SOME INVESTIGATIONS IN LOW ENERGY SUPERSYMMETRY

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I, **P. FRANCIS PAULRAJ**, hereby declare that the subject matter of this thesis is the record of work done by me, that the contents of this thesis did not form the award of any previous degree to me or to the best of my knowledge to anybody else, and that the thesis has not been submitted by me for any research degree in any other University/Institute.

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Publications

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3. P. N. Pandita and P. Francis Paulraj, *Renormalization Group Evolution in Non-Minimal Supersymmetric Standard Model with R-parity Violation*, submitted to The Euro. Phys. J. C.

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Chapter 1

Introduction

1.1 Standard Model

The Standard Model [1] is enormously successful in describing all the current experimental data [2]. However, despite its tremendous successes,[1] SM is not satisfactory, and many present and future experiments will look at the issues raised by the Standard Model. Perhaps the most important among these is: what breaks electroweak symmetry? Related to the nature of breakdown of electroweak symmetry is the question: is there supersymmetry? Nevertheless, the Standard Model remains the basis on which our quest for new physics must be built, so we start with a brief review of its basic features and examine whether its successes offer any hint of the direction in which to search for new physics.

The Standard Model (SM) of electroweak interactions is a gauge theory based[2] on the gauge group $SU(2)_L \times U(1)_Y$. The electroweak Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{\mathrm{symm}} + \mathcal{L}_{\mathrm{Higgs}}, \tag{1.1}$$

[1] For the first time, clear evidence for new physics beyond the Standard Model may be emerging from non-accelerator neutrino experiments [3]

[2] Here we shall not discuss the strong interactions which are described by an unbroken $SU(3)_C$ gauge theory.

where $\mathcal{L}_{\text{symm}}$ involves only gauge bosons and fermions [4], and can be written as

$$\mathcal{L}_{\text{symm}} = -\frac{1}{4}\sum_{A=1}^{3} F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{\psi}_L i\gamma^\mu D_\mu \psi_L + \bar{\psi}_R i\gamma^\mu D_\mu \psi_R. \tag{1.2}$$

This is the Yang-Mills Lagrangian for the gauge group $SU(2)_L \times U(1)_Y$ with fermion matter fields. Here

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad F^A_{\mu\nu} = \partial_\mu W^A_\nu - \partial_\nu W^A_\mu - g\epsilon_{ABC}W^B_\mu W^C_\nu, \tag{1.3}$$

are the gauge antisymmetric tensors constructed out of the gauge field B_μ associated with $U(1)$, and W^A_μ corresponding to the three $SU(2)$ generators; ϵ_{ABC} are the gauge structure constants which, for $SU(2)$, coincide with totally antisymmetric Levi-Civita tensor. The normalization of the $SU(2)$ gauge coupling g is therefore specified by Eq.(1.3).

The fermion fields are described by their left-hand and right hand components:

$$\psi_{L,R} = \left[\frac{(1 \mp \gamma_5)}{2}\right]\psi, \quad \bar{\psi}_{L,R} = \bar{\psi}\left[\frac{(1 \pm \gamma_5)}{2}\right], \tag{1.4}$$

with γ_5 and other Dirac matrices defined as in Bjorken and Drell [5]. In SM the left and right handed fermions have different transformation properties under the gauge group. Thus, mass terms for fermions (of the form $\bar{\psi}_L\psi_R + h.c.$) are forbidden in the symmetric limit. In particular all ψ_R are singlets in the Standard Model. The spectrum of known fermions in the Standard Model is shown below:

$$q_L = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \begin{pmatrix} c_i \\ s_i \end{pmatrix}_L, \begin{pmatrix} t_i \\ b_i \end{pmatrix}_L \sim (3,\ 2,\ \frac{1}{6}), \tag{1.5}$$

$$u_R = u_{iR},\ c_{iR},\ t_{iR} \sim (3,\ 1,\ \frac{2}{3}), \tag{1.6}$$

$$d_R = d_{iR},\ s_{iR},\ b_{iR} \sim (3,\ 1,\ -\frac{1}{3}), \tag{1.7}$$

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \sim (1,\ 2,\ -\frac{1}{6}), \tag{1.8}$$

$$l_R = e_R,\ \mu_R,\ \tau_R \sim (1,\ 1,\ -1), \tag{1.9}$$

where $i = 1, 2, 3$ denotes the $SU(3)_C$ index, and numbers in the parentheses denote the transformation properties under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. The q_L are left-handed $SU(2)$ doublets of quarks, whereas u_R, d_R are right handed quark singlets. Similarly, l_L are lepton doublets and l_R are singlets.

In the absence of mass terms, there are only vector and axial vector interactions in the Lagrangian that have the property of not mixing ψ_L and ψ_R. Fermion masses will be introduced, together with $W^\pm$ and Z mass, by the mechanism of spontaneous symmetry breaking. The covariant derivatives D_μ in Eq.(1.2) are given by (g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively)

$$D_\mu = \left(\partial_\mu + ig \sum_{A=1}^{3} t^A W_\mu^A + ig' \frac{Y}{2} B_\mu \right), \tag{1.10}$$

where t^A and $\frac{Y}{2}$ are the $SU(2)_L$ and $U(1)_Y$ generators, respectively in the appropriate representation of the fermions. The $SU(2)$ generators satisfy the commutation relations

$$[t^A, t^B] = i\epsilon_{ABC} t^C, \tag{1.11}$$

with $tr(t^A t^B) = \frac{\delta^{AB}}{2}$ in the fundamental representation. The $SU(2)$ piece in Eq.(1.10) appears only for the left-handed fermions ψ_L, which are isospin doublets, while the right handed fermions ψ_R are isospin singlets, and hence couple only to the $U(1)$ gauge boson B_μ, via the hypercharge $\frac{Y}{2}$. The electric charge generator Q (in units of e, the positron charge) is given by

$$Q = t^3 + \frac{Y}{2}. \tag{1.12}$$

We note that the normalization of the $U(1)$ gauge coupling g' in Eq.(1.10) is now specified as a consequence of Eq.(1.12).

All fermion couplings to the gauge bosons can be derived directly from Eqs.(1.2) and (1.10). The charged current (CC) couplings are

$$\begin{aligned} g\left(t^1 W_\mu^1 + t^2 W_\mu^2\right) &= g\left\{\left[\frac{(t^1 + it^2)}{\sqrt{2}}\right] \frac{(W_\mu^1 - iW_\mu^2)}{\sqrt{2}} + h.c.\right\} \\ &= g\left\{\left[\frac{(t^+ W_\mu^-)}{\sqrt{2}}\right] + h.c.\right\}, \end{aligned} \tag{1.13}$$

where $t^{\pm} = t^1 \pm i t^2$ and $W^{\pm} = \frac{(W^1 \pm i W^2)}{\sqrt{2}}$. We, thus, obtain the charged current vertex

$$V_{\bar{\psi}\psi W} = g\bar{\psi}\gamma_\mu \left[\left(\frac{t^+}{\sqrt{2}}\right)\frac{(1-\gamma_5)}{2}\right]\psi W_\mu^- + h.c. \tag{1.14}$$

In the neutral current (NC) sector, the photon A_μ and Z_μ, the mediator of weak neutral currents (NC), are orthogonal and normalized linear combinations of B_μ and W_μ^3:

$$\begin{aligned} A_\mu &= \cos\theta_W B_\mu + \sin\theta_W W_\mu^3, \\ Z_\mu &= -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3, \end{aligned} \tag{1.15}$$

where θ_W is the weak mixing angle. The photon is characterized by equal couplings to left and right fermions with a strength equal to the electric charge. From Eq.(1.12) for the charge matrix Q, we immediately obtain

$$g\sin\theta_W = g'\cos\theta_W = e, \tag{1.16}$$

or equivalently

$$\tan\theta_W = \frac{g'}{g}. \tag{1.17}$$

Given Eq.(1.16), one can easily derive the Z couplings:

$$\Gamma_{\bar{\psi}\psi Z} = \frac{g}{2\cos\theta_W}\bar{\psi}\gamma_\mu[t^3(1-\gamma_5) - 2Q\sin^2\theta_W]\psi Z^\mu, \tag{1.18}$$

where $\Gamma_{\bar{\psi}\psi Z}$ is the notation for the vertex. In the Standard Model $t^3 = \pm\frac{1}{2}$.

In order to derive the effective four-fermion interactions that are equivalent, at low energies, to the CC and NC couplings given in Eqs.(1.14) and (1.18), we note that large masses, as experimentally observed, are provided for $W^{\pm}$ and Z by $\mathcal{L}_{\mathrm{Higgs}}$ (see later). For the left - left CC couplings, when the momentum transfer squared can be neglected in comparison to m_W^2 in the propagator of Born diagrams with single W exchange, we can write, using Eq.(1.14),

$$\mathcal{L}_{eff}^{CC} \simeq \left(\frac{g^2}{8m_W^2}\right)\left[\bar{\psi}\gamma_\mu(1-\gamma_5)t^+\psi\right]\left[\bar{\psi}\gamma_\mu(1-\gamma_5)t^-\psi\right]. \tag{1.19}$$

By specializing further to the case of doublet fields such as $\nu_e - e^-$ or $\nu_\mu - \mu^-$, we obtain the tree-level relation of g with the Fermi coupling constant G_F measured from μ decay

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}. \tag{1.20}$$

Using the fact that $g\sin\theta_W = e$, we can write this as

$$m_W = \left(\frac{\pi\alpha}{\sqrt{2}G_F}\right)^{\frac{1}{2}} \frac{1}{\sin\theta_W} \simeq \frac{37.2802\ \ GeV}{\sin\theta_W}. \tag{1.21}$$

In the same manner, we obtain from Eq.(1.18) in the Born approximation the effective four-fermion NC interaction given by

$$\mathcal{L}_{eff}^{NC} \simeq \sqrt{2}G_F\rho_0\bar{\psi}\gamma_\mu[t^3(1-\gamma_5) - 2Q\sin^2\theta_W]\psi$$
$$\times\, \bar{\psi}\gamma^\mu[t^3(1-\gamma_5) - 2Q\sin^2\theta_W]\psi, \tag{1.22}$$

where

$$\rho_0 = \frac{m_W^2}{m_Z^2\cos^2\theta_W}. \tag{1.23}$$

All couplings given above are obtained at tree level and are modified in higher orders of perturbation theory. In particular the relations between m_W and $\sin\theta_W$ as given in Eq.(1.21), as well as the observed value of ρ ($\rho = \rho_0$ at tree level) in different neutral current processes, are altered by radiative corrections which can be calculated in perturbation theory [6].

We now come to the Higgs part of the electroweak Lagrangian in Eq.(1.1), and the phenomenon of the generation of mass for gauge bosons and fermions. The Higgs Lagrangian is specified by gauge invariance and renormalizibility to be

$$\mathcal{L}_{Higgs} = (D_\mu\phi)^\dagger(D_\mu\phi) - V\left(\phi^\dagger\phi\right) - \bar{\psi}_L\Gamma\psi_R\phi - \bar{\psi}_R\Gamma^\dagger\psi_L\phi, \tag{1.24}$$

where ϕ is the Higgs multiplet. In the Standard Model, where all ψ_L transform as doublets and all right handed fermions ψ_R transform as singlets, ϕ is a $SU(2)_L$ doublet. The quantities Γ (which include coupling constants) are matrices that make the Yukawa couplings invariant under the Lorentz and gauge groups. The gauge invariant potential $V(\phi^\dagger\phi)$ contains at most quartic terms in ϕ so that the theory is renormalizable:

$$V\left(\phi^\dagger\phi\right) = -\frac{1}{2}\mu^2\phi^\dagger\phi + \frac{1}{4}\lambda\left(\phi^\dagger\phi\right)^2. \qquad (1.25)$$

Spontaneous gauge symmetry breaking is induced if the minimum of V, which is the classical analogue of quantum mechanical vacuum state (both are states of minimum energy) is obtained for non-vanishing values of the scalar field ϕ. We denote the vacuum expectation value of ϕ, i.e. the position of minimum, by v:

$$\langle 0|\phi|0\rangle = v \neq 0. \qquad (1.26)$$

The fermion mass matrix is obtained from the Yukawa couplings by replacing $\phi(x)$ by v:

$$M = \bar{\psi}_L\mathcal{M}\psi_R + \bar{\psi}_R\mathcal{M}^\dagger\psi_L, \qquad (1.27)$$

with

$$\mathcal{M} = \Gamma.v. \qquad (1.28)$$

We note that by a suitable change of basis we can always make the matrix $\mathcal{M}$ Hermitian, γ_5-free, and diagonal. Indeed, we can make separate unitary transformations on ψ_L and ψ_R according to

$$\psi_L^{'} = U\psi_L, \qquad \psi_R^{'} = V\psi_R, \qquad (1.29)$$

and consequently

$$\mathcal{M} \to \mathcal{M}' = U^\dagger\mathcal{M}V. \qquad (1.30)$$

This transformation does not alter the general structure of the fermion couplings in $\mathcal{L}_{\mathbf{symm}}$.

If only one Higgs doublet is present, the change of basis that makes $\mathcal{M}$ diagonal will at the same time diagonalize also the fermion-Higgs Yukawa couplings. Thus, in this case, no flvour-changing neutral Higgs exchanges are present. However, when there are several Higgs doublets, there would be flavour-changing neutral current couplings induced by Higgs exchanges. On the other hand one Higgs doublet for each electric charge sector, i.e. one doublet coupled only to u-type quarks, one doublet to d-type quarks, one doublet to charged leptons, would not lead to flavour-changing neutral currents, because the mass matrices of fermions with different charges are diagonalized separately. For several Higgs doublets in a given charge sector it is also possible to generate CP violation by complex phases in the Higgs couplings. In the presence of six quark flavours [7], this CP-violation mechanism is not necessary. In fact, at present, the simplest model with only one Higgs doublet seems adequate for describing all observed phenomenon.

We now turn to the gauge-boson masses and their couplings to the Higgs boson. These effects are induced by the $(D_\mu\phi)^\dagger(D_\mu\phi)$ term in $\mathcal{L}_{\text{Higgs}}$ in Eq.(1.24), where

$$D_\mu\phi = \left[\partial_\mu + ig\sum_{A=1}^{3} t^A W_\mu^A + ig'\left(\frac{Y}{2}\right)B_\mu\right]\phi. \tag{1.31}$$

Here t^A and $\frac{Y}{2}$ are the $SU(2)_L$ and $U(1)_Y$ generators in the representation spanned by ϕ. Not only doublets but all non-singlet Higgs representations can contribute to gauge-boson masses. The condition that photon remain massless is equivalent to the condition that vacuum is electrically neutral:

$$Q|v\rangle = \left(t^3 + \frac{Y}{2}\right)|v\rangle = 0. \tag{1.32}$$

The charged W mass is given by the quadratic terms in the W field arising from $\mathcal{L}_{\text{Higgs}}$, when ϕ is replaced by v. We obtain

$$m_W^2 W_\mu^+ W^{-\mu} = g^2\left|\left(\frac{t^+v}{\sqrt{2}}\right)\right|^2 W_\mu^+ W^{-\mu}, \tag{1.33}$$

whereas for the Z mass we get (using the definition (1.15))

$$\frac{1}{2}m_Z^2 Z_\mu Z^\mu = \left|\left[g\cos\theta_W t^3 - g'\sin\theta_W\left(\frac{Y}{2}\right)\right]v\right|^2 Z^\mu Z_\mu, \tag{1.34}$$

where the factor $\frac{1}{2}$ on the left hand side is for the proper normalization for the definition of a neutral vector field. Using (1.32) and (1.17) we obtain for the mass of the Z boson

$$\frac{1}{2}m_Z^2 = (g\cos\theta_W + g'\sin\theta_W)|t^3v|^2 = \left(\frac{g^2}{\cos^2\theta_W}\right)|t^3v|^2. \tag{1.35}$$

For a Higgs doublet,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad v = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{\frac{\mu^2}{\lambda}}, \tag{1.36}$$

we have

$$|t^+ v|^2 = v^2, \quad |t^3 v|^2 = \frac{1}{4} v^2, \tag{1.37}$$

so that

$$m_W^2 = \frac{1}{2} g^2 v^2, \quad m_Z = \frac{1}{2} \frac{g^2 v^2}{\cos^2 \theta_W}. \tag{1.38}$$

From (1.20), it follows that

$$v = 2^{-\frac{3}{4}} G_F^{-\frac{1}{2}} = 174.1 GeV. \tag{1.39}$$

For Higgs doublets we have

$$\rho_0 = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1. \tag{1.40}$$

This relation is typical of one or more Higgs doublets [8] and would be spoiled if there were Higgs triplets in the model. In general

$$\rho_0 = \frac{\sum_i \left((t_i)^2 - (t_i^3)^2 + t_i \right) v_i^2}{\sum_i 2 (t_i^3)^2 v_i^2}, \tag{1.41}$$

for several Higgs multiplets with vacuum expectation values (VEVs) v_i, weak isospin t_i, and z-component of isospin t_i^3. These results are valid at the tree level and are modified by calculable electroweak radiative corrections.

In the minimal version of the SM only one Higgs doublet is present. Then the fermion-Higgs couplings are proportional to the fermion masses. In fact, from the Yukawa couplings $g_{\phi\bar{\psi}\psi}(\bar{\psi}_L \phi \psi_R + h.c.)$, the mass m_ψ of the fermion is obtained by replacing ϕ by v, so that $m_\psi = g_{\phi\bar{\psi}\psi} v$. In the minimal SM three out of four Hermitian fields in ϕ are removed from the physical spectrum by the Higgs mechanism and become the third polarization state of W^+, W^-, and Z. The fourth neutral Higgs boson is physical and should be found. If more doublets are present, two more charged and two more neutral Higgs scalars should be found for each additional doublet.

The couplings of the physical Higgs h to the gauge bosons can be simply obtained from $\mathcal{L}_{\mathbf{Higgs}}$ by the replacement

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ v + \left(\frac{h}{\sqrt{2}}\right) \end{pmatrix}, \tag{1.42}$$

with the result

$$\begin{aligned} \mathcal{L}[h, W, Z] &= g^2 \left(\frac{v}{\sqrt{2}}\right) W_\mu^+ W^{-\mu} h + \left(\frac{g^2}{4}\right) W_\mu^+ W^{-\mu} h^2 \\ &+ \left[\frac{g^2 v Z_\mu Z^\mu}{2\sqrt{2}\cos^2\theta_W}\right] h + \left[\frac{g^2}{8\cos^2\theta_W}\right] Z_\mu Z^\mu h^2. \end{aligned} \tag{1.43}$$

Once the vacuum expectation value $\langle 0|\phi(x)|0\rangle = v$ of the neutral Higgs boson is fixed in the minimal SM, the mass of the remaining physical Higgs boson is given by

$$m_h^2 = \mu^2 = \lambda v^2, \tag{1.44}$$

which is a free parameter in the Standard Model. The present direct experimental limit [9] on m_h from LEP is $m_h > 89.7\ GeV/c^2$.

In the SM with only one Higgs doublet a lower limit on m_h can be derived from the requirement of vacuum stability [10, 11, 12]. The limit is a function of the mass of the top-quark (m_t) and of the energy scale where the model breaks down and new physics appears. If one requires that λ remain positive up to $\Lambda = 10^{15} - 10^{19}$ GeV, then the resulting bound on m_h in the SM with only one Higgs doublet can be written as [11]

$$m_h > 134 + 2.1[m_t - 173.8] - 4.5\frac{\alpha_3(m_Z) - 0.119}{0.006}, \tag{1.45}$$

where $\alpha_3 = g_3^2/4\pi$, with g_3 the $SU(3)_C$ gauge coupling constant. We see that the discovery of a Higgs particle with $m_h \lesssim 100\ GeV$ (the discovery limit at LEP II) would imply that the SM breaks down at a scale Λ of the order of a few TeV. It can be shown that the lower limit is not much relaxed even if strict vacuum stability is replaced by some sufficiently long metastability.

On the other hand an upper limit on the Higgs mass in the SM is important for assessing the chances of success of the Large Hadron Collider (LHC) as an accelerator designed to detect the Higgs boson. The upper limit [13] arises from the requirement that the Landau pole associated with the non-asymptotically free behaviour of the $\lambda\phi^4$ theory does not occur below the scale Λ. The initial value of λ at the weak scale increases with m_h and the derivative is positive at large m_h. Thus if m_h is too large the Landau pole occurs at too low an energy. The upper limit on m_h for $m_t \sim 175$ GeV is given by $m_h \lesssim 180$ GeV for $\Lambda \sim m_{GUT} - m_{Pl}$, and $m_h \lesssim 0.5 - 0.8$ TeV for $\Lambda = 1$ TeV [14]. Thus, for $m_t \sim 174$ GeV, only a small range of values for m_h is allowed, $130 < m_h < 200$ GeV, if the Standard Model holds up to $\Lambda \sim m_{GUT}$ or m_{Pl}.

1.2 Why Supersymmetry?

At present there is no confirmed experimental evidence against the Standard Model. Nevertheless, there are several questions raised by the Standard Model which point toward physics beyond it. Central among these is the question of the generation of masses: do particle masses really originate from the Higgs mechanism, and, if so, why are these masses so small compared to the Planck mass $m_{Pl} \simeq 10^{19}$ GeV? In other words why is $m_W \ll m_{Pl}$? This is known as the mass hierarchy problem [15]. The Planck mass is the only candidate we have for a fundamental mass scale in physics, where gravity is expected to become as strong as other particle interactions. The hierarchy problem can also be rephrased as: Why is $G_F \gg G_N$ (the Newton's constant)?, since $G_F \sim \frac{1}{m_W^2}$ and $G_N \sim \frac{1}{m_{Pl}^2}$. We could set $m_W \ll m_{Pl}$ by hand, and ignore the problem. However, there is the question of radiative corrections. The radiative corrections to the squared Higgs mass in the Standard Model can be written as

$$\delta m_h^2 = \mathcal{O}\left(\frac{g^2}{16\pi^2}\right)\int^{\Lambda} d^4k \frac{1}{k^2} = \mathcal{O}\left(\frac{\alpha}{\pi}\right)\Lambda^2, \tag{1.46}$$

where the cut off Λ in integral represents the scale up to which the Standard Model remains valid, and beyond which new physics sets in. If $\Lambda \simeq m_{Pl}$ or the grand unification scale, the quantum correction Eq.(1.46) is much larger than the physical value of $m_h \sim 100$ GeV. This is not a problem for renormalization theory: there could be large bare contribution with the opposite sign, and one could fine tune its value to many significant figures so that the physical value m_h^2 comes out to be of the right order of magnitude. However, this seems unnatural, and would have to be repeated order by order in perturbation theory. In contrast, the one-loop radiative corrections to a fundamental fermion mass m_f are proportional to m_f itself, and only logarithmically divergent:

$$\delta m_f = \mathcal{O}\left(\frac{g^2}{16\pi^2}\right) m_f \int^\Lambda d^4k \frac{1}{k^4} = \mathcal{O}\left(\frac{\alpha}{\pi}\right) m_f \ln \frac{\Lambda}{m_f}. \tag{1.47}$$

This correction is no larger than the physical value, for any $\Lambda \lesssim m_{Pl}$. The reason for this is that there is an underlying chiral symmetry which is reflected in the m_f factor in (1.47) that keeps the quantum corrections naturally (logarithmically) small. The hope is to find a corresponding symmetry principle to make a small Higgs boson mass natural: $\delta m_h^2 \lesssim m_h^2$.

Supersymmetry [16] is at present the only known symmetry which can make the small Higgs boson mass natural, exploiting the fact that boson and fermion loop diagrams have opposite signs (see Fig.(1.1)). If there are equal number of fermions and bosons, and if they have equal couplings as in a supersymmetric theory, the quadratic divergences (1.46) cancel:

$$\begin{aligned}\delta m_h^2 &= -\left(\frac{g_F^2}{16\pi^2}\right)(\Lambda^2 + m_F^2) + \left(\frac{g_B^2}{16\pi^2}\right)(\Lambda^2 + m_B^2) \\ &= \mathcal{O}\left(\frac{\alpha}{4\pi}\right)|m_B^2 - m_F^2|, \end{aligned} \tag{1.48}$$

where $g_F^2/4\pi = g_B^2/4\pi = \alpha$ is the common coupling of bosons and fermions. This is no larger than the physical value, $\delta m_h^2 \lesssim m_h^2$, and hence naturally small, if [3]

$$|m_B^2 - m_F^2| \lesssim 1\ TeV^2. \tag{1.49}$$

This naturalness argument is the only available theoretical motivation for thinking that supersymmetry may manifest itself at an accessible energy scale.

We note that the above argument is qualitative, and it does not tell us whether the supersymmetric partners of the known particles appear at 500 GeV, 1 TeV or 2 TeV.

[3]There is a logarithmic multiplicative factor in the right hand side of Eq.(1.48) which is not explicitly shown here.

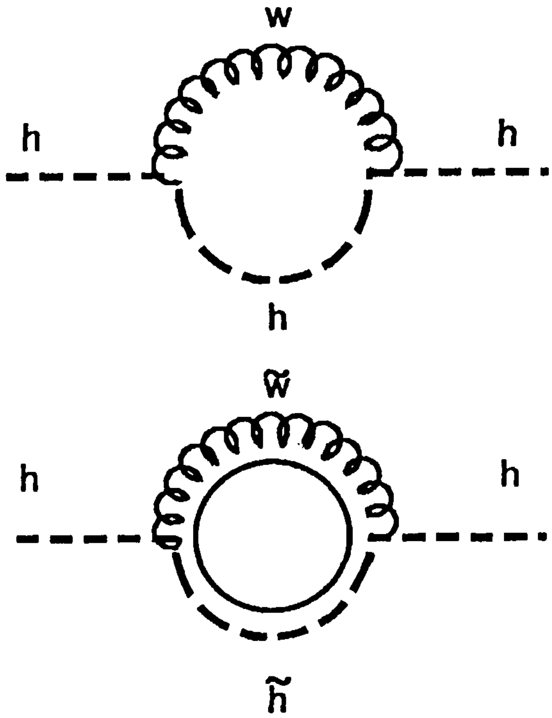

Figure 1.1: Contributions to the Higgs boson self-energy. Contributions from the individual graphs to the Higgs self-energy are separately quadratically divergent, but when both are included the divergence is removed. In models with broken supersymmetry a finite residual piece remains.

Supersymmetry is a symmetry that links bosons to fermions via spin-$\frac{1}{2}$ charges Q_α (where α is a spinorial index). It is the last possible symmetry of the particle scattering matrix [17]. All previously-known symmetries are generated by bosonic charges, which are, apart from the momentum operator P_μ associated with Lorentz invariance, scalar charges Q that relate different particles of the same spin J: $Q|J\rangle = |J'\rangle$, $Q \in U(1), SU(2), SU(3), ...$ Indeed Coleman and Mandula [18] showed that it was impossible to mix such internal symmetries with Lorentz invariance using bosonic charges.

The possible algebra of spinorial charges connecting bosons to fermions can be easily explored [17]. Let $Q^i_\alpha, i = 1, 2, ..., N$ be a set of spinorial charges. If they are to be symmetry generators, they must commute with the Hamiltonian

$$[Q^i_\alpha, H] = 0. \tag{1.50}$$

Hence, their anticommutator (which is bosonic) must also commute with H

$$[\{Q^i_\alpha, Q^j_\beta\}, H] = 0. \tag{1.51}$$

By the Coleman-Mandula theorem [18], this anticommutator must be a combination of the conserved Lorentz vector charge P_μ and some scalar charge Z^{ij}. The only possible form is in fact

$$\{Q^i_\alpha, Q^j_\beta\} = 2\delta^{ij}(\gamma^\mu C)_{\alpha\beta} P_\mu + Z^{ij}, \tag{1.52}$$

where we have used four-component spinors, C is the charge conjugation matrix and Z^{ij} is antisymmetric in the supersymmetry indices. Thus, this so called "central charge" vanishes for $N = 1$ supersymmetry which is of phenomenological interest.

The basic building blocks of $N = 1$ supersymmetric theories are supermultiplets containing the following helicity states [19]:

$$chiral: \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}, \quad gauge: \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}, \quad graviton: \begin{pmatrix} 2 \\ \frac{3}{2} \end{pmatrix}, \tag{1.53}$$

which are used to describe matter and Higgs bosons, gauge fields and gravity, respectively. We note that $N = 2$ supersymmetric theories will have left- and right-handed particles (helicity $\mp\frac{1}{2}$) in an identical representation of the gauge group, and hence cannot accomodate parity violation, and are not suitable for phenomenology. We shall denote the chiral supermultiplet in (1.53) by $\hat{\Phi}$. It contains a fermion (ψ), a scalar (ϕ), and the auxiliary field (F). Similarly, a vector or gauge supermultiplet will be denoted by $\hat{W}$, which contains the gauge boson (W), the gauge fermion (ω) and the auxiliary field (D), etc.

The simplest $N = 1$ supersymmetric theory contains a free fermion and a free boson [16, 20]:

$$\mathcal{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) + i\psi^+ \bar{\sigma} \cdot \partial \psi, \tag{1.54}$$

where we work with two-component spinors, and $\sigma_\mu = (1, \underline{\sigma})$, $\bar{\sigma}_\mu = (1, -\underline{\sigma})$ with $\underline{\sigma}$ being the Pauli matrices. The supersymmetry transformation laws are

$$\delta_\xi \phi = \sqrt{2}\xi^T C\psi, \quad \delta_\xi \psi = \sqrt{2} i\sigma \cdot \partial\phi C\xi^*, \tag{1.55}$$

where ξ is an infinitesimal spinor parameter, and C is the charge conjugation matrix: $C = -i\sigma^2 = C^*$, $C^{-1} = C^T = -C$. Under the transformation (1.55) the Lagrangian (1.54) changes by a total derivative, and hence the action $A = \int d^4x\ \mathcal{L}$ is invariant. We also see in (1.55) a reflection of the supersymmetry algebra (1.52): after two supersymmetry transformations, the fields (ϕ, ψ) are transformed by derivatives $(\partial\phi, \partial\psi)$, corresponding to the action of the momentum operator $P_\mu = i\partial_\mu$.

The example (1.54) can be extended to include the interactions:

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi + i\psi^+\bar{\sigma}\cdot\partial\psi + F^\dagger F + \left(F\frac{\partial W}{\partial\phi} - \frac{1}{2}\psi^T C\psi\frac{\partial^2 W}{\partial\phi^2} + h.c.\right), \qquad (1.56)$$

with supersymmetry transformation:

$$\begin{aligned} \delta_\xi\phi &= \sqrt{2}\xi^T C\psi, \quad \delta_\xi\psi = \sqrt{2}i\sigma\cdot\partial\phi C\xi^*, \\ \delta_\xi F &= -\sqrt{2}i\xi^\dagger\bar{\sigma}\cdot\partial\psi. \end{aligned} \qquad (1.57)$$

The field F is called an auxiliary field; it has no kinetic term, and may be eliminated by using the equation of motion

$$F^\dagger = -\frac{\partial W}{\partial\phi}. \qquad (1.58)$$

Thus, all matter interactions are charecterized by the analytic function $W(\hat{\Phi})$, which is called the superpotential. Renormalizibility of the field theory requires the superpotential to be a cubic function of chiral superfields: for $W = \lambda\hat{\Phi}_1\hat{\Phi}_2\hat{\Phi}_3$, one obtains from (1.56) the following particle interactions:

$$\lambda\left[(\psi_1^T C\psi_2)\phi_3 + (\psi_2^T C\psi_3)\phi_1 + (\psi_3^T C\psi_1)\phi_2\right] + |\lambda\phi_1\phi_2|^2 + |\lambda\phi_2\phi_3|^2 + |\lambda\phi_3\phi_1|^2, \quad (1.59)$$

where the last three terms provide a quartic potential for the scalar fields ϕ_i and are called "F-terms". Furthermore, in addition to the gauge interactions of the chiral fermions and their bosonic partners, there are gaugino interactions

$$\sqrt{2}g\left[(\psi_i^T C(T^a)^i_j V_a)\phi^{j*} + h.c.\right], \qquad (1.60)$$

where $(T^a)^i_j$ is the gauge representation matrix for the chiral fields. There is also another quartic potential term for the scalars:

$$V = \frac{g^2}{2}\sum_a |\phi^{i*}(T^a)^j_i\phi_j|^2, \qquad (1.61)$$

which are called "D-terms". We also note that the conventional gauge-boson kinetic term and the gauge interactions of fermions in the adjoint representation of the gauge group, such as the gauginos, are automatically supersymmetric.

1.3 The Supersymmetric Standard Model

In order to construct the minimal supersymmetric version of the Standard Model the first natural question is: can one construct it out of Standard Model particles alone? One can easily see that this is impossible, because the known bosons and fermions have different quantum numbers [21]. For example, gluons are in the octet ($\underline{8}$) representation of $SU(3)_C$, whereas quarks are in the triplet ($\underline{3}$) representation of $SU(3)_C$. Similarly, there are no weak isotriplet fermions as would be needed to partner the electroweak gauge bosons. The known leptons are isodoublets like the Higgs boson, but carry lepton number, so they cannot be the supersymmetric partners of the Higgs boson. For these reasons, new particles must be postulated [21] as supersymmetric partners of known particles (see Table 1.1). The spectrum of

Particle	Spin	Spartner	Spin
quark: q	$\frac{1}{2}$	squark	0
lepton: l	$\frac{1}{2}$	slepton	0
photon: γ	1	photino: $\tilde{\gamma}$	$\frac{1}{2}$
W	1	wino: $\tilde{W}$	$\frac{1}{2}$
Z	1	zino: $\tilde{Z}$	$\frac{1}{2}$
Higgs: H	0	higgsino: $\tilde{h}$	$\frac{1}{2}$

Table 1.1: Particles in the Standard Model and their supersymmetric partners

chiral superfields of the minimal supersymmetric standard model (MSSM) is shown in Table 1.2. We note that each superfield has a family index i (i = 1, 2, 3), representing the three known families of quarks and leptons, and their superpartners. This index is not explicitly shown.

Superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Particle Content
$\hat{Q}$	3	2	$\frac{1}{6}$	$(u_L, d_L), (\tilde{u}_L, \tilde{d}_L)$
$\hat{U}^c$	$\bar{3}$	1	$-\frac{2}{3}$	$\bar{u}_R, \tilde{u}_R^*$
$\hat{D}^c$	$\bar{3}$	1	$\frac{1}{3}$	$\bar{d}_R, \tilde{d}_R^*$
$\hat{L}$	1	2	$-\frac{1}{2}$	$(\nu_L, e_L), (\tilde{\nu}_L, \tilde{e}_L)$
$\hat{E}^c$	1	1	1	$\bar{e}_R, \tilde{e}_R^*$
$\hat{H}_1$	1	2	$-\frac{1}{2}$	$(H_1, \tilde{h}_1)$
$\hat{H}_2$	1	2	$\frac{1}{2}$	$(H_2, \tilde{h}_2)$

Table 1.2: Chiral Superfields of the MSSM

The minimal supersymmetric extension of the SM has the same gauge interactions as the Standard Model. In addition, there are couplings of the form (1.59) derived from the following superpotential:

$$W = h_U \hat{Q}\hat{U}^c\hat{H}_2 + h_D\hat{Q}\hat{D}^c\hat{H}_1 + h_L\hat{L}\hat{E}^c\hat{H}_1 + \mu\hat{H}_1\hat{H}_2. \tag{1.62}$$

Here $\hat{Q}(\hat{L})$ denote isodoublets of supermultiplets containing $(u,d)_L$ $[(\nu,e)_L]$, $\hat{D}^c$ $[\hat{U}^c, \hat{E}^c]$ are singlets containing the left-handed conjugates d_L^c $[u_L^c, e_L^c]$ of the right-handed d_R $[u_R, e_R]$, and the superpotential couplings h_D $[h_U, h_L]$ correspond to the Yukawa couplings of the SM that give masses to the d $[u, l^-]$, respectively:

$$m_d = h_D\langle H_1\rangle, \quad m_u = h_U\langle H_2\rangle, \quad m_l = h_L\langle H_1\rangle. \tag{1.63}$$

Each of these should be understood as a 3×3 matrix in generation space, which is to be diagonalized as in the Standard Model.

One feature of Table 1.2 needs an explanation. The SM contains a single $SU(2)_L$ doublet of Higgs bosons. In the supersymmetric (SUSY) extension of the SM, this scalar doublet acquires a SUSY partner which is an $SU(2)_L$ doublet of Majorana fermion fields, $\tilde{h}_1$ (the Higgsino), which contribute to the triangle $SU(2)_L$ and $U(1)_Y$ gauge anomalies. Since the fermions of the Standard Model have exactly the right quantum numbers to cancel the triangle anomalies among themselves [22], it follows that the contribution from the fermionic partner of the Higgs doublet remains uncancelled. These contributions must be cancelled somehow if the SUSY theory is to be sensible. The simplest way is to add a second Higgs doublet with precisely the opposite $U(1)_Y$ quantum numbers from the first Higgs doublet. Then the contribution from the fermions of the second Higgs doublet will cancel the anomalies from the first doublet, leaving an anomaly free theory. We see from Table 1.2 that the fermions satisfy the conditions for anomaly cancellation:

$$Tr\ (Y^3) = Tr\ (T_{3L}^2\ Y) = 0. \tag{1.64}$$

We further note that two Higgs doublets are also needed to give masses to both the up- and down- quarks and leptons in a supersymmetric theory, since one cannot use the complex conjugate of a Higgs superfield in the superpotential, as this would violate supersymmetry. Note also that the ratio of Higgs vacuum expectation values

$$\tan\beta \equiv \frac{\langle H_2 \rangle}{\langle H_1 \rangle}, \tag{1.65}$$

is undetermined and should be treated as a free parameter.

In addition to the chiral superfields, the MSSM will contain the massless vector superfields corresponding to the gauge bosons of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ and their Majorana fermion partners. These are shown in Table 1.3, where $\hat{G}^a$ contains the gluons (g^a), and their supersymmetric partners, the gluinos ($\tilde{g}^a$); $\hat{W}^i$ contains

Superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Particle Content
$\hat{G}^a$	8	1	0	g, $\tilde{g}$
$\hat{W}^i$	1	3	0	W_i, $\tilde{\omega}_i$
$\hat{B}$	1	1	0	B, $\tilde{b}$

Table 1.3: Vector Superfields of the MSSM

$SU(2)_L$ gauge bosons (W^i), and their fermionic partners, winos ($\tilde{\omega}^i$); and $\hat{B}$ contains the $U(1)$ gauge field, B, and its fermionic partner, $\tilde{b}$ (bino).

In addition to the Standard-Model-like superpotential terms (1.62), the following superpotential couplings are also allowed [23] by gauge invariance, supersymmetry and renormalizibility:

$$W' = \lambda \hat{L}\hat{L}\hat{E}^c + \lambda' \hat{L}\hat{Q}\hat{D}^c + \lambda'' \hat{U}^c\hat{D}^c\hat{D}^c + \mu_i \hat{L}^i \hat{H}_2. \tag{1.66}$$

In general, $\lambda, \lambda', and\ \lambda''$ could all be matrices which could mix the interactions of the 3 generations. Each of these violate conservation of either lepton number L or baryon number B. These couplings can mediate proton decay at tree level through the exchange of the scalar partner of the down quark. If SUSY partners of the SM particles have masses in the TeV region, then these interactions are severely restricted by experimental measurements [24]. The usual strategy is to require that all of these undesirable lepton and baryon number violating terms be forbidden by a symmetry [25], called R-parity (R_p). It is defined as a multiplicative quantum number such that all particles of the SM have $R_p = +1$, while their SUSY partners have $R_p = -1$. R-parity can also be defined as

$$R_p \equiv (-1)^{3(B-L)+2S}, \tag{1.67}$$

for a particle of spin S. Such a symmetry forbids the lepton and baryon number violating terms of the superpotential (1.66). The assumption of R_p conservation has profound experimental consequences which go beyond the details of a specific model. Conservation of R-parity implies that (i) SUSY partners can only be pair produced from SM particles; (ii) models with R_p conservation will have a lightest SUSY particle (LSP) which is stable, and (iii) the LSP will interact very weakly with ordinary matter, and a generic signal for R-parity conserving SUSY theories is missing transverse energy from the non-observed LSP.

Since nature is not supersymmetric, we must have a mechanism for supersymmetry breaking. This mechanism is not well understood at present. It is typically assumed that the SUSY breaking occurs at a high scale, say m_{Pl}, and results from some complete theory encompassing gravity. At the moment the usual approach is to assume that the MSSM, which is a theory at the electroweak scale, is an effective low energy theory [26]. The supersymmetry breaking is implemented by including explicit "soft" mass terms for the scalar members of the chiral multiplets and for the gaugino members of the vector supermultiplets in the Lagrangian. These interactions are termed soft because they do not re-introduce the quadratic divergences which motivated the introduction of supersymmetry in the first place. The dimension of soft operators in the Lagrangian must be 3 or less, which means that the possible soft operators are mass terms, bilinear mixing terms ("B" terms), and trilinear scalar mixing terms ("A" terms). The origin of these supersymmetry breaking terms is left unspecified. The complete set of soft SUSY breaking terms (which respect R parity and the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry) is given by the Lagrangian [27]

$$\begin{aligned}
-\mathcal{L}_{\rm soft} \;=\;& m_1^2\,|H_1|^2 + m_2^2\,|H_2|^2 - B\mu\,\epsilon_{ij}(H_1^i\,H_2^j + h.c.) \\
&+\tilde{M}_Q^2(\tilde{u}_L^*\,\tilde{u}_L + \tilde{d}_L^*\,\tilde{d}_L) + \tilde{M}_u^2\tilde{u}_R^*\,\tilde{u}_R + \tilde{M}_d^2\tilde{d}_R^*\,\tilde{d}_R \\
&+\tilde{M}_L^2(\tilde{e}_L^*\,\tilde{e}_L + \tilde{\nu}_L^*\,\tilde{\nu}_L) + \tilde{M}_e^2\tilde{e}_R^*\,\tilde{e}_R \\
&+\frac{1}{2}[M_3\,\bar{\tilde{g}}\,\tilde{g} + M_2\,\bar{\tilde{\omega}}_i\,\tilde{\omega}_i + M_1\,\bar{\tilde{b}}\,\tilde{b}] \\
&+\frac{g}{\sqrt{2}\,M_W}\epsilon_{ij}\Big[\frac{M_d}{\cos\beta}A_dH_1^i\tilde{Q}^j\tilde{d}_R^* + \frac{M_u}{\sin\beta}A_uH_2^j\tilde{Q}^i\tilde{u}_R^* \\
&+\frac{M_e}{\cos\beta}A_eH_1^i\tilde{L}^j\tilde{e}_R^* + h.c.\Big]. \qquad (1.68)
\end{aligned}$$

We note that terms of the type $m_\psi\psi^TC\psi$ which would give mass to the fermions in the chiral supermultiplets, and non-analytic trilinear scalar couplings are not allowed in (1.68).

One usually makes the hypothesis that the soft supersymmetry-breaking parameters $m_i, \tilde{M}_i, M_a, A_\lambda$, and B originate at some high GUT or gravity scale, perhaps from some supergravity or superstring mechanism. The physical values of these soft supersymmetry-breaking parameters are then subject to logarithmic renormalizations that may be calculated and resummed using the renormalization-group techniques [28]. It is usually assumed that the soft supersymmetry-breaking parameters are universal at the GUT or supergravity scale: $m_i^2 \equiv M_i^2 \equiv m_0^2,\;\; M_a \equiv m_{1/2},\;\; A_\lambda \equiv A$, although such a universality is not very well motivated, since in particular, general supergravity models give no theoretical hint why they should be universal. If one assumes universality, the parameters $\mu, \tan\beta, m_0, m_{1/2}, A$ suffice to characterize MSSM phenomenology.

Assuming universality, typical renormalization group calculations [29] imply that the scalar masses are generally renormalized to larger values as the scale is reduced, but this is not necessarily the case if there are large Yukawa interactions such as those of the top quark. Such effects of the top quark Yukawa couplings must certainly be taken into account. Also the effects of bottom quark and τ lepton Yukawa couplings may be important if $\tan\beta$ is large. The potential significance of these Yukawa interactions is that they tend to drive $m_h^2 = (m_0^2 + \mu^2)$ to smaller values at smaller renormalization scales μ [30] via the renormalization group evolution

$$\mu\frac{\partial m_h^2}{\partial(\ln\mu)} = \frac{1}{(4\pi)^2}\left(3h_t^2(m_h^2 + m_{\tilde{q}}^2 + m_t^2) + \cdots\right), \tag{1.69}$$

where $m_{\tilde{q}}$ is a squark mass.

It is then possible to generate electroweak symmetry breaking dynamically, even if $m_h^2 > 0$ at the input scale along with the other scalar mass-squared parameters [30]. The appropriate renormalization scale for discussing the effective Higgs potential of the MSSM is $Q \lesssim 1\ TeV$, and the electroweak gauge symmetry will be broken if either or both of $m_{h_{1,2}}^2(Q) < 0$, as in the SM Higgs potential (1.25). Here $m_{h_1}^2 = m_1^2 + \mu^2$ and $m_{h_2}^2 = m_2^2 + \mu^2$, respectively. This is certainly possible for $m_t \sim 175\ GeV$ as observed.

The superpotential (1.62) of the MSSM contains a bilinear term $\mu\hat{H}_1\hat{H}_2$ coupling the two Higgs superfields, with μ having the dimensions of mass. We note that this term is allowed by supersymmetry. With the bilinear term in the superpotential one would also expect a bilinear supersymmetry-breaking term $B\mu H_1 H_2$ in the scalar potential (see Eq.(1.68)), where B is expected to be of the order of supersymmetry breaking scale ($\lesssim$ 1 TeV). With a term of this form the mass of the only CP-odd Higgs boson in MSSM is given by

$$m_A^2 = \frac{2B\mu}{\sin 2\beta}. \tag{1.70}$$

There would seem to be two natural options for the parameter μ: either $\mu = 0$ due to some symmetry, or it acquires an extremely large mass. For $\mu = 0$, there is a Peccei-Quinn symmetry [31] in the potential of the MSSM, leading to a massless axion ($m_A = 0$), which is ruled out by experiment. The second option of having a very large μ effectively obliterates the weak scale: the Higgs doublets are removed to the high mass scale, or (worse) electroweak symmetry is broken at high scale. Thus, a non-zero $\mu = \mathcal{O}(m_W)$ is required in order to break $SU(2)_L \times U(1)_Y$ sucessfully without producing an unacceptable axion. Softly broken supersymmetry ensures that the radiative corrections to μ are now under control so that $\mu = \mathcal{O}(m_W) \ll m_{Pl}$ is technically natural, thus solving the easy part of the hierarchy problem. However, it does not provide any dynamical reason why μ should be so small in the first place. Thus, it is a serious problem, why μ is of the order of electroweak scale. It endangers the whole idea of low energy supersymmetry. The simplest mechanism that provides a dynamical source for a term of the form $\mu \hat{H}_1 \hat{H}_2$ is the inclusion of an additional [32] singlet Higgs field $\hat{N}$. Then, if the superpotential contains a trilinear term $W \ni \lambda \hat{H}_1 \hat{H}_2 \hat{N}$ and if the scalar component of $\hat{N}$ develops a vacuum expectation value $\langle N \rangle \equiv x$, a bilinear $\mu \hat{H}_1 \hat{H}_2$ mixing term with $\mu \equiv \lambda x$ is generated. In the presence of soft supersymmetry breaking, one would expect $x \lesssim \mathcal{O}$ (1 TeV), and hence $\mu \lesssim \mathcal{O}$ (1 TeV) $\ll m_{Pl}$, thereby solving the μ problem. We shall study [33, 34] some aspects of such an extension of MSSM, called the non-minimal supersymmetric standard model (NMSSM), in Chapter 2. In particular, we shall study the renormalization group evolution and infra-red fixed points of the Yukawa couplings of NMSSM, and carry out a detailed study of the of the stability of these fixed points.

The minimal supersymmetric standard model contains two Higgs doublet superfields $\hat{H}_1$ and $\hat{H}_2$ with opposite hypercharges so as to generate masses for up- and down-quarks (and leptons), and to cancel triangle gauge anomalies. After spontaneous symmetry breaking induced by the neutral components of H_1 and H_2 obtaining vacuum expectation values, the MSSM contains two neutral CP-even (h, H), one neutral CP-odd (A), and two charged Higgs bosons [35]. Because of underlying gauge invariance and supersymmetry, the lightest CP-even Higgs boson in MSSM has tree level upper bound of m_Z on its mass. Although radiative corrections to this result are appreciable, these are under control because of underlying softly broken supersymmetry [36]. This results in an upper bound of $m_h \lesssim 125\ GeV$ on the radiatively corrected lightest Higgs boson mass in MSSM.

Although there are two distinct scales, the electroweak breaking scale and the supersymmetry (SUSY) breaking scale, in the MSSM, the (tree level) upper bound on the mass of the lightest Higgs boson is independent of the SUSY breaking scale (radiative correction induce only a logarithmic dependence on the SUSY breaking scale). The existence of such an upper bound on the mass of the lightest Higgs boson in MSSM has been investigated in situations where the underlying supersymmetric model respects baryon (B) and lepton (L) number conservation. However, as noted earlier, gauge invariance, supersymmetry, and renormalizibility allow B and L violating terms (1.66) in the superpotential of MSSM. In MSSM these terms are eliminated by invoking the discrete R-parity symmetry (1.67). However, the assumption of R_p conservation appears to be *ad hoc*, since it is not required for the internal consistency of the MSSM. It is, therefore, more appealing to have a supersymmetric theory where R-parity is related to a gauge symmetry, and ite conservation is automatic because of the invariance of the underlying theory under this gauge symmetry. Indeed, R_p conservation follows automatically in certain theories with gauged $(B - L)$, as is suggested by the appearance of $(B - L)$ in (1.67). It has been noted by several authors [37, 38] that if the gauge symmetry of MSSM is extended to left-right symmetry, $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the theory becomes automatically R-parity conserving. Such a left-right symmetry solves the problem of explicit B and L violation of MSSM, and has received [39] much attention recently . Such a naturally R-parity conserving theory necessarily involves the extension of the Standard Model gauge group, and since the extended gauge symmetry has to be broken, it involves "a new scale", the scale of left-right symmetry breaking beyond the SUSY and $SU(2)_L \times U(1)_Y$ breaking scales of MSSM. It is, therefore, important to ask whether the upper bound on the lightest Higgs boson mass in such R-parity conserving theories depends on the scale of breakdown of the extended gauge group. It has been shown [40] that in the supersymmetric left-right model with minimal particle content the upper bound on the mass of the lightest neutral Higgs boson depends only on the gauge couplings and those vacuum expectation values (VEVs) which break $SU(2)_L \times U(1)_Y$. The upper bound does not depend on any other scales (vacuum expectation value) that exist in such models. In Chapter 3 we shall study [41] higher order radiative corrections to the lightest Higgs boson mass in the minimal version of the supersymmetric left-right model in order to arrive at a precise value for the upper bound on the lightest Higgs boson mass in these models. Since the one-loop logarithmic correction proportional to m_t^4 is domonant, it is sufficient to consider two-loop leading and next-to-leading log contributions proportional to $m_t^4\alpha_3$ and $m_t^4\alpha_t$, where $\alpha_3 \equiv g_3^2/4\pi, \alpha_t \equiv h_t^2/4\pi$, respectively. We shall also compare our result with the corresponding result on the lightest Higgs boson mass in MSSM. In particular, we shall show that the upper bound on the lightest Higgs mass in this class of models lies considerably above the corresponding upper bound in the MSSM.

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Chapter 2

Non-Minimal Supersymmetric Standard Model with R-Parity Violation

2.1 The Non-Minimal supersymmetric Standard Model

As discussed in Chapter 1, a light Higgs sector ($m_H^2 \ll m_{Pl}^2$) is natural in a softly broken supersymmetric model since the quadratically divergent loop contributions to the mass of the Higgs boson cancel, leaving a finite correction of the form $\delta m_H^2 = \mathcal{O}(\alpha/\pi)(m_B^2 - m_F^2)$, where $m_{B,F}$ are the masses of the bosonic and fermionic partner particles circulating in the loops, and α is square of a generic coupling constant. The minimal supersymmetric version of the Standard Model is constructed by simply doubling the particle species, i.e. by including a boson for each fermion of the Standard Model, and vice versa. Furthermore, in order to give masses to all the quarks and leptons, and to cancel gauge anomalies, at least two-Higgs doublets H_1, H_2 are required in the minimal supersymmetric standard model (MSSM). The superpotential of MSSM is given by [1]

$$W = h_U \hat{Q}\hat{U}^c \hat{H}_2 + h_D \hat{Q}\hat{D}^c \hat{H}_1 + h_L \hat{L}\hat{E}^c \hat{H}_1 + \mu \hat{H}_1 \hat{H}_2, \tag{2.1}$$

and contains a bilinear term $\mu\hat{H}_1\hat{H}_2$ coupling the two Higgs doublet superfields, with μ having dimensions of mass. The spectrum of physical Higgs bosons of MSSM then includes two CP-even neutral Higgs bosons (h^0, H^0), one CP-odd Higgs boson (A^0), and a complex charged Higgs boson $(H^\pm)$. With the bilinear term in the superpotential one would also expect a bilinear supersymmetry-breaking term in the scalar potential:

$$V_1 = -B\mu\epsilon_{ij} H_1^i H_2^j + h.c., \tag{2.2}$$

where B is expected to be of the order of supersymmetry breaking scale ($\lesssim 1\ TeV$). With a term of the form given in (2.2), the mass of the CP-odd Higgs boson is given by

$$m_A^2 = \frac{2B\mu}{\sin 2\beta}, \quad \tan\beta = \frac{v_2}{v_1}, \quad v_i = \langle 0|H_i^0|0\rangle. \tag{2.3}$$

For $\mu = 0$, the CP-odd Higgs boson A^0 is massless (the axion), which is in contradiction with experiment. Thus, a non-zero μ is required in order to break $SU(2)_L \times U(1)_Y$ without producing an unacceptable electroweak axion. Softly broken supersymmetry ensures that the radiative corrections to μ are now under control so that $\mu = \mathcal{O}(m_W) \ll m_{Pl}$ is technically natural, thus solving the easy part of the hierarchy problem. However, it does not provide any dynamical reason why μ should be so small in the first place. There would seem to be two natural options: either there is a symmetry which forbids the bilinear coupling, or it acquires an extremely large mass. The first possibility leads to an electroweak axion, which is excluded by experiment. The second option effectively obliterates the weak scale: the Higgs doublets are removed to the high mass scale, or (worse) electroweak symmetry is broken at the high scale. Thus, it is a serious problem, why μ is of the order of the electroweak scale. It endangers the whole idea of low energy supersymmetry.

The simplest mechanism that provides a dynamical source for a term of the form $\mu\hat{H}_1\hat{H}_2$ is the inclusion of an additional [2] singlet Higgs field $\hat{N}$. Then, if the superpotential contains a trilinear term

$$W \ni \lambda\epsilon_{ij}\hat{N}\hat{H}_1^i\hat{H}_2^j, \tag{2.4}$$

and if N develops a vacuum expectation value $\langle N\rangle \equiv x$, a bilinear $\hat{H}_1\hat{H}_2$ mixing term with

$$\mu = \lambda x, \tag{2.5}$$

is generated. In the presence of soft supersymmetry breaking, one would expect $x \lesssim \mathcal{O}(1\text{TeV})$, and hence $\mu \lesssim \mathcal{O}(1\text{ TeV}) \ll m_{Pl}$, thereby solving the μ problem.

It must be mentioned, for the sake of completeness, that alternative mechanisms for generating the term (2.2) have been proposed [3, 4, 5]. If μ and B vanish at the tree level due to some global symmetry, then non-zero values for these parameters cannot be generated by higher order corrections. However, such a symmetry could be broken. If the breaking is spontaneous, it should occur at a scale of the order $10^9 - 10^{12}$ GeV, so that the couplings of associated Goldstone bosons are sufficiently weak to be phenomenologically acceptable. Then, once supersymmetry is broken, radiative corrections could generate [3] a $B\mu$ of order $(\alpha/\pi)(m_B^2 - m_F^2)$. On the other hand if the breaking is explicit, a radiative generation of $B\mu$ could still occur [4]. Another mechanism [3], which operates at the tree level, proceeds via non-renormalizable terms in the superpotential of the form $\left(\frac{1}{m_{Pl}}\right)^{n-1}\hat{\Phi}^n\hat{H}_1\hat{H}_2$, where $\hat{\Phi}$ is a gauge singlet acquiring a vacuum expectation value at some intermediate mass scale $\langle\hat{\Phi}\rangle = M < m_{Pl}$. Another alternative [5] is to start from a supergravity model whose superpotential is purely trilinear in the observable sector, but contains an explicit mass scale $M \sim 10^{10} - 10^{11}$ GeV in the hidden sector which breaks local supersymmetry. If the Kähler potential mixes the hidden and observable sectors in a peculiar way, then a $\mu \sim (M^2/m_{Pl}) \sim m_W$ is generated in the corresponding low-energy theory with softly broken global supersymmetry. However, the mixing of the Higgs doublets $\hat{H}_1$ and $\hat{H}_2$ induced by a singlet via (2.4) is the most appealing of all the mechanisms. Considerable attention [6, 7, 8, 9, 10] has been devoted to the study of such a supersymmetric model containing two Higgs doublets and a singlet Higgs superfield $\hat{N}$. This chapter is devoted to the study of some aspects of such a non-minimal supersymmetric standard model (NMSSM) based on the gauge group $SU(2)_L \times U(1)_Y$.

The non-minimal supersymmetric standard model is charecterized by the superpotential

$$W = h_U \hat{H}_2 \hat{Q} \hat{U}^c + h_D \hat{H}_1 \hat{Q} \hat{D}^c + h_L \hat{H}_1 \hat{L} \hat{E}^c + \lambda \hat{H}_1 \hat{H}_2 \hat{N} - \frac{1}{3} k \hat{N}^3, \tag{2.6}$$

where gauge and generation indices are not explicitly displayed, and the sign of the $k\hat{N}^3$ term is conventional. In comparision with MSSM, the Higgs mixing term $\mu \hat{H}_1 \hat{H}_2$ has been replaced by the trilinear coupling $\lambda \hat{H}_1 \hat{H}_2 \hat{N}$, where $\hat{N}$ is a $SU(2)_L \times U(1)_Y$ singlet. The form of the superpotential (2.6) is scale-independent: if bilinear terms are absent at tree-level, they cannot be generated by renormalization. We note that for $k = 0$, the Lagrangian of the NMSSM (2.6) has a global $U(1)$ symmetry corresponding to

$$N \rightarrow N e^{i\theta}, \quad H_1 H_2 \rightarrow H_1 H_2 e^{i\theta}, \tag{2.7}$$

which is spontaneously broken by the vacuum expectation values of the Higgs fields. This would produce a phenomenologically unacceptable axion. The introduction of the term $\frac{k}{3}\hat{N}^3$ explicitly breaks the symmetry (2.7), thereby avoiding this problem. Furthermore, if the following two conditions are satisfied: (i) there are renormalizable couplings between the singlet N and superheavy chiral multiplets, and (ii) supersymmetry-breaking mass splittings inside the superheavy chiral multiplets are of order $\sqrt{m_W M}$, where M is some superheavy scale, then the hierarchy $m_W \ll M$ tends to be destabilized by radiative corrections [11]. However, there is no reason to expect that both of these conditions are satisfied [12]. Only in the presence of a specific mechanism for supersymmetry breaking can one make a definite statement.

The Lagrangian associated with the superpotential of (2.6), even after the addition of the most general soft supersymmetry breaking terms, has a discrete Z_3 symmetry, corresponding to a phase transformation of $\hat{N}$ of the form $\hat{N} \rightarrow \alpha \hat{N}, \alpha^3 = 1$, accompanied by suitable transformations of the remaining fields. This symmetry, if spontaneously broken by the ground state, could create a serious domain wall problem. However, it has been shown [13] that a nonrenormalizable term $\sim \hat{N}^4 / m_{Pl}$ in (2.6) would prevent the density of domain walls from becoming large enough to cause cosmological problems, while being much too small to impact the low energy phenomenology of the model that we are interested in here.

In addition to the superpotential (2.6), the model is specified by the following soft supersymmetry-breaking terms:

$$
\begin{aligned}
-\mathcal{L}_{\rm soft} &= [m_{H_1}^2|H_1|^2 + m_{H_2}^2|H_2|^2 + m_N^2|N|^2 + \cdots\cdot] \\
&+[(A_U h_U H_2 \tilde{Q}\tilde{U}^c + A_D h_D H_1 \tilde{Q}\tilde{D}^c + A_L h_L H_1 \tilde{L}\tilde{E}^c + h.c.) \\
&-(\lambda A_\lambda H_1 H_2 N + h.c.) - (\frac{1}{3} k A_k N^3 + h.c.) + \cdots\cdot], \qquad (2.8)
\end{aligned}
$$

where the dots stand for other terms that are not relevant to our discussion. Here m_{H_1}, m_{H_2} and m_N^2 are soft supersymmetry breaking scalar masses, and $A_U, A_D, \cdots$ are soft supersymmetry breaking trilinear couplings. The absence of an explicit soft supersymmetry breaking term of the form (2.2) is a natural assumption. If such a term is absent at the grand-unification scale then the renormalization group equations imply [14] that such a term is not generated in evolving down to the low energy scale. The spectrum of physical particles of NMSSM has fermions and gauge bosons and their supersymmetric partners (as in MSSM), and an additional neutralino, and the following Higgs bosons (and their supersymmetric partners): three CP-even neutral Higgs bosons ($S_i, i = 1, 2, 3$), two CP-odd neutral Higgs bosons ($P_i, i = 1, 2$), and the charged Higgs bosons ($H^\pm$). It is in the spectrum of Higgs bosons that the model differs crucially from the minimal supersymmetric standard model [6, 7, 8, 9, 10].

2.1.1 R-parity violation

Supersymmetry, gauge invariance, and the particle content of the non-minimal supersymmetric standard model allow the following renormalizable terms in the superpotential [15]:

$$
W_B = \frac{1}{2}\lambda''_{ijk}\hat{D}^{ci}\hat{D}^{cj}\hat{U}^{ck}, \qquad (2.9)
$$

$$
W_L = \lambda_i \hat{L}^i \hat{H}_2 \hat{N} + \frac{1}{2}\lambda_{ijk}\hat{L}^i\hat{L}^j\hat{E}^{ck} + \lambda'_{ijk}\hat{L}^i\hat{Q}^j\hat{D}^{ck}. \qquad (2.10)
$$

where i, j, k are generation indices. The superpotential terms (2.9) violate baryon number B, while those of (2.10) violate lepton number L. The Yukawa couplings λ_{ijk} and λ''_{ijk} are antisymmetric in first two indices due to $SU(2)_L$ and $SU(3)_C$ invariance, respectively. Thus, there are 9 independent λ_{ijk}, and same number of independent

λ''_{ijk} couplings. The term $\lambda_i \hat{L}^i \hat{H}_2 \hat{N}$ in the superpotential W_L can be rotated away into the B and L conserving term $\lambda \hat{H}_1 \hat{H}_2 \hat{N}$ via an $SU(4)$ rotation between the superfields $\hat{H}_1$ and $\hat{L}^i$. However, this operation must be performed at some energy scale, and the mixing is regenerated at other scales through the renormalization group equations. Usually, a discrete symmetry [16], called R parity (R_p) is assumed to forbid the superpotential terms (2.9) and (2.10). However, since R_p conservation is not theoretically motivated by any known principle, the possibility of R_p nonconservation deserves a serious consideration. As such, considerable attention has been paid to the study of R_p violating couplings in the minimal supersymmetric standard model [17]. Phenomenological studies of R-parity violating couplings in the MSSM have placed constraints on the various couplings $\lambda_{ijk}, \lambda'_{ijk}, \lambda''_{ijk}$, but considerable latitude remains for these couplings [18]. We note that the simultaneous presence of the terms in (2.9) and (2.10) is essentially ruled out by the stringent constraints [19] implied by the lack of observation of nucleon decay. To escape these lifetime constraints, it is sufficient that only one of these classes be absent or very highly suppressed.

Studies of the renormalization group equations (RGEs), relating couplings at the electroweak scale to their values at the grand unified theory (GUT) scale, have led to new insights and constraints on the observable low-energy parameters in the MSSM as well as the NMSSM with R-parity conservation [14, 20]. It seems, therefore, natural to see what can be learned from similar studies with R-parity violation. Much attention has been paid to the study of MSSM with R-parity violation, including the study of quasi infra-red fixed points [21], and the true fixed points [22]. Since NMSSM is a viable alternative to MSSM, it is important to study the renormalization group evolution of this model with R-parity violation. In this chapter we undertake a detailed study of the RGEs for the non-minimal supersymmetric standard model with R-parity violation. To this end we first derive the RGEs for the NMSSM with R-parity violation in the next section.

2.2 The Renormalization Group equations

We are interested in the one-loop renormalization group equations for the dimensionless trilinear Yukawa couplings in the superpotential (2.6), (2.9) and (2.10). For a general N=1 supersymmetric theory with a trilinear term $d_{abc}\hat{\Phi}^a\hat{\Phi}^b\hat{\Phi}^c$ in the superpotential involving chiral superfields $\hat{\Phi}^a$, $\hat{\Phi}^b$, $\hat{\Phi}^c$, the evolution of the couplings d_{abc} with the scale μ is given by the RGEs [23]

$$16\pi^2\frac{\partial d_{abc}}{\partial(\ln\mu)} = \gamma_a^e d_{ebc} + \gamma_b^e d_{aec} + \gamma_c^e d_{abe}, \tag{2.11}$$

where γ_a^e are the elements of the anomalous dimension matrix, and sum over repeated indices is understood. The anomalous dimensions are given by

$$\gamma_a^e = \frac{1}{2}\sum_{b,c} d_{abc}d^{ebc} - 2\delta_a^e g_A^2 C_a^A, \tag{2.12}$$

to one loop order. The sum over A represents a sum over all dominant gauge couplings, and C_a^A is the quadratic Casimir of the representation of $\hat{\Phi}_a$ under the gauge group with coupling g_A:

$$(T_R^A\, T_R^A)_a^b = C_R^A\, \delta_a^b. \tag{2.13}$$

Here, T_R^A is a matrix in the R representation for the group labelled by A. Pictorially, the RG evolution of the trilinear coupling can be described as an insertion of the anomalous dimension correction on each external leg [24]. We have calculated the anomalous dimension [25] for the various superfields for the NMSSM with R-parity violating couplings. These are summarized in Table 2.1. The renormalization group equation for the Yukawa couplings h_U, h_D, h_L of the superpotential (2.6) are obtained from (2.11) with the index c belonging to a Higgs field. The general form of the RGEs are

$$\begin{aligned}
16\pi^2\frac{\partial}{\partial\ln\mu}(h_U)_{ab} &= (h_U)_{ib}\gamma_{Q_a}^{Q_i} + (h_U)_{ai}\gamma_{\bar{U}_b}^{\bar{U}_i} + (h_U)_{ab}\gamma_{H_2}^{H_2}, &(2.14)\\
16\pi^2\frac{\partial}{\partial\ln\mu}(h_D)_{ab} &= (h_D)_{ib}\gamma_{Q_a}^{Q_i} + (h_D)_{ai}\gamma_{\bar{D}_i}^{\bar{D}_i} + (D)_{ab}\gamma_{H_1}^{H_1} + \lambda'_{iab}\gamma_{H_1}^{L_i}, &(2.15)\\
16\pi^2\frac{\partial}{\partial\ln\mu}(h_L)_{ab} &= (h_L)_{ib}\gamma_{L_a}^{L_i} + (h_L)_{ai}\gamma_{\bar{E}_b}^{\bar{E}_i} + (h_L)_{ab}\gamma_{H_1}^{H_1} + \lambda_{iab}\gamma_{H_1}^{L_i}. &(2.16)
\end{aligned}$$

$\phi_{i,j}$	NMSSM	L violation	B violation
$\hat{N}, \hat{N}$	$4\lambda^2 + 4k^2$	$\lambda^i \lambda_i$	–
$\hat{L}_i, \hat{H}_1$	–	$\lambda^{iab}(h_E)_{ab} + 3\lambda'^{iab}(h_D)_{ab} + \lambda\lambda^i$	–
$\hat{L}_{i,j}$	$h_L h_L^\dagger - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2$	$\lambda_{iab}\lambda^{jab} + 3\lambda'_{iab}\lambda'^{jab} + \lambda_i \lambda^i \delta^i_j$	–
$\hat{E}^c_{i,j}$	$2h_E^\dagger h_E - \frac{6}{5}g_1^2$	$\lambda^{abi}\lambda_{abj}$	–
$\hat{D}^c_{i,j}$	$2h_D^\dagger h_D - \frac{8}{3}g_3^2 - \frac{2}{15}g_1^2$	$2\lambda'^{abi}\lambda'_{abj}$	$2\lambda''^{iab}\lambda''_{jab}$
$\hat{U}^c_{i,j}$	$2h_U^\dagger h_U - \frac{8}{3}g_3^2 - \frac{8}{15}g_1^2$	–	$\lambda''^{abi}\lambda''_{abj}$
$\hat{Q}_{i,j}$	$h_U h_U^\dagger + h_D h_D^\dagger - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{1}{30}g_1^2$	$\lambda'_{aib}\lambda'^{ajb}$	–
$\hat{H}_1, \hat{H}_1$	$Tr(h_E h_E^\dagger + 3h_D h_D^\dagger) + \lambda^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2$	–	–
$\hat{H}_2, \hat{H}_2$	$3Tr(h_U h_U^\dagger) + \lambda^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2$	$\lambda_i \lambda^i$	–

Table 2.1: The anomalous dimensions $\gamma^{\phi_i}_{\phi_j}$ in the non-minimal supersymmetric standard model with lepton and baryon number violating couplings. Here i, j are flavour indices.

The evolution of gauge couplings g_i ($i = 1, 2, 3$ denoting the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gauge groups, with GUT normalization for $U(1)_Y$ gauge group) in NMSSM with R-parity violation is same as in MSSM, as this evolution is unaffected at the one-loop level by the presence of a singlet, or R-parity violating couplings. These evolution equations are

$$16\pi^2 \frac{dg_i}{d(\ln \mu)} = b_i g_i^3, \quad i = 1, 2, 3, \tag{2.17}$$

where b_i are the beta functions for the respective gauge couplings with $b_1 = 33/5,\ b_2 = 1,\ b_3 = -3$.

The third generation Yukawa couplings are dominant couplings, so we shall retain in the anomalous dimensions only the $(3,3)$ elements $h_t \equiv (h_U)_{33}, h_b \equiv (h_D)_{33}, h_\tau \equiv (h_L)_{33}$ in $h_U,\ h_D,\ h_L$, setting all other elements to zero. Apart from these there are 39 independent R_p violating couplings λ_i, λ_{ijk} *and* λ'_{ijk} in the lepton number violating sector, and 9 independent couplings λ''_{ijk} in the baryon number violating sector. Thus, we shall have to study 42 coupled nonlinear evolution equations in L-violating case, and 12 in the B-violating case. We need some radical simplification in the R_p-violating sector to make the system of evolution equations tractable.

It is possible that there may exist a generational hierarchy among the R_p violating couplings, similar to that of conventional Higgs couplings. The R_p violating couplings to higher generations evolve more strongly because of larger Higgs couplings in their RGEs, and hence have the potential to take larger values than the R_p-violating couplings to the lower generations. Thus, we shall retain only the couplings λ_3, λ_{233} and λ'_{333}, or λ''_{233}, and neglect all others. This assumption is also motivated by the fact that the experimental upper limits are stronger for the couplings with lower indices. With these simplification the RGEs for the Yukawa couplings and the R-parity violating couplings in the NMSSM can be written as [15, 25]

$$
\begin{aligned}
\frac{dh_t}{dt} &= \frac{h_t}{16\pi^2}\left[6h_t^2 + h_b^2 + \lambda^2 + \lambda_3^2 + \lambda'^2_{333} + 2\lambda''^2_{233}\right. \\
&\quad \left. - \left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2\right)\right], && (2.18) \\
\frac{dh_b}{dt} &= \frac{1}{16\pi^2}\left[\left(h_t^2 + 6h_b^2 + h_\tau^2 + 6\lambda'^2_{333} + 2\lambda''^2_{233} + \lambda^2\right) h_b + \lambda\lambda_3\lambda'_{333}\right. \\
&\quad \left. - \left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2\right) h_b\right], && (2.19) \\
\frac{dh_\tau}{dt} &= \frac{h_\tau}{16\pi^2}\left[3h_b^2 + 4h_\tau^2 + \lambda^2 + \lambda_3^2 + 4\lambda_{233}^2 + 3\lambda'^2_{333} - \left(3g_2^2 + \frac{9}{5}g_1^2\right)\right], && (2.20) \\
\frac{d\lambda}{dt} &= \frac{1}{16\pi^2}\left[\left(3h_t^2 + 3h_b^2 + h_\tau^2 + 4\lambda^2 + 2\kappa^2 + 4\lambda_3^2\right)\lambda + 3h_b\lambda_3\lambda'_{333}\right. \\
&\quad \left. - \left(3g_2^2 + \frac{3}{5}g_1^2\right)\lambda\right], && (2.21) \\
\frac{dk}{dt} &= \frac{k}{16\pi^2}\left[6\lambda^2 + 6k^2 + 6\lambda_3^2\right], && (2.22) \\
\frac{d\lambda''_{233}}{dt} &= \frac{\lambda''_{233}}{16\pi^2}\left[\left(2h_t^2 + 2h_b^2 + 2\lambda'^2_{333} + 6\lambda''^2_{233}\right) - \left(8g_3^2 + \frac{4}{5}g_1^2\right)\right], && (2.23) \\
\frac{d\lambda_3}{dt} &= \frac{1}{16\pi^2}\left[\left(3h_t^2 + h_\tau^2 + 4\lambda^2 + 2k^2 + 4\lambda_3^2 + \lambda_{233}^2 + 3\lambda'^2_{333}\right)\lambda_3 + 3h_b\lambda\lambda'_{333}\right. \\
&\quad \left. - \left(3g_2^2 + \frac{3}{5}g_1^2\right)\lambda_3\right], && (2.24) \\
\frac{d\lambda_{233}}{dt} &= \frac{\lambda_{233}}{16\pi^2}\left[4h_\tau^2 + \lambda_3^2 + 4\lambda_{233}^2 + 3\lambda'^2_{333} - \left(3g_2^2 + \frac{9}{5}g_1^2\right)\right], && (2.25) \\
\frac{d\lambda'_{333}}{dt} &= \frac{1}{16\pi^2}\left[\left(h_t^2 + 6h_b^2 + h_\tau^2 + \lambda_3^2 + \lambda_{233}^2 + 6\lambda'^2_{333} + 2\lambda''_{233}\right)\lambda'_{333} + h_b\lambda\lambda_3\right. \\
&\quad \left. - \left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2\right)\lambda'_{333}\right], && (2.26)
\end{aligned}
$$

where it is understood that one takes either $\lambda_3 = \lambda_{233} = \lambda^{'}_{333} = 0$, or $\lambda^{''}_{233} = 0$. We simplify the form of the RGEs by using the notation

$$\begin{aligned}
R_t &= \frac{h_t^2}{g_3^2},\quad R_b = \frac{h_b^2}{g_3^2},\quad R_\tau = \frac{h_\tau^2}{g_3^2},\quad R_\lambda = \frac{\lambda^2}{g_3^2},\quad R_k = \frac{k^2}{g_3^2}, && (2.27)\\
R^{''} &= \frac{\lambda^{''2}_{233}}{g_3^2}, && (2.28)\\
R_3 &= \frac{\lambda_3^2}{g_3^2},\quad R = \frac{\lambda^2_{233}}{g_3^2},\quad R^{'} = \frac{\lambda^{'}_{333}}{g_3^2}. && (2.29)
\end{aligned}$$

We shall first consider the evolution of Yukawa couplings arising from superpotential terms (2.6) and (2.9), which involve baryon number violation. With the above definitions, and retaining only the $SU(3)_C$ gauge coupling, the one-loop renormalization group equations for h_t, h_b, h_τ, λ, k *and* $\lambda^{''}_{233}$ can be written in the form [15]

$$\begin{aligned}
\frac{dR^{''}}{d(-\ln\mu^2)} &= \tilde{\alpha}_3 R^{''}[(8+b_3) - 6R^{''} - 2R_b - 2R_t], && (2.30)\\
\frac{dR_b}{d(-\ln\mu^2)} &= \tilde{\alpha}_3 R_b\left[\left(\frac{16}{3}+b_3\right) - 2R^{''} - 6R_b - R_\tau - R_t - R_\lambda\right], && (2.31)\\
\frac{dR_\tau}{d(-\ln\mu^2)} &= \tilde{\alpha}_3 R_\tau[b_3 - 3R_b^2 - 4R_\tau - R_\lambda], && (2.32)\\
\frac{dR_t}{d(-\ln\mu^2)} &= \tilde{\alpha}_3 R_t\left[\left(\frac{16}{3}+b_3\right) - 2R^{''} - R_b - 6R_t - R_\lambda\right], && (2.33)\\
\frac{dR_\lambda}{d(-\ln\mu^2)} &= \tilde{\alpha}_3 R_\lambda[b_3 - 3R_b - R_\tau - 3R_t - 4R_\lambda - 2R_k], && (2.34)\\
\frac{dR_k}{d(-\ln\mu^2)} &= \tilde{\alpha}_3 R_k[b_3 - 6R_\lambda - 6R_k], && (2.35)
\end{aligned}$$

where $b_3 = -3$ is the beta function for g_3 in MSSM (or NMSSM), and $\tilde{\alpha}_3 = g_3^2/(16\pi^2)$. Ordering the ratios as $R_i = (R^{''}, R_b, R_\tau, R_t, R_\lambda, R_k)$, we can rewrite the RG equations (2.30) - (2.35) in the form ($t = -\ln\mu^2$)

$$\frac{dR_i}{dt} = \tilde{\alpha}_3 R_i[(r_i + b_3) - \sum_j S^B_{ij} R_j], \qquad (2.36)$$

where $r_i = \sum_k 2C_R$, C_R is the QCD quadratic Casimir for the various fields ($C_Q = C_{U^c} = C_{D^c} = 4/3$) and the sum is over the representation of the three fields associated with the trilinear coupling that enters the definition of R_i, and S^B is a matrix whose entries are the numerical coefficients (the anomalous dimensions) of R_i's in the evolution equations (2.30) - (2.35). We shall be considering the renormalization group evolution and infra-red fixed points of the couplings h_t, h_b, λ, k *and* λ''_{233} only, and shall ignore the evolution equation (2.32) for h_τ. However, the coupling h_τ does enter the evolution equation (2.31) for h_b, but it can be related to h_b at the weak scale (which we take to be the top-quark mass), since

$$h_\tau(m_t) = \frac{\sqrt{2}m_\tau(m_t)}{\eta_\tau v \cos\beta}, \tag{2.37}$$

and

$$h_\tau(m_t) = \frac{m_\tau(m_\tau)}{m_b(m_b)}\frac{\eta_b}{\eta_\tau}h_b(m_t) = 0.6\ h_b(m_t), \tag{2.38}$$

where η_b gives the QCD or QED running [26] of the b-quark mass $m_b(\mu)$ between $\mu = m_b$ and $\mu = m_t$ (similarly for η_τ), and $\tan\beta = v_2/v_1$ is the usual ratio of the Higgs vacuum expectation values, with $v = (2\sqrt{2}G_F)^{-1/2} = 174.1$ GeV. The anomalous dimension matrix can then be written as

$$S^B = \begin{bmatrix} 6 & 2 & 2 & 0 & 0 \\ 2 & 6+\eta & 1 & 1 & 0 \\ 2 & 1 & 6 & 1 & 0 \\ 0 & 3+\eta & 3 & 4 & 2 \\ 0 & 0 & 0 & 6 & 6 \end{bmatrix}, \tag{2.39}$$

where

$$\eta = h_\tau^2(m_t)/h_b^2(m_t) \simeq 0.36, \tag{2.40}$$

is the factor coming from Eq.(2.38). The ordering of the various couplings in (2.36) and (2.39) is now $R_i = (R'', R_b, R_t, R_\lambda, R_k)$.

For the case of lepton number violating couplings arising from the superpotential (2.10), we shall consider three different cases, i.e. we shall take $\lambda'_{333} \gg \lambda_3, \lambda_{233}$, or $\lambda_{233} \gg \lambda_3, \lambda'_{333}$, or $\lambda_3 \gg \lambda_{233}, \lambda'_{333}$, respectively. In the case when λ'_{333} is the dominant of the lepton number violating couplings, we reorder the couplings as $R_i = (R', R_b, R_t, R_\lambda, R_k)$, so that the relevant RGEs (2.18), (2.19), (2.21), (2.22) and (2.26) can be written as [15]

$$\frac{dR_i}{dt} = \tilde{\alpha}_3 R_i[(r_i + b_3) - \sum S^L_{ij}R_j], \tag{2.41}$$

where S^L is the anomalous dimension matrix for this case:

$$S^L = \begin{bmatrix} 6 & 6+\eta & 1 & 0 & 0 \\ 6 & 6+\eta & 1 & 1 & 0 \\ 1 & 1 & 6 & 1 & 0 \\ 0 & 3+\eta & 3 & 4 & 2 \\ 0 & 0 & 0 & 6 & 6 \end{bmatrix}, \tag{2.42}$$

and other quantities are defined in a manner similar to the baryon number violating case (2.36), with $\eta \simeq 0.36$ as in (2.40).

If on the other hand the coupling λ_{233} is the dominant of the lepton number violating couplings, we reorder the Yukawa couplings as $(R, R_b, R_t, R_\lambda, R_k)$, so that the RGEs can once again be written in the form

$$\frac{dR_i}{dt} = \tilde{\alpha}_3 R_i[(r_i + b_3) - \sum_j S^{L'}_{ij} R_j], \tag{2.43}$$

with the anomalous dimension matrix

$$S^{L'} = \begin{bmatrix} 4 & 4\eta & 0 & 0 & 0 \\ 0 & 6+\eta & 1 & 1 & 0 \\ 0 & 1 & 6 & 1 & 0 \\ 0 & 3+\eta & 3 & 4 & 2 \\ 0 & 0 & 0 & 6 & 6 \end{bmatrix}, \tag{2.44}$$

with the by now usual definitions of the various quantities in (2.43) and (2.44).

Finally, if the lepton number violating coupling λ_3 is the dominant coupling, we reorder the couplings as $(R_3, R_b, R_t, R_\lambda, R_k)$, so that the RGEs can be put in the form

$$\frac{dR_i}{dt} = \tilde{\alpha}_3 R_i[(r_i + b_3) - \sum_j S^{L''}_{ij} R_j], \tag{2.45}$$

with the anomalous dimension matrix

$$S^{L''} = \begin{bmatrix} 4 & \eta & 3 & 4 & 2 \\ 0 & 6+\eta & 1 & 1 & 0 \\ 1 & 1 & 6 & 1 & 0 \\ 4 & 3+\eta & 3 & 4 & 2 \\ 6 & 0 & 0 & 6 & 6 \end{bmatrix}. \tag{2.46}$$

In the following, we shall study these renormalization group equations and their Infra-red fixed points in detail [15, 25].

2.3 Infra-red Fixed Points

There is considerable interest in the study of infra-red (IR) stable fixed points of the Standard Model (SM) and its extensions, especially those of the minimal supersymmetric standard model (MSSM). This interest follows from the fact that in SM (and in the MSSM) there are large number of unknown dimensionless Yukawa couplings, as a consequence of which the fermion masses cannot be predicted. One may attempt to relate the Yukawa couplings to the gauge couplings via the Pendleton-Ross infra-red stable fixed point (IRSFP) for the top-quark Yukawa coupling [27], or via the quasi-fixed point behaviour [28]. The predictive power of different models is enhanced if the renormalization group (RG) running of parameters is dominated by the IRSFPs. Typically, these fixed points are for ratios like Yukawa coupling to the gauge coupling, or, in the context of supersymmetric models, the supersymmetry breaking trilinear A-parameter to the gaugino mass, etc. These ratios do not always attain their fixed point values at the weak scale, the range between the GUT (or Planck) scale and the weak scale being too small for the ratios to closely approach the fixed point. Nevertheless, the couplings may be determined by the quasi-fixed point behaviour [28], where the value of the Yukawa coupling at the weak scale is independent of its value at the GUT scale, provided the Yukawa couplings at the unification scale are large. For the fixed point or quasi-fixed point to be successful, it is necessary that these fixed points be stable [22].

In this, and the subsequent sections, we shall study the infra-red fixed points of the Yukawa couplings of the non-minimal supersymmetric standard model (NMSSM) with R-parity (R_p) violation. In the previous section we have shown that the RGEs for the Yukawa couplings of NMSSM with R_p violation can be cast into the form

$$\frac{dR_i}{dt} = \tilde{\alpha}_3 R_i[(r_i + b_3) - \sum_j S_{ij} R_j], \quad i = 1, \cdots, 5, \tag{2.47}$$

where $r_i = \sum_R 2C_R$, C_R the quadratic QCD Casimir, with the sum being over the representations of the three fields associated with the trilinear coupling h_i that enters the definition of R_i, and S_{ij} is the matrix whose numerical value is completely specified by the wave function anomalous dimensions. The fixed point of the ratios R_i is then reached when the right hand side of the Eq.(2.47) is zero for all i. Thus, writing the fixed point solutions as R_i^* there are two fixed point values for each coupling: $R_i^* = 0$ or the solution [22] corresponding to

$$[(r_i + b_3) - \sum_j S_{ij} R_j^*] = 0. \tag{2.48}$$

The non-trivial fixed point solution, following from (2.48), is

$$R_i^* = \sum_{j=1}^{n} (S^{-1})_{ij} (r_j + b_3). \tag{2.49}$$

We shall first consider only the non-trivial fixed points, and later extend our analysis to the general case where some of the couplings are zero at the fixed point.

To determine the infra-red stability of the system of Eq.(2.47), we Taylor expand it around the fixed point (2.49). For this we make a change of variables to $\rho_i(t) \equiv R_i(t) - R_i^*$. The RGE (2.47) then becomes

$$\frac{d\rho_i(t)}{dt} = \tilde{\alpha}_3(t)(\rho_i(t) + R_i^*)\left[(r_i + b_3) - \sum_{j=1}^{n} S_{ij}(\rho_j(t) + R_j^*)\right], \tag{2.50}$$

where we have substituted the fixed point values of R_i^* from (2.49). To study the stability we drop the quadratic terms in (2.50) and change the independent variable from t to $\tilde{\alpha}_3$ by using the RG equation for $g_3(\tilde{\alpha}_3 = g_3^2/16\pi^2,\ t = -\ln \mu^2)$

$$16\pi^2 \frac{dg_3}{d(\ln \mu)} = b_3 g_3^2, \tag{2.51}$$

to get the linearized system of equation (no sum over i)

$$\frac{d\rho_i(t)}{d(\ln \tilde{\alpha}_3(t))} \simeq \frac{1}{b_3} R_i^* \sum_{j=1}^{5} S_{ij} \rho_j(t), \tag{2.52}$$

which describes the behaviour of the trajectories as they approach the fixed point. The general solution of (2.52) is

$$\rho_i(t) \simeq \sum_{k=1}^{n} a_k x_i^{(k)} (\tilde{\alpha}_3(t))^{\lambda_k}, \tag{2.53}$$

where $x_j^{(k)}, \lambda_k$ are the $k = 1, 2, \cdots, n$ eigenvectors and eigenvalues of the eigenvalue equation

$$\sum_{j=1}^{n} A_{ij} x_j^{(k)} = \lambda_k x_i^{(k)}, \tag{2.54}$$

$$A_{ij} \equiv \frac{1}{b_3} R_i^* S_{ij}. \tag{2.55}$$

The a_k in the solution are constants which describe the particular linear combinations of the eigenvectors and are set by the initial conditions. The infra-red stability properties of the solutions in Eq.(2.53) are independent of the sign of one-loop gauge beta function b_3, as we now show.

For $b_3 > 0$, $\tilde{\alpha}$ decreases with decreasing renormalization scale μ. We require every eigenvalue λ_k of the matrix $\frac{1}{|b_3|} R_i^* S_{ij}$ to have a positive real part for infra-red stability, so that $\rho_i \to 0$ as μ decreases. For $b_3 < 0$, $\tilde{\alpha}_3$ increases with decreasing renormalization scale μ, so in this case we require all the eigenvalues of the matrix $\frac{-1}{|b_3|} R_i^* S_{ij}$ to have negative real parts, which is equivalent to the previous condition for $b_3 > 0$, so the stability condition is independent of the sign of the beta function. The situation $\lambda_k = 0$ corresponds to a direction in coupling space which is neither attracted to, nor repelled by the fixed point (to first order). On the other hand, if the non-trivial fixed point solution leads to values of R_i with unphysical negative values, such fixed points must be rejected. In such cases we are led to consider other fixed points where some of the couplings take zero values at the fixed point.

As remarked earlier, in general for each value of i there are two possible fixed point solutions to consider: $R_i^* = 0$ or the non-zero fixed point value considered above. There are thus 2^n fixed points contained in the theory, some of which could be degenerate. So far we have considered only the possibility of non-zero fixed point value for each coupling. For each of the remaining possibilities the fixed points can be determined in a straightforward manner. Suppose in general that we are considering a possibility with m zero fixed point solutions and $n-m$ non-zero solutions. First we reorder the n couplings such that

$$\begin{aligned} [R_i^*]_{i=1,\cdots,m} &= 0, \\ [R_i^*]_{i=m+1,\cdots,n} &= \sum_{m+1}^{n} (S^{-1})_{ij}(r_j + b_3). \end{aligned} \tag{2.56}$$

We note that the non-zero couplings are now determined by the lower right hand $(n-m)\times(n-m)$ block of the re-ordered matrix S. The procedure for the non-zero solutions turns out to be similar to that considered above, while that of zero fixed point is even simpler.

For the general case including m zero couplings, we need to Taylor expand around the fixed point (2.56). We make a change of variables to $\rho_i(t) \equiv R_i(t) - R_i^*$, where $R_i^* = 0$ for $i = 1, 2, \cdots, m$. With this change of variables, the RGE (2.47) becomes

$$\left[\frac{d\rho_i(t)}{dt}\right]_{i=1,\cdots,m} = \tilde{\alpha}_3(t)\rho_i(t)\left[(r_i + b_3) - \sum_{j=1}^{m} S_{ij}\rho_j(t) - \sum_{j=m+1}^{n} S_{ij}(\rho_j(t) + R_j^*)\right], \tag{2.57}$$

$$\left[\frac{d\rho_i(t)}{dt}\right]_{i=m+1,\cdots,n} = \tilde{\alpha}_3(t)\left(\rho_i(t) + R_i^*\right)\left[(r_i + b) - \sum_{j=1}^{m} S_{ij}\rho_j(t) - \sum_{j=m+1}^{n} S_{ij}(\rho_j(t) + R_j^*)\right], \tag{2.58}$$

where we have substituted the fixed point values of R_i^* from (2.56). We drop the quadratic terms, change the independent variable from t to $\tilde{\alpha}_3$ using the RGE for g_3, to obtain a linearised system of equations:

$$\left[\frac{d\rho_i(t)}{d\ln\tilde{\alpha}_3(t)}\right]_{i=1,\cdots,m} \simeq \frac{-\rho_i(t)}{b_3}\left[(r_i + b_3) - \sum_{j=m+1}^{n} S_{ij}R_j^*\right], \tag{2.59}$$

$$\left[\frac{d\rho_i(t)}{d\ln\tilde{\alpha}_3(t)}\right]_{i=m+1,\cdots,n} \simeq \frac{1}{b_3}R_i^*\sum_{j=1}^{n} S_{ij}\rho_j(t). \tag{2.60}$$

Equations (2.59) and (2.60) describe the behaviour of the trajectories as they approach the fixed point.

For $i = 1, \cdots, m$ the equations (2.59) are of the simple form

$$\left[\frac{d\rho_i(t)}{d\ln\tilde{\alpha}_3(t)}\right]_{i=1,\cdots,m} \simeq \lambda_i\rho_i(t), \tag{2.61}$$

where

$$\lambda_i = \frac{1}{b_3}\left[\sum_{j=m+1}^{n} S_{ij}R_j^* - (r_i + b_3)\right], \quad i = 1,\cdots,m, \tag{2.62}$$

with the solution

$$[\rho_i(t)]_{i=1,\cdots,m} = a_i(\tilde{\alpha})^{\lambda_i}. \tag{2.63}$$

As in the previous argument, for $b_3 > 0$, in order to have $\rho_i \to 0$ in the infra-red we require $\lambda_i > 0$, where λ_i is given by Eq.(2.62). Similarly for $b_3 < 0$, we require $\lambda_i < 0$ in order that $\rho_i \to 0$ in this case.

For $i = m+1,\cdots,n$, assuming that the m zero solutions are infra-red stable, as discussed above, we may simplify the procedure by working in the infra-red region where $[\rho_i]_{i=1,\cdots,m} \to 0$, so that Eqs.(2.60) reduce to a simple set of equations:

$$\left[\frac{d\rho_i(t)}{d\ln\tilde{\alpha}_3(t)}\right]_{i=m+1,\cdots,n} \simeq \frac{1}{b_3}R_i^* \sum_{j=m+1}^{n} S_{ij}\rho_j(t). \tag{2.64}$$

The argument now parallels to the one given previously for the non-zero solutions, as in Eq.(2.52) and the discussion following it, with attention now being focussed on the $(n-m)\times(n-m)$ lower right hand block of the re-ordered matrix S.

2.4 Infra-red Fixed Points with Baryon number Violation

We now apply the formalism of the previous section to the non-minimal supersymmetric standard model with baryon number violation [15].The anomalous dimension matrix for this case is given by (2.39).The non-trivial infra-red fixed points are given by (2.49) with the ordering $R_i^* = (R''^*, R_b^*, R_t^*, R_\lambda^*, R_k^*)$, $r_i = (8, \frac{16}{3}, \frac{16}{3}, 0, 0)$, and $b_3 = -3$. In order to obtain the fixed points we need to obtain the inverse of the matrix S in

(2.39). This is calculated to be

$$(S^B)^{-1} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{8} & -\frac{1}{8} & \frac{1}{8} & -\frac{1}{24} \\ -\frac{5}{4(10+\eta)} & \frac{23}{8(10+\eta)} & \frac{7}{8(10+\eta)} & -\frac{15}{8(10+\eta)} & \frac{5}{8(10+\eta)} \\ -\frac{5+\eta}{4(10+\eta)} & \frac{7+3\eta}{8(10+\eta)} & \frac{23+3\eta}{8(10+\eta)} & -\frac{15+3\eta}{8(10+\eta)} & \frac{5+\eta}{8(10+\eta)} \\ \frac{15+4\eta}{4(10+\eta)} & -\frac{45+16\eta}{8(10+\eta)} & -\frac{45+8\eta}{8(10+\eta)} & \frac{85+16\eta}{8(10+\eta)} & -\frac{85+16\eta}{24(10+\eta)} \\ -\frac{15+4\eta}{4(10+\eta)} & \frac{45+16\eta}{8(10+\eta)} & \frac{45+8\eta}{8(10+\eta)} & -\frac{85+16\eta}{8(10+\eta)} & \frac{5(25+4\eta)}{24(10+\eta)} \end{bmatrix}. \tag{2.65}$$

Using (2.65) in Eq.(2.49) we obtain the non-trivial fixed point values:

$$\begin{aligned} R''^* &= \frac{5}{12}, \\ R_b^* &= \frac{25}{4(10+\eta)}, \quad R_t^* = \frac{5(5+\eta)}{4(10+\eta)}, \\ R_\lambda^* &= -\frac{(115+24\eta)}{4(10+\eta)}, \quad R_k^* = \frac{95+22\eta}{4(10+\eta)}. \end{aligned} \tag{2.66}$$

Since R_λ^* is negative, this is not an acceptable fixed point solution. Thus, there is no infra-red fixed point solution for NMSSM with baryon number violation where all the trilinear superpotential couplings attain non-zero fixed point values.

We next try to find a fixed point solution with $R_b^* = 0$, with R''^*, R_t^*, R_λ^* and R_k^* being given by their non-zero solutions, which is relevant for low values of the parameter $\tan\beta$. For this purpose we reorder the couplings as (R_b^*, R''^*, R_t^*, R_λ^*, R_k^*) and consider the appropriate 4×4 submatrix of the matrix S^B in Eq.(2.39) to obtain the fixed point solutions for R''^*, R_t^*, R_λ^*, R_k^* in this case. This sub-matrix is

$$S^{B1} = \begin{pmatrix} 6 & 2 & 0 & 0 \\ 2 & 6 & 1 & 0 \\ 0 & 3 & 4 & 2 \\ 0 & 0 & 6 & 6 \end{pmatrix}, \tag{2.67}$$

with the inverse

$$(S^{B1})^{-1} = \begin{bmatrix} \frac{9}{46} & -\frac{2}{23} & \frac{1}{23} & -\frac{1}{69} \\ -\frac{2}{23} & \frac{6}{23} & -\frac{3}{23} & \frac{1}{23} \\ \frac{3}{23} & -\frac{9}{23} & \frac{16}{23} & -\frac{16}{69} \\ -\frac{3}{23} & \frac{9}{23} & -\frac{16}{23} & \frac{55}{138} \end{bmatrix}, \tag{2.68}$$

resulting in the fixed point solution

$$\begin{aligned} R_b^* &= 0, \\ R''^* &= \frac{95}{138}, \quad R_t^* = \frac{10}{23}, \\ R_\lambda^* &= -\frac{38}{23}, \quad R_k^* = \frac{53}{46}. \end{aligned} \tag{2.69}$$

This is not a theoretically acceptable fixed point solution, as all the fixed point values are not positive.

We now try to find an infra-red fixed point solution with the trilinear coupling $R_\lambda^* = 0$. Proceeding in the same manner as in the case of $R_b^* = 0$, we get the fixed point solution

$$\begin{aligned} R_\lambda^* &= 0, \quad R_k^* = -\frac{1}{2}, \\ R''^* &= \frac{385 + 76\eta}{12(85 + 16\eta)}, \quad R_b^* = \frac{44}{3(85 + 16\eta)}, \quad R_t^* = \frac{10 + 2\eta}{85 + 16\eta}. \end{aligned} \tag{2.70}$$

Since $R_k^* < 0$, this fixed point must be rejected. We can also try to find a fixed point solution with $R_k^* = 0$ with the result

$$\begin{aligned} R_k^* &= 0, \quad R_\lambda^* = -\frac{435 + 96\eta}{30(25 + 4\eta)}, \\ R''^* &= \frac{455 + 83\eta}{30(25 + 4\eta)}, \quad R_b^* = \frac{37}{6(25 + 4\eta)}, \quad R_t^* = \frac{51(5 + \eta)}{30(25 + 4\eta)}. \end{aligned} \tag{2.71}$$

which is not an acceptable fixed point solution either. Finally, we try a fixed point solution with baryon number, and R-parity, conservation, i.e. $R''^* = 0$, and all other Yukawa couplings attaining non-zero fixed point values. This is the case of non-minimal supersymmetric standard model with R-parity conservation. In this case we find the solution

$$\begin{aligned} R''^* &= 0, \\ R_k^* &= \frac{90 + 19\eta}{6(10 + \eta)}, \quad R_\lambda^* = -\frac{105 + 23\eta}{3(10 + \eta)}, \\ R_b^* &= \frac{5}{3(10 + \eta)}, \quad R_t^* = \frac{5(5 + \eta)}{3(10 + \eta)}. \end{aligned} \tag{2.72}$$

which must also be rejected. We have, thus, shown that there is no infra-red fixed point solution with one of the trilinear couplings being zero, with all others attaining a non-zero fixed point value.

Having failed to find an acceptable fixed point solution with one of the couplings attaining a value zero in the infra-red region, we now try to find a solution with two of the trilinear couplings attaining a zero fixed point value. If we take trivial fixed point values for $R''^* = R_b^* = 0$, then the matrix S relevant for the case is

$$S^{B2} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & 6 & 6 \end{bmatrix}, \tag{2.73}$$

with the inverse

$$(S^{B2})^{-1} = \begin{bmatrix} \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{6} \\ 0 & -\frac{1}{2} & \frac{1}{3} \end{bmatrix}, \tag{2.74}$$

resulting in the fixed point solution

$$\begin{aligned} R''^* &= R_b^* = 0, \\ (R_t^*,\ R_\lambda^*,\ R_k^*) &= (\frac{20}{27},\ -\frac{19}{9},\ \frac{29}{18}), \end{aligned} \tag{2.75}$$

which is not a physically acceptable solution. Similarly taking $R_b^* = R_\lambda^* = 0$, we find the fixed point solution

$$\begin{aligned} R_b^* &= R_\lambda^* = 0, \\ (R''^*,\ R_t^*,\ R_k^*) &= (\frac{19}{24},\ \frac{1}{8},\ -\frac{1}{2}), \end{aligned} \tag{2.76}$$

which must also be rejected as being unphysical. Continuing in this manner, we have found only the following physically acceptable fixed point solution with two of the couplings attaining a zero fixed point value:

$$\begin{aligned} R_\lambda^* &= R_k^* = 0, \\ (R''^*,\ R_b^*,\ R_t^*) &= \left(\frac{385+76\eta}{3(170+32\eta)},\ \frac{10}{85+16\eta},\ \frac{10+2\eta}{85+16\eta}\right). \end{aligned} \tag{2.77}$$

We note that the fixed point values for the couplings R''^*, R_b^* and R_t^* are same as in the minimal supersymmetric standard model with third generation baryon number violation [29].

We can also try to find fixed point solutions where three of the trilinear couplings attain zero fixed point values, whereas the remaining two attain non-trivial fixed point values. In this case we find the following theoretically acceptable fixed point solutions:

$$R_\lambda^* = R_k^* = R_b^* = 0, \quad (R''^*,\ R_t^*) = \left(\frac{19}{24},\ \frac{1}{8}\right), \tag{2.78}$$

$$R_\lambda^* = R_k^* = R''^* = 0, \quad (R_b^*,\ R_t^*) = \left(\frac{35}{3(35+6\eta)},\ \frac{35+7\eta}{3(35+6\eta)}\right), \tag{2.79}$$

$$R_\lambda^* = R_k^* = R_t^* = 0, \quad (R_b^*,\ R''^*) = \left(\frac{2}{16+3\eta},\ \frac{76+15\eta}{6(16+3\eta)}\right). \tag{2.80}$$

All the infra-red fixed points (2.77) - (2.80) have one thing in common, namely the trilinear couplings λ and k approach the fixed point value zero in the infra-red region. Furthermore, other trilinear couplings approach the same infra-red fixed point values as in MSSM [29].

Having obtained more than one theoretically acceptable infra-red fixed points in the NMSSM with baryon number violation, it is important to determine which, if any, is likely to be realized in nature. To this end we examine the stability [15] of each of the fixed point solutions (2.77) - (2.80).

We first consider the stability of the fixed point solution (2.77). Since in this case the fixed points of the couplings $R_\lambda^* = R_k^* = 0$, we have to obtain the behaviour of these couplings around the origin. This behaviour is determined by the eigenvalues

$$\lambda_i = \frac{1}{b_3}\left[\sum_{j=3}^{5} \tilde{S}_{ij}^B R_j^* - (r_i + b_3)\right], \quad i = 1,2, \tag{2.81}$$

of Eq.(2.62). Here $r_1 = r_2 = 0$, since the fields entering the trilinear superpotential couplings λ and k are invariant under $SU(3)_C$, with the fixed points $R_j^*,\ j = 3,4,5$ corresponding to fixed point values of R''^*, R_b^*, and R_t^* in Eq.(2.77). The matrix $\tilde{S}^B$ in (2.81) is matrix S^B of (2.39) but now reordered according to $R_i =$

$(R_\lambda,\ R_k,\ R'',\ R_b,\ R_t)$, and can be written as

$$\tilde{S}^B = \begin{bmatrix} 4 & 2 & 0 & 3+\eta & 3 \\ 6 & 6 & 0 & 0 & 0 \\ 0 & 0 & 6 & 2 & 2 \\ 1 & 0 & 2 & 6+\eta & 1 \\ 1 & 0 & 2 & 1 & 6 \end{bmatrix} . \tag{2.82}$$

Inserting these values in Eq.(2.81), we find

$$[\lambda_i]_{i=1,2} = \left[-\frac{2517 + 494\eta}{9(85 + 16\eta)}, -1 \right] , \tag{2.83}$$

thereby indicating that the fixed point is attractive in the infra-red direction. The behaviour of couplings R'', R_b and R_t around their respective fixed points is governed by the sign of the eigenvalues of the matrix A (i not summed over)

$$A_{ij} = \frac{1}{b_3} R_i^* \tilde{S}_{ij}^{B1} , \tag{2.84}$$

given in Eqs.(2.55) and (2.64), with $\tilde{S}_{ij}^{B1}$ being the lower right corner 3×3 submatrix of the matrix (2.82). For stability we require all the eigenvalues of the matrix (2.84) to have negative real parts (note that the QCD β-function $b_3 = -3$ is negative). The eigenvalues of the matrix (2.84) are calculated to be

$$[\lambda_k]_{k=3,4,5} = [-1.6, -0.2, -0.2] \tag{2.85}$$

which shows that the fixed point (2.77) is an infra-red stable fixed point. We note that the eigenvalue λ_3 is greater in magnitude as compared to other eigenvalues in (2.85), indicating that the non-trivial fixed point for λ''_{233} is more attractive, and hence more relevant.

Next we consider the stability of the fixed point solution (2.78). Since in this case the fixed point of the couplings $R_\lambda^* = R_k^* = R_b^* = 0$, we have to obtain the behaviour of these couplings around the origin. This behaviour is determined by the eigenvalues (see Eq.(2.62))

$$\lambda_i = \frac{1}{b_3} \left[\sum_{j=4}^{5} \tilde{S}_{ij}^B R_j^* - (r_i + b_3) \right] , \quad i = 1, 2, 3, \tag{2.86}$$

where the matrix $\tilde{\tilde{S}}^B$ is obtained by appropriately reordering the matrix (2.39), with the ordering of the ratio $R_i = (R_\lambda,\ R_k,\ R_b,\ R'',\ R_t)$:

$$\tilde{\tilde{S}}^B = \begin{bmatrix} 4 & 2 & 3+\eta & 0 & 3 \\ 6 & 6 & 0 & 0 & 0 \\ 1 & 0 & 6+\eta & 2 & 1 \\ 0 & 0 & 2 & 6 & 2 \\ 1 & 0 & 1 & 2 & 6 \end{bmatrix}, \tag{2.87}$$

and $r_1 = r_2 = 0$, $r_3 = 2(C_Q + C_{\bar{D}}) = 16/3$, with the fixed points R_j^*, $j = 4,5$ corresponding to the fixed point values of R''^* and R_t^* in Eq.(2.78). Inserting these values in Eq.(2.86), we find

$$[\lambda_i]_{i=1,2,3} = \left[-\frac{9}{8}, -1, \frac{5}{24}\right], \tag{2.88}$$

showing thereby that the fixed point (2.78) is unstable in the infra-red region. Similarly, it can be shown [15] that the fixed point solutions (2.79) and (2.80) are unstable fixed points.

One may also consider the case where the couplings $\lambda''_{233}, h_b, \lambda$ and k attain trivial fixed point values, whereas h_t attains a non-trivial fixed point value. In this case we get [15]

$$\begin{aligned} R''^* &= R_b^* = R_\lambda^* = R_k^* = 0 \\ R_t^* &= 7/18 \end{aligned} \tag{2.89}$$

which is the same as the Pendleton-Ross [27] top-quark fixed point of the MSSM. We must, of course, study the stability of this solution in the present context. To do so, we must consider the eigenvalues

$$\lambda_i = \frac{1}{b_3}(S_{i5}R_5^* - (r_i + b_3)), \quad i = 1,2,3,4, \tag{2.90}$$

where $R_5^* \equiv R_t^* = 7/18$, and S_{i5} are read from the matrix (2.39), but now reordered according to $R_i = (R'',\ R_b,\ R_\lambda,\ R_k,\ R_t)$, thereby yielding

$$[\lambda_i]_{i=1,2,3,4} = \left[\frac{38}{27}, \frac{35}{54}, -\frac{25}{18}, -1\right]. \tag{2.91}$$

Since the signs of each of λ_1 and λ_2 is positive, this solution is also unstable in the infra-red region. Nevertheless, from our discussion of the infra-red fixed point solution (2.77), it is clear that the Pendleton-Ross fixed point would be stable in the NMSSM in case h_b and λ''_{233} are small, though negligible at the GUT scale (with $\lambda = k = 0$). In this case, these would, of course, evolve away from zero at the weak scale, though realistically they would still be small (but not zero) at the weak scale. Thus, the only true infra-red stable fixed point solution is the baryon number, and R_p, violating solution (2.77). We note that this fixed point solution is identical to the corresponding fixed point solution in the MSSM with baryon number violation [29]. This is one of the main conclusions of this chapter. We note that the value of R_t^* in (2.77) is lower than the corresponding value [27] of 7/18 in MSSM and NMSSM with R_p conservation.

It is now appropriate to examine the implications [15] of the value of $h_t(m_t)$ predicted by our fixed point analysis for the top-quark mass. From (2.77), and $\alpha_3(m_t) \simeq 0.1$, the fixed point value for the top-quark Yukawa coupling is predicted to be $h_t(m_t) \simeq 0.4$, and for the baryon number violating Yukawa coupling to be $\lambda''_{233} \simeq 0.98$. This translates into a top-quark (pole) mass of about $m_t \simeq 70 \sin\beta$ GeV, which is incompatible with the measured value [30] of the top quark mass, $m_t^{pole} \simeq 174$ GeV, for any value of $\tan\beta$. It follows the true fixed point obtained here provides only a qualitative understanding of the top quark mass in NMSSM with R_p violation.

2.5 Infra-red fixed points with Lepton number violation

We now turn to the study of the renormalization group equations for the lepton number [15], and R_p, violating couplings in the superpotential (2.10). Here we shall consider the dimensionless couplings λ_3, λ_{233} and λ'_{333} only. Furthermore, we shall restrict our attention to one kind of lepton number violation at a time. Thus, we shall consider three different cases, i.e. we shall take $\lambda'_{333} \gg \lambda_3$, λ_{233}, or $\lambda_{233} \gg \lambda_3$, λ'_{333}, or $\lambda_3 \gg \lambda_{233}$, λ'_{333}, respectively.

2.5.1 Infra-red fixed points with λ'_{333}

In the situation when λ'_{333} is the dominant of the lepton number violating couplings, we define $R' = \lambda'^2_{333}/g_3^2$ (see Eq.(2.29)) and reorder the couplings as $R_i = (R', R_b, R_t, R_\lambda, R_k)$, so that the RGEs for this case can be written as (see Eq.(2.41))

$$\frac{dR_i}{dt} = \tilde{\alpha}_3 R_i \left[(r_i + b_3) - \sum_j S^L_{ij} R_j \right], \tag{2.92}$$

where the anomalous dimension matrix S^L is given by (2.42), $r_i = (\frac{16}{3}, \frac{16}{3}, \frac{16}{3}, 0, 0)$, and $b_3 = -3$ is the QCD beta function. The non-trivial fixed points are given by (2.49), and to obtain these we need the inverse of the matrix S^L in (2.42). This is calculated to be

$$(S^L)^{-1} = \begin{bmatrix} -\frac{12(10+\eta)}{315+114\eta} & \frac{165+30\eta}{315+114\eta} & \frac{45+6\eta}{315+114\eta} & -\frac{35+6\eta}{105+38\eta} & \frac{35+6\eta}{315+114\eta} \\ \frac{55}{105+38\eta} & -\frac{52}{105+38\eta} & -\frac{18}{105+38\eta} & \frac{35}{105+38\eta} & -\frac{35}{315+114\eta} \\ \frac{15+7\eta}{105+38\eta} & -\frac{18+8\eta}{105+38\eta} & \frac{18+6\eta}{105+38\eta} & \frac{\eta}{105+38\eta} & -\frac{\eta}{315+114\eta} \\ -1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & \frac{1}{6} \end{bmatrix}. \tag{2.93}$$

Using (2.93) in Eq.(2.49), we obtain the non-trivial fixed point solution:

$$\begin{aligned} R'^* &= \frac{420 + 92\eta}{315 + 114\eta}, \\ R_b^* &= -\frac{315}{315 + 114\eta}, \quad R_t^* = \frac{105 + 29\eta}{315 + 114\eta}, \\ R_\lambda^* &= 0, \quad R_k^* = -2. \end{aligned} \tag{2.94}$$

Note that $R_\lambda^* = 0$ in this case. Since R_b^* and R_k^* are negative, this is not an acceptable fixed point solution. We, thus, conclude that a simultaneous non-trivial fixed point for all the couplings λ'_{333}, h_b, h_t, λ and k does not exist.

We next try to find a fixed point solution with $R_b^* = 0$, with R'^*, R_t^*, R_λ^* and R_k^* being given by their non-zero solutions, which is relevant for low values of $\tan\beta$. For this purpose, we appropriately reorder the couplings as $(R_b^*, R'^*, R_t^*, R_\lambda^*, R_k^*)$ and consider the appropriate 4×4 submatrix of the matrix S^L in (2.42) to obtain the fixed point solution for R'^*, R_t^*, R_λ^* and R_k^* in this case. This sub-matrix is

$$S^{L1} = \begin{bmatrix} 6 & 1 & 0 & 0 \\ 1 & 6 & 1 & 0 \\ 0 & 3 & 4 & 2 \\ 0 & 0 & 6 & 6 \end{bmatrix}, \tag{2.95}$$

with the inverse given by

$$(S^{L1})^{-1} = \begin{bmatrix} \frac{9}{52} & -\frac{1}{26} & \frac{1}{52} & -\frac{1}{156} \\ -\frac{1}{26} & \frac{3}{13} & -\frac{3}{26} & \frac{1}{26} \\ \frac{3}{52} & -\frac{9}{26} & \frac{35}{52} & -\frac{35}{156} \\ -\frac{3}{52} & \frac{9}{26} & -\frac{35}{52} & \frac{61}{156} \end{bmatrix}, \tag{2.96}$$

leading to the fixed point solution

$$\begin{aligned} R_b^* &= 0, \;\; R'^* = \frac{10}{39}, \;\; R_t^* = \frac{53}{78}, \\ R_\lambda^* &= -\frac{305}{156}, \;\; R_k^* = \frac{237}{156}, \end{aligned} \tag{2.97}$$

which contains a non-physical negative value, and hence is not an acceptable fixed point solution. Similarly, we have checked that there is no acceptable fixed point solution with any one of the couplings R', or R_k, or R_λ, or R_t attaining trivial fixed point value, with rest of the couplings approaching non-trivial fixed point values.

Having failed to find an acceptable fixed point solution with one of the couplings attaining a value zero in the infra-red region, we now try to find a solution with two of the trilinear couplings attaining a zero fixed point value. If we take trivial fixed point values for $R_\lambda^* = R_k^* = 0$, then the matrix S relevant for this case is

$$S^{L2} = \begin{bmatrix} 6 & 6+\eta & 1 \\ 6 & 6+\eta & 1 \\ 1 & 1 & 6 \end{bmatrix}, \tag{2.98}$$

which is singular. Hence, there are no fixed points in this case. Proceeding in this manner, we find that there are no acceptable fixed points with any of the two couplings approaching a trivial fixed point, with the rest attaining non-trivial fixed point solution.

We next try to obtain a fixed point solution with three of the couplings attaining a trivial fixed point value. If we take the trivial fixed point value for $R_\lambda^* = R_k^* = R_b^* = 0$, then the matrix relevant for the case is

$$S^{L3} = \begin{pmatrix} 6 & 1 \\ 1 & 6 \end{pmatrix}, \tag{2.99}$$

with the inverse

$$(S^{L3})^{-1} = \begin{pmatrix} \frac{6}{35} & -\frac{1}{35} \\ -\frac{1}{35} & \frac{6}{35} \end{pmatrix}, \tag{2.100}$$

leading to the fixed point solution

$$\begin{aligned} R_b^* &= R_\lambda^* = R_k^* = 0, \\ R'^* &= R_t^* = 1/3, \end{aligned} \tag{2.101}$$

which is an acceptable infra-red fixed point. We must check the stability of the solution (2.101). For this purpose we reorder the couplings as $R_i = (R_b,\ R_\lambda,\ R_k,\ R',\ R_t)$, so that the relevant matrix that enters the RG evolution is

$$\tilde{S}^L = \begin{bmatrix} 6+\eta & 1 & 0 & 6 & 1 \\ 3+\eta & 4 & 2 & 0 & 3 \\ 0 & 3 & 3 & 0 & 0 \\ 6+\eta & 0 & 0 & 6 & 1 \\ 1 & 1 & 0 & 1 & 6 \end{bmatrix}, \tag{2.102}$$

with the behaviour of the couplings R_b^*, R_λ^* and R_k^* around the origin determined by the sign of the eigenvalues

$$\lambda_i = \frac{1}{b_3}\left[\sum_{j=4}^{5} \tilde{S}_{ij}^L R_j^* - (r_i + b_3)\right], \quad i = 1,2,3, \tag{2.103}$$

where $r_1 = \frac{16}{3}$, $r_2 = r_3 = 0$, and the fixed points R_j^*, $j = 4,5$ corresponding to fixed point values of R'^* and R_t^* in (2.101). Inserting these values in Eq.(2.103), we find

$$[\lambda_i]_{i=1,2,3} = \left[0, -\frac{9}{3}, -1\right], \tag{2.104}$$

from which we conclude that the fixed point (2.101) will never be reached in the infra-red region. This fixed point is either a saddle point or an ultra-violet fixed point. Proceeding in the same manner, we find that there are no acceptable infra-red fixed point solutions with four of the couplings attaining trivial fixed point values, with the remaining coupling approaching a non-trivial fixed point. In particular, trivial fixed points for λ'_{333}, h_b, λ and k, and Pendleton-Ross type fixed point for top-quark Yukawa coupling is unstable in the infra-red region. We, thus, conclude that there are no non-trivial stable fixed points [15] in the infra-red region for the non-minimal supersymmetric standard model with lepton number, and R-parity, violation governed by the coupling λ'_{333}.

2.5.2 Infra-red fixed points with λ_{233}

If on the other hand λ_{233} is the dominant of the lepton number couplings, we define the ratio $R = \lambda_{233}^2/g_3^2$ (see Eq.(2.29)), and reorder the Yukawa couplings as $R_i = (R,\ R_b,\ R_t,\ R_\lambda,\ R_k)$, so that the relevant RGEs can be written as

$$\frac{dR_i}{dt} = \tilde{\alpha}_3 R_i\left[(r_i + b_3) - \sum_j S_{ij}^{L'} R_j\right], \quad i = 1,2,3,4,5, \tag{2.105}$$

where the anomalous dimension matrix is given by (2.44), $r_i = (0,\ \frac{16}{3},\ \frac{16}{3},\ 0,\ 0)$ and $b_3 = -3$ is the QCD beta function. The non-trivial fixed points are given by (2.49), and to obtain these we need the inverse of the matrix $S^{L'}$ in (2.44). This inverse is given by

$$(S^{L'})^{-1} = \begin{bmatrix} \frac{1}{4} & -\frac{27\eta}{12(10+\eta)} & -\frac{\eta}{4(10+\eta)} & \frac{5\eta}{4(10+\eta)} & -\frac{5\eta}{12(10+\eta)} \\ 0 & \frac{27}{12(10+\eta)} & \frac{1}{4(10+\eta)} & -\frac{5}{4(10+\eta)} & \frac{5}{12(10+\eta)} \\ 0 & \frac{1+\eta}{4(10+\eta)} & \frac{9+\eta}{4(10+\eta)} & -\frac{5+\eta}{4(10+\eta)} & \frac{5+\eta}{12(10+\eta)} \\ 0 & -\frac{15+6\eta}{4(10+\eta)} & -\frac{15+2\eta}{4(10+\eta)} & \frac{35+6\eta}{4(10+\eta)} & -\frac{35+6\eta}{12(10+\eta)} \\ 0 & \frac{15+6\eta}{4(10+\eta)} & \frac{15+2\eta}{4(10+\eta)} & -\frac{35+6\eta}{4(10+\eta)} & \frac{55+8\eta}{12(10+\eta)} \end{bmatrix}. \tag{2.106}$$

Using (2.106) in (2.49), we obtain the following non-trivial fixed point solution for the Yukawa couplings:

$$R^* = -\frac{(90 + 109\eta)}{12(10+\eta)},$$

$$
\begin{aligned}
R_b^* &= \frac{25}{3(10+\eta)}, \quad R_t^* = \frac{5(5+\eta)}{3(10+\eta)}, \\
R_\lambda^* &= -\frac{(105+23\eta)}{3(10+\eta)}, \quad R_k^* = \frac{(30+43\eta)}{6(10+\eta)}.
\end{aligned} \tag{2.107}
$$

Since R^* and R_λ^* are negative, this is not a physically acceptable fixed point solution. We conclude that a simultaneous non-trivial fixed point for all the couplings λ_{233}, h_b, h_t, λ and k does not exist.

We now try to find a fixed point solution with $R_b^* = 0$, and other couplings being given by their non-zero fixed point values, a case which is relevant for low values of $\tan\beta$. We reorder the couplings as $R_i = (R_b,\ R,\ R_t,\ R_\lambda,\ R_k)$, and consider the appropriate 4×4 submatrix of the matrix $S^{L'}$ in (2.44) to obtain the fixed point solution. This sub-matrix is

$$
S^{L'1} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 6 & 1 & 0 \\ 0 & 3 & 4 & 2 \\ 0 & 0 & 6 & 6 \end{bmatrix}, \tag{2.108}
$$

with the inverse calculated to be

$$
(S^{L'1})^{-1} = \begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{2}{9} & -\frac{1}{9} & \frac{1}{27} \\ 0 & -\frac{1}{3} & \frac{2}{3} & -\frac{2}{9} \\ 0 & \frac{1}{3} & -\frac{2}{3} & \frac{7}{18} \end{bmatrix}, \tag{2.109}
$$

leading to the fixed point solution

$$
\begin{aligned}
R_b^* &= 0, \quad R^* = -\frac{3}{4}, \quad R_t^* = \frac{20}{27}, \\
R_\lambda^* &= -\frac{19}{9}, \quad R_k^* = \frac{29}{18}.
\end{aligned} \tag{2.110}
$$

Since R^* and R_λ^* are negative, this fixed point solution must be rejected. In a similar manner we find that there is no physically acceptable fixed point solution with any one of the couplings R, R_k, R_λ, R_t attaining trivial fixed point value with rest of the couplings approaching non-trivial fixed point values.

We next try to find a infra-red fixed point solution with two of the Yukawa couplings approaching a trivial fixed point value. If we take the trivial fixed point values for $R_\lambda^* = R_k^* = 0$, then the matrix relevant for the case is

$$S^{L'2} = \begin{pmatrix} 4 & 4\eta & 0 \\ 0 & 6+\eta & 1 \\ 0 & 1 & 6 \end{pmatrix}, \tag{2.111}$$

with the inverse given by

$$(S^{L'2})^{-1} = \begin{pmatrix} \frac{1}{4} & -\frac{6\eta}{35+6\eta} & \frac{\eta}{35+6\eta} \\ 0 & \frac{6}{35+6\eta} & -\frac{1}{35+6\eta} \\ 0 & -\frac{1}{35+6\eta} & \frac{24+4\eta}{4(35+6\eta)} \end{pmatrix}, \tag{2.112}$$

leading to the fixed point values

$$\begin{aligned} R_\lambda^* &= R_k^* = 0, \\ R^* &= -\frac{(315+194\eta)}{12(35+6\eta)}, \quad R_b^* = \frac{35}{3(35+6\eta)}, \quad R_t^* = \frac{7(5+\eta)}{3(35+6\eta)}, \end{aligned} \tag{2.113}$$

which must be rejected as being unphysical. Continuing in this manner, we find there are no acceptable fixed point solutions with any two of the couplings approaching a non-trivial fixed point, with the rest attaining a non-trivial fixed point solution.

We now try to obtain a fixed point solution with three of the couplings attaining a trivial fixed point value. If we take $R_\lambda^* = R_k^* = R_b^* = 0$, then the relevant matrix that we have to consider is

$$S^{L'3} = \begin{pmatrix} 4 & 0 \\ 0 & 6 \end{pmatrix}, \tag{2.114}$$

with the inverse

$$(S^{L'3})^{-1} = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{6} \end{pmatrix}, \tag{2.115}$$

leading to the fixed point solution

$$\begin{aligned} R_b^* &= R_\lambda^* = R_k^* = 0, \\ R^* &= -3/4, \quad R_t^* = 7/16, \end{aligned} \tag{2.116}$$

which is not an acceptable solution. Furthermore, there are no acceptable fixed point solutions with any three of the couplings attaining trivial fixed point values, and the remaining two approaching a non-trivial fixed point.

We have also checked that trivial fixed points for $\lambda_{233}, h_b, \lambda$ and k and the Pendleton-Ross type fixed point for the top-quark Yukawa coupling is unstable in the infra-red region. We conclude that there are no acceptable fixed point solutions [15] for the lepton number, and R_p, violating coupling λ_{233}.

2.5.3 Infra-red fixed points with λ_3

Finally, if λ_3 is the dominant of the lepton number violating couplings [15], we define $R_3 = \lambda_3^2/g_3^2$ (see Eq.(2.29)) and reorder the relevant Yukawa couplings as $R_i = (R_3,\ R_b,\ R_t,\ R_\lambda,\ R_k)$, so that the RGEs for this case can be written as

$$\frac{dR_i}{dt} = \tilde{\alpha}_3 R_i \left[(r_i + b_3) - \sum_j S_{ij}^{L''} R_j\right], \quad i = 1,2,3,4,5, \tag{2.117}$$

where the anomalous dimension matrix $S^{L''}$ is given by (2.46), $r_i = (0,\ \frac{16}{3},\ \frac{16}{3},\ 0,\ 0)$ with $b_3 = -3$ the QCD beta function. The non-trivial fixed points are calculated from (2.49), with

$$(S^{L''})^{-1} = \begin{bmatrix} -\frac{40+4\eta}{27} & -1 & -\frac{1}{9} & \frac{55+4\eta}{27} & -\frac{5}{27} \\ -\frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ -\frac{1+\eta}{27} & 0 & \frac{6}{27} & -\frac{2-\eta}{27} & \frac{1}{27} \\ \frac{55+10\eta}{27} & 1 & -\frac{6}{27} & -\frac{52+10\eta}{27} & -\frac{1}{27} \\ -\frac{5+2\eta}{9} & 0 & \frac{1}{3} & -\frac{1-2\eta}{9} & \frac{7}{18} \end{bmatrix}. \tag{2.118}$$

Using (2.118) in Eq.(2.49), we obtain the non-trivial fixed point solution

$$\begin{aligned} R_3^* &= -\frac{100}{27}, \\ R_b^* &= 0, \quad R_t^* = \frac{20}{27}, \\ R_\lambda^* &= \frac{43}{27}, \quad R_k^* = \frac{29}{18}, \end{aligned} \tag{2.119}$$

which is not an acceptable solution. Note that $R_b^* = 0$ in this case. This is different from the situation that arose in other cases discussed previously. Thus, in this case also we don't have a physically acceptable non-trivial fixed point solution for all the Yukawa couplings.

We next try a fixed point solution with $R_b^* = 0$, which is relevant for low values of $\tan\beta$. The relevant matrix for finding the fixed points in this case is

$$S^{L''1} = \begin{pmatrix} 4 & 3 & 4 & 2 \\ 1 & 6 & 1 & 0 \\ 1 & 3 & 4 & 2 \\ 6 & 0 & 6 & 6 \end{pmatrix}, \tag{2.120}$$

with the inverse

$$(S^{L''1})^{-1} = \begin{pmatrix} \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ -\frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{27} \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{2}{9} \\ -\frac{2}{3} & \frac{1}{3} & 0 & \frac{7}{18} \end{pmatrix}, \tag{2.121}$$

leading to the fixed point solution

$$\begin{aligned} R_b^* &= 0, \quad R_3^* = 0, \\ R_t^* &= \frac{20}{27}, \quad R_\lambda^* = -\frac{19}{9}, \quad R_k^* = \frac{29}{18}, \end{aligned} \tag{2.122}$$

which is unacceptable. We note that in this case $R_b^* = R_3^* = 0$. We have also tried to obtain a fixed point solution with $R_\lambda^* = R_k^* = 0$, with other couplings having non-zero fixed point values. We find the infra-red fixed point solution

$$\begin{aligned} R_\lambda^* &= R_k^* = 0, \\ R_3^* &= -\frac{(210+55\eta)}{3(61+11\eta)}, \quad R_b^* = \frac{55}{3(61+11\eta)}, \quad R_t^* = \frac{97+22\eta}{3(61+11\eta)}, \end{aligned} \tag{2.123}$$

which must be rejected as unphysical. Similarly, we have checked that there are no physically acceptable fixed point solution with any two couplings having a zero fixed point value, with the rest having a non-trivial fixed point. Also there are no fixed point solutions with any of the three couplings attaining a trivial fixed point with the remaining two approaching a non-trivial fixed point value.

Finally, we try to obtain a fixed point with $R_3^* = R_b^* = R_\lambda^* = R_k^* = 0$, and R_t^* approaching a non-zero value. We find in this case

$$\begin{aligned} R_3^* &= R_b^* = R_\lambda^* = R_k^* = 0, \\ R_t^* &= \frac{7}{18}, \end{aligned} \tag{2.124}$$

which is the Pendleton-Ross type fixed point for this case of lepton number violation. We must check the stability of this fixed point to see whether this is an acceptable fixed point. To do so we must solve for the eigenvalues in Eq.(2.62) for this case. We obtain

$$\lambda_i = \frac{1}{b_3}[S_{i5}R_5^* - (r_i + b_3)], \quad i = 1,2,3,4, \tag{2.125}$$

with $R_5^* = R_t^* = 7/18$, and S_{i5} are read from the matrix (2.46) but now reordered according to $R_i = (R_3,\ R_b,\ R_\lambda,\ R_k,\ R_t)$. We get

$$[\lambda_i]_{i=1,2,3,4} = \left(-\frac{25}{18},\ \frac{35}{54},\ -\frac{25}{18},\ -\frac{7}{9}\right), \tag{2.126}$$

making the fixed point (2.124) an unstable fixed point. We conclude that in this case of lepton number violation also there are no acceptable infra-red fixed points.

Thus, there are no non-trivial, infra-red stable fixed points for any of the lepton number violating couplings in the non-minimal supersymmetric standard model [15].

2.6 Conclusions

We have investigated in detail the renormalization group evolution and infra-red fixed point structure of the non-minimal supersymmetric standard model with baryon and lepton number violation (and R-parity violation). We have considered the B and L violating couplings of the highest generation, taking into account only one of these couplings at a time. The analysis of the model yields the surprising and important result [15] that only the simultaneous non-trivial fixed point for the baryon number violating coupling λ''_{233} and the top-quark and b-quark Yukawa couplings h_t and h_b, and the trivial fixed point for λ and k, is stable in the infra-red region. However, the fixed point value for the top quark coupling here is lower than the corresponding value of 7/18 in the MSSM, and NMSSM, with R_p conservation, and is incompatible with the measured value of the top-quark mass. Thus, it appears that the baryon number, and R_p, violating coupling has the effect of reducing the infra-red fixed point value of the top quark Yukawa coupling to the same extent in MSSM [29] and NMSSM [15]. The R_p conserving solution with λ''_{233} attaining its trivial fixed point, with h_t and h_b attaining non-trivial fixed points, is infra-red unstable, as is the case for trivial fixed points for λ''_{233} and h_b, with a nontrivial fixed point for h_t (with $\lambda = k = 0$). Our analysis shows that the usual Pendleton-Ross type of infra-red fixed point for h_t is unstable in the presence of R_p violation, though for small, but negligible, values of h_b and λ''_{233} it could be stable. We have also found that there are no non-trivial infra-red stable fixed points for the lepton number, and R-parity, violating couplings in the NMSSM; the solutions we have found are either unphysical or unstable. Our results have placed strong theoretical constraints on the nature of R_p violating couplings in the NMSSM from fixed point and stability considerations: the fixed points that are unstable, or the fixed point that is a saddle point, cannot be realized in the infra-red region. The fixed points that we have obtained are true fixed points, and serve as a lower bound on the relevant R_p violating Yukawa couplings. Their structure is essentially the same as in MSSM with R_p violation [29]. In particular, from our analysis [15] of the simultaneous (stable) fixed point for the baryon number violating coupling λ''_{233} and the top and bottom Yukawa couplings we infer a lower bound on $\lambda''_{233} \gtrsim 1.0$.

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Chapter 3

The Left-Right Supersymmetric Model and the Mass of the Lightest Higgs Boson

The Higgs potential of the Standard Model (SM), which is crucial in implenting the mechanism of spontaneous symmetry breaking, contains the unknown quartic coupling of the Higgs field. As a consequence, the mass of the only Higgs boson in the SM, which is determined by this quartic coupling, is not known [1]. Furthermore, the single aspect of the SM which has not been verified experimentally is the Higgs sector. On the other hand, supersymmetry is at present the only known framework in which the Higgs sector of the SM, so crucial for its internal consistency, is natural [2]. The minimal version of the supersymmetric standard model (MSSM) contains two Higgs doublets (H_1, H_2) with opposite hypercharges: $Y(H_1) = -1/2$, $Y(H_2) = +1/2$, so as to generate masses for the up- and down-type quarks (and leptons), and to cancel gauge anomalies. After spontaneous symmetry breaking induced by the neutral components of H_1 and H_2 obtaining vacuum expectation values (VEVs), $\langle H_1 \rangle = v_1$, $\langle H_2 \rangle = v_2$, $tan\beta = v_2/v_1$, the MSSM contains two neutral CP-even (h, H), one neutral CP-odd (A), and two charged $(H^{\pm})$ Higgs bosons [1]. Although gauge invariance and supersymmetry fix the quartic couplings of the Higgs bosons in the MSSM in terms of $SU(2)_L$ and $U(1)_Y$ gauge couplings, g and g', respectively, there still remain two independent parameters that describe the Higgs sector of MSSM. These are usually chosen to be $\tan\beta$ and m_A, the mass of the CP-odd Higgs boson. All the Higgs boson masses and the Higgs couplings in the MSSM can be described (at the tree level) in terms of these two parameters.

Because of underlying gauge invariance and supersymmetry (SUSY), the lightest Higgs boson in MSSM has a tree level upper bound of m_Z (the mass of the Z boson) on its mass [3]. Although radiative corrections [4] to the tree level result can be appreciable, these depend only logarithmically on the SUSY breaking scale, and are, therefore, under control. This results in an upper bound of about $125 - 150\ GeV$ on the one-loop radiatively corrected mass [5] of the lightest Higgs boson mass in MSSM. Furthermore, two-loop radiative corrections to the Higgs boson mass matrix in the MSSM are significant, and can reduce the lightest Higgs boson mass by up to $\sim 20\ GeV$ as compared to its one-loop value [6].

Because of the presence of the additional trilinear Yukawa couplings, such a tight constraint on the mass of the lightest Higgs boson need not *a priori* hold in extensions of MSSM based on the gauge group $SU(2)_L \times U(1)_Y$ with an extended Higgs sector. Nevertheless, it has been shown that the upper bound on the lightest Higgs boson mass in these models depends only on the weak scale, and dimensionless couplings (and only logarithmically on the SUSY breaking scale), and is calculable if all the couplings remain perturbative below some scale [7, 8, 9, 10, 11, 12, 13]. This upper bound can vary between $150\ GeV$ and $200\ GeV$ depending on the Higgs structure of the underlying supersymmetric model. Thus, nonobservation of such a light Higgs boson below this upper bound will rule out an entire class of supersymmetric models based on the gauge group $SU(2)_L \times U(1)_Y$.

The exixtence of the upper bound on the lightest Higgs boson mass in MSSM, and in models with arbitrary Higgs sectors, has been investigated in a situation where the underlying supersymmetric model respects baryon (B) and lepton (L) number conservation. However, gauge invariance, supersymmetry and renormalizability allow B and L violating terms in the superpotential of the MSSM [14]:

$$W' = \lambda \hat{L}\hat{L}\hat{E}^c + \lambda' \hat{L}\hat{Q}\hat{D}^c + \lambda'' \hat{U}^c\hat{D}^c\hat{D}^c + \mu_i \hat{L}^i \hat{H}_2. \tag{3.1}$$

Each of these terms violate either baryon number or lepton mumber. These couplings can mediate proton decay at tree level through the exchange of the scalar partner of the down quark.

The strength of these lepton and baryon number violating terms is, however, severely limited by phenomenological [15] and cosmological [16] constraints. Indeed, unless the strength of the baryon-number violating term is less than 10^{-13}, it will lead to contradiction with the present lower limits on the lifetime of the proton. In MSSM, the appearance of these B and L violating couplings is prevented by invoking a discrete Z_2 symmetry known as matter parity, or R-parity [17]. The matter parity of each superfield may be defined as

$$(\text{matter parity}) \equiv (-1)^{3(B-L)}. \tag{3.2}$$

The multiplicative conservation of matter parity forbids all renormalizable B and L violating terms in the superpotential of MSSM. Equivalently, the R-parity of any component *field* is defined by $R_p = (-1)^{3(B-L)+2S}$, where S is the spin of the field. Since $(-1)^{2S}$ is conserved in any Lorentz-invariant interaction, matter parity conservation and R-parity conservation are equivalent. Conservation of R-parity then immediately implies that superpartners can be produced only in pairs, and that the lightest supersymmetric particle (LSP) is absolutely stable.

Although the minimal supersymmetric standard model with R_p conservation can provide a description of nature which is consistent with all known observations, the assumption of R_p conservation appears to be *ad hoc*, since it is not required for the internal consistency of the MSSM. Furthermore, all global, symmetries, discrete or continuous, could be violated by Planck scale physics effects [18]. The problem becomes acute for low energy supersymmetric models, because B and L are no longer automatic symmetries of the Lagrangian, as they are in the Standard Model. It is, therefore, more appealing to have a supersymmetric theory where R-parity is related to a gauge symmetry, and its conservation is automatic because of the invariance of the underlying theory under this gauge symmetry. Fortunately, there is a compelling scenario which does provide for exact R-parity conservation due to a deeper principle. Indeed, R_p conservation follows automatically in certain theories with gauged $(B-L)$, as is suggested by the fact that matter parity is simply a Z_2 subgroup of $(B - L)$. It has been noted by several authors [19, 20] that if the gauge symmetry of MSSM is extended to $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$, or $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the theory becomes automatically R-parity conserving. Such a left-right supersymmetric theory (SLRM) solves the problem of explicit B and L violation of MSSM, and has received much attention recently [21, 22, 23, 24, 25, 26]. We note here that left-right symmetric theories are interesting in their own right, because, among other appealing features, they offer a simple and natural explanation for the smallness of the neutrino mass through the see-saw mechanism [27].

Such a naturally R-parity conserving theory necessarily involves the extension of the Standard Model gauge group, and since the extended gauge symmetry has to be broken, it involves "a new scale", the scale of left-right symmetry breaking, beyond the SUSY and $SU(2)_L \times U(1)_Y$ breaking scales of MSSM. It is, therefore, important to ask whether the upper bound on the lightest Higgs boson mass in naturally R-parity conserving theories depends on the scale of the breakdown of the extended gauge group. In [28] it was shown that in the supersymmetric left-right model with minimal particle content the upper bound on the mass of the lightest neutral Higgs boson depends only on the gauge couplings and those vacuum expectation values (VEVs) which break the $SU(2)_L \times U(1)_Y$ symmetry. The upper bound does not depend on vacuum expectation values which break the left-right gauge symmetry. These results were also generalized to supersymmetric left-right models with an extended Higgs sector, and those with non-renormalizable couplings [29].

In this chapter we shall study higher order radiative corrections to the lightest Higgs boson mass in the minimal version of the supersymmetric left-right model so as to arrive a precise value of the upper bound on the lightest Higgs mass in such a naturally R-parity conserving supersymmetric model. We shall also compare the results of our calculations with the corresponding upper bound on the lightest Higgs boson mass in the minimal supersymmetric standard model.

We shall start with a brief review of the Higgs sector of MSSM in order to set the notations, and, later, to allow a comparison with the results obtained for the lightest Higgs mass in the left-right supersymmetric model.

3.1 The Higgs sector of MSSM

3.1.1 Tree Level Masses of Higgs Bosons

The minimal supersymmetric standard model (MSSM) contains two complex Higgs doublet superfields $(\hat{H}_1, \hat{H}_2)$ with the following $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers:

$$H_1 \equiv \begin{pmatrix} \hat{H}_1^0 \\ \hat{H}_1^- \end{pmatrix} \sim \left(1, 2, -\frac{1}{2}\right), \quad H_2 \equiv \begin{pmatrix} \hat{H}_2^+ \\ \hat{H}_2^0 \end{pmatrix} \sim \left(1, 2, +\frac{1}{2}\right). \tag{3.3}$$

The superpotential of the MSSM is (generation indices suppressed)

$$W = h_U \hat{Q}\hat{U}^c\hat{H}_2 + h_D \hat{Q}\hat{D}^c\hat{H}_1 + h_L \hat{L}\hat{E}^c\hat{H}_1 + \mu \hat{H}_1\hat{H}_2, \tag{3.4}$$

and the soft SUSY breaking terms are given by the Lagrangian (see Chapter 1)

$$\begin{aligned} -\mathcal{L}_{\rm soft} \;=\; & m_1^2\,|H_1|^2 + m_2^2\,|H_2|^2 - B\mu\,\epsilon_{ij}(H_1^i\,H_2^j + h.c.) \\ & + \tilde{M}_Q^2(\tilde{u}_L^*\,\tilde{u}_L + \tilde{d}_L^*\,\tilde{d}_L) + \tilde{M}_u^2\tilde{u}_R^*\,\tilde{u}_R + M_d^2\tilde{d}_R^*\,\tilde{d}_R \\ & + \frac{g}{\sqrt{2}\,M_W}\epsilon_{ij}\left[\frac{M_d}{\cos\beta}A_d H_1^i\tilde{Q}^j\tilde{d}_R^* + \frac{M_u}{\sin\beta}A_u H_2^j\tilde{Q}^i\tilde{u}_R^*\right] + \cdots \end{aligned} \tag{3.5}$$

where the ellipses denote terms which are not relevant to our discussion of the Higgs sector. In Eq.(3.5) generation indices have not been shown explicitly. The spin-0 components of the Higgs superfields are denoted by H_1 and H_2, respectively, whereas the scalar partner of a fermion f is denoted by $\tilde{f}$, and M_u and M_d are the up- and down-type quark mass matrices. The scalar potential can be calculated from

$$V = V_F + V_D + V_{soft}, \tag{3.6}$$

where V_F, V_D and V_{soft} represent the contributions of F-terms, the D-terms, and the soft supersymmetry breaking terms, respectively:

$$V_F = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2, \tag{3.7}$$

$$V_D = \frac{g^2}{2}\sum_a |\phi^{i*}(T_i^{aj})\phi_j|^2, \tag{3.8}$$

$$V_{soft} = -\mathcal{L}_{\text{soft}}, \tag{3.9}$$

where ϕ_i are all the scalar fields of the theory, and T^a are the generators of the gauge group. We assume that the scalar partners of quarks and leptons do not obtain vacuum expectation values so that $SU(3)_C$ and lepton number remain unbroken. We, therfore, consider only the Higgs part of the scalar potential. The tree level potential for the Higgs bosons H_1 and H_2 can be written as [1]

$$\begin{aligned} V_0(Higgs) &= m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_3^2\epsilon_{ij}(H_1^i H_2^j + h.c.) \\ &+\frac{1}{8}g^2(H_2^\dagger\vec{\sigma}H_2 + H_1^\dagger\vec{\sigma}H_1)^2 + \frac{1}{8}g'^2(|H_2|^2 - |H_1|^2)^2, \end{aligned} \tag{3.10}$$

where σ_i are the Pauli matrices, and ϵ_{ij} is the totally antisymmetric tensor in two dimensions. We have redefined the parameters of Eq.(3.4) and Eq.(3.5) in Eq.(3.10) as

$$\begin{aligned} m_1^2 + |\mu|^2 &\rightarrow m_1^2, \\ m_2^2 + |\mu|^2 &\rightarrow m_2^2, \\ m_3^2 &= -B\mu. \end{aligned} \tag{3.11}$$

For simplicity we take m_3^2 to be real and choose a field basis such that $m_3^2 \leq 0$. We denote the VEVs of the neutral components of H_1 and H_2 by v_1 and v_2, respectively, which, without loss of generalization, are chosen to be real and non-negative. We introduce the auxiliary variables

$$v^2 \equiv v_1^2 + v_2^2, \quad \tan\beta \equiv \frac{v_2}{v_1}. \tag{3.12}$$

The minimization conditions for (3.10) can be written as

$$\sin 2\beta = -\frac{2m_3^2}{m_1^2 + m_2^2}, \quad v^2 = \frac{4}{g^2 + g'^2}\frac{m_1^2 - m_2^2\tan^2\beta}{\tan^2\beta - 1}, \tag{3.13}$$

which combined with the tree-level expression for the Z mass

$$m_Z^2 = \frac{1}{2}(g^2 + g'^2)v^2, \tag{3.14}$$

gives the masses for CP-even neutral Higgs bosons (h, H)

$$m_{h,H}^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2\cos^2 2\beta}\right], \tag{3.15}$$

where

$$m_A^2 = -\frac{2m_3^2}{\sin 2\beta}, \tag{3.16}$$

is the mass of the CP-odd Higgs boson. From Eq.(3.15) we obtain the tree level results [3]

$$\begin{aligned} m_h &< m_Z|\cos 2\beta| < m_Z < m_H, \\ m_h &< m_A|\cos 2\beta| < m_A < m_H. \end{aligned} \tag{3.17}$$

Furthermore, the mass of the charged Higgs bosons is given by

$$m_{H^\pm}^2 = m_W^2 + m_A^2, \tag{3.18}$$

so that

$$m_{H^\pm} > m_W. \tag{3.19}$$

At tree level, the Higgs sector of MSSM is described by two parameters, which can be conveniently chosen as m_A and $\tan\beta$. There are, however, substantial radiative corrections to the CP-even neutral Higgs boson masses. These radiative corrections [4, 5] have been calculated using various methods of differing complexity. We shall here follow the method of one-loop effective potential [30] for the calculation of the radiative corrections.

3.1.2 Radiative Corrections to Neutral Higgs Masses

The tree level potential of Eq.(3.10) is subject to radiative corrections. Indeed, one of the good features of MSSM is the possibility of describing $SU(2)_L \times U(1)_Y$ breaking as an effect of radiative corrections [31], via a generalization of the Coleman-Weinberg [30] mechanism. This is implemented in the following manner. One starts with boundary conditions at the unification scale M_U

$$m_1^2 = m_2^2 = \mu_0^2 + m_0^2, \quad m_3^2 = -B_0\mu_0, \tag{3.20}$$

such that the potential is bounded from below:

$$\mathcal{A} \equiv m_1^2 + m_2^2 + 2m_3^2 \geq 0, \tag{3.21}$$

and the symmetric point $v_1 = v_2 = 0$ is a minimum:

$$\mathcal{B} \equiv m_1^2 m_2^2 - m_3^4 > 0. \tag{3.22}$$

Then, one defines, at scales Q around the electroweak scale, a renormalization-group-improved tree-level Higgs potential, $V_0(Q)$, which incorporates the large logarithmic corrections, proportional to $\propto \ln(M_U/Q)$, into running masses $[m_i^2(Q),\ i = 1,2,3]$ and gauge couplings $[g^2(Q),\ g'^2(Q)]$. If the running parameters are such that $\mathcal{A}(Q) \geq 0$ but $\mathcal{B}(Q) < 0$, one would say that the gauge symmetry is broken and one can then compute v_1, v_2 and the associated gauge and Higgs boson masses according to Eqs.(3.12)-(3.18).

However, it is not clear that, in the presence of a rich particle spectrum of MSSM, the use of $V_0(Q)$ is adequate for the determination of VEVs v_1 and v_2 and the gauge and Higgs boson masses. Indeed, it has been shown [32] that the naive use of the minimization condition of Eq.(3.13) applied to the running parameters of $V_0(Q)$ at the scale $Q \sim m_Z$ is in general a bad approximation for the determination of v_1 and v_2. A much more reliable procedure is the use of the full one-loop effective potential

$$V_1(Q) = V_0(Q) + \Delta V_1(Q), \tag{3.23}$$

where

$$\Delta V_1(Q) = \frac{1}{64\pi^2} Str\mathcal{M}^4 \left(\ln\frac{\mathcal{M}^2}{Q^2} - \frac{3}{2}\right), \tag{3.24}$$

where field-independent contributions proportional to $Str\mathcal{M}^2$ have been neglected, since MSSM has softly broken supersymmetry. In the above formula, $\mathcal{M}^2$ is the field-dependent generalized squared mass matrix of the model. It contains mass terms for all spin-0, spin-$\frac{1}{2}$ and spin-1 particles of MSSM, and one is interested in its dependence on the fields H_1 and H_2. The supertrace is defined according to

$$Strf(\mathcal{M}^2) = \sum_i (-1)^{2J_i}(2J_i + 1) f(m_i^2), \tag{3.25}$$

where m_i^2 is the field dependent mass eigenvalue of the ith particle of spin J_i. The logarithmic terms in Eq.(3.24) play a crucial role in making $V_1(Q)$ independent of the renormalization scale Q, up to higher order corrections. The same applies to the vacuum expectation values v_1 and v_2 if one neglects small wave function renormalization effects. Since the use of full one-loop effective potential is essential for the determination of the VEVs v_1 and v_2, it is natural to use it to address the related issue of the neutral Higgs boson masses. The radiatively corrected Higgs mass matrix is given, to a good approximation, by the matrix of the second derivatives of V_1 with respect to the Higgs fields. In the neutral Higgs sector, one has a 4×4 matrix of second derivatives of V_1, that breaks up into a pair of 2×2 matrices for CP-even and CP-odd sectors, respectively, when CP is conserved. It has been pointed out [4] that the dominant contributions to the supertrace in Eq.(3.24) is that due to top-stop system. However, for very large values of $\tan\beta$, also the bottom-sbottom contributions can be non-negligible. Furthermore, if D-terms are not neglected, $Str\mathcal{M}^2$ is field independent only when all the above contributions are included. Therefore, we will include both the top-stop and the bottom-sbottom contributions in our calculations of the radiative corrections.

In order to calculate the radiatively corrected Higgs boson masses, we need the mass matrices for the stop and sbottom states. The 2×2 $\tilde{t}_L - \tilde{t}_R$ and $\tilde{b}_L - \tilde{b}_R$ mass squared matrices are (keeping only the dependence on H_1^0 and H_2^0)

$$\begin{pmatrix} \tilde{M}_Q^2 + h_t^2|H_2^0|^2 + (\frac{g^2}{4} - \frac{g'^2}{12})D & h_t(A_t H_2^0 + \mu H_1^{0*}) \\ h_t(A_t H_2^{0*} + \mu H_1^0) & \tilde{M}_u^2 + h_t^2|H_2^0|^2 + \frac{g'^2}{3}D \end{pmatrix}, \tag{3.26}$$

and

$$\begin{pmatrix} \tilde{M}_Q^2 + h_b^2|H_1^0|^2 - (\frac{g^2}{4} + \frac{g'^2}{12})D & h_b(A_b H_1^0 + \mu H_2^{0*}) \\ h_b(A_b H_1^{0*} + \mu H_2^0) & \tilde{M}_d^2 + h_b^2|H_1^0|^2 - \frac{g'^2}{6}D \end{pmatrix}, \tag{3.27}$$

where (see Eqs.(3.4) and (3.5))

$$(h_U)_{33} \equiv h_t, \quad (h_D)_{33} \equiv h_b, \quad (A_u)_{33} \equiv A_t, \quad (A_d)_{33} \equiv A_b, \tag{3.28}$$

$$D \equiv (|H_1^0|^2 - |H_2^0|^2), \tag{3.29}$$

and $\tilde{M}_Q$, $\tilde{M}_u$, and $\tilde{M}_d$ stand for field-independent soft supersymmetry breaking masses. Furthermore, we note that the top and bottom quark masses squared are given by

$$m_t^2 = h_t^2|H_2^0|^2, \quad m_b^2 = h_b^2|H_1^0|^2, \tag{3.30}$$

respectively. We assume that A_t, A_b and μ are real. Using Eqs.(3.26) and (3.27) in $\Delta V_1(Q)$ as given by Eq.(3.24), we can calculate the radiatively-corrected squared masses in the CP-odd neutral sector. Denoting $\phi_i \equiv Im H_i^0$ $(i = 1,2)$, and imposing the appropriate one-loop minimization conditions, we can write in the following form the 2×2 matrix of the second derivatives of $V_1(Q)$ with respect to ϕ_i [33]

$$\left(\frac{\partial^2 V_1}{\partial\phi_i \partial\phi_j}\right)_{v_1,v_2} = \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} \Delta, \tag{3.31}$$

where

$$\begin{aligned} \Delta &= -2m_3^2 - \frac{3g^2}{32\pi^2 \sin^2\beta} \frac{m_t^2}{m_W^2} \frac{A_t \mu}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} [f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)] \\ &\quad - \frac{3g^2}{32\pi^2 \cos^2\beta} \frac{m_b^2}{m_W^2} \frac{A_b \mu}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} [f(m_{\tilde{b}_1}^2) - f(m_{\tilde{b}_2}^2)], \end{aligned} \tag{3.32}$$

$$f(m^2) \equiv 2m^2 \left(\ln\frac{m^2}{Q^2} - 1\right). \tag{3.33}$$

The eigenvalues $m_{\tilde{t}_{1,2}}^2$, $m_{\tilde{b}_{1,2}}^2$ of the stop and sbottom mass-squared matrices are given by

$$\begin{aligned} m_{\tilde{t}_{1,2}}^2 &= m_t^2 + \frac{1}{2}(\tilde{M}_Q^2 + \tilde{M}_u^2) + \frac{1}{4} m_Z^2 \cos 2\beta \\ &\pm \sqrt{\left[\frac{1}{2}(\tilde{M}_Q^2 - \tilde{M}_u^2) + \frac{1}{12}(8m_W^2 - 5m_Z^2)\cos 2\beta\right]^2 + m_t^2 \tilde{A}_t^2}, \end{aligned} \tag{3.34}$$

$$\begin{aligned} m_{\tilde{b}_{1,2}}^2 &= m_b^2 + \frac{1}{2}(\tilde{M}_Q^2 + \tilde{M}_d^2) - \frac{1}{4} m_Z^2 \cos 2\beta \\ &\pm \sqrt{\left[\frac{1}{2}(\tilde{M}_Q^2 - \tilde{M}_d^2) - \frac{1}{12}(4m_W^2 - m_Z^2)\cos 2\beta\right]^2 + m_b^2 \tilde{A}_b^2}, \end{aligned} \tag{3.35}$$

where

$$\tilde{A}_t = A_t + \mu \cot\beta, \tag{3.36}$$

$$\tilde{A}_b = A_b + \mu \tan\beta. \tag{3.37}$$

We note that the determinant of the mass matrix (3.31) vanishes, corresponding to the masslessness of the Goldstone boson which goes into making the Z^0 boson massive via the Higgs mechanism. The mass of the remaining CP-odd Higgs boson is given by

$$m_A^2 = \frac{\Delta}{\sin 2\beta}. \tag{3.38}$$

We note that the expression (3.38) for m_A^2 has the same form as the tree level one, Eq.(3.16), if either $\mu = 0$ or $A_t = A_b = 0$, as one would expect from the structure of the mass-squared matrices (3.26) and (3.27). The explicit Q-dependence in the one-loop expression (3.38) for m_A^2 is actually cancelled by the implicit Q-dependence of m_3^2 and $\tan\beta$ to leading order, so that the physical value of m_A^2 can be renormalization-scale-independent as it should be. Residual Q dependence in (3.38) is a two-loop effect that would cancel in a full calculation of m_A using the two-loop effective potential.

We next derive the mass-squared matrix for the CP-even Higgs bosons in terms of the one-loop corrected mass m_A as given by Eq.(3.38). Defining $\psi_i \equiv ReH_i^0$ $(i = 1,2)$, we get for the second derivatives of $V_1(Q)$

$$\begin{aligned}
\left(\frac{\partial^2 V_1}{\partial\psi_i \partial\psi_j}\right)_{v_1,v_2} &= \begin{pmatrix} \cot\beta & -1 \\ -1 & \tan\beta \end{pmatrix} m_Z^2 \sin 2\beta + \begin{pmatrix} \tan\beta & -1 \\ -1 & \cot\beta \end{pmatrix} \Delta \\
&\quad + \frac{3g^2}{8\pi^2 m_W^2} \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{12} & \Delta_{22} \end{pmatrix},
\end{aligned} \tag{3.39}$$

where Δ is given by Eq.(3.32), and

$$\begin{aligned}
\Delta_{11} &= \frac{m_b^4}{\cos^2\beta}\left(\log\frac{m_{\tilde{b}_1}^2 m_{\tilde{b}_2}^2}{m_b^4} + \frac{2A_b\tilde{A}_b}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2}\log\frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_2}^2}\right) \\
&\quad + \frac{m_b^4}{\cos^2\beta}\frac{A_b^2\tilde{A}_b^2}{(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)^2} g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \\
&\quad + \frac{m_t^4}{\sin^2\beta}\frac{\mu^2\tilde{A}_t^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2),
\end{aligned} \tag{3.40}$$

$$\begin{aligned}
\Delta_{22} &= \frac{m_t^4}{\sin^2\beta}\left(\log\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4} + \frac{2A_t\tilde{A}_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\log\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right) \\
&\quad + \frac{m_t^4}{\sin^2\beta}\frac{A_t^2\tilde{A}_t^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \\
&\quad + \frac{m_b^4}{\cos^2\beta}\frac{\mu^2\tilde{A}_b^2}{(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)^2} g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2),
\end{aligned} \tag{3.41}$$

$$\Delta_{12} = \frac{m_t^4}{\sin^2\beta}\frac{\mu\tilde{A}_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\left(\log\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} + \frac{A_t\tilde{A}_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)\right)$$
$$+\frac{m_b^4}{\cos^2\beta}\frac{\mu\tilde{A}_b}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2}\left(\log\frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_2}^2} + \frac{A_b\tilde{A}_b}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2}g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2)\right), \quad (3.42)$$

and

$$g(m_1^2, m_2^2) = 2 - \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2}\log\frac{m_1^2}{m_2^2}. \quad (3.43)$$

Using Eqs.(3.38)-(3.43), it is straightforward to compute $m_{h,H}^2$ and the mixing angle α, which specifies the eigenvectors of the matrix (3.39), as functions of $m_A^2 = \Delta/\sin 2\beta$. The angle α (together with β) determines all the couplings of the Higgs bosons. We note that the terms proportional to A_t, A_b and μ in Eqs.(3.40)-(3.42) were obtained neglecting the D-terms in the squark masses. Actually D-terms must be neglected everywhere in our formulae in order to gain approximate independence of the renormalization scale Q, since we are including only the quark-squark contributions to ΔV_1. Indeed, it is clear from (3.34) and (3.35) that D-term contributions to the squark masses are indeed very small, especially when the soft supersymmetry-breaking mass parameters are large with respect to m_W and m_Z, and/or $\tan\beta$ is not very large. In actual numerical evaluations of the Higgs boson masses it is conventional to assume common soft supersymmetry-breaking masses for the squarks, as well as common trilinear couplings:

$$A \equiv A_t = A_b,$$
$$\tilde{m} \equiv \tilde{M}_Q = \tilde{M}_u = \tilde{M}_d. \quad (3.44)$$

The one-loop radiatively corrected masses ($m_h, m_H; m_h < m_H$) of the CP-even Higgs bosons (h, H) can be obtained by diagonalizing the 2×2 mass matrix

$$\mathcal{M}^2 = \frac{1}{2}\left(\frac{\partial^2 V_1}{\partial\psi_i\partial\psi_j}\right)_{v_1,v_2}, \quad (3.45)$$

which can be written as

$$\mathcal{M}^2 = \begin{bmatrix} m_A^2\sin^2\beta + m_Z^2\cos^2\beta & -(m_Z^2 + m_A^2)\sin\beta\cos\beta \\ -(m_Z^2 + m_A^2)\sin\beta\cos\beta & m_A^2\cos^2\beta + m_Z^2\sin^2\beta \end{bmatrix}$$

$$+\frac{3g^2}{16\pi^2 m_W^2}\begin{bmatrix}\Delta_{11} & \Delta_{12}\\ \Delta_{12} & \Delta_{22}\end{bmatrix}, \tag{3.46}$$

where the second matrix in (3.46) represents the one-loop radiative corrections. The radiative corrections are, in general, positive, and they shift the mass of the lightest Higgs boson upwards from its tree level value. We show [34] in Fig.3.1 the resulting mass of the lightest Higgs boson, m_h, as a function of μ and $\tan\beta$, for two values of A, and for $\tilde{m} = 1$ TeV. With a wider range of parameter values, or when the squark mass scale is taken to be smaller, the dependence [35] on μ and $\tan\beta$ can be dramatic.

The lightest Higgs boson mass falls off rapidly at small values of $\tan\beta$. Since the CERN e^+e^- collider LEP experiments are obtaining lower bounds on the mass of the lightest Higgs boson, they are begining to rule out significant parts of the small-$\tan\beta$ parameter space depending on the model assumptions. For $\tan\beta > 1$, ALEPH finds $m_h > 62.5$ GeV at 95 % C.L. [36]. Recently, a new study has been presented, with a lower limit of $m_h > 72.2$ GeV, irrespective of the value of $\tan\beta$, and a limit of ~ 88 GeV for $1 < \tan\beta \leq 2$ [37].

From Eq.(3.46) we can write an upper bound on the one-loop radiatively corrected mass of the lightest Higgs boson as

$$m_h^2 \leq m_Z^2\cos^2 2\beta + \frac{3g^2}{16\pi^2 m_W^2}(\Delta_{11}\cos^2\beta + \Delta_{22}\sin^2\beta + \Delta_{12}\sin 2\beta). \tag{3.47}$$

If we ignore the b-quark mass effects in Δ_{ij} in our calculations, which is a reasonable approximation for moderate values of $\tan\beta \lesssim 20 - 30$, then we can approximate the part that arises due to radiative corrections in (3.47) as

$$\begin{aligned}&\Delta_{11}\cos^2\beta + \Delta_{22}\sin^2\beta + \Delta_{12}\sin 2\beta\\ &\approx m_t^4 \ln\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right) + \frac{m_t^4\tilde{A}_t^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\left[\tilde{A}_t^2 g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + 2\ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right)\right].\end{aligned} \tag{3.48}$$

Furthermore, in the limit

$$\frac{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2} \ll 1,$$

we obtain from (3.48)

$$
\begin{aligned}
&\Delta_{11}\cos^2\beta + \Delta_{22}\sin^2\beta + \Delta_{12}\sin 2\beta \\
&\approx 2m_t^4\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{\tilde{A}_t^2}{M_S^2}\left(1 - \frac{\tilde{A}_t^2}{12M_S^2}\right)\right],
\end{aligned}
\tag{3.49}
$$

so that the upper bound (3.47) can be written as

$$
m_h^2 \lesssim m_Z^2\cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 m_W^2}\left[2\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{2\tilde{A}_t^2}{M_S^2}\left(1 - \frac{\tilde{A}_t^2}{12M_S^2}\right)\right],
\tag{3.50}
$$

where $M_S^2 = (m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)/2$ is the supersymmetry breaking scale. We note that the upper bound (3.50) is maximized for $\tilde{A}_t = \sqrt{6}M_S$.

For $m_t = 175$ GeV, and $M_S = 1$ TeV, the upper bound (3.50) becomes

$$
\begin{aligned}
m_h &\lesssim 124\ GeV, \quad \tilde{A}_t = 0, \\
&\lesssim 150\ GeV, \quad \tilde{A} = \sqrt{6}\ M_S.
\end{aligned}
\tag{3.51}
$$

We see that the radiative corrections to the tree level result are large. The radiative corrections are proportional to m_t^4, and also to $\ln(M_S^2)$. For $M_S \gg m_Z$, the logarithmic term in (3.51) can be large, and can potentially spoil the perturbation expansion. In this case it is necessary to perform a RG-improvement which resums the leading logs to all orders in perturbation theory. The resultant RG-improved perturbation expansion is better behaved and more reliable. The numerical effects of the RG improvement can be significant for values of M_S as low as 500 GeV. A reliable prediction for the lightest Higgs boson mass is crucial for its detection at LEP II and Tevatron.

From Eq.(3.46) ikt is clear that one can write the one-loop radiatively corrected mass matrix for the CP-even Higgs bosons in the form

$$
\mathcal{M}^2 = \mathcal{M}_0^2 + \Delta M_{1LL}^2 + \Delta\mathcal{M}_{mix}^2,
\tag{3.52}
$$

where the subscript 0 refers to the tree level result, the subscript $1LL$ refers to the one-loop leading logarithmic approximation to the full one-loop calculation, and the subscript mix refers to the contribution arising from the mixing effects of the third generation squarks (which are due to nonzero to A and μ). Due to radiative corrections, the matrix $\mathcal{M}^2_{1LL}$ depends explicitly on top-quark mass. But which top-quark mass should one use? In a diagrammatic analysis, working in an on-shell scheme, we would use pole mass. The analysis based on RG-running would naturally use running mass, $m_t(m_t)$. The choice between the pole mass and the running mass cannot be decided based on one-loop considerations alone. Since the dependence on m_t enters only at one loop, the distinction between various definitions of m_t is a two-loop effect. Because the one-loop logarithmic correction proportional to m_t^4 is dominant, it is sufficient to consider the two-loop leading and next-to-leading log contributions proportional to $m_t^4\alpha_3$ and $m_t^4\alpha_t$, where $\alpha_3 \equiv g_3^2/4\pi$, $\alpha_t \equiv h_t^2/4\pi$, respectively. A simple analytical formula which incorporates these dominant effects via the RG-improvement can then be written as [6]

$$\begin{aligned}
\mathcal{M}^2_{1RG} &\simeq \bar{\mathcal{M}}^2_{1LL} + \Delta\bar{\mathcal{M}}^2_{mix} \\
&\equiv \mathcal{M}^2_{1LL}(m_t(\mu_t), m_b(\mu_b)) + \Delta\mathcal{M}^2_{mix}(m_t(\mu_{\tilde{t}}), m_b(\mu_{\tilde{b}})),
\end{aligned} \tag{3.53}$$

where

$$\begin{aligned}
\mu_t &\equiv \sqrt{m_t M_S}, \quad \mu_b \equiv \sqrt{m_Z M_S}, \\
\mu_{\tilde{q}} &= M_S, \quad q = t,\ b.
\end{aligned} \tag{3.54}$$

That is the numerically integrated RG-improved CP-even Higgs squared-mass matrix, $\mathcal{M}^2_{1RG}$, is well approximated by replacing all occurences of m_t and m_b in $\mathcal{M}^2_{1LL}(m_t, m_b)$ by the corresponding running masses. We note that the scales at which one evaluates m_t and m_b are different in $\mathcal{M}^2_{1LL}$ and $\Delta\mathcal{M}^2_{mix}$, respectively. Intutively, the squark mixing correction arises from integrating out heavy squarks that appear in one-loop corrections to Higgs scalar four-point functions. As a result one should choose $\mu_{\tilde{q}}$ to coincide with the mass of the heaviest squark.

Since the leading m_t^4 term provides the dominant source of the neutral Higgs mass radiative correction, it follows that the algorithm of replacing m_t with $m_t(\mu)$ in the one loop formula for $\mathcal{M}^2_{1LL}$ successfully reproduces the most important aspects of the RG-improvement while minimizing the effects of the non-leading logarithmic two-loop effects, which are numerically small. We note that the running top-quark mass at the scale of μ_t is given in terms of the running mass $m_t(m_t)$ via the solution of the relevant RG equations:

$$m_t(\mu_t) = m_t(m_t)\left[1 - \frac{1}{\pi}\left(\alpha_3 - \frac{3}{16}(\alpha_t^{SM} - \alpha_b^{SM})\right)\ln\left(\frac{\mu_t^2}{m_t^2}\right)\right],\ m_A \simeq \mathcal{O}(M_S), \tag{3.55}$$

$$m_t(\mu_t) = m_t(m_t)\left[1 - \frac{1}{\pi}\left(\alpha_3 - \frac{1}{16}(\alpha_b + 3\alpha_t)\right)\ln\left(\frac{\mu_t^2}{m_t^2}\right)\right],\ m_A \simeq \mathcal{O}(M_Z), \tag{3.56}$$

where α_3 and α_t have been defined earlier, and $\alpha_b = h_b^2/4\pi$, and all the couplings on the right hand side are evaluated at m_t. Similar considerations apply to the mass of the b-quark. Note further that for $m_A = \mathcal{O}(M_S)$, the effective theory of Higgs sector at mass scales below M_S is that of the one-Higgs doublet Standard Model, and hence the notation α_t^{SM} and α_b^{SM} in the above equations. Furthermore, $m_t(m_t)$ is expressed in terms of top-quark pole mass using

$$m_t(m_t) = \frac{m_t^{pole}}{1 + \frac{g_3^2}{3\pi^2}} = (165 \pm 5)\ GeV, \tag{3.57}$$

where we have used the recent experimental result [38] $m_t^{pole} = (173.8 \pm 5.2)$ GeV.

From (3.53) one can evaluate the upper bound on the lightest Higgs boson mass including the two-loop leading and next-to-leading contributions. In the large m_A limit, neglecting the b-quark mass, the result can be written as [6]

$$\begin{aligned} m_h^2 \ \lesssim \ & m_Z^2\cos^2 2\beta\left(1 - \frac{3m_t^2 t}{8\pi^2 v^2}\right) + \frac{3}{4\pi^2}\frac{m_t^2}{v^2}\left[t + \frac{X_t}{2}\right] \\ & + \frac{3}{4\pi^2}\frac{m_t^2}{v^2}\left[\frac{1}{16\pi^2}\left(\frac{3}{2}\frac{m_t^2}{v^2} - 32\pi\alpha_3\right)(X_t t + t^2)\right], \end{aligned} \tag{3.58}$$

where

$$t = \ln\frac{M_S^2}{m_t^2},\quad X_t = \frac{2\tilde{A}_t^2}{M_S^2}\left(1 - \frac{\tilde{A}_t^2}{12M_S^2}\right), \tag{3.59}$$

and $v = 174.1$ GeV is the weak scale vacuum expectation value. We note that Eq.(3.58) includes the leading D-term correction $\mathcal{O}(m_Z^2 m_t^2)$. This contribution cannot be neglected, since, for the experimental m_t value, it can account for a negative shift as large as $\sim$ 5 GeV. The analytical approximation (3.58) reproduces the all-loop renormalization-group improved leading-log result, including two-loop leading-log effects, within an error of less than 2 GeV in all cases. We note that the upper bound (3.58) is maximized for the values of mixing parameters such that $\tilde{A}_t = \sqrt{6}\, M_S$ (or $X_t = 6$). From the upper bound (3.58) we obtain

$$\begin{aligned} m_h &\lesssim 110 GeV, \quad X_t = 0, \\ &\lesssim 122 GeV, \quad X_t = 6, \end{aligned} \tag{3.60}$$

for the case of no mixing ($X_t = 0$) and maximal mixing ($X_t = 6$), respectively. We note that the two-loop leading log effects significantly decrease the mass of the lightest Higgs boson from the one loop result (3.51). We shall come back to the result (3.60) later in this Chapter.

3.2 The Minimal Supersymmetric Left-Right Model

We now come to the discussion of the minimal supersymmetric left-right model (SLRM) which has B and L number conservation built in due to underlying gauge symmetry. The SLRM is based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The matter fields of this model consist of the three families of quark and lepton chiral superfields with the following transformation properties under the gauge group:

$$\begin{aligned} \hat{Q} &= \begin{pmatrix} \hat{U} \\ \hat{D} \end{pmatrix} \sim (3,2,1,\frac{1}{3}), \qquad \hat{Q}^c = \begin{pmatrix} \hat{D}^c \\ \hat{U}^c \end{pmatrix} \sim (3^*,1,2,-\frac{1}{3}), \\ \hat{L} &= \begin{pmatrix} \hat{\nu} \\ \hat{E} \end{pmatrix} \sim (1,2,1,-1), \qquad \hat{L}^c = \begin{pmatrix} \hat{E}^c \\ \hat{\nu}^c \end{pmatrix} \sim (1,1,2,1), \end{aligned} \tag{3.61}$$

where the numbers in the brackets denote the quantum numbers under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The Higgs sector consists of the bidoublet and triplet Higgs superfields:

$$\hat{\Phi} = \begin{pmatrix} \hat{\Phi}_1^0 & \hat{\Phi}_1^+ \\ \hat{\Phi}_2^- & \hat{\Phi}_2^0 \end{pmatrix} \sim (1,2,2,0),$$

$$
\begin{aligned}
\hat{\chi} &= \begin{pmatrix} \hat{\chi}_1^0 & \hat{\chi}_1^+ \\ \hat{\chi}_2^- & \hat{\chi}_2^0 \end{pmatrix} \sim (1,2,2,0), \\
\hat{\Delta}_R &= \begin{pmatrix} \frac{1}{\sqrt{2}}\hat{\Delta}_R^- & \hat{\Delta}_R^0 \\ \hat{\Delta}_R^{--} & -\frac{1}{\sqrt{2}}\hat{\Delta}_R^- \end{pmatrix} \sim (1,1,3,-2), \\
\hat{\delta}_R &= \begin{pmatrix} \frac{1}{\sqrt{2}}\hat{\delta}_R^+ & \hat{\delta}_R^{++} \\ \hat{\delta}_R^0 & -\frac{1}{\sqrt{2}}\hat{\delta}_R^+ \end{pmatrix} \sim (1,1,3,2), \\
\hat{\Delta}_L &= \begin{pmatrix} \frac{1}{\sqrt{2}}\hat{\Delta}_L^- & \hat{\Delta}_L^0 \\ \hat{\Delta}_L^{--} & -\frac{1}{\sqrt{2}}\hat{\Delta}_L^- \end{pmatrix} \sim (1,3,1,-2), \\
\hat{\delta}_L &= \begin{pmatrix} \frac{1}{\sqrt{2}}\hat{\delta}_L^+ & \hat{\delta}_L^{++} \\ \hat{\delta}_L^0 & -\frac{1}{\sqrt{2}}\hat{\delta}_L^+ \end{pmatrix} \sim (1,3,1,2). \qquad (3.62)
\end{aligned}
$$

There are two bidoublet superfields in order to implement the $SU(2)_L \times U(1)_Y$ breaking, and to generate a nontrivial Kobayashi-Maskawa matrix. Furthermore, two $SU(2)_R$ Higgs triplet superfields $\hat{\Delta}_R$ and $\hat{\delta}_R$ with opposite $(B-L)$ are necessary to break the left-right symmetry spontaneously, and to cancel triangle gauge anomalies due to the fermionic superpartners. The gauge symmetry is supplemented by a discrete left-right symmetry under which the fields can be chosen to transform as

$$
\begin{aligned}
&\hat{Q} \leftrightarrow \hat{Q}^c,\ \hat{L} \leftrightarrow \hat{L}^c,\ \hat{\Phi} \leftrightarrow -\tau_2\hat{\Phi}^T\tau_2,\ \hat{\chi} \leftrightarrow -\tau_2\hat{\chi}^T\tau_2, \\
&\hat{\Delta}_R \leftrightarrow \hat{\delta}_L,\ \hat{\delta}_R \leftrightarrow \hat{\Delta}_L. \qquad (3.63)
\end{aligned}
$$

Thus, the $SU(2)_L$ triplets $\hat{\Delta}_L$ and $\hat{\delta}_L$ are needed in order to make the Lagrangian fully symmetric under $L \leftrightarrow R$ transformation, although these are not needed phenomenologically for symmetry breaking, or the see-saw mechanism.

The most general gauge invariant superpotential involving these superfields can be written as (generation indices suppressed)

$$
\begin{aligned}
W_{min} &= h_{\Phi Q}\hat{Q}^T i\tau_2\hat{\Phi}\hat{Q}^c + h_{\chi Q}\hat{Q}^T i\tau_2\hat{\chi}\hat{Q}^c \\
&+ h_{\Phi L}\hat{L}^T i\tau_2\hat{\Phi}\hat{L}^c + h_{\chi L}\hat{L}^T i\tau_2\hat{\chi}\hat{L}^c + h_{\delta_L}\hat{L}^T i\tau_2\hat{\delta}_L\hat{L} + h_{\Delta_R}\hat{L}^{cT} i\tau_2\hat{\Delta}_R\hat{L}^c \\
&+ \mu_1 \mathrm{Tr}(\mathrm{i}\tau_2\hat{\Phi}^{\mathrm{T}}\mathrm{i}\tau_2\hat{\chi}) + \mu_1' \mathrm{Tr}(\mathrm{i}\tau_2\hat{\Phi}^{\mathrm{T}}\mathrm{i}\tau_2\hat{\Phi}) + \mu_1'' \mathrm{Tr}(\mathrm{i}\tau_2\hat{\chi}^{\mathrm{T}}\mathrm{i}\tau_2\hat{\chi}) \\
&+ \mathrm{Tr}(\mu_{2\mathrm{L}}\hat{\Delta}_{\mathrm{L}}\hat{\delta}_{\mathrm{L}} + \mu_{2\mathrm{R}}\hat{\Delta}_{\mathrm{R}}\hat{\delta}_{\mathrm{R}}). \qquad (3.64)
\end{aligned}
$$

The scalar potential can be calculated from

$$V = V_F + V_D + V_{soft}, \tag{3.65}$$

where V_F, V_D, and V_{soft} represent the contribution of F-terms, the D-terms, and the soft SUSY breaking terms, respectively. The different components of the scalar potential (3.65) for the minimal left-right supersymmetric model can be written as follows (g_L, g_R and g_{B-L} are the three gauge couplings):

$$\begin{aligned}
V_F &= |h_{\Phi L} i\tau_2 \Phi L^c + h_{\chi L} i\tau_2 \chi L^c + 2h_{\delta_L} L^T i\tau_2 \delta_L|^2 \\
&+ |h_{\Phi L} L^T i\tau_2 \Phi + h_{\chi L} L^T i\tau_2 \chi + 2h_{\Delta_R} L^{cT} i\tau_2 \Delta_R|^2 \\
&+ |h_{\Delta_R} L^c L^{cT}(i\tau_2) + \mu_{2R}\delta_R|^2 + |h_{\delta_L} L L^T(i\tau_2) + \mu_{2L}\Delta_L|^2 \\
&+ |h_{\Phi Q} Q^c Q^T(i\tau_2) + h_{\Phi L} L^c L^T(i\tau_2) + \mu_1(i\tau_2)\chi^T(i\tau_2) + 2\mu_1'(i\tau_2)\Phi^T(i\tau_2)|^2 \\
&+ |h_{\chi Q} Q^c Q^T(i\tau_2) + h_{\chi L} L^c L^T(i\tau_2) + \mu_1(i\tau_2)\Phi^T(i\tau_2) + 2\mu_1''(i\tau_2)\chi^T(i\tau_2)|^2 \\
&+ |(i\tau_2)(h_{\Phi Q}\Phi + h_{\chi Q}\chi)Q^c|^2 + |Q^T(i\tau_2)(h_{\Phi Q}\Phi + h_{\chi Q}\chi)|^2 \\
&+ |\mu_{2R}\Delta_R|^2 + |\mu_{2L}\delta_L|^2,
\end{aligned} \tag{3.66}$$

$$\begin{aligned}
V_D &= \frac{1}{8} g_L^2 \sum_a \Big[\mathrm{Tr}(\Phi^\dagger \tau_a \Phi) + \mathrm{Tr}(\chi^\dagger \tau_a \chi) + 2\mathrm{Tr}(\Delta_L^\dagger \tau_a \Delta_L) + 2\mathrm{Tr}(\delta_L^\dagger \tau_a \delta_L) \\
&+ L^\dagger \tau_a L + Q^\dagger \tau_a Q\Big]^2 + \frac{1}{8} g_R^2 \sum_a \Big[-\mathrm{Tr}(\Phi \tau_a \Phi^\dagger) - \mathrm{Tr}(\chi \tau_a \chi^\dagger) \\
&+ 2\mathrm{Tr}(\Delta_R^\dagger \tau_a \Delta_R) + 2\mathrm{Tr}(\delta_R^\dagger \tau_a \delta_R) + \mathrm{L}^{c\dagger} \tau_a \mathrm{L}^c + \mathrm{Q}^{c\dagger} \tau_a \mathrm{Q}^c\Big]^2 \\
&+ \frac{1}{8} g_{B-L}^2 \Big[2\mathrm{Tr}(-\Delta_R^\dagger \Delta_R + \delta_R^\dagger \delta_R - \Delta_L^\dagger \Delta_L + \delta_L^\dagger \delta_L) \\
&- L^\dagger L + L^{c\dagger} L^c + \frac{1}{3} Q^\dagger Q - \frac{1}{3} Q^{c\dagger} Q^c\Big]^2,
\end{aligned} \tag{3.67}$$

$$\begin{aligned}
V_{soft} &= + m_Q^2 |Q|^2 + m_{Q^c}^2 |Q^c|^2 + (Q^T i\tau_2 (h_{\Phi Q} A_{\Phi Q} \Phi + h_{\chi Q} A_{\chi Q} \chi) Q^c + h.c.). \\
&+ m_{L^c}^2 |L^c|^2 + m_L^2 |L|^2 \\
&+ (L^T i\tau_2 (A_\Phi \Phi + A_\chi \chi) L^c + A_{\Delta_R} L^{cT} i\tau_2 \Delta_R L^c + A_{\delta_L} L^T i\tau_2 \delta_L L + h.c.) \\
&+ m_\Phi^2 \mathrm{Tr}|\Phi|^2 + \mathrm{m}_\chi^2 \mathrm{Tr}|\chi|^2 \\
&- (m_{\Phi\chi}^2 \mathrm{Tr}(\mathrm{i}\tau_2 \Phi^\mathrm{T} \mathrm{i}\tau_2 \chi) + \mathrm{m}_{\Phi\Phi}^2 \mathrm{Tr}(\mathrm{i}\tau_2 \Phi^\mathrm{T} \mathrm{i}\tau_2 \Phi) + \mathrm{m}_{\chi\chi}^2 \mathrm{Tr}(\mathrm{i}\tau_2 \chi^\mathrm{T} \mathrm{i}\tau_2 \chi) + \mathrm{h.c.})
\end{aligned}$$

$$
\begin{aligned}
&+m_{\Delta_R}^2|\Delta_R|^2+m_{\delta_R}^2|\delta_R|^2-(m_{\Delta\delta}^2\mathrm{Tr}\Delta_\mathrm{R}\delta_\mathrm{R}+\mathrm{h.c.})\\
&+m_{\Delta_L}^2|\Delta_L|^2+m_{\delta_L}^2|\delta_L|^2-(m_{\Delta\delta}{}'^2\mathrm{Tr}\Delta_\mathrm{L}\delta_\mathrm{L}+\mathrm{h.c.}).
\end{aligned}
\tag{3.68}
$$

The general form of the vacuum expectation values of the various scalar fields which preserve the $U(1)_{em}$ gauge invariance can be written as

$$
\begin{aligned}
\langle\Phi\rangle &= \begin{pmatrix} \kappa_1 & 0 \\ 0 & e^{i\phi_1}\kappa_1' \end{pmatrix}, & \langle\chi\rangle &= \begin{pmatrix} e^{i\phi_2}\kappa_2' & 0 \\ 0 & \kappa_2 \end{pmatrix},\\
\langle\Delta_R\rangle &= \begin{pmatrix} 0 & v_{\Delta_R} \\ 0 & 0 \end{pmatrix}, & \langle\delta_R\rangle &= \begin{pmatrix} 0 & 0 \\ v_{\delta_R} & 0 \end{pmatrix},\\
\langle\Delta_L\rangle &= \begin{pmatrix} 0 & v_{\Delta_L} \\ 0 & 0 \end{pmatrix}, & \langle\delta_L\rangle &= \begin{pmatrix} 0 & 0 \\ v_{\delta_L} & 0 \end{pmatrix},\\
\langle L\rangle &= \begin{pmatrix} \sigma_L \\ 0 \end{pmatrix}, & \langle L^c\rangle &= \begin{pmatrix} 0 \\ \sigma_R \end{pmatrix}.
\end{aligned}
\tag{3.69}
$$

We note that the triplet vacuum expectation values v_{Δ_R} and v_{δ_R} represent the scale of $SU(2)_R$ breaking and are, according to the lower bounds [38] on heavy W- and Z-boson masses, in the range $v_{\Delta_R}, v_{\delta_R} \gtrsim 1$ TeV. These represent a new scale, the right-handed breaking scale, which we shall generically denote as M_R. We note that κ_1' and κ_2' contribute to the mixing of the charged gauge bosons and to the flavour changing neutral currents, and are usually assumed to vanish. Furthermore, since the electroweak ρ parameter is close to unity, $\rho = 0.9998 \pm 0.0008$ [38], the triplet vacuum expectation values v_{Δ_L} and v_{δ_L} must be small.

The Yukawa coupling $h_{\chi L}$ is proportional to the neutrino Dirac mass m_D. The light neutrino mass in the see-saw mechanism is given by $\sim m_D^2/m_M$, where $m_M = h_{\Delta_R}v_{\Delta_R}$ is the Majorana mass. The magnitude of $h_{\chi L}$ is not accurately determined given the present upper limit on the light neutrino masses. On the other hand the Yukawa coupling $h_{\Phi L}$ is proportional to the electron mass and is, thus, small.

In the minimal model described above, parity cannot be spontaneously broken at the renormalizable level without spontaneous breaking of R-parity. This may be cured by adding more fields to the theory. In [23, 39] it was suggested that a parity-odd singlet, coupled appropriately to triplet fields, be introduced so as to ensure proper symmetry breaking. This leads to a set of degenerate minima connected by a flat direction, all of them breaking parity. When soft SUSY breaking terms are switched on, the degeneracy is lifted, but the global minimum that results breaks $U(1)_{em}$. Because of the flat direction connecting the minima, there is no hope that the fields remain in the phenomenologically acceptable vacuum, which rolls down to global minimum after SUSY is softly broken. The only option that is left is to have a relatively low $SU(2)_R$ breaking scale, with spontaneously broken R-parity ($\langle\hat{\nu^c}\rangle \equiv \sigma_R$ is non-zero). We note that present experiments allow for a low $SU(2)_R$ breaking scale. Furthermore, since R-parity is broken only spontaneously, the breaking is under control as compared to the explicit breaking that arises in MSSM based on the gauge group $SU(2)_L \times U(1)_Y$.

There is an alternative to the minimal left-right supersymmetric model which involves the addition of a couple of triplet fields, $\Omega_L(1,3,1,0)$ and $\Omega_R(1,1,3,0)$, instead of a singlet Higgs superfield, to the minimal model [40]. In these extended models the breaking of $SU(2)_R$ is achieved in two stages. In the first stage the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is broken to an intermediate symmetry group $SU(2)_L \times U(1)_R \times U(1)_{B-L}$, and at the second stage $U(1)_R \times U(1)_{B-L}$ is broken to $U(1)_Y$ at a lower scale. In this theory there is only one parity-breaking minimum, in contrast to the minimal model, which respects the electromagnetic gauge invariance. The see-saw mechanism takes its canonical form with $m_\nu \simeq m_D^2/M_{BL}$, where m_D is the neutrino Dirac mass. In this case the low-energy effective theory is the MSSM with unbroken R-parity, and contains besides the usual MSSM states, a triplet of Higgs scalars much lighter than the $B-L$ breaking scale.

A second option [41, 42] is to add non-renormalizable terms to the Lagrangian of the minimal left-right supersymmetric model, while retaining the minimal Higgs content. It has been shown that the addition of non-renormalizable terms suppressed by a high scale such as Planck mass, $M \sim M_{Pl} \sim 10^{19}$ GeV, with the minimal field content ensures the correct pattern of symmetry breaking in the supersymmetric left-right model. In particular the scale of parity breakdown is predicted to be in the intermediate region $M_R \gtrsim 10^{10} - 10^{11}$ GeV, and R-parity remains exact. This theory contains singly charged and doubly charged Higgs scalars with a low mass of order M_R^2/M_{Pl}, which are experimentally accessible. However, what is different is the nature of see-saw mechanism. Whereas in the renormalizable version the see-saw mechanism takes its canonical form, in the non-renormalizable case it takes a form similar to what occurs in the non-supersymmetric left-right models, with the neutrino mass depending on the unknown parameters of the Higgs potential. This in general leads to different neutrino mass spectra, which can be experimentally distinguished.

3.3 The Tree-level Upper Bound on the Lightest Higgs Mass in SLRM

In this section we shall discuss the lightest Higgs boson mass in the minimal supersymmetric left-right model. From (3.65), (3.66), (3.67) and (3.68) it is straightforward to derive the mass matrix for the CP-even Higgs scalars, whose eigenvalues will provide the masses of the physical scalar Higgs bosons. Given the fact that the Higgs sector of SLRM models contain a large number of Higgs multiplets, and the VEVs of some of the Higgs fields involve possibly large mass scales compared to the electroweak and SUSY breaking scales, it is important to ask what is the mass of the lightest Higgs boson in these models. The CP-even Higgs boson mass matrix is a 10×10 matrix in the minimal supersymmetric left-right model. We shall not write down the full 10×10 mass matrix here, since for the specific purpose of the determination of a general bound on the lightest Higgs mass, the problem is much simpler. The upper bound on the lightest Higgs boson mass in the minimal model was derived in [28] using the fact that for any Hermitean matrix the smallest eigenvalue must be smaller than that of its upper left corner 2×2 submatrix. In the basis in which the first two indices correspond to (Φ_1^0, χ_2^0), we find for these upper left corner matrix elements $m_{11}^2, m_{22}^2, m_{12}^2$

$$
\begin{aligned}
m_{11}^2 &= -m_{\Phi\chi}^2 \frac{\kappa_2}{\kappa_1} + \frac{1}{2}(g_L^2 + g_R^2)\kappa_1^2, \\
m_{22}^2 &= -m_{\Phi\chi}^2 \frac{\kappa_1}{\kappa_2} + \frac{1}{2}(g_L^2 + g_R^2)\kappa_2^2,
\end{aligned}
$$

$$m_{12}^2 \;=\; m_{\Phi_X}^2 - \frac{1}{2}(g_L^2 + g_R^2)\kappa_1\kappa_2. \qquad (3.70)$$

It follows that the upper bound on the lightest Higgs boson mass in the minimal supersymmetric left-right model can be written as [28]:

$$m_h^2 \leq \frac{1}{2}(g_L^2 + g_R^2)(\kappa_1^2 + \kappa_2^2)\cos^2 2\beta = \left(1 + \frac{g_R^2}{g_L^2}\right) m_{W_L}^2 \cos^2 2\beta, \qquad (3.71)$$

where $\tan\beta = \kappa_2/\kappa_1$. The upper bound (3.71) is not only independent of the supersymmetry breaking parameters (as in the case of supersymmetric models based on $SU(2)_L \times U(1)_Y$), but it is also independent of the $SU(2)_R$ breaking scale, which, *a priori*, can be large. The upper bound is controlled by the weak scale vacuum expectation value, $\kappa_1^2 + \kappa_2^2$, and the dimensionless gauge couplings (g_L and g_R) only. Since the former is essentially fixed by the electroweak scale, the gauge couplings g_L and g_R determine the bound[1].

3.4 Radiative Corrections to the Lightest Higgs Mass in SLRM

As we have seen, the radiative corrections to the lightest Higgs boson mass are significant in the MSSM. It is, therefore, important to consider the radiative corrections to the upper bound on the lightest Higgs boson mass in the minimal supersymmetric left-right model. In this section we discuss the leading and next-to-leading log radiative corrections to the tree level upper bound on the lightest Higgs mass in the minimal supersymmetric left-right model, which was obtained in last section. As in the case of MSSM, we shall use the method of one-loop effective potential [30] for the calculation of leading one-loop radiative corrections, where the effective potential may be expressed as the sum of the tree-level potential plus a correction coming from the sum of one-loop diagrams with external lines having zero momenta

$$V_1(Q) = V_0(Q) + \Delta V_1(Q), \qquad (3.72)$$

[1]The upper bound (3.71) is valid when $\kappa_1', \kappa_2', \sigma_L, v_{\Delta_L}, v_{\delta_L} \rightarrow 0$. However, even when $\kappa_1', \kappa_2', \sigma_L, v_{\Delta_L}, v_{\delta_L}$ are non-zero, the upper bound on the lightest Higgs mass at the tree level does not depend on either the right-handed breaking scale or the SUSY breaking scale [29].

where V_0 is the tree level potential (3.65) evaluated at the appropriate running scale Q, and ΔV_1 is the one loop correction given by

$$\Delta V_1 = \frac{1}{64\pi^2}\sum_i (-1)^{2J_i}(2J_i+1)m_i^4\left(\ln\frac{m_i^2}{Q^2} - \frac{3}{2}\right), \tag{3.73}$$

where m_i is the field dependent mass eigenvalue of the ith particle of spin J_i. The dominant contribution to (3.73) comes from top-stop $(t-\tilde{t})$ system. However, under certain conditions the contribution of bottom-sbottom $(b-\tilde{b})$ can be nonneglible. We shall include both these contributions in the calculations of the one-loop radiative corrections.

In order to evaluate the contributions of top-stop and bottom-sbottom systems to (3.73), we need the stop and sbottom mass matrices for the SLRM. From (3.66), (3.67) and (3.68), it is straightforward to calculate squark mass matrices [26]. Ignoring the interfamily mixing, the part of the potential containing the stop and sbottom mass terms can be written as

$$V_{squark} = \begin{pmatrix} U_L^* & U_R^* \end{pmatrix} \tilde{M}_U \begin{pmatrix} U_L \\ U_R \end{pmatrix} + \begin{pmatrix} D_L^* & D_R^* \end{pmatrix} \tilde{M}_D \begin{pmatrix} D_L \\ D_R \end{pmatrix}, \tag{3.74}$$

where the mass matrix elements for the stop are

$$\begin{aligned}
(\tilde{M}_U)_{U_L^* U_L} &= \tilde{m}_Q^2 + m_u^2 + \frac{1}{4}g_L^2\omega_\kappa^2 - \frac{1}{6}g_{B-L}^2\omega_R^2, \\
(\tilde{M}_U)_{U_R^* U_L} &= h_{\Phi Q}A_{\Phi Q}\kappa_1' + h_{\chi Q}A_{\chi Q}\kappa_2 - \mu_1(h_{\phi Q}\kappa_2' + h_{\chi Q}\kappa_1) - 2h_{\phi Q}\mu_1'\kappa_1 \\
&\quad -2h_{\chi Q}\mu_1''\kappa_2' + (h_{\phi L}h_{\phi Q} + h_{\chi L}h_{\chi Q})\sigma_L\sigma_R \\
&= \left[(\tilde{M}_U)_{U_L^* U_R}\right]^*, \\
(\tilde{M}_U)_{U_R^* U_R} &= m_{Q^c}^2 + m_u^2 + \frac{1}{4}g_R^2(\omega_\kappa^2 - 2\omega_R^2) + \frac{1}{6}g_{B-L}^2\omega_R^2,
\end{aligned} \tag{3.75}$$

while for the sbottom these are

$$\begin{aligned}
(\tilde{M}_D)_{D_L^* D_L} &= m_Q^2 + m_d^2 - \frac{1}{4}g_L^2\omega_\kappa^2 - \frac{1}{6}g_{B-L}^2\omega_R^2, \\
(\tilde{M}_D)_{D_R^* D_L} &= -h_{\Phi Q}A_{\Phi Q}\kappa_1 - h_{\chi Q}A_{\chi Q}\kappa_2' + \mu_1(h_{\phi Q}\kappa_2 + h_{\chi Q}\kappa_1') \\
&\quad +2h_{\phi Q}\mu_1'\kappa_1' + 2h_{\chi Q}\mu_1''\kappa_2 \\
&= \left[(\tilde{M}_D)_{D_L^* D_R}\right]^*, \\
(\tilde{M}_D)_{D_R^* D_R} &= m_{Q^c}^2 + m_d^2 - \frac{1}{4}g_R^2(\omega_\kappa^2 - 2\omega_R^2) + \frac{1}{6}g_{B-L}^2\omega_R^2,
\end{aligned} \tag{3.76}$$

where top and bottom squared masses are given by $(h_{\chi Q}\kappa_2)^2$ and $(h_{\Phi Q}\kappa_1)^2$, respectively, m^2_Q, $m^2_{Q^c}$, $A_{\Phi Q}$ and $A_{\chi Q}$ are soft supersymmetry breaking parameters (see Eq. (3.68)), and

$$\omega_R^2 = v_{\Delta_R}^2 - v_{\delta_R}^2 - \frac{1}{2}\sigma_R^2, \quad \omega_\kappa^2 = \kappa_1^2 + \kappa_2'^2 - \kappa_2^2 - \kappa_1'^2. \tag{3.77}$$

In order that $SU(3)_C \times U(1)_{\rm em}$ is unbroken, none of the physical squared masses of squarks can be negative. Necessarily then all the diagonal elements of the squark mass matrices should be non-negative. Combining the diagonal elements of the stop and sbottom mass matrices leads to the inequality

$$m_Q^2 + m_{Q^c}^2 \geq |\frac{1}{2}g_R^2\omega_R^2| = \frac{1}{2}g_R^2|v_{\Delta_R}^2 - v_{\delta_R}^2 - \frac{1}{2}\sigma_R^2|. \tag{3.78}$$

where we have ignored terms which are of the order of the weak scale or less.

The eigenvalues $m^2_{\tilde t_{1,2}}, m^2_{\tilde b_{1,2}}$ of the stop and sbottom mass squared matrices are given by ($m^2_{\tilde t_1} > m^2_{\tilde t_2}, m^2_{\tilde b_1} > m^2_{\tilde b_2}$)

$$\begin{aligned} m^2_{\tilde t_{1,2}} &= m^2_{\tilde t} \pm \Delta^2_{\tilde t}, \\ m^2_{\tilde b_{1,2}} &= m^2_{\tilde b} \pm \Delta^2_{\tilde b}, \end{aligned} \tag{3.79}$$

where

$$\begin{aligned} m^2_{\tilde t} = &\ \tfrac{1}{2}\left[m_Q^2 + m_{Q^c}^2 + 2m_t^2 + \tfrac{1}{4}(g_L^2 + g_R^2)\omega_\kappa^2 - \tfrac{1}{2}g_R^2\omega_R^2\right], \\ \Delta^2_{\tilde t} = &\ \tfrac{1}{2}\Big\{\left[m_Q^2 - m_{Q^c}^2 + \tfrac{1}{4}(g_L^2 - g_R^2)\omega_\kappa^2 + \tfrac{1}{2}g_R^2\omega_R^2 - \tfrac{1}{3}g_{B-L}^2\omega_R^2\right]^2 \\ & +4\left[h_{\chi Q}A_{\chi Q}\kappa_2 - (\mu_1 h_{\chi Q} + 2\mu_1' h_{\Phi Q})\kappa_1\right]^2\Big\}^{\frac{1}{2}}, \end{aligned} \tag{3.80}$$

and

$$\begin{aligned} m^2_{\tilde b} = &\ \tfrac{1}{2}\left[m_Q^2 + m_{Q^c}^2 + 2m_b^2 - \tfrac{1}{4}(g_L^2 + g_R^2)\omega_\kappa^2 + \tfrac{1}{2}g_R^2\omega_R^2\right], \\ \Delta^2_{\tilde b} = &\ \tfrac{1}{2}\Big\{\left[m_Q^2 - m_{Q^c}^2 - \tfrac{1}{4}(g_L^2 - g_R^2)\omega_\kappa^2 - \tfrac{1}{2}g_R^2\omega_R^2 - \tfrac{1}{3}g_{B-L}^2\omega_R^2\right]^2 \\ & +4\left[h_{\Phi Q}A_{\Phi Q}\kappa_1 - (\mu_1 h_{\Phi Q} + 2\mu_1'' h_{\chi Q})\kappa_2\right]^2\Big\}^{\frac{1}{2}}. \end{aligned} \tag{3.81}$$

Using eqs. (3.79), (3.80) and (3.81) in (3.73), the radiatively-corrected expressions for the matrix elements of the upper left corner 2×2 submatrix of the 10×10 CP-even Higgs mass matrix can be calculated. After imposing the appropriate one-loop minimization conditions, we find the following form for the radiatively corrected upper left corner 2×2 submatrix of CP-even Higgs mass matrix:

$$\frac{1}{2}\begin{pmatrix} (g_L^2+g_R^2)\kappa_1^2 & -(g_L^2+g_R^2)\kappa_1\kappa_2 \\ -(g_L^2+g_R^2)\kappa_1\kappa_2 & (g_L^2+g_R^2)\kappa_2^2 \end{pmatrix}$$
$$+\begin{pmatrix} \tan\beta & -1 \\ -1 & \cot\beta \end{pmatrix}\left(\frac{\Delta}{2}\right)+\frac{3g_L^2}{16\pi^2 m_{W_L}^2}\begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{12} & \Delta_{22} \end{pmatrix}, \tag{3.82}$$

where

$$\begin{aligned}\Delta = & \left[-2m_{\Phi\chi}^2+\frac{3A_t\tilde{\mu}_t}{32\pi^2}\left(\frac{g_L^2}{\sin^2\beta}\frac{m_t^2}{m_W^2}\right)\frac{f(m_{\tilde{t}_1}^2)-f(m_{\tilde{t}_2}^2)}{m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2}\right. \\ & \left.+\frac{3A_b\tilde{\mu}_b}{32\pi^2}\left(\frac{g_L^2}{\cos^2\beta}\frac{m_b^2}{m_W^2}\right)\frac{f(m_{\tilde{b}_1}^2)-f(m_{\tilde{b}_2}^2)}{m_{\tilde{b}_1}^2-m_{\tilde{b}_2}^2}\right],\end{aligned} \tag{3.83}$$

$$f(x^2)=2x^2(\ln(x^2/Q^2)-1), \tag{3.84}$$

$$\tilde{\mu}_t \equiv \mu_1+2\mu_1'\frac{m_b}{m_t}\tan\beta,\ \ \tilde{\mu}_b \equiv \mu_1+2\mu_1''\frac{m_t}{m_b}\cot\beta, \tag{3.85}$$

$$A_t \equiv A_{\chi Q},\ \ A_b \equiv A_{\phi Q}, \tag{3.86}$$

and

$$\begin{aligned}\Delta_{11} = & \frac{m_b^4}{\cos^2\beta}\left[\ln\left(\frac{m_{\tilde{b}_1}^2 m_{\tilde{b}_2}^2}{m_b^4}\right)+\frac{2A_b(A_b-\tilde{\mu}_b\tan\beta)}{m_{\tilde{b}_1}^2-m_{\tilde{b}_2}^2}\ln\left(\frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_2}^2}\right)\right] \\ & +\frac{m_b^4}{\cos^2\beta}\frac{A_b^2(A_b-\tilde{\mu}_b\tan\beta)^2}{\left(m_{\tilde{b}_1}^2-m_{\tilde{b}_2}^2\right)^2}g(m_{\tilde{b}_1}^2,m_{\tilde{b}_2}^2) \\ & +\frac{m_t^4}{\sin^2\beta}\frac{(A_t-\tilde{\mu}_t\cot\beta)^2\tilde{\mu}_t^2}{(m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2)^2}g(m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2),\end{aligned} \tag{3.87}$$

$$\begin{aligned}\Delta_{22} = & \frac{m_t^4}{\sin^2\beta}\left[\ln\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right)+\frac{2A_t(A_t-\tilde{\mu}_t\cot\beta)}{m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2}\ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right)\right] \\ & +\frac{m_t^4}{\sin^2\beta}\frac{A_t^2(A_t-\tilde{\mu}_t\cot\beta)^2}{\left(m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2\right)^2}g(m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2) \\ & +\frac{m_b^4}{\cos^2\beta}\frac{(A_b-\tilde{\mu}_b\tan\beta)^2\tilde{\mu}_b^2}{\left(m_{\tilde{b}_1}^2-m_{\tilde{b}_2}^2\right)^2}g(m_{\tilde{b}_1}^2,m_{\tilde{b}_2}^2),\end{aligned} \tag{3.88}$$

$$\Delta_{12} = \frac{m_t^4}{\sin^2\beta}\frac{(A_t-\tilde{\mu}_t\cot\beta)(-\tilde{\mu}_t)}{m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2}\times\left[\ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right)+\frac{A_t(A_t-\tilde{\mu}_t\cot\beta)}{m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2}g(m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2)\right]$$
$$+\frac{m_b^4}{\cos^2\beta}\frac{(A_b-\tilde{\mu}_b\tan\beta)(-\tilde{\mu}_b)}{m_{\tilde{b}_1}^2-m_{\tilde{b}_2}^2}\times\left[\ln\left(\frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_2}^2}\right)+\frac{A_b(A_b-\tilde{\mu}_b\tan\beta)}{m_{\tilde{b}_1}^2-m_{\tilde{b}_2}^2}g(m_{\tilde{b}_1}^2,m_{\tilde{b}_2}^2)\right], \tag{3.89}$$

with

$$g(m_1^2,m_2^2) = 2-\frac{m_1^2+m_2^2}{m_1^2-m_2^2}\ln\left(\frac{m_1^2}{m_2^2}\right). \tag{3.90}$$

We have neglected D-terms in the squark masses, because these are small, and, since we are including only the quark-squark contributions to ΔV_1, in order to gain approximate independence of the renormalization scale Q (see also the inequality (3.78)).

Using Eqs. (3.82) and (3.83), the one-loop radiatively corrected upper bound on the lightest Higgs boson mass in the SLRM can be written as

$$m_h^2 \le \frac{1}{2}(g_L^2+g_R^2)\left(\kappa_1^2+\kappa_2^2\right)\cos^2 2\beta + \frac{3g_L^2}{16\pi^2 m_{W_L}^2}\left(\Delta_{11}\cos^2\beta+\Delta_{22}\sin^2\beta+\Delta_{12}\sin 2\beta\right). \tag{3.91}$$

For $\tan\beta \lesssim 20$, one can neglect the b-quark contribution in the radiative corrections. Then, in the approximation [6]

$$|m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2| \ll |m_{\tilde{t}_1}^2+m_{\tilde{t}_2}^2|, \tag{3.92}$$

the upper bound (3.91) on the lightest Higgs mass reduces to

$$m_h^2 \le \frac{1}{2}(g_L^2+g_R^2)\left(\kappa_1^2+\kappa_2^2\right)\cos^2 2\beta + \frac{3g_L^2 m_t^4}{16\pi^2 m_{W_L}^2}\left(2\ln(\frac{M_s^2}{m_t^4})+2\frac{\tilde{A}_t^2}{M_s^2}(1-\frac{\tilde{A}_t^2}{12M_s^2})-8\frac{\mu_1''^4}{3M_s^4}\right), \tag{3.93}$$

where $\tilde{A}_t = A_t - \mu_1\cot\beta$, and M_s is the supersymmetry breaking scale ($2M_s^2 = m_{\tilde{t}_1}^2+m_{\tilde{t}_2}^2$). In this limit the upper bound Eq. (3.93) on the lightest Higgs mass in the supersymmetric left-right model is similar in form to the corresponding MSSM upper bound except for the term proportional to μ_1''. Since the right-handed gauge coupling g_R is not known, the upper bound on the right-hand side of (3.93) comes from the requirement that the left-right supersymmetric model remains perturbative below some scale Λ. In order to implement this requirement we need to solve the renormalization group equations for the gauge couplings of the theory.

The renormalization group equation for the model can be written as [28]

$$16\pi^2 \frac{dg_{L,R}}{dt} = 6g_{L,R}^3, \tag{3.94}$$

$$16\pi^2 \frac{dg_{B-L}}{dt} = 16g_{B-L}^3. \tag{3.95}$$

Requiring that g_R remains perturbative upto a scale Λ, i.e.

$$\frac{g_i(Q^2)}{4\pi} \leq 1, \quad for \;\; Q^2 \leq \Lambda^2, \tag{3.96}$$

where equality holds for $Q^2 = \Lambda^2$, we can evaluate the upper bound (3.91). We note that the $U(1)_Y$ coupling is related to the g_R and g_{B-L} through $g' = g_R g_{B-L}/\sqrt{(}g_R^2 + g_{B-L})$.

The radiatively corrected upper bound (3.91) on the mass of the lightest Higgs boson is plotted [28, 29] in Fig.3.2 as a function of the large scale Λ up to which the supersymmetric left-right model remains perturbative. The upper bound comes from the requirement that all the gauge couplings of the SLRM remain perturbative below the scale Λ. We note that the $U(1)_Y$ coupling g' is related to g_R and g_{B-L} through $g' = g_L g_{B-L}/\sqrt{(}g_L^2 + g_L^2)$. We have taken into account the dominant one-loop radiative corrections coming from the quark and squark loops in our calculations. In Fig.3.2 we have taken two values of $\tan\beta = 2$ and $\tan\beta = 20$. In the figure the upper bound is shown for two different values of the $SU(2)_R$ breaking scale, $M_R = 10$ TeV and $M_R = 10^{10}$ GeV, respectively, and for two values of soft supersymmetry breaking mass parameter, $M_s = 1$ TeV and $M_s = 10$ TeV. It is seen from this figure that if the difference between the $SU(2)_R$ breaking scale and the large scale Λ is more than two orders of magnitude, the radiatively corrected upper bound on the mass of the lightest Higgs boson remains below 250 GeV. For large values of Λ the upper bound is below 200 GeV. The upper bound increases with increasing M_R and with increasing soft supersymmetry breaking parameters. It is considerably larger than the corresponding upper bound in the MSSM.

Because the one-loop leading logarithmic correction proportional to m_t^4 is dominant, it is sufficient to consider the two-loop leading and next-to-leading log contributions proportional to $m_t^4\alpha_3$ and $m_t^4\alpha_t$ to obtain an accurate prediction for the lightest Higgs boson mass in SLRM as in the case of MSSM. We shall now consider these effects and calculate their contribution to the lightest Higgs boson mass. We shall be interested in the maximum value of the upper bound for the Higgs mass which is obtained in the decoupling limit, i.e. all the Higgs bosons except the lightest one are as heavy as the supersymmetry breaking scale, so that the effective Higgs sector at scales below M_S is the one-Higgs-doublet Standard Model which is characterized by one Higgs self-coupling parameter λ. We shall work in the limit $m_b = 0$ and $\beta = \pi/2$. Supersymmetry and gauge invariance fixes the value of λ above the supersymmetry breaking scale (M_S) to be

$$\lambda(M_S) = \frac{1}{2}\left(g_L^2(M_S) + g_R^2(M_S)\right). \tag{3.97}$$

The physical mass of the Higgs boson is determined at the scale m_Z, which is assumed to be much less than M_S, via

$$m_h = \lambda(m_Z)v^2(m_Z), \tag{3.98}$$

$$v^2(m_Z) = (\kappa_1^2(m_Z) + \kappa_2^2(m_Z)). \tag{3.99}$$

If supersymmetry were unbroken, then λ would have the value given by (3.97), which would imply $m_h = \frac{1}{2}(g_L^2 + g_R^2)(\kappa_1^2 + \kappa_2^2)$. This is the expected result for $\beta = \pi/2$ in the decoupling regime (see Eq.(3.71)). In the case of broken supersymmetry Eq.(3.97) is taken as a boundary condition for the one-Higgs-doublet Standard Model evolution of λ from M_S to m_Z. We shall work in the approximation $h_b = g_L = g' = 0$, since a more precise computation is not needed for the effects that we are interested in. In this approximation the RG equations for the effective theory of one-Higgs-doublet Standard Model can be written as [43]

$$\beta_\lambda \equiv \frac{d\lambda}{d\ln\mu^2} = \frac{3}{8\pi^2}\left[\lambda^2 - h_t^4\right] - 2\lambda\gamma_v + \frac{1}{(16\pi^2)^2}\left[30h_t^6 - 32h_t^2 g_3^2\right], \tag{3.100}$$

$$\beta_{h_t^2} \equiv \frac{dh_t^2}{d\ln\mu^2} = \frac{1}{16\pi^2}\left[\frac{9}{2}h_t^2 - 8g_3^2\right]h_t^2, \tag{3.101}$$

where

$$\gamma_v \equiv \frac{1}{v^2}\frac{dv^2}{d\ln\mu^2} = -\frac{3}{16\pi^2}h_t^2. \tag{3.102}$$

Solving iteratively the RGE for λ we obtain

$$\lambda(M_S) = \lambda(m_t) + \beta_\lambda(M_S)\ln\left(\frac{M_S^2}{m_t^2}\right), \tag{3.103}$$

where

$$\begin{aligned}
\beta_\lambda(M_S) &\simeq -\frac{3}{8\pi^2}h_t^4(M_S) + \frac{1}{(16\pi^2)^2}\left[30h_t^6 - 32h_t^2g_3^2\right] && (3.104)\\
&\simeq -\frac{3}{8\pi^2}h_t^4(m_t)\left[1 + \frac{\beta_{h_t^2}}{h_t^2}\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{1}{3\pi^2}g_3^2 - \frac{5}{16\pi^2}h_t^2\right], && (3.105)
\end{aligned}$$

where the second step follows from solving Eq.(3.101), and substituting the solution in $\beta_\lambda(M_S)$. Substituting Eq.(3.105) in Eq.(3.103), and using the boundary condition (3.97), we get

$$\begin{aligned}
\frac{1}{2}(g_L^2 + g_R^2)v^2 &= \frac{\lambda(m_t)v^2(m_t)}{\left(1 - \gamma_v\ln\frac{M_S^2}{m_t^2}\right)} - \frac{3}{8\pi^2}h_t^4(m_t)v^2(m_t)\ln\frac{M_S^2}{m_t^2}\\
&\quad\times\left[1 + \left(\gamma_v + \frac{\beta_{h_t^2}}{h_t^2}\right)\ln\frac{M_S^2}{m_t^2} + \frac{4}{3}\frac{\alpha_3}{\pi} - \frac{5}{4}\frac{\alpha_t}{\pi}\right], \qquad (3.106)
\end{aligned}$$

where we have used the solution of RGE (3.102):

$$v^2(M_S) = \frac{v^2(m_t)}{\left[1 - \gamma_v\ln\frac{M_S^2}{m_t^2}\right]}. \tag{3.107}$$

We must now take into account two modifications of the result (3.106). First we must distinguish between the Higgs pole mass (henceforth denoted by m_h with no argument) and the running Higgs mass evaluated at m_h. For this we can use the results of Sirlin and Zucchini [44]

$$m_h = \frac{4m_W^2\lambda(m_h)}{g_L^2}\left[1 + \frac{1}{8}\left(\frac{\alpha_t}{\pi}\right)\right], \tag{3.108}$$

which is valid in the limit of $h_b = g_L = g' = 0$ and $\lambda \ll h_t$. Second, we must take into account a finite correction between $v^2(m_t)$ and $v^2 = 4m^2_{W_L}/g^2_L$,

$$v^2(m_t) = \frac{4m_W^2}{g_L^2}\left[1 - \frac{3}{8}\frac{\alpha_t}{\pi}\right], \tag{3.109}$$

which can be obtained from Ref.[45]. Using (3.108) and (3.109) in (3.106) we obtain

$$\begin{aligned} m_h^2 &\simeq \frac{1}{2}(g_L^2 + g_R^2)(\kappa_1^2 + \kappa_2^2) + \frac{3}{8\pi^2}\left(\frac{g_L^2}{m_W^2}\right) m_t^4(m_t) \ln\frac{M_S^2}{m_t^2} \\ &\quad \times \left[1 + \left(\frac{3}{32\pi^2}\frac{m_t^2}{v^2} - \frac{2\alpha_3}{\pi}\right)\ln\frac{M_S^2}{m_t^2} + \frac{4}{3}\frac{\alpha_3}{\pi} - \frac{3}{8}\frac{\alpha_t}{\pi}\right], \end{aligned} \tag{3.110}$$

where we have used $m_h^2 = \lambda(m_Z)v^2(m_Z)$. We note that in the approximation where $h_b = g_L = g' = 0$ and $\lambda \ll h_t$, both $\lambda(\mu)$ and $v(\mu)$ do not run for $\mu \leq m_t$.

So far we have focussed our attention on the radiative corrections under the assumption that one mass scale, M_S, characterizes the supersymmetric masses. This is not a realistic assumption. We now take into account the effects arising from the mass splittings and $\tilde{q}_L - \tilde{q}_R$ mixing in the third generation squark sector. This can generate additional squared-mass shifts proportional to m_t^4 and thus can have profound effect on the radiatively corrected lightest Higgs boson mass. We shall use the method of [46] to evaluate these effects. The squark mass eigenstates of the third family are obtained from Eq.(3.79). We shall work in the simplified limit of decoupling, with $m_b = 0$ and $\beta = \pi/2$. Since we shall ignore the effects of b-quark, we shall not take into account the effects of bottom squarks either. Then the one-loop effective potential (3.72) is dominated by the stop contribution. The stop masses are given by Eq.(3.79) in the appropriate limit, where $m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2} = 2M_S^2$. Thus, we identify the scale Q in the effective potential at which we are evaluating the threshold effects with M_S.

From the one-loop effective potential (3.72) it is straightforward to obtain the contribution to the ϕ^4 operator, where ϕ is the Standard Model Higgs doublet (we are working in the decoupling region, where all except one Higgs boson is light). This contribution is

$$\frac{3}{32\pi^2}\left\{\left(\frac{2A_t^2}{M_S^2}\right)h_t^4 - \left(\frac{A_t^4}{6M_S^2}\right)h_t^4\right\}\phi^4, \tag{3.111}$$

where h_t is the usual top Yukawa coupling in the Standard Model. This gives a threshold contribution to the SM quartic coupling

$$\Delta\lambda_{SM} = \frac{3h_t^4}{16\pi^2}\frac{A_t^2}{M_S^2}\left(2 - \frac{A_t^2}{6M_S^2}\right). \tag{3.112}$$

This means that Eq.(3.97) is modified to

$$\lambda(M_S) = \frac{1}{2}(g_L^2 + g_R^2) + \frac{3A_t^2}{8\pi^2 M_S^2}\left(1 - \frac{A_t^2}{12M_S^2}\right)h_t^4(M_S). \tag{3.113}$$

We can now proceed as in the previous case where we had ignored the threshold contributions, but now using the boundary condition (3.113). This leads to an additional contribution to the upper bound for the lightest Higgs boson mass

$$(\Delta m_h^2)_{mix} = \frac{3g^2 m_t^4 A_t^2}{8\pi^2 m_W^2 M_S^2}\left(1 - \frac{A_t^2}{12M_S^2}\right)\left[1 + 2\left(\gamma_v + \frac{\beta_{h_t^2}}{h_t^2}\right)\ln\frac{M_S^2}{m_t^2}\right]. \tag{3.114}$$

We note that there is an additional factor of 2 multiplying the logarithmic term as compared to Eq.(3.106). We can also put in the $\tan\beta$ dependence, while still ignoring the b-quark and b-squark contribution, an approximation that is valid for moderate values of $\tan\beta \lesssim 20-30$. Furthermore, we note that the one-loop result is maximized for $\mu_1'' = 0$, and since we are interested in the maximum value of the upper bound, we shall take $\mu_1'' = 0$. This leads to the renormalization-group improved leading log result, including two-loop leading-log effects, for the upper bound on the mass of the lightest Higgs boson in the minimal supersymmetric left-right model:

$$\begin{aligned} m_h^2 \quad \lesssim \quad & \frac{1}{2}(g_L^2 + g_R^2)\left(1 - \frac{3m_t^2 t}{8\pi^2 v^2}\right)v^2\cos^2 2\beta + \frac{3}{4\pi^2}\frac{m_t^2}{v^2}\left[t + \frac{X_t}{2}\right] \\ & + \frac{3}{4\pi^2}\frac{m_t^2}{v^2}\left[\frac{1}{16\pi^2}\left(\frac{3}{2}\frac{m_t^2}{v^2} - 32\pi\alpha_3\right)(X_t t + t^2)\right], \end{aligned} \tag{3.115}$$

where

$$\begin{aligned} \tilde{A}_t &= A_t - \mu_1\cot\beta \\ X_t &= \frac{2\tilde{A}_t^2}{M_S^2}\left(1 - \frac{\tilde{A}_t^2}{12M_S^2}\right), \end{aligned} \tag{3.116}$$

and all other quantities have been defined previously. Here we have included the leading D-term contributions [47] which is $\mathcal{O}(m_Z^2 m_t^2)$ as these cannot be ignored for the experimental m_t range. We have ignored the contribution proportional to $(\frac{4}{3}\frac{\alpha_3}{\pi} - \frac{5}{4}\frac{\alpha_t}{\pi})$ in Eq.(3.106) as this accounts for only a 3% correction. We note that the upper bound (3.115) is maximized for $X_t = 6$.

The two-loop radiatively corrected upper bound on the mass of the lightest Higgs boson in the minimal supersymmetric left-right model is plotted in Fig.([?]) for different values of the $SU(2)_R \times U(1)_{B-L}$ breaking scale M_R as a function of $\tan\beta$. We have taken $X_t = 0$ (no mixing), and the scale upto which the model remains perturbative to be $\Lambda = 10^{19}$ GeV. Also plotted in Fig.(3.3) is the corresponding upper bound on the lightest Higgs boson mass in MSSM, Eq.(3.58). We see from this figure that the SLRM upper bound is higher as compared to the MSSM bound for all values of $\tan\beta$. The upper bound increases with increasing M_R, and increase with decreasing M_U. In Fig.(3.4) we show the corresponding results for the case of maximal mixing in the squark sector, $X_t = 6$. We see that the upper bound on the lightest Higgs boson mass can go upto about 145 GeV for $M_R = 10^{10}$ GeV for moderate values of $\tan\beta$. We can summarize these results as

$$\begin{aligned} m_h &\lesssim 135\ GeV, \quad X_t = 0, \\ &\lesssim 145\ GeV, \quad X_t = 6, \end{aligned} \tag{3.117}$$

for $M_S = 1$ TeV, $M_R = 10^{10}$ GeV, and $M_U = 10^{19}$ GeV. This should be compared with the corresponding result for MSSM, Eq.(3.60). We see that the upper bound on the lightest Higgs boson mass in SLRM can be appreciably larger than the corresponding upper bound in the MSSM.

3.5 Conclusions

The spectrum of Higgs bosons in supersymmetric models is quite rich. In MSSM we need at least two Higgs doublets with opposite hypercharge to cancel triangle gauge anomalies, and to generate masses for the up- and down-quarks (and leptons). In MSSM there are five physical Higgs bosons, two neutral CP-even, one neutral CP-odd, and two charged, that remain in the spectrum after spontaneous symmetry breaking. Despite the extended nature of the Higgs spectrum, and despite two independent scales, $SU(2)_L \times U(1)_Y$ and supersymmetry breaking, in the model, there is a (tree level) upper bound on the mass of the lightest Higgs boson which is controlled by weak scale, and is independent of the supersymmetry breaking scale. Although radiative corrections to this result are large, they depend only on the logarithm of the SUSY breaking scale, and are under control. This leads to a definite prediction, $m_h \lesssim 125$ GeV, on the mass of the lightest Higgs boson in MSSM. Similar results holds in models based on $SU(2)_L \times U(1)_Y$ with an extended Higgs sector, with the lightest Higgs mass bound being independent of SUSY breaking scale at the tree level, and radiative corrections similar to those in MSSM.

The MSSM suffers from the problem of baryon (B) abd lepton (L) number nonconservation at the renormalizable level. Although the appearance of all the dimension 4 operators violating B and L can be prevented by involving a discrete R-parity symmetry, such a mechanism is *ad hoc*, and is not required for the internal consistency of MSSM. It is, therefore, more appropriate to invoke gauge symmetries for preventing baryon and lepton number violation to occur at an unacceptable level in supersymmetric models. We have studied one such extended supersymmetric model based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in which B and L violating operators are forbidden due to invariance of the theory under $U(1)_{B-L}$. We have studied the mass of lightest Higgs boson in such a supersymmetric model in detail. Since the extended gauge symmetry must be broken at a scale higher than the electroweak scale, the left-right supersymmetric model has at least three scales associated with it: (i) the electroweak scale; (ii) the SUSY breaking scale; and (iii) the left-right symmetry breaking scale. The lightest Higgs boson mass in these models depends only on the electroweak scale, and is not only independent of the SUSY breaking scale, but also the left-right breaking scale. We have calculated leading and next-to-leading logarithmic radiative corrections to the upper bound on the lightest Higgs boson mass in this class of models. We have shown that this upper bound has a value which is considerably higher than the corresponding upper bound in MSSM. If a Higgs boson is detected which is too heavy to be the Higgs boson of the MSSM, then in the context of supersymmetry the left-right model studied in this Chapter may be a viable alternative.

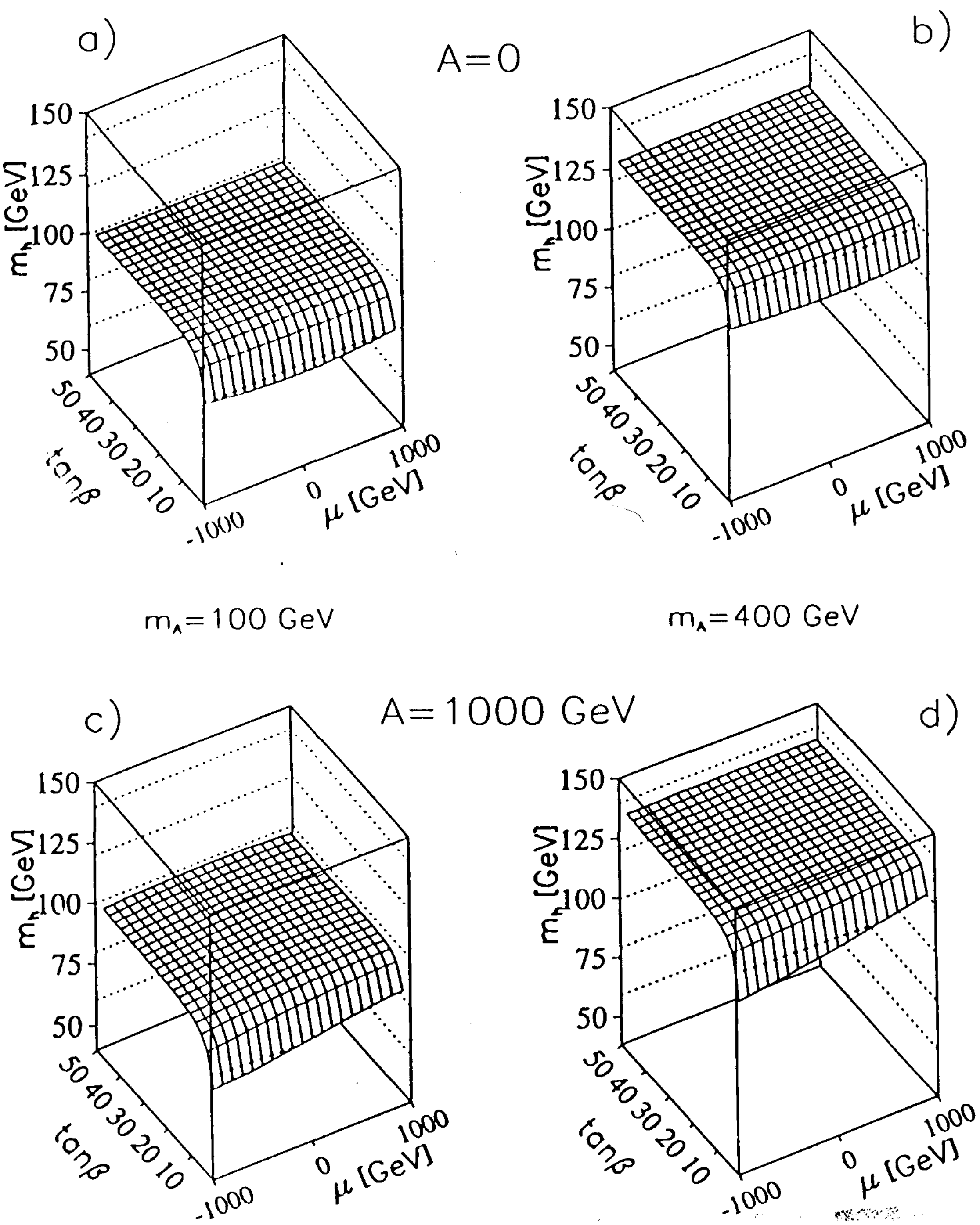

Figure 3.1: Mass of the lightest Higgs boson m_h as a function of μ and $\tan\beta$. Two values of m_A and two values of A are considered: a) $m_A = 100\ GeV$, $A = 0$, b) $m_A = 400\ GeV$, $A = 0\ GeV$, c) $m_A = 100\ GeV$, $A = 1\ TeV$, d) $m_A = 400\ GeV$, $A = 1\ TeV$. We have taken $\tilde{m} = 1\ TeV$.

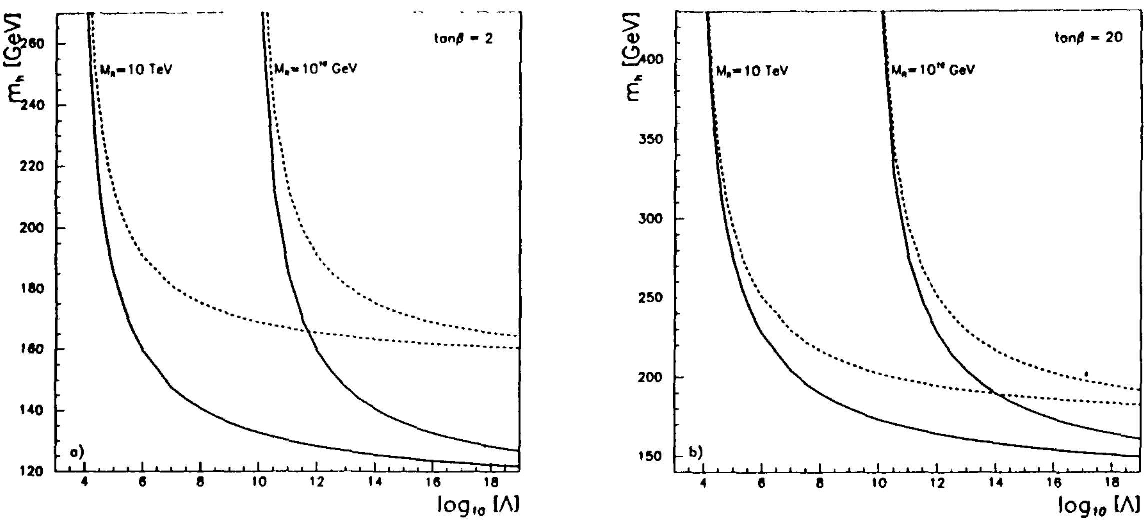

Figure 3.2: The upper bound on the one-loop radiatively corrected mass of the lightest neutral Higgs boson for two different values of $\tan\beta$. The right-handed scale M_R is indicated in the Figure. The bi- and trilinear soft supersymmetry breaking parameters are 1 TeV (solid line) and 10 TeV (dashed line). Supersymmetric Higgs mixing parameters are assumed to vanish, and $m_t^{pole} = 175$ GeV.

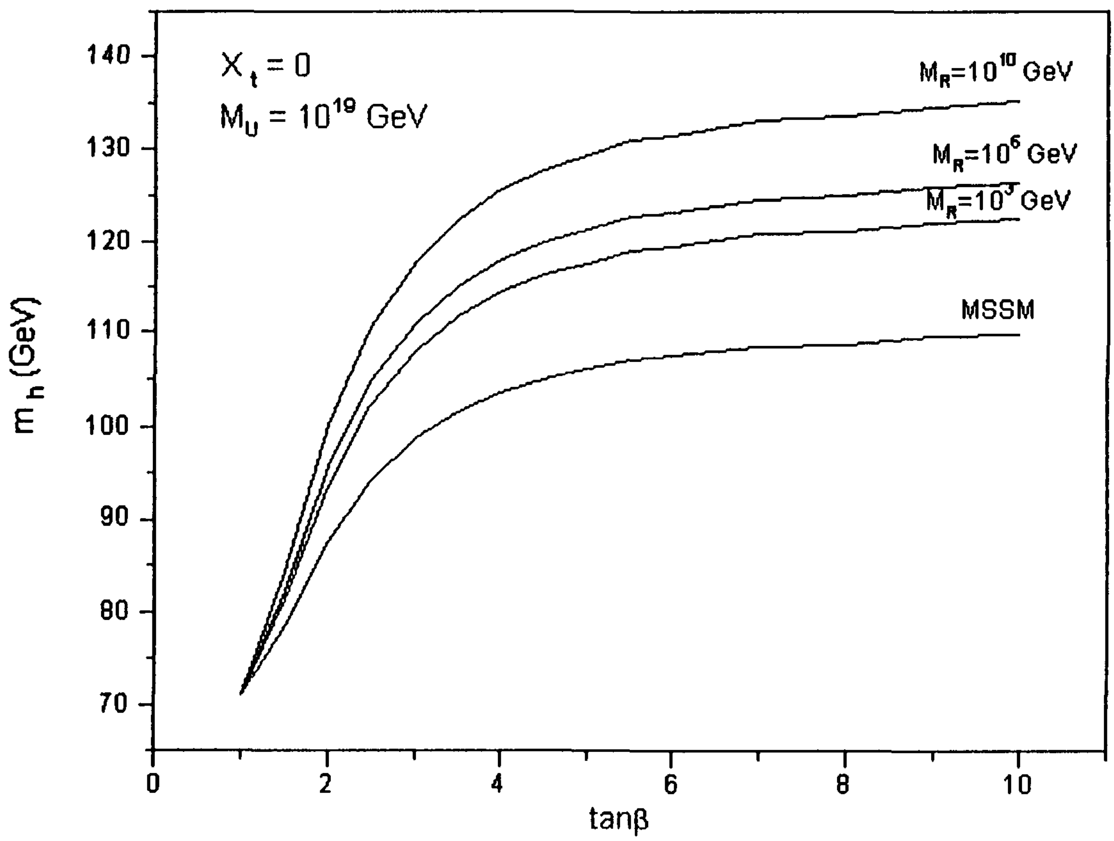

Figure 3.3: The upper bound on the two-loop radiatively corrected mass of the lightest neutral CP-even Higgs boson in the minimal supersymmetric left-right model as a function of $\tan\beta$, for different values of right-handed scale M_R, and no mixing in the squark sector. The supersymmetry breaking scale is taken to be M_S = 1 TeV, and $m_t(m_t)$ = 165 GeV. Also shown is the corresponding upper bound for the lightest Higgs boson mass of MSSM.

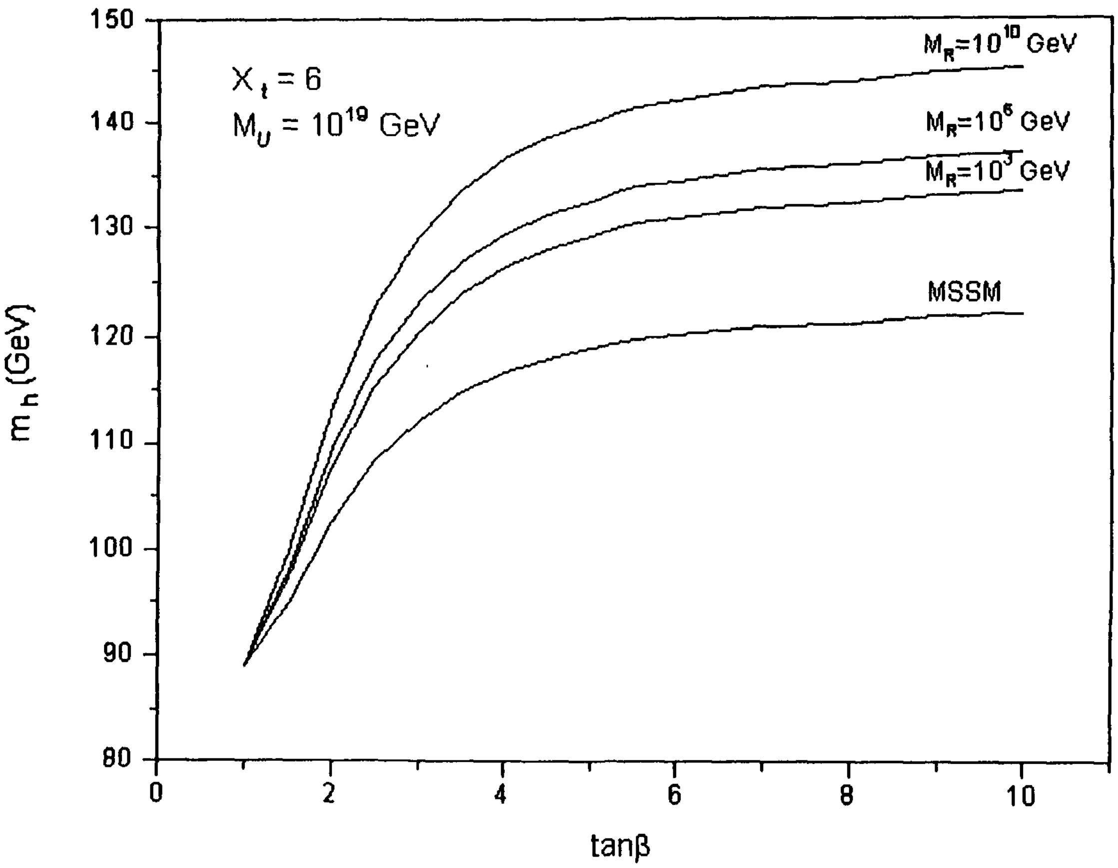

Figure 3.4: Same as in Fig.(3.3), but with maximal mixing in the squark sector.

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Chapter 4

Summary and Concluding Remarks

The Higgs potential of the Standard Model (SM), which is crucial in implenting the mechanism of spontaneous symmetry breaking, and consequently the generation of the masses of gauge bosons and fermions, contains the unknown quartic coupling of the Higgs field. As a consequence, the mass of the only Higgs boson in the SM, which is determined by this quartic coupling, is not known [1]. The Higgs sector is at present the only sector of the SM which is not verified experimentally. If a Higgs boson is discovered and its mass measured, the Higgs potential of the SM can be uniquely determined.

On the other hand the Higgs sector of SM is unstable under one-loop radiative corrections due to quadratic divergences in the Higgs mass parameter. This makes the mass hierarchy $m_W \ll m_{Pl}$ unnatural. In contrast the one-loop radiative corrections to a fermion mass does not exhibt the quadratic divergences, and we, therefore, say that a fermion mass is "natural". It is only when we attempt to understand electroweak symmetry breaking by including the Higgs sector in the SM that we face the problem of quadratic divergences.

The one-loop radiative corrections to the fermion mass m_f are proportional to m_f itself, and only logarithmically divergent. This is because there is a chiral symmetry that keeps the quantum corrections naturally (logarithmically) small. The hope is to find a corresponding symmetry principle to make small boson masses natural, $\delta m^2_{h,W} \lesssim m^2_{h,W}$.

Supersymmetry is at present the only known symmetry which can make small boson masses natural. Supersymmetry exploits the fact that the boson and fermion loop diagrams have opposite signs. If there are equal numbers of fermions and bosons, and if they have couplings as in a supersymmetric theory, the quadratic divergences cancel, with the result that

$$\delta m^2_{h,W} = \mathcal{O}\left(\frac{\alpha}{4\pi}\right) |m^2_B - m^2_F|, \tag{4.1}$$

where m_B and m_F are the masses of the boson and its fermionic partner in a supersymmetric theory. Thus, $\delta m^2_{h,W} \lesssim m^2_{h,W}$, and hence naturally small, if

$$|m^2_B - m^2_F| \lesssim 1\ TeV^2. \tag{4.2}$$

The naturalness argument is the only available theoretical motivation for thinking that supersymmetry may manifest itself at an accessible energy scale. We note that this argument is qualitative, since it does not tell us whether sparticles should appear at 500 GeV, 1 TeV or 2 TeV. However, the naturalness argument tells us that a large hierarchy is intrinsically unstable, and supersymmetry is the most plausible way of stablizing it. Furthermore, many logarithmic divergences are absent in a supersymmetric theory, which stablizes the possible grand unified theory(GUT) Higgs corrections to the mass of the Higgs boson, which is also important for stablizing the hierarchy $m_W \ll m_{GUT}$.

The minimal supersymmetric version of the Standard Model is constructed by simply doubling the particle spectrum, by including a boson for each fermion, and vice versa, of the SM. In addition the minimal supersymmetric standard model (MSSM) contains two Higgs doublets (H_1, H_2) with opposite hypercharges: $Y(H_1) = -\frac{1}{2}$, $Y(H_2) = +\frac{1}{2}$, so as to generate masses for the up- and down-type quarks (and leptons), and to cancel the triangle gauge anomalies. Supersymmetry, gauge invariance and renormalizibility then dictate the superpotential of the MSSM to be

$$W = h_U \hat{Q}\hat{U}^c\hat{H}_2 + h_D\hat{Q}\hat{D}^c\hat{H}_1 + h_L\hat{L}\hat{E}^c\hat{H}_1 + \mu\hat{H}_1\hat{H}_2, \tag{4.3}$$

and contains a bilinear term $\mu\hat{H}_1\hat{H}_2$ coupling the two Higgs doublets, with μ having dimensions of mass. For $\mu = 0$, the Higgs potential of the MSSM has a Peccei-Quinn symmetry [3] leading to a massless axion, which is in contradiction with experiment. Thus, a non-zero μ is needed to break $SU(2)_L \times U(1)_Y$ in the MSSM. However, it is natural for $\mu = 0$ (because of Peccei-Quinn symmetry), or μ to be very large, since the bilinear term is supersymmetric invariant. The first option is ruled out by experiment, whereas the second option effectively obliterates the weak scale. So it is a serious problem why $\mu = \mathcal{O}(m_W)$. Of course, softly broken supersymmetry ensures that the radiative corrections to μ are under control so that small μ is technically natural, thus solving the easy part of the hierarchy problem. However, it does not provide any dynamical reason why μ should be small in the first place. Thus, a small μ in the context of minimal supersymmetric standard model is a serious problem which can endanger the whole idea of low scale supersymmetry.

The simplest mechanism that provides a dynamical source for a term of the form $\mu\hat{H}_1\hat{H}_2$ is the inclusion of an additional [4] singlet field $\hat{N}$. Then, if the superpotential contains a trilinear term of the form $W = \lambda\hat{N}\hat{H}_1\hat{H}_2$, and if N develops a vacuum expectation value $\langle N\rangle \equiv x$, a bilinear $\mu\hat{H}_1\hat{H}_2$ mixing term with $\mu = \lambda x$ is generated. With soft supersymmetry breaking, we expect $x \lesssim \mathcal{O}$ $(1\ TeV)$, and hence $\mu \lesssim \mathcal{O}$ $(1\ TeV) \ll m_{Pl}$, thereby solving the μ problem. Such a singlet field appears in grand unified supersymmetric models [2, 5], in the massless mode sector of many superstring models [6], as well as in superstring models based on E_6 [7], and $SU(5) \times U(1)$ gauge groups [8]. Thus, the minimal extension of the supersymmetric standard model with an additional Higgs singlet superfield, the so called non-minimal supersymmetric standard model, is an appealing alternative to MSSM, and could be an appropriate model for testing the general assumptions of low energy supersymmetry.

As in the Standard Model and its minimal supersymmetric extension, there are a large number of unknown dimensionless Yukawa couplings in the non-minimal supersymmetric standard model with a singlet field $\hat{N}$. Because of the unknown Yukawa couplings, the fermion masses cannot be predicted either in the SM or its supersymmetric extensions. Thus, there is considerable interest in the study of the infra-red

(IR) stable fixed points of SM, and its supersymmetric extensions. The reason is that one may attempt to relate the Yukawa couplings to the gauge couplings via the Pendleton-Ross infra-red stable fixed point (IRSFP) for the top quark Yukawa coupling [9], or via the quasi-fixed point behaviour [10]. The predictive power of various models may, thus, be enhanced if the renormalization group (RG) running of the parameters is dominated by IRSFPs. Typically, these fixed points are for ratios like Yukawa coupling to the gauge coupling, or, in the context of supersymmetric models, the supersymmetry breaking trilinear A-parameter to the gaugino mass, etc. These ratios do not always attain their fixed point values at the weak scale, the range between the GUT (or Planck) scale and the weak scale being too small for the ratios to closely approach the fixed point. Nevertheless, the couplings may be determined by quasi-fixed point behaviour [10], where the value of the Yukawa coupling at the weak scale is independent of its value at the GUT scale, provided the Yukawa couplings at the unification scale are large. For fixed point or quasi-fixed point scenarios to be successful, it is necessary that these fixed points be stable [11].

We have carried out a detailed analysis of the infra-red stable fixed points [12, 13] of the non-minimal supersymmetric standard model (NMSSM). The NMSSM is a viable low energy alternative to the minimal supersymmetric standard model. We have included the possible baryon (B) and lepton (L) number violating interactions (and hence R_p violating interactions) of the highest generation that are allowed by gauge invariance, supersymmetry, renormalizibility and the particle content of the underlying model. We have derived and analyzed the one-loop renormalization group equations for the evolution of Yukawa couplings and R_p violating couplings, taking into account B and L violating couplings one at a time. The analysis of the model yields the surprising and important result that only the simultaneous non-trivial fixed point for the baryon number violating coupling λ''_{233} and the top-quark and b-quark Yukawa couplings h_t and h_b, and the trivial fixed points for λ and k , is stable in the infra-red region. These fixed point values are

$$\lambda''^{*}_{233} \simeq 1.0,$$
$$h_t^* \simeq 0.4,$$

$$
\begin{aligned}
h_b^* &\simeq 0.4, \\
\lambda^* &= 0, \\
k^* &= 0.
\end{aligned}
\tag{4.4}
$$

The fixed point value for the top-quark Yukawa coupling here is lower than its corresponding value of $\simeq 0.7$ in MSSM, and NMSSM, with R-parity (and B and L) conservation, and is incomptable with the measured value of the top-quark mass, $m_t \simeq 174$ GeV. The fixed point values for λ''^*_{233}, h_t^* and h_b^* in (4.4) are identical to the corresponding fixed point solution in the MSSM with baryon number violation [14]. Thus, the true fixed point obtained here provides only a qualitative understanding of the top-quark mass in NMSSM with R_p violation. Furthermore, the baryon number, and R_p, violating coupling has the effect of reducing the infra-red fixed point value of the top quark Yukawa coupling to the same extent in MSSM and NMSSM. The R_p conserving solution with λ''_{233} attaining its trivial fixed point, with h_t and h_b attaining non-trivial fixed points, is infra-red unstable, as is the case for trivial fixed points for λ''_{233} and h_b, with a non-trivial fixed point for h_t. Our analysis shows that the usual Pendleton-Ross type of infra-red fixed point is unstable in the presence of R_p violation, though for small, but negligible, values of h_b and λ''_{233} it could be stable. We have also found that there are no non-trivial infra-red stable fixed points for the lepton number, and R-parity, violating couplings in the NMSSM. Our results are the first in placing strong theoretical constraints on the nature of R_p violating couplings in the NMSSM from fixed-point and stability considerations: the fixed points that are unstable, or the fixed point that is a saddle point, cannot be realized in the infra-red region. The fixed points obtained here are true fixed points, and serve as a lower bound on the relevant R_p violating Yukawa couplings. In particular from our analysis of the simultaneous (stable) fixed point for the baryon number violating coupling λ''_{233} and the top and bottom Yukawa couplings, we infer a lower bound on $\lambda''_{233} \gtrsim 1.0$.

Due to gauge invariance, supersymmetry, and the particle content, the Higgs sector of MSSM, with two Higgs doublets of opposite hypercharge, is highly constrained. The quartic couplings of the Higgs bosons in MSSM are functions of $SU(2)_L$ and $U(1)_Y$ gauge couplings. As a consequence only two independent parameters, usually chosen to be m_A (mass of CP-odd Higgs boson) and $\tan\beta$ ($\equiv v_2/v_1$), are needed to describe the Higgs sector of MSSM at the tree level. Furthermore, the lightest Higgs boson in MSSM has a tree level upper bound of m_Z on its mass [1]. Although radiative corrections to the tree level result can be appreciable, these depend only logarithmically on the SUSY breaking scale, and are, therefore, under control. This results in an upper bound of $\lesssim 125$ GeV on the two-loop radiatively corrected mass of the lightest Higgs boson in MSSM [15].

If the Higgs content of the supersymmetric model based on $SU(2)_L \times U(1)_Y$ is enlarged, there are additional trilinear Yukawa couplings in the superpotential involving the additional Higgs fields and the two doublets H_1 and H_2 of the MSSM. Therefore, a tight constraint on the mass of the lightest Higgs boson, as in MSSM, need not *a priori* hold in such extensions of MSSM. Nevertheless, it has been shown that the upper bound on the lightest Higgs boson mass in these extended models depends only on the weak scale, and dimensionless couplings, and only logarithmically on the SUSY breaking scale. The upper bound is calculable if all the Yukawa couplings remain perturbative below some scale. This upper bound can vary between 150 GeV and 200 GeV, depending on the Higgs structure of the underlying SUSY model [16]. Nonobservation of such a light Higgs boson below this upper bound will rule out an entire class of supersymmetric models based on the gauge group $SU(2)_L \times U(1)_Y$.

The existence of the upper bound on the lightest Higgs boson mass in MSSM, and in models with arbitrary Higgs sectors, has been investigated in a situation where the underlying supersymmetric model respect baryon (B) and lepton (L) number conservation. However, gauge invariance, supersymmetry and renormalizibility allow B and L violating terms in the superpotential of MSSM [17]:

$$W' = \lambda \hat{L}\hat{L}\hat{E}^c + \lambda' \hat{L}\hat{Q}\hat{D}^c + \lambda'' \hat{U}^c\hat{D}^c\hat{D}^c + \mu_i \hat{L}^i \hat{H}_2. \tag{4.5}$$

Each of these terms violate either baryon number or lepton number. These couplings can mediate proton decay at tree level through the exchange of scalar partner of the down quark. The strength of these B and L violating terms is severely constrained by phenomenological constraints. In MSSM, the appearance of these B and L violating couplings is prevented by invoking the discrete R-parity symmetry [18]:

$$R_p = (-1)^{3(B-L)+2S}. \tag{4.6}$$

The multiplicative conservation of R_p forbids all renormalizable B and L violating terms in the superpotential of MSSM. The conservation of R-parity has profound experimental consequences. It implies that superpartners can be produced only in pairs, and that the lightest supersymmetric particle (LSP) is absolutely stable, resulting in the missing energy signature for supersymmetry.

Although the MSSM with R_p conservation is consistent all known observations, the assumption of R_p conservation is *ad hoc*, since it is not required for the internal consistency of the MSSM. Furthermore, all global symmetries, discrete or continuous, tend to be violated by the Planck scale physics effects. It is, therefore, more appealing to have a supersymmetric theory where R-parity is related to a gauge symmetry, and its conservation is automatic because of the invariance of the underlying theory under this gauge symmetry. Indeed R_p conservation follows automatically in certain gauge theories with gauged $(B-L)$, as is suggested by the fact that R_p is simply a Z_2 subgroup of B-L. Thus, if the gauge symmetry of MSSM is extended to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the theory becomes automatically R-parity conserving theory [19]. Such a supersymmetric left-right theory (SLRM) solves the problem of explicit B and L violation of MSSM, and has received much attention recently [20]. Moreover, left-right symmetric theories are interesting in their own right, because, among other appealing features, they offer a simple and natural explanation for the smallness of neutrino mass through the see-saw mechanism.

Such a naturally R-parity conserving theory necessarily involves the extension of the Standard Model gauge group, and since the extended gauge symmetry has to be broken, it involves a "new scale", the scale of left-right gauge symmetry breaking. It is, therefore, important to ask whether the upper bound on the lightest Higgs boson mass in a naturally R-parity conserving theory depends on the scale of the breakdown of the extended gauge group. It has been shown [21] that in supersymmetric left-right model with minimal particle content the upper bound on the mass of the lightest Higgs boson depends only on the gauge couplings and those vacuum expectation values

(VEVs) which break the $SU(2)_L \times U(1)_Y$ symmetry. The upper bound does not depend on VEVs which break the left-right gauge symmetry, which could, *a priori*, be large. Thus, the mass of the lightest Higgs boson in such theory is controlled (at the tree level) by the weak scale and the dimensionless gauge couplings only. Since the former is essentially known, the gauge couplings determine the upper bound on the lightest Higgs boson mass. The one-loop radiative corrections to the upper bound on the lightest Higgs mass are similar to those in the MSSM. Since the one-loop radiative corrections to the lightest Higgs mass are large, we have calculated two-loop leading and next-to-leading log contributions proportional to $m_t^4\alpha_3$ and $m_t^4 h_t^2$, respectively [22] to the upper bound on the lightest Higgs mass in the minimal left-right supersymmetric model (SLRM).

The upper bound on the lightest Higgs boson mass is saturated when there is only one light Higgs boson with the mass of the electroweak breaking scale. Then, at the tree level one can write $m_h^2 < m_{h_{max}}^2 = \lambda_{SM}(\kappa_1^2+\kappa_2^2)$, where $\lambda_{SM} = \frac{1}{2}(g_L^2 + g_R^2)\cos^2 2\beta$, where κ_1 and κ_2 are doublet vacuum expectation values, and g_L and g_R are $SU(2)_L$ and $SU(2)_R$ gauge couplings, respectively, with $\tan\beta = \kappa_2/\kappa_1$. The radiative corrections to this result can be subdivided into three parts:

a) Corrections coming from the one-loop renormalization of λ_{SM}

b) one-loop corrections coming from stop mixing effects

c) the next-to-leading (two-loop order) corrections.

This leads to the two-loop radiatively corrected upper bound on the mass of the lightest Higgs boson in the minimal supersymmetric left-right model [22]:

$$\begin{aligned} m_h^2 &\lesssim \frac{1}{2}(g_L^2+g_R^2)v^2\left(1-\frac{3m_t^2 t}{8\pi^2 v^2}\right) \\ &+\frac{3}{4\pi^2}\frac{m_t^2}{v^2}\left[t+\frac{X_t}{2}+\frac{1}{16\pi^2}\left(\frac{3}{2}\frac{m_t^2}{v^2}-32\pi\alpha_3\right)(X_t t+t^2)\right], \end{aligned} \tag{4.7}$$

where

$$\begin{aligned} t &= \ln\frac{M_S^2}{m_t^2}, \quad v^2 \equiv (\kappa_1^2+\kappa_2^2), \\ \tilde{A}_t &= A_t - \mu_1\cot\beta, \quad X_t = \frac{2\tilde{A}_t^2}{M_S^2}\left(1-\frac{\tilde{A}_t^2}{12M_S^2}\right). \end{aligned} \tag{4.8}$$

Since the gauge coupling corresponding to $SU(2)_R$ group is not known, the upper bound comes from the requirement that the left-right symmetric model remains perturbative below some scale Λ. This fixes the $SU(2)_R$ gauge coupling at the weak scale and allows a numerical evaluation of the upper bound (4.7). The radiative corrections to the upper bound are similar in form to the corresponding radiative corrections in MSSM. The radiatively corrected upper bound on the mass of the lightest Higgs boson in SLRM increases with increasing scale(M_R) of $SU(2)_R$ breaking. For $M_R = 1$ TeV, this bound remains below 135 GeV, whereas for $M_R = 10^{10}$ GeV it remains below 145 GeV, where we take the scale $\Lambda = 10^{19}$ GeV. The upper bound becomes less restrictive for smaller values of Λ or larger values of M_R. It is considerably larger [22] than the corresponding upper bound in MSSM.

In the minimal supersymmetric left-right model that we have considered, R-parity is ultimately broken, albeit spontaneously. There are alternative supersymmetric left-right models in which R-parity is conserved. One of the alternative is to add a couple of triplet fields, $\hat{\Omega}_L(1,3,1,0)$ and $\hat{\Omega}_R(1,1,3,0)$, to the minimal model [23]. The superpotential of this model can be written as

$$\begin{aligned} W_\Omega &= W_{min} + \frac{1}{2}\mu_{\Omega_L}\mathrm{Tr}\,\hat{\Omega}_L^2 + \frac{1}{2}\mu_{\Omega_R}\mathrm{Tr}\,\hat{\Omega}_R^2 + a_L\mathrm{Tr}\,\hat{\Delta}_L\hat{\Omega}_L\hat{\delta}_L + a_R\mathrm{Tr}\,\hat{\Delta}_R\hat{\Omega}_R\hat{\delta}_R \\ &\quad +\mathrm{Tr}\,\hat{\Omega}_L\left(\alpha_L\hat{\Phi}i\tau_2\hat{\chi}^Ti\tau_2 + \alpha_L'\hat{\Phi}i\tau_2\hat{\Phi}^Ti\tau_2 + \alpha_L''\hat{\chi}i\tau_2\hat{\chi}^Ti\tau_2\right) \\ &\quad +\mathrm{Tr}\,\hat{\Omega}_R\left(\alpha_R i\tau_2\hat{\Phi}^Ti\tau_2\hat{\chi} + \alpha_R' i\tau_2\hat{\Phi}^Ti\tau_2\hat{\Phi} + \alpha_R'' i\tau_2\hat{\chi}^Ti\tau_2\hat{\chi}\right), \end{aligned} \tag{4.9}$$

where W_{min} is the superpotential of the minimal supersymmetric left-right model. In this model the breaking of $SU(2)_R$ is achieved in two stages. In the first stage the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is broken to an intermediate symmetry group $SU(2)_L \times U(1)_R \times U(1)_{B-L}$, and at the second stage $U(1)_R \times U(1)_{B-L}$ is broken to $U(1)_Y$ at a lower scale. In this case the low energy effective theory is MSSM with unbroken R-parity. In this model also there is an upper bound on the mass of the lightest Higgs boson which is independent of the $SU(2)_R \times U(1)_{B-L}$ and $U(1)_R \times U(1)_{B-L}$ breaking scales, and depends only on the gauge couplings and the weak scale vacuum expectation values [24]. We, therefore, expect that this bound is numerically similar to the upper bound on the lightest Higgs boson mass in the minimal supersymmetric left-right model.

A second option is to add non-renormalizable terms to the Lagrangian of the minimal left-right supersymmetric model [25], while retaining the minimal Higgs content. The superpotential for this class of models can be written as

$$\begin{aligned}
W_{NR} \;=\; & W_{min} + \frac{a_L}{2M}(\mathrm{Tr}\,\hat{\Delta}_L\hat{\delta}_L)^2 + \frac{a_R}{2M}(\mathrm{Tr}\,\hat{\Delta}_R\hat{\delta}_R)^2 + \frac{c}{M}\mathrm{Tr}\,\hat{\Delta}_L\hat{\delta}_L\mathrm{Tr}\,\hat{\Delta}_R\hat{\delta}_R \\
& + \frac{b_L}{2M}\mathrm{Tr}\,\hat{\Delta}_L^2\mathrm{Tr}\,\hat{\delta}_L^2 + \frac{b_R}{2M}\mathrm{Tr}\,\hat{\Delta}_R^2\mathrm{Tr}\,\hat{\delta}_R^2 \\
& + \frac{1}{M}\left[d_1\mathrm{Tr}\,\hat{\Delta}_L^2\mathrm{Tr}\,\hat{\delta}_R^2 + d_2\mathrm{Tr}\,\hat{\delta}_L^2\mathrm{Tr}\,\hat{\Delta}_R^2\right] \\
& + \frac{\lambda_{ijkl}}{M}\mathrm{Tr}\,i\tau_2\hat{\Phi}_i^T i\tau_2\hat{\Phi}_j\mathrm{Tr}\,i\tau_2\hat{\Phi}_k^T i\tau_2\hat{\Phi}_l + \frac{\alpha_{ijL}}{M}\mathrm{Tr}\,\hat{\Delta}_L\hat{\delta}_L\hat{\Phi}_i i\tau_2\hat{\Phi}_j^T i\tau_2 \\
& + \frac{\alpha_{ijR}}{M}\mathrm{Tr}\,\hat{\Delta}_R\hat{\delta}_R i\tau_2\hat{\Phi}_i^T i\tau_2\hat{\Phi}_j \\
& + \frac{1}{M}\mathrm{Tr}\,\tau_2\hat{\Phi}_i^T\tau_2\hat{\Phi}_j[\beta_{ijL}\mathrm{Tr}\,\hat{\Delta}_L\hat{\delta}_L + \beta_{ijR}\mathrm{Tr}\,\hat{\Delta}_R\hat{\delta}_R] \\
& + \frac{\eta_{ij}}{M}\mathrm{Tr}\,\hat{\Phi}_i\hat{\Delta}_R i\tau_2\hat{\Phi}_j^T i\tau_2\hat{\delta}_L + \frac{\overline{\eta}_{ij}}{M}\mathrm{Tr}\,\hat{\Phi}_i\hat{\delta}_R i\tau_2\hat{\Phi}_j^T i\tau_2\hat{\Delta}_L \\
& + \frac{k_{ql}}{M}\hat{Q}^T i\tau_2\hat{L}\,\hat{Q}^{cT}i\tau_2\hat{L}^c + \frac{k_{qq}}{M}\hat{Q}^T i\tau_2\hat{Q}\,\hat{Q}^{cT}i\tau_2\hat{Q}^c + \frac{k_{ll}}{M}\hat{L}^T i\tau_2\hat{L}\,\hat{L}^{cT}i\tau_2\hat{L}^c \\
& + \frac{1}{M}[j_L\hat{Q}^T i\tau_2\hat{Q}\,\hat{Q}^T i\tau_2\hat{L} + j_R\hat{Q}^{cT}i\tau_2\hat{Q}^c\,\hat{Q}^{cT}i\tau_2\hat{L}^c]. \qquad (4.10)
\end{aligned}$$

It has been shown that the addition of non-renormalizable terms suppressed by a high scale such as Planck mass, $M \sim m_{Pl} \sim 10^{19}$ GeV, with the minimal field content ensures the correct pattern of symmetry breaking in the supersymmetric left-right model. In particular the scale of parity breakdown is predicted to be in the intermediate region, $M_R \gtrsim 10^{10} - 10^{11}$ GeV, and R-parity remains exact. This theory contains singly charged and doubly charged Higgs scalars with a low mass of order M_R^2/m_{Pl}, which are experimentally accessible. However, what is different is the nature of see-saw mechanism. Whereas in the renormalizable version the see-saw mechanism takes its canonical form, in the non-renormalizable case it takes a form similar to what occurs in the non-supersymmetric left-right models, with the neutrino mass depending on the unknown parameters of the Higgs potential. This in general leads to different neutrino mass spectra, which can be experimentally distinguished.

The non-renormalizable terms give an extra contribution to the tree level upper bound on the lightest Higgs boson mass that obtains in the minimal supersymmetric left-right model. This contribution can be written as [24]

$$\mathcal{O}\left(\frac{v_R^2}{m_{Pl}^2}\right)\langle\Phi^0\rangle^2+\mathcal{O}\left(\frac{1}{m_{Pl}^4}\right)\langle\Phi^0\rangle^4, \tag{4.11}$$

where v_R is the vacuum expectation value which breaks the left-right symmetry in these models, and $\langle\Phi^0\rangle$ is the usual doublet vacuum expectation value. For the right handed breaking scale $\sim 10^{10}$ GeV, the contribution (4.11) is numerically negligible, so that the upper bound on the lightest Higgs mass for this class of models is essentially the same as in the case of minimal model that we have studied. Therefore, our result on the light Higgs mass bound in the minimal supersymmetric left-right model is a representative of a wide class of left-right supersymmetric models.

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Curriculum Vitae

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