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Reconstruction of Potentials as Well as Dynamics of Scalar Fields in DGP Braneworld Model

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Abstract Here, we investigate the cosmological implications of Holographic Dark Energy (HDE) in the DGP braneworld model of the universe. Taking HDE in DGP braneworld, we investigate the model of non-interacting dark energy and derive its equation of state. Subsequently, we study the correspondence between k-essence, tachyon, dilaton, hessence and DBI-essence dark energy with the non-interacting HDE in a flat DGP braneworld and reconstruct the corresponding scalar potentials which describe the dynamics of the scalar fields. Also we study the correspondence between above mentioned scalar potentials and effective dark energy coming from DGP braneworld in the absence of HDE and in this situation, the potentials are reconstructed.

Keywords DGP braneworld · Tachyon · k-Essence · DBI-essence · Hessence · Scalar fields · Dilaton · Holographic dark energy (HDE)

1 Introduction

In recent observations it is strongly believed that the universe is experiencing an accelerated expansion. The observation from type Ia supernovae [1–3] is associated with Large scale Structure and Cosmic Microwave Background anisotropies have shown the evidences

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to support cosmic acceleration. It is found that cosmic acceleration is driven by some unknown fluid having its gravitational effect in the very late universe. This unknown fluid has distinguishing feature of violating strong energy condition (SEC) being called dark energy (DE) [4]. DE is characterized by negative pressure ($p < 0$) and positive energy density ($\rho > 0$) which is related by the equation of state (EoS) $p = w\rho$. The combined astrophysical observations suggests that universe is spatially flat and consists of about 70% DE, 30% dust matter (cold dark matter plus baryons) and negligible radiation. Various models have been proposed to solve this problem but nature of this DE still remains a source of doubt. A comprehensive review of these models is available in [4]. There are different candidates which obey the property of dark energy given by—quintessence [5, 6], k-essence [7], tachyon [8], phantom [9], ghost condensate [10, 11] and quintom [12, 13], interacting dark energy models [14, 15], brane world models [16] and Chaplygin gas models [17].

A simple and well studied model of brane-gravity (BG) is the Dvali-Gabadadze-Porrati (DGP) braneworld model [18–20]. In this model our 4-dimensional world is a FRW brane embedded in a 5-dimensional Minkowski bulk. On the 4-dimensional brane the action of gravity is proportional to M_P^2 whereas in the bulk it is proportional to the corresponding quantity in 5-dimensions. The model is then characterized by a cross over length scale

$$r_c = \frac{M_P^2}{2M_5^3} \tag{1.1}$$

so that gravity is 4-dimensional theory at scales $a \ll r_c$ where matter behaves as pressure less dust but gravity *leaks out* into the bulk at scales $a \gg r_c$ where matter approaches the behaviour of a cosmological constant [21]. A review on brane-gravity and its various applications with special attention to cosmology is available in [22–25].

The another way to approach to the problem of DE arises from holographic principle which states that the number of degrees of freedom for a system within a finite region should be finite and is bounded by the area of its boundary. As in [26] one obtains HDE as

$$\rho_D = 3c^2 M_P^2 L^{-2} \tag{1.2}$$

where L is an IR cut-off and $M_P^2 = 1/\sqrt{8\pi G}$. Li [27] showed that if we choose L as the radius of the event horizon, we can get the correct equation of state and get the desired accelerating universe. In the above expression of ρ_D, c is any free dimensionless parameter characterizing all of the uncertainties of the theory, whose value can only be determined by observations. The models of HDE, to some extent has advantage comparing to other dynamical dark energy models because they originate from fundamental principles of quantum gravity.

On the basis of the holographic principle proposed by [29] several authors have studied holographic model for dark energy [30, 31]. Employment of Friedman equation $\rho = 3M_p^2 H^2$ where ρ is the total energy density and taking $L = H^{-1}$ one can find $\rho_m = 3(1-c^2)M_p^2 H^2$. Thus either ρ_m or ρ_D behaves like H^2. If we take L as the size of the current universe, say, the Hubble radius $\frac{1}{H}$ then the dark energy density will be close to the observational result. So, in the last few years, the HDE models have [27, 28] received considerable interest.

It may be noted that in literature, standard DGP model has been generalized to (i) LDGP model by adding a cosmological constant [32], (ii) QDGP model by adding quiescence perfect fluid [33], (iii) CDGP by Chaplygin gas [34] and (iv) SDGP by a scalar field [35]. In [36, 37] the DGP model has been analyzed by adding HDE. In [38], DGP brane cosmology with a brane scalar field is introduced. The aim of this paper is to investigate the correspondence between HDE and other dark energy candidates namely k-essence field [39], tachyonic field [40], dilaton field [11, 41, 42], Hessence [43] and DBI-essence [44] in DGP braneworld model of the universe without any considerable interaction. In General Relativity set up this type of correspondence have been extensively studied [45–48]. We suggest holographic description of the scalar fields in DGP braneworld and reconstruct the potential and dynamics of various scalar fields which describe DE.

The paper is organized as follows: Sect. 2 deals with HDE in the DGP braneworld model while in Sect. 3, we establish the correspondence between HDE and other dark energy namely, k-essence, tachyonic field, dilaton, hessence and DBI-essence. Also Sect. 4 deals with the correspondence between above types of dark energy models and effective DE coming from DGP brane. The paper ends with a short discussion in Sect. 5.

2 HDE in the DGP Braneworld Model

In flat, homogeneous and isotropic brane, the Friedmann equation in DGP braneworld model [18–20] is given by

$$H^2=\left(\sqrt{\frac{\rho_{total}}{3}+\frac{1}{4r_c^2}}+\epsilon\frac{1}{2r_c}\right)^2 \tag{2.1}$$

or, equivalently (squaring r.h.s of (2.1) and eliminating square root term)

$$H^2-\epsilon\frac{H}{r_c}=\frac{\rho_{total}}{3} \tag{2.2a}$$

where $H=\frac{\dot{a}}{a}$ is the Hubble parameter, ρ_{total} is the total cosmic fluid energy density and $r_c=\frac{M_P^2}{2M_5^2}$ is the crossover scale which determines the transition from 4D to 5D behavior and $\epsilon=\pm 1$ (choosing $8\pi G=1$ i.e., $M_P^2=1$). For $\epsilon=1$, we have standard DGP(+) model which is self accelerating model without any form of dark energy, and effective w is always non phantom. However for $\epsilon=-1$, we have DGP(−) model which does not self accelerate but requires dark energy on the brane. It experiences 5D gravitational modifications to its dynamics which effectively screen dark energy.

The Friedmann equation (2.2a) can be written as

$$H^2=\frac{1}{3}(\rho_m+\rho_{eff}) \tag{2.2b}$$

where ρ_{eff}, the effective energy density is given by

$$\rho_{eff}=\rho_D+\epsilon\frac{3H}{r_c} \tag{2.2c}$$

Here we take $\rho_{total}=\rho_m+\rho_D$ where ρ_m is the energy density of CDM and ρ_D is the energy density of DE. The energy conservation equation is given by

$$\dot{\rho}_{total}+3H(\rho_{total}+p_{total})=0 \tag{2.2d}$$

where $p_{total}=p_D$ which is the thermodynamic pressure of DE. Now assume that there is no interaction between dark matter and DE. As the two component matter system is non-interacting so they satisfy energy conservation separately, i.e.

$$\dot{\rho}_m+3H\rho_m=0 \tag{2.3}$$

and

$$\dot{\rho}_D + 3H(\rho_D + p_D) = 0 \tag{2.4}$$

Our choice for HDE density is

$$\rho_D = \frac{3c^2}{R_E^2} \tag{2.5}$$

where c is a constant and R_E is the future event horizon, given by

$$R_E = a\int_t^\infty \frac{dt}{a} = a\int_a^\infty \frac{da}{Ha^2} \tag{2.6}$$

The Friedmann equation (2.1) can be rewritten as

$$\frac{H}{H_0} = \sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}} \tag{2.7}$$

where the dimensionless density parameters are defined as

$$\Omega_m = \frac{\rho_m}{3H_0^2}, \qquad \Omega_D = \frac{\rho_D}{3H_0^2}, \qquad \Omega_{r_c} = \frac{1}{4r_c^2H_0^2} \tag{2.8}$$

and H_0 is Hubble parameter at redshift $z = 0$. Substituting the value of ρ_D from (2.5) in (2.8) and then differentiating the resulting equation w.r.t redshift $z = 1/a - 1$, we get the evolution equation of Ω_D as

$$\frac{d\Omega_D}{dz} = \frac{2\Omega_D^{3/2}}{c(1+z)}\left(\frac{c}{\sqrt{\Omega_D}} - \frac{1}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}}}\right) \tag{2.9}$$

From (2.2a), setting $z = 0$ we get the initial condition of the above differential equation as

$$\Omega_D(0) = 1 - 2\epsilon\sqrt{\Omega_{r_c}} - \Omega_m(0) \tag{2.10}$$

From conservation equation (2.4) of HDE, we obtain the equation of state parameter of HDE,

$$w_D = -1 + (1+z)\frac{1}{3\omega_D}\frac{d\Omega_D}{dz} \tag{2.11}$$

Eliminating $\frac{d\Omega_D}{dz}$ from (2.9) and (2.11) we finally get

$$w_D = -\frac{1}{3} - \frac{2}{3c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}}} \tag{2.12}$$

As in GR based theory here also $w_D < -1/3$.

3 Correspondence Between HDE and Other Dark Energies

In the following subsections, we consider several types of dark energy models namely, k-essence, tachyon, dilaton, hessence and DBI-essence. Then we investigate the correspondence between HDE in DGP braneworld model and above mentioned candidates of DE.

After that we reconstruct the potentials as well as dynamics of scalar fields of DE candidates in DGP model.

3.1 Holographic k-Essence Model

In the kinetically driven scalar field theory, we have *non-canonical* kinetic energy term with *no* potential. Scalars, modelling this theory, are popularly known as *k-essence*. First time, it was used for kinetically driven inflation. Later on, it was used as a source of dark energy. Motivated by Born-Infeld action of String Theory [49], it was used as a source to explain the mechanism for producing the late time acceleration of the universe. This model is given by the action [39]

$$S=\int d^4x\sqrt{-g}\tilde{\mathcal{L}}(\tilde{\phi},\tilde{X}) \tag{3.1}$$

with

$$\tilde{\mathcal{L}}(\tilde{\phi},\tilde{X})=K(\tilde{\phi})\tilde{X}+L(\tilde{\phi})\tilde{X}^2 \tag{3.2}$$

ignoring higher order terms of

$$\tilde{X}=\frac{1}{2}g^{ij}\partial_i\tilde{\phi}\partial_j\tilde{\phi} \tag{3.3}$$

Using the following transformations, $\phi=\int d\tilde{\phi}\sqrt{|L(\tilde{\phi})|/K(\tilde{\phi})}$, $X=\frac{|L|}{K}\tilde{X}$ and $f(\phi)=K^2/|L|$, the action (3.1) can be rewritten as

$$S=\int d^4x\sqrt{-g}f(\phi)\mathcal{L}(X) \tag{3.4}$$

with

$$\mathcal{L}(X)=X^2-X. \tag{3.5}$$

From the action (3.4), the energy-momentum tensor components can be written as

$$T_{ij}=f(\phi)\Big[\frac{d\mathcal{L}}{dX}\partial_i\phi\partial_j\phi-g_{ij}\mathcal{L}\Big]. \tag{3.6}$$

These equations yield energy density as

$$\rho_k=f(\phi)[-X+3X^2], \tag{3.7}$$

and the pressure density as

$$p_k=f(\phi)[-X+X^2]. \tag{3.8}$$

where $X=(1/2)\dot{\phi}^2$ for homogeneous ϕ.

Therefore, from (3.7) and (3.8), the EoS parameter for k-essence scalar field is given as

$$w_k=p_k/\rho_k=\frac{(X-1)}{(3X-1)} \tag{3.9}$$

Equating (3.9) with the HDE equation of state parameter (2.12), we have an expression of X in the form

$$X=\frac{\frac{2}{3}+\frac{1}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}}{1+\frac{1}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}} \tag{3.10}$$

From (3.7) we get

$$f(\phi)=\frac{3H_0^2\Omega_D}{X(3X-1)} \tag{3.11}$$

Using the relation $\dot{\phi}^2=2X$ and the above expression, the evolutionary form of the k-essence scalar field is obtained as

$$\dot{\phi}=\left(\frac{\frac{4}{3}+\frac{2}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}}{1+\frac{1}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}}\right)^{1/2} \tag{3.12}$$

Using the relation $\dot{\phi}=aH\phi'$, where the dot and the prime stand for the derivative with respect to the cosmic time and the derivative with respect to a respectively, and integrating we obtain the evolutionary form of k-essence scalar field as

$$\phi(a)-\phi(a_0)=\int_{a_0}^{a}\frac{1}{Ha}\left(\frac{\frac{4}{3}+\frac{2}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}}{1+\frac{1}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}}\right)^{1/2}da \tag{3.13}$$

3.2 HDE Tachyon Model

An action for tachyon scalar ϕ is given by Born-Infeld like action [40]

$$S=-\int d^4x\sqrt{-g}V(\phi)\sqrt{1-g^{ij}\partial_i\phi\partial_j\phi}, \tag{3.14}$$

where $V(\phi)$ is the tachyon potential.

Energy-momentum tensor components for tachyon scalar ϕ are obtained as

$$T_{ij}=V(\phi)\left[\frac{\partial_i\phi\partial_j\phi}{\sqrt{1-g^{ij}\partial_i\phi\partial_j\phi}}+g_{ij}\sqrt{1-g^{kl}\partial_k\phi\partial_l\phi}\right]. \tag{3.15}$$

Equations (3.14) yield respectively the energy density and pressure for the tachyon as

$$\rho_t=T_0^0=\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \tag{3.16}$$

$$p_t=-T_1^1=-T_2^2=-T_3^3=-V(\phi)\sqrt{1-\dot{\phi}^2} \tag{3.17}$$

for which the EoS reads as

$$w_t=\frac{p_t}{\rho_t}=\dot{\phi}^2-1 \tag{3.18}$$

Now we suggest a correspondence between HDE and tachyon scalar field, i.e. we identify ρ_t with ρ_D. Using $\rho_t = \rho_D = 3H_0^2\Omega_D$ and (3.15), we have

$$V(\phi) = \rho_t\sqrt{1-\dot{\phi}^2} = 3H_0^2\Omega_D\left[\frac{1}{3} + \frac{2}{3c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}\right]^{1/2} \tag{3.19}$$

Equating (3.17) with the HDE equation of state parameter (2.12), we reconstruct the scalar field as

$$\dot{\phi} = \left[\frac{2}{3}\left(1 - \frac{1}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}\right)\right]^{1/2} \tag{3.20}$$

which implies

$$\phi' = \frac{1}{aH}\left[\frac{2}{3}\left(1 - \frac{1}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}\right)\right]^{1/2} \tag{3.21}$$

Evolutionary form of the tachyon field is obtained by integrating the above equation

$$\phi(a) - \phi(a_0) = \int_{a_0}^{a}\frac{1}{Ha}\left[\frac{2}{3}\left(1 - \frac{1}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}\right)\right]^{1/2} da \tag{3.22}$$

3.3 HDE Dilaton Model

The pressure density and the energy density of the dilaton scalar are given by [41]

$$p_d = -X + ce^{\lambda\phi}X^2 \tag{3.23}$$

$$\rho_d = -X + 3ce^{\lambda\phi}X^2 \tag{3.24}$$

where c and λ are positive constants and $\dot{\phi}^2 = 2X$. Consequently, the EoS parameter for the dilaton scalar field can be written as

$$w_d = \frac{p_d}{\rho_d} = \frac{-1+ce^{\lambda\phi}X}{-1+3ce^{\lambda\phi}X} \tag{3.25}$$

Like above two cases, comparison of (2.12) and (3.24) yields

$$3ce^{\lambda\phi}X = \frac{\frac{1}{c}\sqrt{\Omega_D} + 2 + \frac{2}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}}{1 + \frac{1}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}} \tag{3.26}$$

Substituting $X = \dot{\phi}^2/2$, in the above equation we get

$$e^{\frac{\lambda}{2}\phi}\dot{\phi} = \frac{1}{\sqrt{3c}}\left(\frac{\frac{2}{c}\sqrt{\Omega_D} + 4 + \frac{4}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}}{1 + \frac{1}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{r_c}}+\epsilon\sqrt{\Omega_{r_c}}}}\right)^{1/2} \tag{3.27}$$

Integrating the above equation, we get

$$\phi(a) = \frac{2}{\lambda} \ln \left[e^{\frac{\lambda}{2}\phi(a_0)} + \frac{\lambda}{2\sqrt{3c}} \int_{a_0}^{a} \frac{1}{aH} \left(\frac{\frac{2}{c}\sqrt{\Omega_D} + 4 + \frac{4}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{rc}}+\epsilon\sqrt{\Omega_{rc}}}}{1 + \frac{1}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m+\Omega_D+\Omega_{rc}}+\epsilon\sqrt{\Omega_{rc}}}} \right)^{1/2} da \right] \tag{3.28}$$

3.4 HDE Hessence Model

The Lagrangian density of the hessence is given by [43]

$$\mathcal{L}_h = \frac{1}{2}[(\partial_\mu \phi)^2 - \phi^2(\partial_\mu \theta)^2] - V(\phi) \tag{3.29}$$

The pressure and energy density for the hessence model are given by

$$p_h = \frac{1}{2}(\dot{\phi}^2 - \phi^2\dot{\theta}^2) - V(\phi) \tag{3.30}$$

and

$$\rho_h = \frac{1}{2}(\dot{\phi}^2 - \phi^2\dot{\theta}^2) + V(\phi) \tag{3.31}$$

with

$$Q = a^3\phi^2\dot{\theta} = \text{constant} \tag{3.32}$$

The corresponding equation of state parameter for hessence DE is given by

$$w_h = \frac{p_h}{\rho_h} = \frac{(\dot{\phi}^2 - \phi^2\dot{\theta}^2) - 2V(\phi)}{(\dot{\phi}^2 - \phi^2\dot{\theta}^2) + 2V(\phi)} \tag{3.33}$$

By $\rho_h = \rho_D = 3H_0^2\Omega_D$ and equating (2.12) and (3.32), we get the expression of potential as

$$V = H_0^2\Omega_D \left(2 + \frac{1}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}}}\right) \tag{3.34}$$

Also the scalar field ϕ can be found from the following first order non-linear ordinary differential equation

$$a^2H^2\left(\frac{d\phi}{da}\right)^2 - \frac{Q^2}{a^6\phi^2} = 2H_0^2\Omega_D \left(1 - \frac{1}{c}\frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}}}\right) \tag{3.35}$$

3.5 HDE DBI-Essence Model

Consider that the dark energy scalar field is a Dirac-Born-Infeld (DBI) scalar field. In this case, the action of the field be written as [44],

$$S_{dbi} = -\int d^4x a^3(t) \left[T(\phi)\sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}} + V(\phi) - T(\phi) \right] \tag{3.36}$$

where $T(\phi)$ is the warped brane tension and $V(\phi)$ is the DBI potential. From the above expression, the corresponding pressure and the energy density of the scalar field becomes,

$$p_{dbi} = \frac{\gamma - 1}{\gamma} T(\phi) - V(\phi) \quad \text{and} \quad \rho_{dbi} = (\gamma - 1)T(\phi) + V(\phi) \tag{3.37}$$

where γ is reminiscent from the usual relativistic Lorentz factor and is given by,

$$\gamma = \left(1 - \frac{\dot{\phi}^2}{T(\phi)}\right)^{-\frac{1}{2}} \tag{3.38}$$

Thus the equation of state for DBI-essence is given by,

$$w_{dbi} = \frac{(\gamma - 1)T(\phi) - \gamma V(\phi)}{\gamma((\gamma - 1)T(\phi) + V(\phi))} \tag{3.39}$$

Now we consider here two particular cases $\gamma =$ constant and $\gamma \neq$ constant [50].

Case I: γ = constant. In this case, for simplicity, we assume $T(\phi) = n\dot{\phi}^2$ $(n > 1)$. So we have $\gamma = \sqrt{\frac{n}{n-1}}$. In this case the expressions for ϕ, $T(\phi)$ and $V(\phi)$ are given by

$$\phi(a) - \phi(0) = \int_{a_0}^{a} \frac{H_0}{Ha} \left[2\Omega_D \sqrt{\frac{n-1}{n}} \left(1 - \frac{1}{c} \frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}}}\right)\right]^{1/2} da \tag{3.40}$$

$$T = 2\sqrt{n(n-1)} H_0^2 \Omega_D \left(1 - \frac{1}{c} \frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}}}\right) \tag{3.41}$$

and

$$V = H_0^2 \Omega_D \left[3 - 2(n - \sqrt{n(n-1)}) \left(1 - \frac{1}{c} \frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}}}\right)\right] \tag{3.42}$$

Case II: $\gamma \neq$ constant. In this case let us assume $\gamma = \dot{\phi}^s$. So from (3.37) we have $T(\phi) = \frac{\dot{\phi}^{2s+2}}{\dot{\phi}^{2s} - 1}$. In this case the expressions for ϕ, $T(\phi)$ and $V(\phi)$ are given by

$$\phi(a) - \phi(0) = \int_{a_0}^{a} \frac{1}{Ha} \left[2H_0^2 \Omega_D \left(1 - \frac{1}{c} \frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}}}\right)\right]^{\frac{1}{s+2}} da \tag{3.43}$$

$$T = \frac{[2H_0^2 \Omega_D (1 - \frac{1}{c} \frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}}})]^{\frac{2s+2}{s+2}}}{[2H_0^2 \Omega_D (1 - \frac{1}{c} \frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}}})]^{\frac{2s}{s+2}} - 1} \tag{3.44}$$

and

$$V = 3H_0^2 \Omega_D - \frac{[2H_0^2 \Omega_D (1 - \frac{1}{c} \frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}}})]^{\frac{2s+2}{s+2}}}{[2H_0^2 \Omega_D (1 - \frac{1}{c} \frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}}})]^{\frac{s}{s+2}} + 1} \tag{3.45}$$

4 Correspondence Between Effective Dark Energy in DGP Brane and Other Dark Energies

Here we have considered there is no any external DE like HDE in DGP braneworld model. So the DGP braneworld model of the universe is filled with only dark matter. In absence of HDE, put $\rho_D = 0$ in (2.2c), we have the effective density coming from DGP model as

$$\rho_{eff} = \frac{3\epsilon H}{r_c} \tag{4.1}$$

From the energy conservation equation (2.2d) and the field equation (2.2a), we obtain the effective pressure coming from DGP model as

$$p_{eff} = -\frac{3H^2 \frac{\epsilon}{r_c}}{2H - \frac{\epsilon}{r_c}} \tag{4.2}$$

So the EoS parameter for DGP model is given by

$$w_{eff} = \frac{p_{eff}}{\rho_{eff}} = \frac{\epsilon - 2Hr_c}{Hr_c^2} \tag{4.3}$$

Here ϵ must be positive because otherwise ρ_{eff} will be negative, which is not possible. So $\epsilon = +1$. The effective part for DGP brane generates dark energy provided $w_{eff} < -1/3$ i.e., $Hr_c(6 - r_c) > 3$. In the following subsections, we'll discuss the correspondence of effective dark energy coming from DGP brane with other dark energy models. In this situation, we reconstruct the potentials as well as dynamics of scalar fields in k-essence, tachyon, dilaton, hessence and DBI dark energy models.

4.1 k-Essence Model

As before in Sect. 3.1, equating effective EoS of effective energy density (4.3) and w_k from (3.9) in this case, we have

$$X = \frac{1 - 2Hr_c - Hr_c^2}{3 - Hr_c(6 + r_c)} \tag{4.4}$$

From (3.7) we get

$$f(\phi) = \frac{3H}{r_c X(3X - 1)} \tag{4.5}$$

Using the relation $\dot{\phi}^2 = 2X$ and the above expression (4.4), the evolutionary form of the k-essence scalar field is obtained as

$$\dot{\phi} = \left[\frac{2 - 2Hr_c(2 + r_c)}{3 - Hr_c(6 + r_c)}\right]^{1/2} \tag{4.6}$$

which integrates to

$$\phi(a) - \phi(a_0) = \int_{a_0}^{a} \frac{1}{Ha}\left[\frac{2 - 2Hr_c(2 + r_c)}{3 - Hr_c(6 + r_c)}\right]^{1/2} da \tag{4.7}$$

4.2 Tachyon Model

As before in Sect. 3.2, equating $\rho_t = \rho_{eff}$ from (3.15) and (4.1) and also equating $w_t = w_{eff}$ from (3.17) and (4.3), we have

$$V(\phi) = \frac{3H}{r_c^2}\left[2r_c - \frac{1}{H}\right]^{1/2} \tag{4.8}$$

and

$$\dot{\phi} = \frac{1}{r_c}\left[r_c^2 - 2r_c + \frac{1}{H}\right]^{1/2} \tag{4.9}$$

which integrates to

$$\phi(a) - \phi(a_0) = \int_{a_0}^{a} \frac{1}{Ha}\frac{1}{r_c}\left[r_c^2 - 2r_c + \frac{1}{H}\right]^{1/2} da \tag{4.10}$$

4.3 Dilaton Model

As before in Sect. 3.3, equating $w_d = w_{eff}$ from (3.24) and (4.3) and using $X = \dot{\phi}^2/2$, we get

$$e^{\frac{\lambda}{2}\phi}\dot{\phi} = \left(\frac{2}{3c}\right)^{1/2}\left[\frac{3 + 4Hr_c - 3Hr_c^2}{3 - Hr_c(6 + r_c)}\right]^{1/2} \tag{4.11}$$

On integration, we obtain

$$\phi(a) = \frac{2}{\lambda}\ln\left[e^{\frac{\lambda}{2}\phi(a_0)} + \frac{\lambda}{\sqrt{6c}}\int_{a_0}^{a}\frac{1}{aH}\left(\frac{3 + 4Hr_c - 3Hr_c^2}{3 - Hr_c(6 + r_c)}\right)^{1/2} da\right] \tag{4.12}$$

4.4 Hessence Model

As before in Sect. 3.4, equating $\rho_h = \rho_{eff}$ from (3.30) and (4.1) and also equating $w_h = w_{eff}$ from (3.32) and (4.3), we have the expression for potential as

$$V = \frac{3H(3Hr_c - 1)}{2r_c(2Hr_c - 1)} \tag{4.13}$$

and the scalar field can be calculated from the following differential equation

$$a^2H^2\left(\frac{d\phi}{da}\right)^2 - \frac{Q^2}{a^6\phi^2} = \frac{3H(Hr_c - 1)}{r_c(2Hr_c - 1)} \tag{4.14}$$

4.5 DBI-Essence Model

Similar to Sect. 3.5, here we also consider two cases for constant and variable γ for DBI dark energy model and keeping in mind that the choices in cases I and II remain same.

Case I: $\gamma =$ constant. Assume $T(\phi) = n\dot{\phi}^2$ $(n > 1)$. In this case, equating (3.36) with (4.1) and (3.38) with (4.3) and after simplifying we obtain the expressions for ϕ, $T(\phi)$ and $V(\phi)$ as

$$\phi(a) - \phi(0) = \int_{a_0}^{a}\frac{1}{aH}\left[\sqrt{\frac{n-1}{n}}\,\frac{3H(Hr_c - 1)}{r_c(2Hr_c - 1)}\right]^{1/2} da \tag{4.15}$$

$$T=\sqrt{n(n-1)}\,\frac{3H(Hr_c-1)}{r_c(2Hr_c-1)} \tag{4.16}$$

and

$$V=\frac{3H}{r_c}\left[1-(n-\sqrt{n(n-1)})\frac{(Hr_c-1)}{(2Hr_c-1)}\right] \tag{4.17}$$

Case II: $\gamma \neq$ constant. Assume $\gamma=\dot{\phi}^s$. In this case, equating (3.36) with (4.1) and (3.38) with (4.3) and after simplifying we obtain the expressions for ϕ, $T(\phi)$ and $V(\phi)$ as

$$\phi(a)-\phi(0)=\int_{a_0}^{a}\frac{1}{aH}\left[\frac{3H(Hr_c-1)}{r_c(2Hr_c-1)}\right]^{\frac{1}{s+2}}da \tag{4.18}$$

$$T=\frac{[\frac{3H(Hr_c-1)}{r_c(2Hr_c-1)}]^{\frac{2s+2}{s+2}}}{[\frac{3H(Hr_c-1)}{r_c(2Hr_c-1)}]^{\frac{2s}{s+2}}-1} \tag{4.19}$$

and

$$V=\frac{3H}{r_c}-\frac{[\frac{3H(Hr_c-1)}{r_c(2Hr_c-1)}]^{\frac{2s+2}{s+2}}}{[\frac{3H(Hr_c-1)}{r_c(2Hr_c-1)}]^{\frac{s}{s+2}}+1} \tag{4.20}$$

5 Discussions

In this work, we have considered the flat DGP braneworld model of the universe in FRW background. Here we have also considered that the universe is filled with dark matter and holographic dark energy (HDE) and they are non-interacting. The EoS for HDE have been calculated in terms of dimensionless density parameters. We have studied the correspondence of various dark energy models namely, k-essence, tachyon, dilaton, Hessence and DBI-essence with the non-interacting HDE in DGP braneworld model of the universe. In all the DE models, we have reconstructed the potential and dynamics of the scalar fields in DGP braneworld. Here, we assume that scalar field models of dark energy are effective theories of an underlying theory of dark energy. Naturally, scalar fields with holographic feature should be capable of realizing the holographic evolution of the universe. Next, we have considered there is no any external DE like HDE in DGP braneworld model i.e., in this situation the universe is filled with only dark matter. So we have calculated the effective density and pressure coming from extra terms of the DGP braneworld model. In this situation, we have reconstructed the potentials as well as dynamics of scalar fields in k-essence, tachyon, dilaton, hessence and DBI dark energy models also. It will be interesting to reconstruct the potentials for various dark energy models in several versions of modified gravity theories.

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