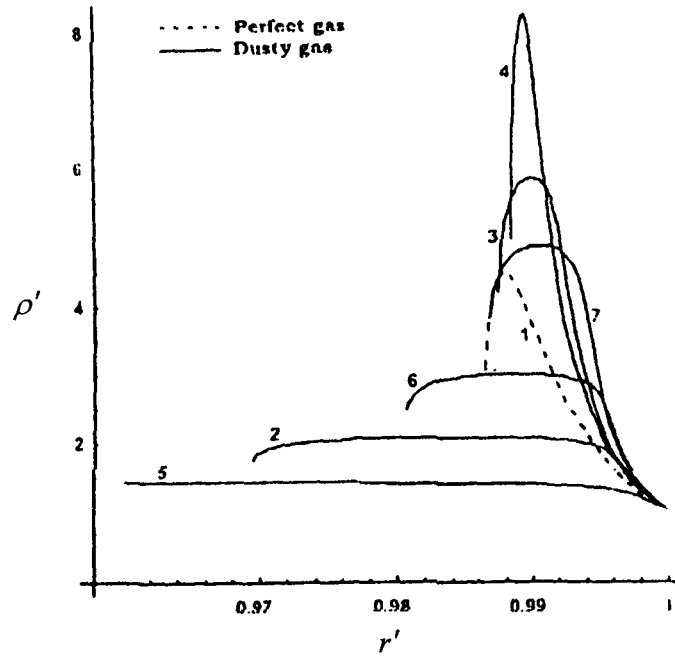


Fig. 1(a) Variation of reduced density  $\rho'$  in the region behind the shock front for  $\Gamma_c = 0.05$ ,  $\Gamma_R = 10$  and  $t/t_0 = 2$  1:  $k_p=0, G_1=0$ ; 2:  $k_p=0.2, G_1=10$ ; 3:  $k_p=0.2, G_1=50$ ; 4:  $k_p=0.2, G_1=100$ ; 5:  $k_p=0.4, G_1=10$ ; 6:  $k_p=0.4, G_1=50$ ; 7:  $k_p=0.4, G_1=100$

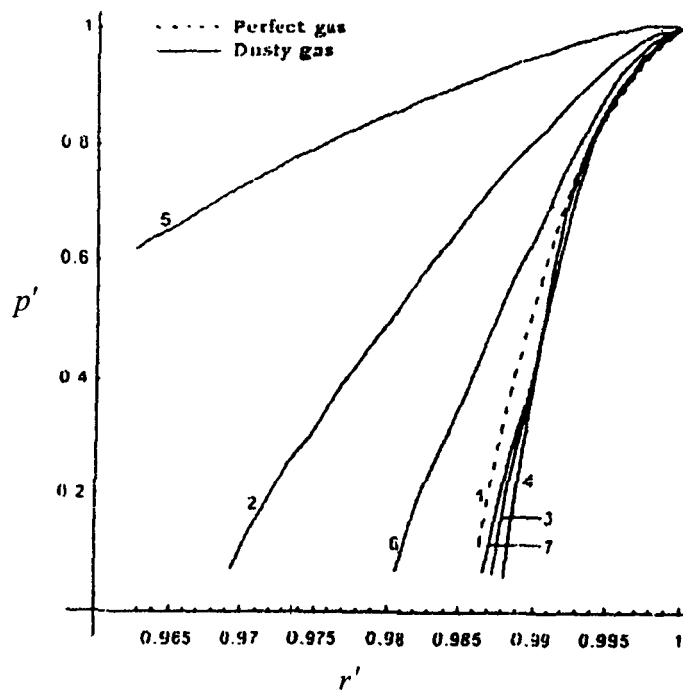


Fig. 1(b) Variation of reduced pressure  $p'$  in region behind the shock front for  $\Gamma_c = 0.05$ ,  $\Gamma_R = 10$  and  $t/t_0 = 2$  1:  $k_p=0, G_1=0$ ; 2:  $k_p=0.2, G_1=10$ ; 3:  $k_p=0.2, G_1=50$ ; 4:  $k_p=0.2, G_1=100$ ; 5:  $k_p=0.4, G_1=10$ ; 6:  $k_p=0.4, G_1=50$ ; 7:  $k_p=0.4, G_1=100$

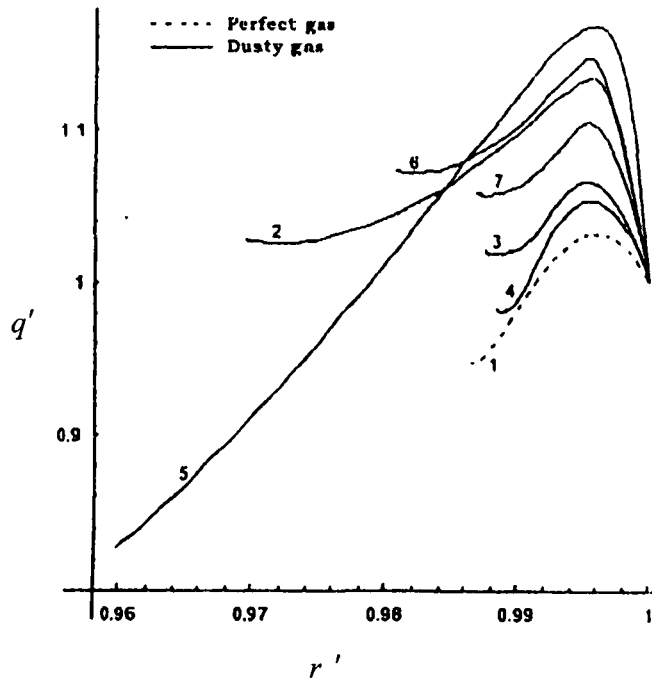


Fig. 1(c) Variation of reduced total heat flux  $q'$  in region behind the shock front for  $\Gamma_{cc}=0.05$ ,  $\Gamma_R=10$  and  $t/t_0=2$  1:  $k_p=0$ ,  $G_1=0$ ; 2:  $k_p=0.2$ ,  $G_1=10$ ; 3:  $k_p=0.2$ ,  $G_1=50$ ; 4:  $k_p=0.2$ ,  $G_1=100$ ; 5:  $k_p=0.4$ ,  $G_1=10$ ; 6:  $k_p=0.4$ ,  $G_1=50$ ; 7:  $k_p=0.4$ ,  $G_1=100$

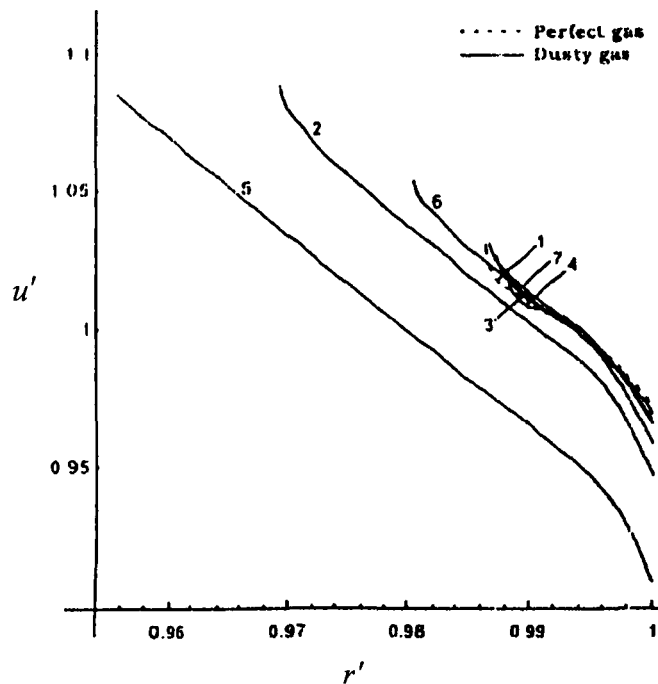


Fig 1(d) Variation of reduced fluid velocity  $u'$  in the region behind the shock front for  $\Gamma_C=0.05$ ,  $\Gamma_R=10$  and  $t/t_0=2$  1:  $k_p=0$ ,  $G_1=0$ ; 2:  $k_p=0.2$ ,  $G_1=10$ ; 3:  $k_p=0.2$ ,  $G_1=50$ ; 4:  $k_p=0.2$ ,  $G_1=100$ ; 5:  $k_p=0.4$ ,  $G_1=10$ ; 6:  $k_p=0.4$ ,  $G_1=50$ ; 7:  $k_p=0.4$ ,  $G_1=100$

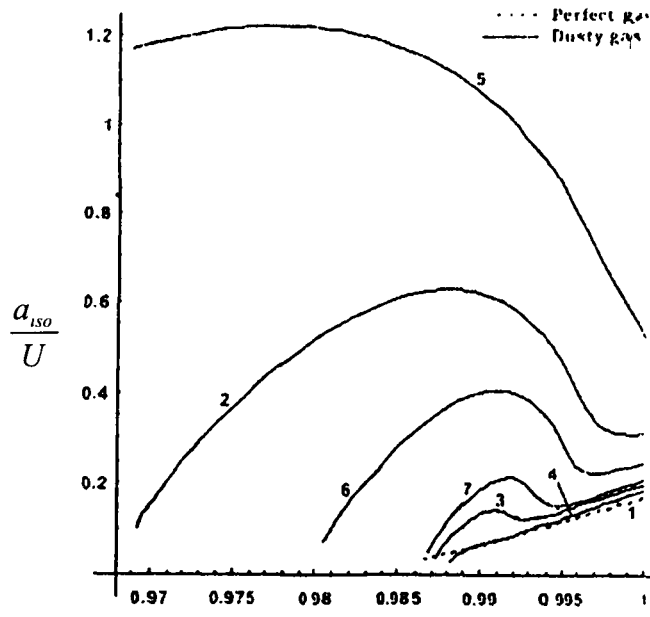


Fig 1 (e) Variation of reduced isothermal sound speed  $\frac{a_{iso}}{U}$  in the region behind the shock front for  $\Gamma_C = 0.05$ ,  $\Gamma_R = 10$  and  $t/t_0 = 2$  1:  $k_p = 0$ ,  $G_1 = 0$ ; 2:  $k_p = 0.2$ ,  $G_1 = 10$ ; 3:  $k_p = 0.2$ ,  $G_1 = 50$ ; 4:  $k_p = 0.2$ ,  $G_1 = 100$ ; 5:  $k_p = 0.4$ ,  $G_1 = 10$ ; 6:  $k_p = 0.4$ ,  $G_1 = 50$ ; 7:  $k_p = 0.4$ ,  $G_1 = 100$

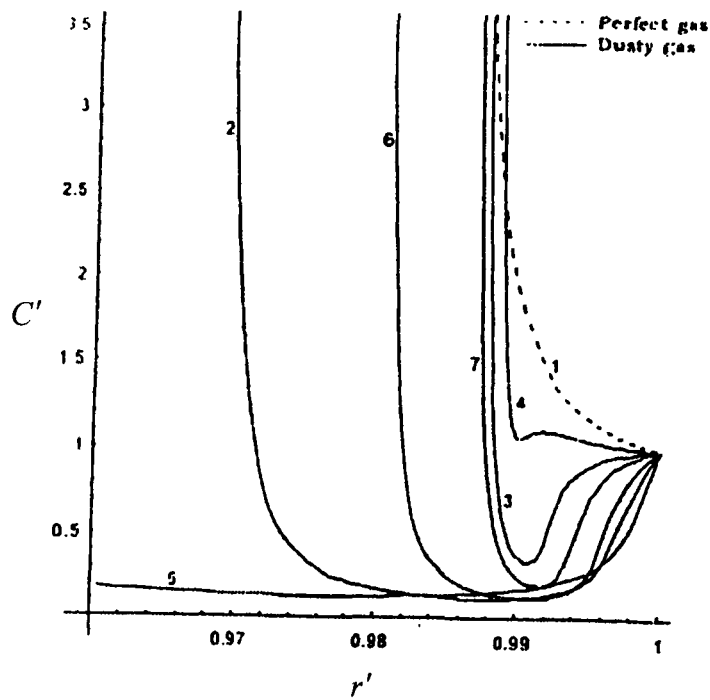


Fig 1 (f) Variation of reduced non-dimensional compressibility  $C'$  in the region behind the shock front for  $\Gamma_C = 0.05$ ,  $\Gamma_R = 10$  and  $t/t_0 = 2$  1:  $k_p = 0$ ,  $G_1 = 0$ ; 2:  $k_p = 0.2$ ,  $G_1 = 10$ ; 3:  $k_p = 0.2$ ,  $G_1 = 50$ ; 4:  $k_p = 0.2$ ,  $G_1 = 100$ ; 5:  $k_p = 0.4$ ,  $G_1 = 10$ ; 6:  $k_p = 0.4$ ,  $G_1 = 50$ ; 7:  $K_p = 0.4$ ,  $G_1 = 100$

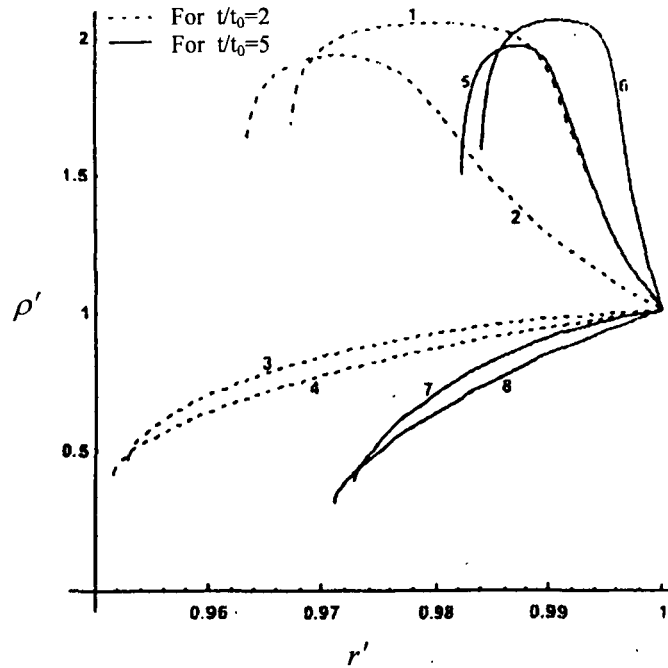


Fig. 2(a) Variation of reduced density  $\rho'$  in the region behind the shock front for  $k_p = 0.02$ ,  $G_1 = 10$  and  $\Gamma_R = 10$ : 1:  $\Gamma_C = 0.05$ ,  $t/t_0 = 2$ ; 2:  $\Gamma_C = 0.1$ ,  $t/t_0 = 2$ ; 3:  $\Gamma_C = 0.8$ ,  $t/t_0 = 2$ ; 4:  $\Gamma_C = \infty$ ,  $t/t_0 = 2$ ; 5:  $\Gamma_C = 0.05$ ,  $t/t_0 = 5$ ; 6:  $\Gamma_C = 0.1$ ,  $t/t_0 = 5$ ; 7:  $\Gamma_C = 0.8$ ,  $t/t_0 = 5$ ; 8:  $\Gamma_C = \infty$ ,  $t/t_0 = 5$

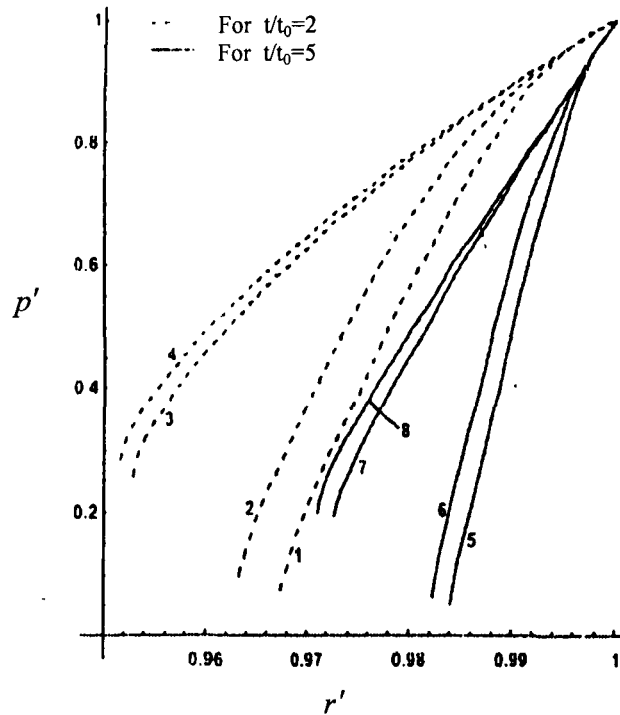


Fig. 2(b) Variation of reduced pressure  $p'$  in the region behind the shock front for  $k_p = 0.02$ ,  $G_1 = 10$  and  $\Gamma_R = 10$ : 1:  $\Gamma_C = 0.05$ ,  $t/t_0 = 2$ ; 2:  $\Gamma_C = 0.1$ ,  $t/t_0 = 2$ ; 3:  $\Gamma_C = 0.8$ ,  $t/t_0 = 2$ ; 4:  $\Gamma_C = \infty$ ,  $t/t_0 = 2$ ; 5:  $\Gamma_C = 0.05$ ,  $t/t_0 = 5$ ; 6:  $\Gamma_C = 0.1$ ,  $t/t_0 = 5$ ; 7:  $\Gamma_C = 0.8$ ,  $t/t_0 = 5$ ; 8:  $\Gamma_C = \infty$ ,  $t/t_0 = 5$

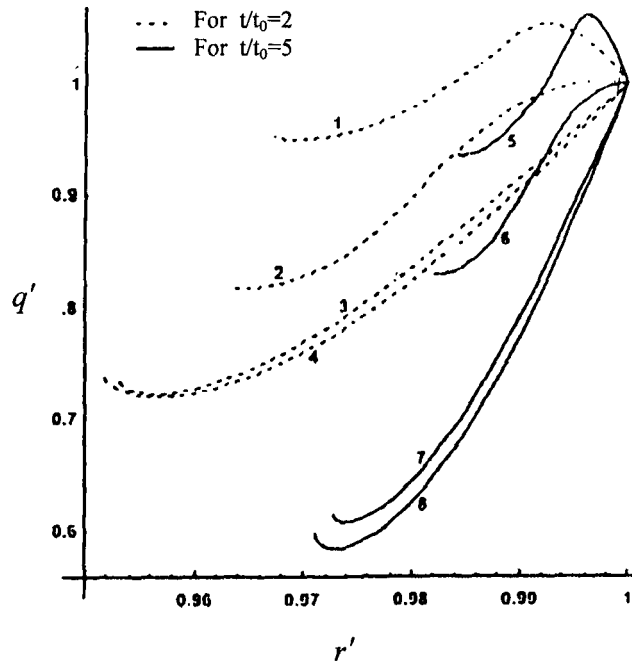


Fig. 2(c) Variation of reduced total heat flux  $q'$  in the region behind the shock front for  $k_p=0.02$ ,  $G_1=10$  and  $\Gamma_R=10$  1:  $\Gamma c=0.05$ ,  $t/t_0=2$ ; 2:  $\Gamma c=0.1$ ,  $t/t_0=2$ ; 3:  $\Gamma c=0.8$ ,  $t/t_0=2$ ; 4:  $\Gamma c=\infty$ ,  $t/t_0=2$ ; 5:  $\Gamma c=0.05$ ,  $t/t_0=5$ ; 6:  $\Gamma c=0.1$ ,  $t/t_0=5$ ; 7:  $\Gamma c=0.8$ ,  $t/t_0=5$ ; 8:  $\Gamma c=\infty$ ,  $t/t_0=5$

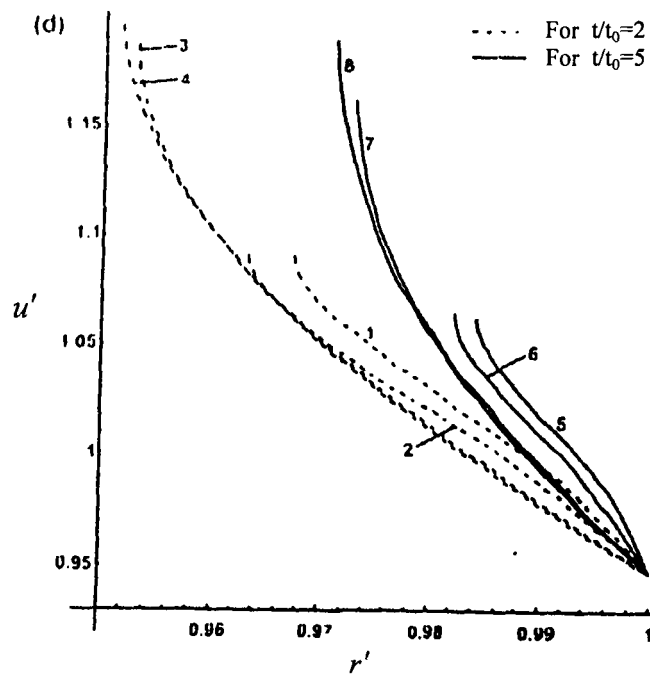


Fig 2 (d) Variation of reduced fluid velocity  $u'$  in the region behind the shock front for  $k_p=0.02$ ,  $G_1=10$  and  $\Gamma_R=10$  1:  $\Gamma c=0.05$ ,  $t/t_0=2$ ; 2:  $\Gamma c=0.1$ ,  $t/t_0=2$ ; 3:  $\Gamma c=0.8$ ,  $t/t_0=2$ ; 4:  $\Gamma c=\infty$ ,  $t/t_0=2$ ; 5:  $\Gamma c=0.05$ ,  $t/t_0=5$ ; 6:  $\Gamma c=0.1$ ,  $t/t_0=5$ ; 7:  $\Gamma c=0.8$ ,  $t/t_0=5$ ; 8:  $\Gamma c=\infty$ ,  $t/t_0=5$

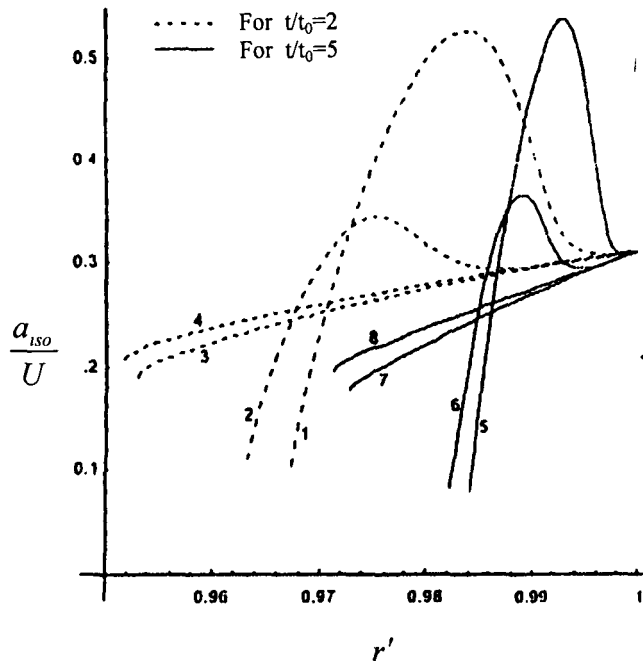


Fig. 2(e) Variation of reduced isothermal sound speed  $a_{iso}/U$  in the region behind the shock front for  $k_p=0.02$ ,  $G_1=10$  and  $\Gamma_r$  For  $t/t_0=2$   $c=0.05$ ,  $t/t_0=2$ ; 2:  $\Gamma c=0.1$ ,  $t/t_0=2$ ; 3:  $\Gamma c=0.8$ ,  $t/t_0=2$ ; 4:  $\Gamma c=\infty$ ,  $t/t_0=2$ ; 5:  $\Gamma c=0.05$ ,  $t/t_0=5$ ; 6:  $\Gamma c=0.1$ ,  $t/t_0=5$ ; 7:  $\Gamma c=0.8$ ,  $t/t_0=5$ ; 8:  $\Gamma c=\infty$ ,  $t/t_0=5$

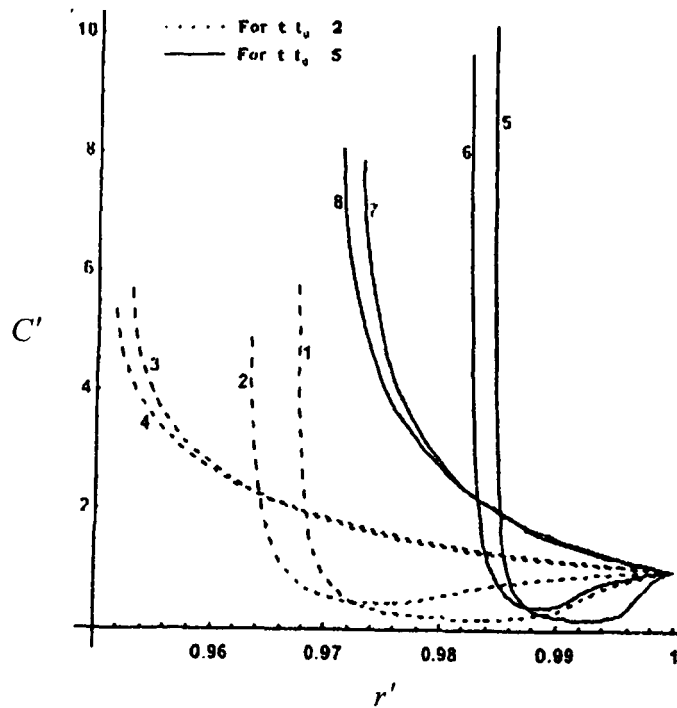


Fig. 2(f) Variation of reduced non-dimensional compressibility  $C'$  in region behind the shock front for  $K_p=0.02$ ,  $G_1=10$  and  $\Gamma_r=10$  1:  $\Gamma c=0.05$ ,  $t/t_0=2$ ; 2:  $\Gamma c=0.1$ ,  $t/t_0=2$ ; 3:  $\Gamma c=0.8$ ,  $t/t_0=2$ ; 4:  $\Gamma c=\infty$ ,  $t/t_0=2$ ; 5:  $\Gamma c=0.05$ ,  $t/t_0=5$ ; 6:  $\Gamma c=0.1$ ,  $t/t_0=5$ ; 7:  $\Gamma c=0.8$ ,  $t/t_0=5$ ; 8:  $\Gamma c=\infty$ ,  $t/t_0=5$

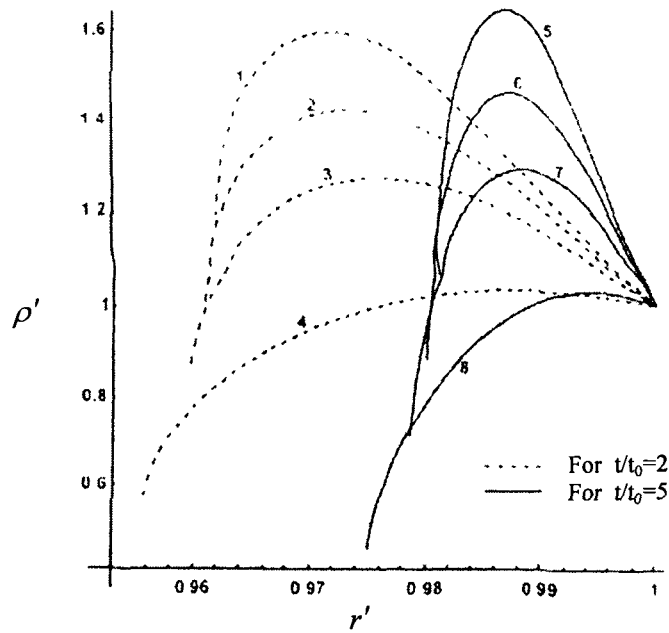


Fig. 3(a) Variation of reduced density  $\rho'$  in the region behind the shock front for  $k_p = 0.02$ ,  $G_1 = 10$  and  $\Gamma c = 0.1$  1:  $\Gamma_R = 50$ ,  $t/t_0 = 2$ ; 2:  $\Gamma_R = 100$ ,  $t/t_0 = 2$ ; 3:  $\Gamma_R = 200$ ,  $t/t_0 = 2$ ; 4:  $\Gamma_R = 1000$ ,  $t/t_0 = 2$ ; 5:  $\Gamma_R = 50$ ,  $t/t_0 = 5$ ; 6:  $\Gamma_R = 100$ ,  $t/t_0 = 5$ ; 7:  $\Gamma_R = 200$ ,  $t/t_0 = 5$ ; 8:  $\Gamma_R = 1000$ ,  $t/t_0 = 5$

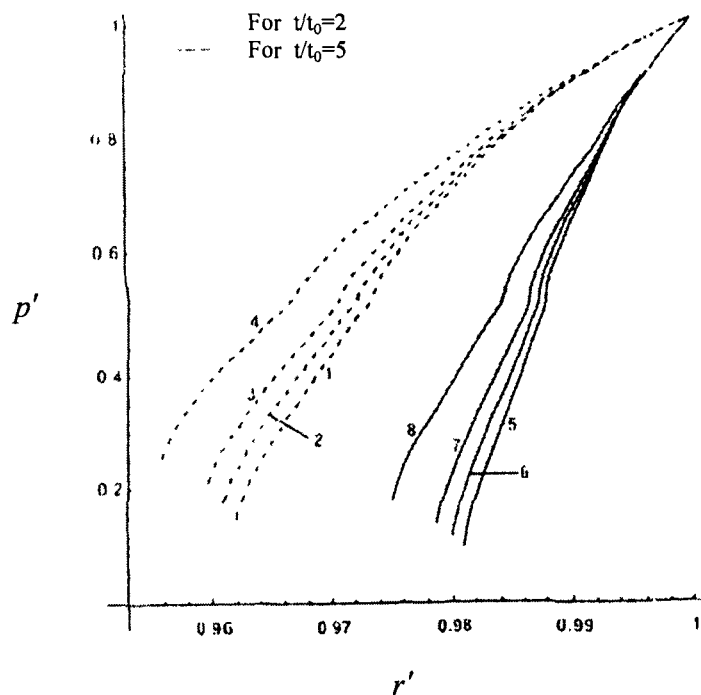


Fig. 3(b) Variation of reduced pressure  $p'$  in the region behind the shock front for  $K_p = 0.02$ ,  $G_1 = 10$  and  $\Gamma c = 0.1$  1:  $\Gamma_R = 50$ ,  $t/t_0 = 2$ ; 2:  $\Gamma_R = 100$ ,  $t/t_0 = 2$ ; 3:  $\Gamma_R = 200$ ,  $t/t_0 = 2$ ; 4:  $\Gamma_R = 1000$ ,  $t/t_0 = 2$ ; 5:  $\Gamma_R = 50$ ,  $t/t_0 = 5$ ; 6:  $\Gamma_R = 100$ ,  $t/t_0 = 5$ ; 7:  $\Gamma_R = 200$ ,  $t/t_0 = 5$ ; 8:  $\Gamma_R = 1000$ ,  $t/t_0 = 5$

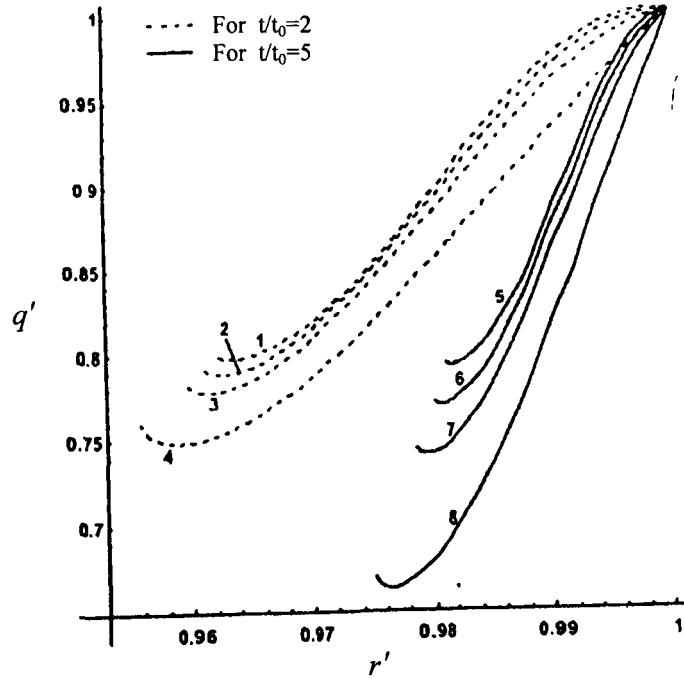


Fig. 3(c) Variation of reduced total heat flux  $q'$  in the region behind the shock front for  $k_p=0.02$ ,  $G_1=10$  and  $\Gamma_c=0.1$  1:  $\Gamma_R=50$ ,  $t/t_0=2$ ; 2:  $\Gamma_R=100$ ,  $t/t_0=2$ ; 3:  $\Gamma_R=200$ ,  $t/t_0=2$ ; 4:  $\Gamma_R=1000$ ,  $t/t_0=2$ ; 5:  $\Gamma_R=50$ ,  $t/t_0=5$ ; 6:  $\Gamma_R=100$ ,  $t/t_0=5$ ; 7:  $\Gamma_R=200$ ,  $t/t_0=5$ ; 8:  $\Gamma_R=1000$ ,  $t/t_0=5$

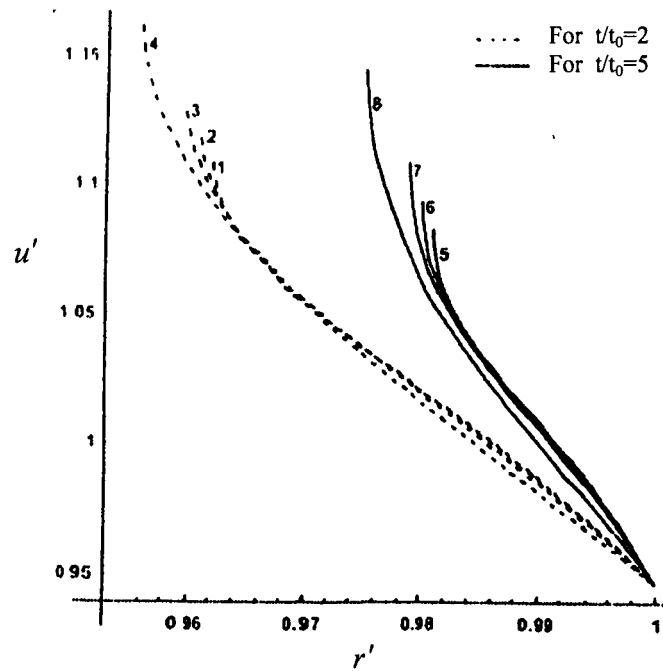


Fig. 3(d) Variation of reduced fluid velocity  $u'$  in the region behind the shock front for  $k_p=0.02$ ,  $G_1=10$  and  $\Gamma_c=0.1$  1:  $\Gamma_R=50$ ,  $t/t_0=2$ ; 2:  $\Gamma_R=100$ ,  $t/t_0=2$ ; 3:  $\Gamma_R=200$ ,  $t/t_0=2$ ; 4:  $\Gamma_R=1000$ ,  $t/t_0=2$ ; 5:  $\Gamma_R=50$ ,  $t/t_0=5$ ; 6:  $\Gamma_R=100$ ,  $t/t_0=5$ ; 7:  $\Gamma_R=200$ ,  $t/t_0=5$ ; 8:  $\Gamma_R=1000$ ,  $t/t_0=5$

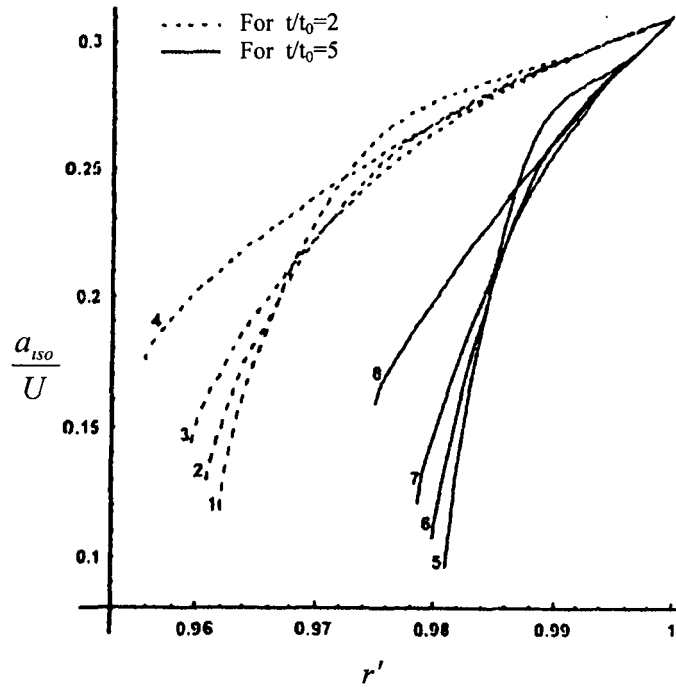


Fig. 3(e) Variation of reduced isothermal sound speed  $a_{iso}/U$  in the region behind the shock front for  $k_p=0.02$ ,  $G_1=10$  and  $\Gamma_c =0.1$  1:  $\Gamma_R=50$ ,  $t/t_0=2$ ; 2:  $\Gamma_R=100$ ,  $t/t_0=2$ ; 3:  $\Gamma_R=200$ ,  $t/t_0=2$ ; 4:  $\Gamma_R=1000$ ,  $t/t_0=2$ ; 5:  $\Gamma_R=50$ ,  $t/t_0=5$ ; 6:  $\Gamma_R=100$ ,  $t/t_0=5$ ; 7:  $\Gamma_R=200$ ,  $t/t_0=5$ ; 8:  $\Gamma_R=1000$ ,  $t/t_0=5$

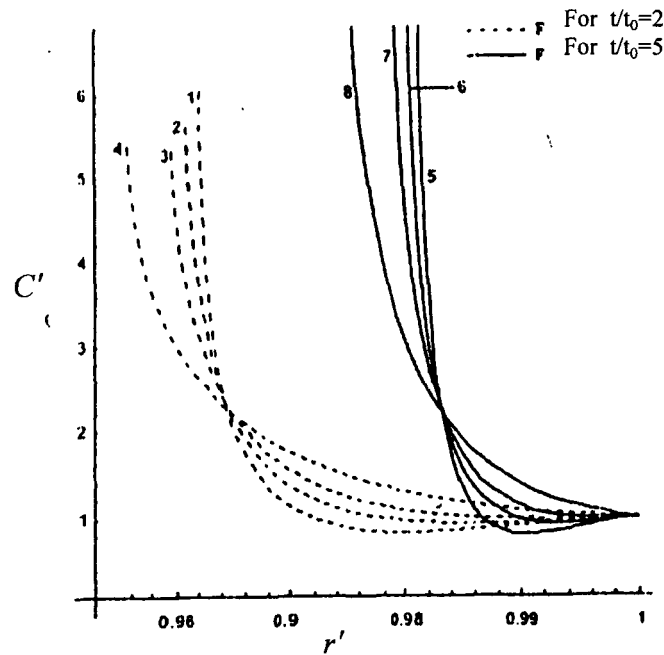


Fig. 3(f) Variation of reduced non-dimensional compressibility  $C'$  in the region behind the shock front for  $k_p=0.02$ ,  $G_1=10$  and  $\Gamma_c =0.1$  1:  $\Gamma_R=50$ ,  $t/t_0=2$ ; 2:  $\Gamma_R=100$ ,  $t/t_0=2$ ; 3:  $\Gamma_R=200$ ,  $t/t_0=2$ ; 4:  $\Gamma_R=1000$ ,  $t/t_0=2$ ; 5:  $\Gamma_R=50$ ,  $t/t_0=5$ ; 6:  $\Gamma_R=100$ ,  $t/t_0=5$ ; 7:  $\Gamma_R=200$ ,  $t/t_0=5$ ; 8:  $\Gamma_R=1000$ ,  $t/t_0=5$

# Bibliography

- [1] Elliott,L.A.: *Similarity methods in radiation hydrodynamics*. Proc. R. Soc. Lord. A 258, 287-301 (1960).
- [2] Wang, K. C.: *Approximate solution of a plane radiating piston problem*. Phys. fluids , 1922-19280 (1966).
- [3] Helliwell, J. B.: *Self-similar piston problem with radiative heat transfer*. J. Fluid Mech. 37, 497 -512 (1969),
- [4] Ghoneim, A. F.,Kamel,M. M, Berger, S. A. and Oppenheim, A. K.: *Effects of internal heat transfer on the structure of self-similar blast waves*. J. Fluid Mech. 117, 473-491 (1982).
- [5] Singh, K. K. and Vishwakarma, J . P.: *Propagation of spherical shock waves in a dusty gas with radiation heat-flux*. J. Theo. Appl. Mech. 45(4), 801-817 (2007).
- [6] Vishwakarma, J. P.: *Propagation of shock waves in a dusty gas with exponentially varying density*. Eur. Phys. J. B 16, 369-372 (2000).

- [7] Singh, J. B. and Srivastava, S. K.: *Propagation of spherical shock waves in an exponential medium with radiation heat flux*. *Astrophys. Space Sci.* 88, 277-282 (1982).
- [8] Pai, S. I. :*Two phase flow(Vieweg Tracts in Pure and Applied Physics)(Braunschweig:Vieweg) Chapter v* (1977).
- [9] Marble, F. E.: *Dynamics of dusty gases, Ann. Rev. Fluid Mech.* 2, 397-446 (1970).
- [10] Zel'dovich, Ya. B. and Raizer, Yu.P.:*Physics of Shock Waves and High Temperature Hydrodynamic Phenomena Vol. II, Academic Press ,New york* (1967).
- [11] Bhowick, J. B.: *An exact analytical solution in radiating gas dynamics, Astrophys Space Sci.* 74, 481-485(1981).
- [12] Laumbach, D. D. and Probst, R. F.: *A point explosion in a cold exponential atmosphere Part II Radiating flow . J. Fluid Mech.* 40, 833-858 (1970).
- [13] Miura, H. and Glass I.I.: *Development of the flow induced by a piston moving impulsively in a dusty gas. Proc R. Soc.Lord.A* 397, 23-39 (1985).
- [14] Naidu, G.H., Venkatanandan, K. and Ranga Rao, M.P.: *Approximate analytical solutions for self similar flows of a dusty gas with variable energy. Int. J.Eng. Sci.* 23 ,23-39 (1985).
- [15] Verma, B.G. and Vishwakarma, J.P.: *Axially symmetric explosion in magnetogasdynamics. Astrophysics.Space Sci.* 69, 177-188 (1980).
- [16] Moelwyn-Hughes, E.A.: *Physical Chemistry Pergamon Press London* (1961).

- [17] Pai, S.I., Menon,S. and Fan,Z.Q.: *Similarity solution of a strong shock wave propagation in a mixture of gas and dusty particles*. Int.J.Eng.Sci. 18,1365-1373 (1980).
- [18] Vishwakarma, J.P., Nath, G .and Singh,K.K.: *Propagation of shock waves in a dusty gas with heat conduction, radiation heat flux and exponentially varying density* . Phys. scr. 78, 11pp (2008).
- [19] Abdel- Raouf, A.M. and Gretler, W.: *Quasi-similar solution for blast wave with internal heat transfer effects*., Fluid Dyn.Res. 8, 273-285 (1991).
- [20] Pomaraming,G.C.: *The Equation of Radiation Hydrodynamics* (Int. Ser. Monographs in Natural Philosophy Vol. 54) (Oxford: Pergamon) (1973).

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