RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 47, NUMBER 11 1 JUNE 1993

Higgs scalar in the grand desert with observable proton lifetime in SU(5) and small neutrino masses in SO(10)

M. L. Kynshi and M. K. Parida
Physics Department, North-Eastern Hill University, Bijni Complex, Laitumkhrah, Shillong-793003, India
(Received 29 December 1992)

We find that the presence of a real scalar in the grand desert transforming as $\zeta(3,0,8)$ under $SU(2)_L \times U(1)_Y \times SU(3)_C$ ensures the agreement of the GUT predictions with the data from CERN LEP and proton lifetime (τ_p). The mass of ζ is predicted to be close to the Peccei-Quinn symmetry-breaking scale. The computation of the threshold effects in SU(5) with Higgs representations **24**, **5**, and **75** shows that the maximum allowed τ_p for reasonable superheavy Higgs boson masses is accessible to experimental tests at low energies. The additional predictions in SO(10) are small neutrino masses compatible with solutions to the solar-neutrino problem and the dark matter of the Universe.

PACS number(s): 12.10.Dm, 12.15.Cc, 12.15.Ff, 13.30.Ce

Precision measurements of $\sin^2\theta_W$ and strong interaction coupling at the CERN e^+e^- collider LEP combined with an improved estimation of the electromagnetic fine structure constant at the Z mass have led to a very accurate determination of the gauge couplings of the standard model (SM). This has revived interest in grand unified theories (GUT's) with or without supersymmetry (SUSY) [1–7]. Minimal SUSY GUT's or nonSUSY GUT's [1,7] with intermediate scales [2–5] are consistent with the LEP data and the existing limit on the proton lifetime (τ_p), but the minimal nonSUSY SU(5) theory predicts $\sin^2\theta_W$ and τ_p significantly lower than the experimental data. The disagreement with the data persists in a number of grand desert models even if threshold effects are taken into consideration. But recently the addition of more than one scalar or fermionic degrees of freedom with low masses [5–7] has been shown to be consistent with the data. When more than one scalar multiplet under the SM are kept light, we need more than one additional fine-tuning of the parameters in the GUT Lagrangian.

In attempting to confine such an unnatural act of additional fine-tuning of parameters to a minimum we demonstrate here that the presence of a single real scalar multiplet $\zeta(3,0,8)$, which we call a colorful bloom [8], ensures complete agreement with the data provided M_ζ is close to the Peccei-Quinn symmetry-breaking scale or the geometrical mean of the Planck and electroweak scales. We discuss the model embedding in SU(5) and SO(10) GUT's and compute threshold effects on the grand unification mass M_U, the ζ mass (M_ζ) and the GUT coupling constant showing that the maximally allowed τ_p in both the GUT's even for a factor of 100 mass splitting among superheavy Higgs-scalar components is accessible to experimental tests at low energies. The dominant induced contributions to Majorana neutrino masses in SO(10) are compatible with values needed for the solar neutrino puzzle in the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism, and dark matter of the Universe.

The renormalization group equations (RGE's) in the mass range $M_Z \le \mu \le M_\zeta$ are written as

$$\mu\frac{\partial\alpha_i(\mu)}{\partial\mu}=\frac{a_i}{2\pi}\alpha_i^2(\mu)+\frac{1}{8\pi}2\sum_{j=1}^{3}b_{ij}\alpha_i^2(\mu)\alpha_j(\mu)\ , \qquad (1)$$

where $\alpha_i=g_i^2/4\pi$ and $i=1$, 2, and 3 refer to the $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$ gauge couplings, respectively. In the mass range $M_\zeta \le \mu \le M_U$ the RGE's in (1) have the coefficients a_i' and b_{ij}' defined below. The GUT-threshold effects on the gauge couplings are included through the matching functions [9] $\lambda_i(M_U)$, $i=1,2,3$:

$$\frac{1}{\alpha_i(M_U)}=\frac{1}{\alpha_G}-\frac{\lambda_i}{12\pi}\ ,\quad i=1,2,3\ . \qquad (2)$$

For $\mu \le M_Z - M_\zeta$, the coefficients a_i and b_{ij} are taken to be due to the standard three fermion generations ($n_g=3$) and one light Higgs doublet ($n_h=1$): $a_1=\frac{41}{10}$, $a_2=-\frac{19}{6}$, $a_3=-7$, $b_{11}=\frac{199}{50}$, $b_{12}=\frac{27}{10}$, $b_{13}=\frac{44}{5}$, $b_{21}=\frac{9}{10}$, $b_{22}=\frac{35}{6}$, $b_{23}=12$, $b_{31}=\frac{11}{10}$, $b_{32}=\frac{9}{2}$, and $b_{33}=-26$. For $\mu \le M_\zeta - M_U$, the additional contribution due to the $\zeta(3,0,8)$ is included to evaluate the coefficients $a_2'=-\frac{1}{2}$, $a_3'=-\frac{11}{2}$, $b_{22}'=\frac{161}{2}$, $b_{23}'=108$, $b_{32}'=\frac{81}{2}$, $b_{33}'=37$, $a_1'=a_1$ and $b_{ij}'=b_{ij}$ for other values of i and j. Using the combinations $\alpha^{-1}(M_Z)-\frac{8}{3}\alpha_2^{-1}(M_Z)$ and $\alpha^{-1}(M_Z)-\frac{8}{3}\alpha_3^{-1}(M_Z)$ we get two simultaneous equations in $\ln(M_U/M_Z)$ and $\ln(M_\zeta/M_Z)$; their solutions yield

$$\ln\frac{M_U}{M_Z}=\frac{16\pi}{187\alpha}\left[\frac{7}{8}-\frac{10\alpha}{3\alpha_s}+\sin^2\theta_W\right]+\frac{5}{1122}[16(P_3^\zeta+P_3^U)-9(P_2^\zeta+P_2^U)-7(P_1^\zeta+P_1^U)]+\frac{5}{3366}[7\lambda_1+9\lambda_2-16\lambda_3]\ , \qquad (3)$$

 ©1993 The American Physical Society

$$\ln\frac{M_\xi}{M_Z}=\frac{4\pi}{187\alpha}\left[15+\frac{23\alpha}{\alpha_s}-63\sin^2\theta_W\right]+\frac{1}{187}[48(P_2^\xi+P_2^U)-23(P_3^\xi+P_3^U)-25(P_1^\xi+P_1^U)]$$
$$+\frac{1}{561}[25\lambda_1-48\lambda_2+23\lambda_3]\ , \qquad (4)$$

where $P_i^\xi=B_{ij}X_j^\xi$, $P_i^U=B'_{ij}X_j^U$, and the repeated index implies summation,

$$X_j^\xi=\ln\frac{\alpha_j(M_\xi)}{\alpha_j(M_Z)}\ ,\quad X_j^U=\ln\frac{\alpha_j(M_U)}{\alpha_j(M_\xi)}\ ,\quad B_{ij}=b_{ij}/a_j\ ,$$

$B'_{ij}=b'_{ij}/a'_j$. The first (second) term containing α, α_3, and $\sin^2\theta_W$ (P_1, P_2, and P_3) represents one- (two-)loop contributions. The third term containing λ_i's represent threshold effects on the respective masses. Thus even though there is no new gauge-boson-mass threshold for $M_Z<\mu<M_U$, the GUT threshold effects induce changes on M_ξ. It has been found by one of us (M.K.P.) [10] that the electromagnetic fine structure constant matching at the electroweak scale is possible only by taking into account the threshold effect on the GUT-coupling constant α_G which is obtained from the evolution equation for $\alpha^{-1}(M_Z)=\frac{5}{3}\alpha_1^{-1}(M_Z)+\alpha_2^{-1}(M_Z)$:

$$\frac{1}{\alpha_G}=\frac{3}{8\alpha}+\frac{1}{187\alpha}\left[\frac{347}{8}+\frac{466\alpha}{3\alpha_s}-271\sin^2\theta_W\right]$$
$$-\frac{1}{4488\pi}[932(P_3^\xi+P_3^U)-945(P_2^\xi+P_2^U)+1135(P_1^\xi+P_1^U)]+\frac{1}{13464\pi}(1135\lambda_1-945\lambda_2+932\lambda_3)\ , \qquad (5)$$

where the λ-dependent (-independent) terms represent threshold (loop) contributions. We use the following values as the input parameters at the Z mass consistent with the LEP data

$$\sin^2\theta_W=0.2333\pm0.0008\ ,$$
$$\alpha_3=0.113\pm0.005\ , \qquad (6)$$
$$\alpha^{-1}=127.9\pm0.2\ ,$$

leading to $\alpha_1(M_Z)=0.0171\pm0.00006$, $\alpha_2(M_Z)=0.03351\pm0.0002$. At first, ignoring threshold effects but including two-loop contributions, we evaluate $\alpha_i^{-1}(\mu)$ as a function of μ by numerically integrating (1) in the range $\mu=M_Z-M_\xi$ and the corresponding equation in the range $\mu=M_\xi-M_U$ while changing M_ξ and M_U around their one-loop values given by (3) and (4) until we obtain an excellent meeting point as shown in Fig. 1. We find that the introduction of $\xi(3,0,8)$ decreases the slope of the $\alpha_2^{-1}(\mu)$ and $\alpha_3^{-1}(\mu)$ trajectories for $\mu\geq M_\xi$ while $\alpha_1^{-1}(\mu)$ remains unaffected. Both numerical solutions and the analytic expressions (3)–(5) ignoring threshold effects are consistent with $M_\xi^0=10^{10.2}$ GeV, $M_U^0=10^{15}$ GeV, $\alpha_G^{-1}=39.05$ with $X_1^\xi=0.330$, $X_2^\xi=-0.336$, $X_3^\xi=-1.411$, $X_1^U=0.081$, $X_2^U=0.067$, and $X_3^U=-0.077$.

The Higgs scalars necessary for spontaneous symmetry breaking (SSB) of a GUT to $U(1)_{em}\times SU(3)_C$ and ξ can be found in suitable representations of popular models such as SU(5), SO(10), E_6, SO(18), $SU(8)_L\times SU(8)_R$, SU(15), and SU(16), etc. When threshold effects are neglected all the models yield almost the same solutions for M_U, M_ξ, and α_G; but the threshold effects on these quantities could differ from one model to the other. In order to look into the possibility of testifying or ruling out the model through proton-lifetime measurements it is essential to compute the maximum allowed value of τ_p including threshold effects. For the sake of simplicity we compute the effects in SU(5) and SO(10) only. In SU(5), in addition to the Higgs representations **24** and **5** necessary for the SSB at the M_U and M_Z scales, ξ is contained in the representation **75**. We specify the superheavy components of these representations with respective transformation properties under SM: $\mathbf{5}\supset C(1,-\frac{2}{3},3)$, $\mathbf{24}\supset D_1(3,0,1)+D_2(1,0,8)$, $\mathbf{75}\supset E_1(1,\frac{10}{3},3)+E_2(2,\frac{5}{3},3)+E_3(1,-\frac{10}{3},\bar{3})+E_4(2,-\frac{5}{3},\bar{3})+E_5(2,-\frac{5}{3},\bar{6})+E_6(2,\frac{5}{3},6)+E_7(1,0,8)$. Contributions to the matching functions including those due to 12 superheavy degenerate gauge-bosons are

$$\lambda_1=5+\tfrac{2}{5}\eta_C+5\eta_{E_1}+5\eta_{E_2}+5\eta_{E_3}+\tfrac{5}{2}\eta_{E_4}+5\eta_{E_5}+5\eta_{E_6}\ ,$$
$$\lambda_2=3+2\eta_{D_1}+\tfrac{3}{2}\eta_{E_2}+\tfrac{3}{2}\eta_{E_4}+3\eta_{E_5}+3\eta_{E_6}\ ,$$
$$\lambda_3=2+\eta_C+3\eta_{D_2}+\tfrac{1}{2}\eta_{E_1}+\eta_{E_2}+\eta_{E_3}+\eta_{E_4}+5\eta_{E_5}+5\eta_{E_6}+3\eta_{E_7}\ ,$$

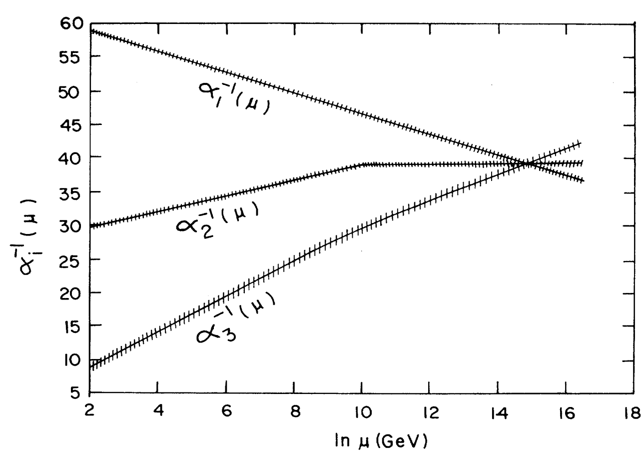

FIG. 1. Evolution of the three inverse gauge couplings in the model where the slope changes correspond to $M_\xi=10^{10.2}$ GeV and the unification at $M_U=10^{15}$ GeV.

where $\eta_i = \ln(M_i/M_u)$. In the degenerate case when al superheavy Higgs boson masses are equal ($\eta_i = \eta = \pm \ln\beta$ we obtain

$$\Delta\ln\left[\frac{M_U}{M_Z}\right] = \frac{5}{3366}(7\lambda_1+9\lambda_2-16\lambda_3) = \frac{25-36\eta}{561}\ ,$$

$$\Delta\ln\left[\frac{M_\xi}{M_Z}\right] = \frac{1}{561}(25\lambda_1-48\lambda_2+23\lambda_3) = \frac{27+567\eta}{561}\ ,$$

$$\Delta\alpha_G^{-1} = \frac{1}{13464\pi}(1135\lambda_1-945\lambda_2+932\lambda_3)$$

$$=(4704+37776\eta)/13464\pi\ ,$$

leading to $M_\xi/M_{\xi^0} = 10^{\pm 1}$, $M_U/M_{U^0} = 10^{{}^{-0.03}_{+0.08}}$ and $\alpha_G^{-1} = 39.05^{+2.1}_{-1.9}$ for $\beta=10$. In the nondegenerate case the occurrence of ten unknown superheavy masses introduces arbitrariness into the theory, but the extremal uncertainties can nevertheless be estimated by assuming that each $\eta_i = +\ln\beta$ or $-\ln\beta$ corresponds to a mass splitting by a factor β^2 among the superheavy-scalar components. In the absence of any precise value of β we extend the arguments used in the SM where the one-loop radiative corrections and unitarity bound on the tree-level amplitudes restrict the scalar boson mass to vary only by a factor 10 on either side of the symmetry-breaking scale. Thus we will assume $|\ln\beta| \leq 2.3025$ saturating these limits. We extremize $\Delta\ln(M_U/M_Z)$ and find that this is possible if $\eta_{D_1} = \eta_{E_1} = \eta_{E_2} = \eta_{E_3} = \eta_{E_4} = \eta^{(+)}$ and $\eta_C = \eta_{D_2} = \eta_{E_5} = \eta_{E_6} = \eta_{E_7} = \eta^{(-)}$ leading to

$$\Delta\ln\left[\frac{M_U}{M_Z}\right] = 5(30+102\eta^{(+)}-145.2\eta^{(-)})/3366\ ,$$

$$\Delta\ln\left[\frac{M_\xi}{M_Z}\right] = (27+204\eta^{(+)}+363\eta^{(-)})/561\ ,$$

$\Delta\alpha_G^{-1} = (4704+15093\eta^{(+)}+22683\eta^{(-)})/13464\pi$. The results of extremal threshold corrections are presented in Table I where the $\pm$ ($\mp$) sign in the exponent of 10 have been obtained with $\eta^{(+)} = -\eta^{(-)} = \ln\beta(-\ln\beta)$ for $\beta = \sqrt{10}-10$. It is to be noted that $\beta=\sqrt{10}$ (10) implies a mass splitting by a factor 10 (100) among the superheavy Higgs-scalar components. In all estimations for the proton lifetime presented in Table I, we have used $\tau_p/\tau_p^0 = (M_U^4 \alpha_G^{0^2}/M_U^{0^4}\alpha_G^2)$. Including threshold effects we have obtained for $\beta=10$ in the nondegenerate case $M_\xi = 10^{10.2\mp 0.3\pm 0.63}$ GeV, $M_U = 10^{15^{+0.38}_{-0.35}\pm 0.20}$ GeV, $\alpha_G^{-1} = 39.05^{-0.5}_{+0.7}\pm 0.70$, and $\tau_p = 10^{31^{+1.6}_{-1.4}\pm 0.80}$ yr, where the second uncertainties are due to the experimental errors in the input parameters and the first are due to threshold effects.

TABLE I. Threshold effects on the mass scales M_U and M_ξ for different factors of mass splitting among the superheavy scalar components in the SU(5) GUT. The first row (D) represents values for the degenerate case.

β	M_ξ/M_ξ^0	M_U/M_U^0	α_G^{-1}	τ_p/τ_p^0
10(D)	$10^{{}^{-1.0}_{+1.0}}$	$10^{{}^{+0.08}_{-0.02}}$	$39.0^{+2.1}_{-1.9}$	$10^{{}^{+0.30}_{-0.08}}$
10	$10^{{}^{-0.26}_{+0.30}}$	$10^{{}^{+0.38}_{-0.35}}$	$39.0^{-0.5}_{+0.7}$	$10^{{}^{+1.6}_{-1.4}}$
5	$10^{{}^{-0.17}_{+0.22}}$	$10^{{}^{+0.27}_{-0.23}}$	$39.0^{-0.3}_{+0.4}$	$10^{{}^{+1.08}_{-0.92}}$
$\sqrt{10}$	$10^{{}^{-0.12}_{+0.16}}$	$10^{{}^{+0.20}_{-0.16}}$	$39.0^{-0.1}_{+0.3}$	$10^{{}^{+0.80}_{-0.64}}$

Thus, even if we allow a maximum factor of 10 due to uncertainties in the estimation of $p \to e^+\pi^0$, matrix elements $\tau_p \leq 10^{34}$ yr, which is accessible to the Super-Kamiokande experiments. As the accepted range for the Peccei-Quinn symmetry breaking scale is 10^9-10^{12} GeV the value of M_ξ predicted by the model is within this range and also close to the geometrical mean between the Planck and the electroweak scales. The neutrino mass in SU(5) is, however, exactly zero.

The additional advantage of the desertlike SO(10) over SU(5) is its potential to yield a nonvanishing neutrino mass. We have checked that the conventional seesaw mechanism [11,12] including radiative corrections yields too small a neutrino mass while the dominant induced contribution [13] accounts for the values needed for the solar neutrino puzzle and the dark matter of the Universe. The symmetry SO(10) is broken to SM through Higgs representations **45** and **126**. The representation **10** contains the standard Higgs doublet that drives the symmetry breaking at the electroweak scale, besides the real scalar $\xi(3,0,8) \subset \mathbf{210}$ of SO(10). Thus, only one additional fine-tuning of the parameters is needed in the GUT Lagrangian to keep ξ light. The superheavy components of these Higgs representations under SM are decomposed as

$$\mathbf{10} \supset T_1(1,-\tfrac{2}{3},\bar{3}) + T_2(1,\tfrac{2}{3},3) + T_3(2,-1,1)\ ,$$

$$\mathbf{45} \supset F_1(3,0,1) + F_2(1,0,8)\ ,$$

$$\begin{aligned}\mathbf{126} \supset\ & L_1(3,2,1)+L_2(3,\tfrac{2}{3},3)+L_3(3,-\tfrac{2}{3},6)+R_1(1,4,1)+R_2(1,2,1)+R_3(1,\tfrac{2}{3},\bar{3})+R_4(1,\tfrac{2}{3},\bar{6})\\ &+R_5(1,\tfrac{4}{3},\bar{3})+R_6(1,-\tfrac{4}{3},\bar{6})+R_7(1,-\tfrac{8}{3},\bar{3})+R_8(1,\tfrac{8}{3},\bar{6})+\xi_1(2,1,1)+\xi_2(2,-1,1)+\xi_3(2,-\tfrac{1}{3},3)\\ &+\xi_4(2,-\tfrac{7}{3},3)+\xi_5(2,\tfrac{7}{3},\bar{3})+\xi_6(2,\tfrac{1}{3},\bar{3})+\xi_7(2,1,8)+\xi_8(2,-1,8)+S_1(1,\tfrac{2}{3},3)+S_2(1,-\tfrac{2}{3},3)\ ,\end{aligned}$$

$$\begin{aligned}\mathbf{210} \supset\ & \Sigma_{R_1}(1,2,1)+\Sigma_{R_2}(1,-2,1)+\Sigma_{R_3}(1,-\tfrac{4}{3},3)+\Sigma_{R_4}(1,\tfrac{2}{3},3)+\Sigma_{R_5}(1,-\tfrac{10}{3},3)+\Sigma_{R_6}(1,\tfrac{10}{3},\bar{3})\\ &+\Sigma_{R_7}(1,-\tfrac{2}{3},\bar{3})+\Sigma_{R_8}(1,\tfrac{4}{3},\bar{3})+\Sigma_{R_9}(1,2,\bar{8})+\Sigma_{R_{10}}(1,-2,8)+\Sigma_{R_{11}}(1,0,8)+\Sigma_{L_1}(3,0,1)+\Sigma_{L_2}(3,-\tfrac{4}{3},3)\\ &+\Sigma_{L_3}(3,\tfrac{4}{3},\bar{3})+\chi_1(2,3,1)+\chi_2(2,1,1)+\chi_3(2,\tfrac{5}{3},3)+\chi_4(2,-\tfrac{1}{3},3)+\chi_5(2,\tfrac{1}{3},6)+\chi_6(2,-\tfrac{5}{3},6)+(\chi_i\to\bar{\chi}_i)\\ &+\rho_1(2,\tfrac{5}{3},3)+\rho_2(2,-\tfrac{1}{3},3)+(\rho_i\to\bar{\rho}_i)+\sigma_1(1,-\tfrac{4}{3},3)+\sigma_2(1,\tfrac{4}{3},\bar{3})+\sigma_3(1,0,8)\ .\end{aligned}$$

There is a natural constraint on some of these masses: along with the left-right discrete symmetry (=parity), the Pati-Salam [14] gauge symmetry

$$SU(2)_L \times SU(2)_R \times SU(4)_C (\equiv G_{224}, g_{2L} = g_{2R})$$

becomes a good symmetry as soon as SO(10) is restored for $\mu \geq M_U$. Since the G_{224} multiplet $(1,3,1) \subset \mathbf{45}$ has been absorbed as would-be Goldstone bosons by the superheavy gauge bosons of mass M_U, its left-handed counterpart $(3,1,1) \supset F_1(3,0,1)$ under the SM must have a mass $M_{F_1} \leq M_U$. Similarly, $M_{T_3} \leq M_U$ since T_3 and the standard doublet are contained in the G_{224} multiplet (2,2,1) of the Higgs representation **10**. Also the G_{224} multiplet $\Sigma_L(3,1,15) \supset \xi(3,0,8)$ and the parity restoration for $\mu \geq M_U$ restricts $M_{\Sigma_{L_i}} \leq M_U$, $M_\Sigma \leq M_U$. For the sake of simplicity we take all the superheavy scalars transforming nontrivially under $SU(2)_L$ or $SU(2)_R$ of G_{224} to be degenerate in masses:

$$M_{L_i} = M_{\Delta_L} \leq M_U \ , \quad M_{R_i} = M_{\Delta_R} \leq M_U \ ,$$

$$M_{\Sigma_{R_i}} = M_{\Sigma_R} \leq M_U \ , \quad M_{\Sigma_{L_i}} = M_{\Sigma_L} \leq M_U \ ,$$

$$M_{\xi_i} = M_\xi \ , \quad M_{\chi_i} = M_\chi \ , \quad M_{\bar{\chi}_i} = M_{\bar{\chi}} \ ,$$

$$M_{\rho_i} = M_\rho \ , \quad M_{\bar{\rho}_i} = M_{\bar{\rho}} \ .$$

It may be noted that the masses M_ξ, M_χ, $M_{\bar{\chi}}$, M_ρ, and $M_{\bar{\rho}}$ need not necessarily be lighter than M_U. Including the scalar contributions we obtain

$$\Delta \ln(M_U/M_Z) = \frac{1}{3366}(-66\eta_{T_1} - 66\eta_{T_2} + 66\eta_{T_3} + 18\eta_{F_1} - 48\eta_{F_2} + 612\eta_{\Delta_L} - 348\eta_{\Delta_R} - 132\eta_\xi - 66\eta_{S_1}$$
$$- 66\eta_{S_2} + 558\eta_{\Sigma_L} - 162\eta_{\Sigma_R} - 62\eta_\chi - 62\eta_{\bar{\chi}} + 66\eta_\rho + 66\eta_{\bar{\rho}} - 12\eta_{\sigma_1} - 12\eta_{\sigma_2} - 240\eta_{\sigma_3}) \ ,$$

$$\Delta \ln\left[\frac{M_\xi}{M_Z}\right] = \frac{1}{6732}\{132(\eta_{T_1} + \eta_{T_2} - \eta_{T_3} + \eta_{S_1} + \eta_{S_2} + 2\eta_\xi - 2\eta_\rho - 2\eta_{\bar{\rho}})$$
$$- 486\eta_{F_1} + 174\eta_{F_2} - 7344\eta_{\Delta_L} + 1044\eta_{\Delta_R} - 4808\eta_{\Sigma_L} + 9828\eta_{\Sigma_R} + 1268(\eta_\chi + \eta_{\bar{\chi}})$$
$$+ 354(\eta_{\sigma_1} + \eta_{\sigma_2}) + 348\eta_{\sigma_3}\} \ .$$

The superheavy degenerate gauge bosons of mass M_U contribute equally to the matching functions $\lambda_1^V = \lambda_2^V = \lambda_3^V = 8$; thus, the constant terms cancel out from the threshold effects on M_U and M_ξ. In the degenerate case $\eta_i = \eta = -\ln\beta$ ($\eta = +\ln\beta$ is not allowed because of parity-restoration constraint) and this gives $\Delta \ln(M_U/M_Z) = -0.0175 \ln\beta$, $\Delta \ln(M_\xi/M_Z) = +0.85 \ln\beta$, and $\Delta\alpha_G^{-1} = 0.212 - 3.62 \ln\beta$, leading to $M_\xi/M_\xi^0 = 10^{+0.85}$, $M_U/M_U^0 = 10^{-0.02}$, $\alpha_G^{-1} = 39.05 - 12$ with $\tau_p/\tau_p^0 = 10^{-0.24}$ for $\beta = 10$. In the nondegenerate case, maximal increase in M_U or τ_p implies

$$\eta_{F_1} = \eta_{T_3} = \eta_{\Delta_L} = \eta_{\Sigma_L} = 0 \ ,$$

$$\eta_{T_1} = \eta_{T_2} = \eta_{F_2} = \eta_{\Delta_R} = \eta_\xi = \eta_{S_1} = \eta_{S_2} = \eta_{\Sigma_R}$$
$$= \eta_\chi = \eta_{\bar{\chi}} = \eta_{\sigma_1} = \eta_{\sigma_2} = \eta_{\sigma_3} = -\ln\beta \ ,$$

$$\eta_\rho = \eta_{\bar{\rho}} = \ln\beta \ .$$

Similarly maximal decrease in M_U or τ_p implies

$$\eta_{\Delta_R} = \eta_{\Sigma_R} = 0 \ ,$$

$$\eta_{T_3} = \eta_{F_1} = \eta_{\Delta_L} = \eta_{\Sigma_L} = \eta_\rho = \eta_{\bar{\rho}} = -\ln\beta \ ,$$

$$\eta_{T_1} = \eta_{T_2} = \eta_{F_2} = \eta_\xi = \eta_{S_1} = \eta_{S_2} = \eta_\chi$$
$$= \eta_{\bar{\chi}} = \eta_{\sigma_1} = \eta_{\sigma_2} = \eta_{\sigma_3} = \ln\beta \ .$$

Table II represents threshold effects on the mass scales and α_G^{-1} for value of $\beta = \sqrt{10} - 10$ and the corresponding effects on τ_p. Again for a factor of 100 mass splitting among the superheavy-scalar components ($\beta = 10$) under the parity restoration constraint the maximal increase in τ_p is found to be $10^{+1.6}$ and this makes the model accessible to experimental test within the SuperKamiokande limit.

The right-handed Majorana neutrino mass in the SO(10) model is $m_{N_R} \simeq (0.1-1)M_U$. The usual seesaw formula [11,12] with appropriate radiative corrections [12] is found to yield negligible contribution as compared to the induced mass of the left-handed neutrinos arising out of the diagram shown in Fig. 2, where $\phi(2,1,1) \subset \mathbf{10}$ is the standard doublet and $\Delta_L(3,1,1)$ and $\Delta_R(1,-1,1)$ originate from the triplets contained in the Higgs representation **126** of SO(10). The induced contribution turns out to be $m_{\nu_i} = \lambda h_i \langle\phi^0\rangle^2 \langle\Delta_R 0\rangle / M_{\Delta_L}^2$ ($i = e, \mu, \tau$). Assuming that the same Yukawa coupling is responsible for the masses of all particles in a given generation of fermions we used $h_i \langle\phi^0\rangle = m_{q_i}$ for the mass of the up quark q_i of ith generation ($i = e, \mu, \tau$), and the quartic Higgs coupling $\lambda \simeq 1$. We then obtain

TABLE II. Same as Table I but for the SO(10) GUT.

β	M_ζ/M_ζ^0	M_U/M_U^0	α_G^{-1}	τ_p/τ_p^0
10(D)	$10^{\substack{+0.85\\-0.0}}$	$10^{\substack{-.02\\+0.0}}$	$39.0^{-12.0}_{+0.0}$	$10^{\substack{-0.24\\+0.0}}$
10	$10^{\substack{+3.6\\-3.7}}$	$10^{\substack{-0.66\\+0.44}}$	$39.0^{+6.3}_{-10.0}$	$10^{\substack{-0.57\\+1.63}}$
5	$10^{\substack{+2.4\\-2.6}}$	$10^{\substack{-0.46\\+0.30}}$	$39.0^{+4.5}_{-7.2}$	$10^{\substack{-1.70\\+0.81}}$
$\sqrt{10}$	$10^{\substack{+1.8\\-1.8}}$	$10^{\substack{-0.33\\+0.22}}$	$39.0^{+3.2}_{-5.0}$	$10^{\substack{-0.90\\+0.84}}$

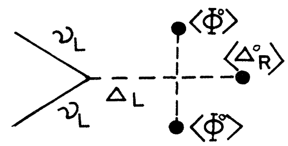

FIG. 2. Feynman diagram for the induced Majorana mass of the left-handed neutrino.

$$m_{\nu_e}=(2.5\times10^{-6}\text{–}2.5\times10^{-4})\text{ eV},$$
$$m_{\nu_\mu}=(7.5\times10^{-4}\text{–}7.5\times10^{-2})\text{ eV},$$
$$m_{\nu_\tau}=(7\times10^{-2}\text{–}7)\text{ eV}.$$

Thus $0.1M_U\leq M_{\Delta_L}\leq 0.2M_U$ is compatible with adiabatic solutions whereas $M_{\Delta_L}\simeq M_U$ is already consistent with nonadiabatic solutions for $\Delta m^2=(m_{\nu_\mu}-m_e)^2$ necessary for the solar neutrino flux using the MSW mechanism. The values of $m_{\nu_\tau}\simeq 1\text{–}7$ eV needed for the interpretation of the dark matter of the Universe are also possible for $M_{\Delta_L}\simeq(0.1\text{–}0.2)M_U$.

The investigations carried out in this Rapid Communication show that the presence of a real $\zeta(3,0,8)$ scalar in the grand desert, which we call a colorful bloom, can ensure complete agreement of the SU(5) and SO(10) model predictions with the LEP data and proton lifetime. In addition, even for a reasonably high choice of a factor 100 mass splitting among the superheavy Higgs-scalar components, the maximal predictions on τ_p can be probed by low-energy experiments, thus ruling out the model or testifying to its consistency. In the SO(10) model, the predicted value of neutrino masses are found to be consistent with adiabatic or nonadiabatic solutions to the solar neutrino flux using the MSW mechanism, while the m_{ν_τ} prediction satisfies the requirement of the cosmological dark matter of the Universe. At present the accepted range of Peccei-Quinn symmetry breaking scale is $M_{PQ}\simeq10^9\text{–}10^{12}$ GeV. Excluding threshold effects in both the GUT's, or including the effect for the degenerate or nondegenerate superheavy scalars in SU(5), or the degenerate case in SO(10), the predicted value of M_ζ is well within this range. However, only in the nondegenerate case of SO(10), the threshold effects might permit M_ζ to be outside this range.

It is well known that the SO(10) GUT's with intermediate breaking [2–4] are consistent with the Weinberg angle, the experimental limit on proton lifetime, and small neutrino masses suggested by the MSW solution to the solar neutrino problem. But without introducing any intermediate gauge symmetry we have investigated consistency of grand desert models with observable proton lifetime or small neutrino masses when the Higgs sector is suitably extended. We have thus found the most economic, but very interesting, of the blooming grand desert models which could be embedded in popular GUT's with just one additional fine-tuning of parameters. The $\zeta(3,0,8)^-$ mass is so high that its direct experimental signature does not seem to be possible in the near future. The most interesting low-energy signatures of the model are experimentally testable proton decay and small neutrino masses needed for solar neutrino flux and cosmology.

This work was supported by a project from the Department of Science and Technology, New Delhi. One of the authors (M.L.K.) would like to acknowledge financial assistance from the Meghalaya Government program.

[1] U. Amaldi, W. de Boer, and H. Furstenau, Phys. Lett. B **260**, 447 (1991); J. Ellis, S. Kelly, and D. V. Nanopoulos, *ibid.* **260**, 131 (1991); F. Anselmo *et al.*, Nuovo Cimento **104A**, 1817 (1992); R. Barbieri and L. Hall, Phys. Rev. Lett. **68**, 752 (1992).

[2] P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991); N. G. Deshpande, E. Keith, and P. B. Pal, *ibid.* **46**, 2261 (1992).

[3] M. K. Parida and P. K. Patra, Phys. Rev. Lett. **66**, 885 (1991); **68**, 768 (1992).

[4] R. N. Mohapatra and M. K. Parida, Phys. Rev. D **47**, 264 (1993).

[5] H. Murayama and T. Yanagida, Tohoku University Report No. TU-370, 1991 (unpublished).

[6] T. G. Rizzo, Phys. Rev. D **45**, 3903 (1992).

[7] U. Amaldi, W. de Boer, H. Furstenau, P. H. Frampton, and J. T. Liu, Phys. Lett. B **281**, 374 (1992).

[8] The word "colorful" is coined to $\zeta(3,0,8)$ because it transforms as an $SU(3)_C$ octet. See, for instance, H. Goldberg, L. Hall, S. F. King, and S. R. Sharpe, Phys. Lett. **151B**, 81 (1985).

[9] L. Hall, Nucl. Phys. **B178**, 75 (1981).

[10] M. K. Parida, Phys. Lett. B **196**, 163 (1987); in *Proceedings of the International Europhysics Conference on High Energy Physics,* Uppsala, Sweden, 1987, edited by O. Botner (European Physical Society, Uppsala, Sweden, 1987).

[11] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity,* Proceedings of the Conference, Stony Brook, New York, 1979, edited by D. Freedman and P. van Nieuwenhuizen (North-Holland, Amsterdam, 1980); T. Yanagida, in *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe,* Tsukuba, Japan, 1979, edited by A. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, 1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[12] S. Bludman, D. Kennedy, and P. Langacker, Phys. Rev. D **45**, 1810 (1992).

[13] D. Chang and R. N. Mohapatra, Phys. Rev. D **32**, 1248 (1985); M. K. Parida, P. K. Patra, and C. C. Hazra, *ibid.* **43**, 2351 (1991).

[14] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974).