



The electronic contribution to the elastic constants in strained layer quantum well superlattices of non-parabolic semiconductors with graded interfaces under magnetic quantization: Simplified theory and suggestion for experimental determination

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Abstract

In this paper, we study the electronic contribution to the elastic constants in strained layer quantum well superlattices of non-parabolic semiconductors with graded structures under strong magnetic quantization and compare the same with that of the constituent materials, by formulating the appropriate dispersion laws. It is found, taking InSb/GaSb quantum well strained superlattices of non-parabolic semiconductors as an example, that the carrier contribution to the second- and third-order elastic constants oscillates both with the electronic concentration and the inverse quantizing magnetic field in different manners together with the fact that the nature of oscillations is totally band structure dependent. We have also suggested an experimental method for determining the electronic contribution to the elastic constants in low-dimensional materials having arbitrary dispersion laws. In addition, the well-known results for bulk specimens of wide-gap stress-free materials have been obtained as special cases from our generalized formulation under certain limiting conditions.

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1. Introduction

With the advent of molecular beam epitaxy, organometallic chemical vapor phase epitaxy, fine line lithography and other experimental techniques, it has become possible to fabricate superlattices (SLs) of non-parabolic semiconductors composed of alternate layers of two different materials with controlled thickness, many of which are currently under study due to their new physical properties [1,2]. The SL, as originally proposed by Esaki and Tsu [3], has found wide applications in many device structures, such as photodetectors [4], avalanche photodiodes [5], transistors [6], tunneling devices [7], light emitters [8], etc.

The most extensively studied SL consists of alternate layers of GaAs and $\text{Ga}_{1-x}\text{Al}_x\text{As}$. In addition to SLs with the usual structure, SLs with more complex structures such as PbTe/SnTe [9], II–VI [10], strained layer [11] and HgTe/CdTe [12], etc. have also been proposed. These complex structures have provided additional degrees of freedom through band gap engineering, so that more parameters could be available for obtaining the desired electronic and optical properties. It is worth remarking that most papers in this vital area of modern nanostructured electronics are based on the assumption that the interface between the layers is sharply defined with zero thickness so as to be devoid of any interface effects: the SL potential distribution may be considered as a one-dimensional array of rectangular potential wells. Advanced experimental techniques may produce SL with physical interface between the two materials crystallographically abrupt, but the bonding environment of the atoms adjoining this interface will change at least on an atomic scale as the potential form changes from a well (barrier) to a barrier (well), an intermediate potential region exists for the charge carriers. Thus, the influence of the finite thickness of the interface on the carrier dispersion law becomes very important since the dispersion relation of the carriers governs all the transport properties. Although the strained layer quantum well SLs of non-parabolic semiconductors are of current interest for both scientific and device purposes, nevertheless, it appears from the literature that the electronic contribution to the

elastic constants for such heterostructures has yet to be investigated for the more practical case which occurs from the consideration of the finite width of the interface in the presence of a quantizing magnetic field.

In this context, we wish to note that the theory for determining the carrier contribution to the elastic constants in ultra thin films of p-type Si already exists [13]. It has been shown that the carrier contribution to the second- and the third-order elastic constants depend on the density-of-states function [14]. Sreedhar and Gupta [14] formulated the same for bulk specimens of non-parabolic compounds whose energy band structures are defined by the two-band model of Kane. It has therefore different values in various materials and varies with the electron concentration, with the thickness of ultra thin films and with the temperature for semiconductors and their heterostructures having various carrier energy spectra. The nature of these variations has been investigated by Ghatak and co-workers [13,15,16] and a few others [14,17]. Some of the significant features which have emerged from these studies are:

- (a) The carrier contribution of the elastic constants increases monotonically with electron concentrations in bulk materials.
- (b) The nature of the variations is significantly affected by the presence of the band non-parabolicity.
- (c) The said contribution has relatively large values in ultra thin films in the presence of size quantization.

The above characteristics are considered as theoretical predictions, and no experimental results are available to the knowledge of the authors in support of these predictions for the present generalized systems. Therefore, it would be of much interest to study the carrier contribution to the elastic constants for the present case and to suggest an experimental method of determining them for materials having arbitrary band structures. In quantum well SLs with graded interfaces under magnetic quantization, the carrier contribution to the elastic constants will be rather significant since the carrier statistics for the present

case exhibits composite oscillations with nano-thickness due to the SdH and the allied SL effects as compared with that of the constituent materials.

In the following, we shall investigate the second- and third-order elastic constants (ΔC_{44} and ΔC_{456}) for the present system. The corresponding results for bulk specimens of stress-free wide-gap materials have also been obtained from our generalized formulation for the purpose of assessing the influence of strain and interface width, respectively. As already remarked, we have also suggested the experimental methods for determining such contribution for materials having arbitrary dispersion laws. We shall study the doping and magnetic field dependences of ΔC_{44} and ΔC_{456} taking the strained quantum well InAs/GaSb SL as an example for the purpose of numerical computations.

2. Theoretical background

The dispersion relation of the conduction electrons in bulk specimens of stressed non-parabolic semiconductors for a stress along the $\langle 110 \rangle$ direction can be expressed [18], in the absence of any quantization, as

$$[k_x/a_i^*(E)]^2 + [k_y/b_i^*(E)]^2 + [k_z/f_i^*(E)]^2 = 1, \quad (1)$$

where

$$[a_i^*(E)]^2 = L_i^*(E)[A_i^*(E) + (1/2)D_i^*(E)]^{-1},$$

$$i = 1, 2, \quad L_i^*(E) = [3E^2 + Eq_i - R_i],$$

$$q_i = [(3E_{gi}/2e_i^2) - (3\epsilon_i c_i^c/e_i^2)],$$

where E_g is the band gap, e_i are the momentum matrix elements, c_i^c is the conduction band deformation potential constant. And ϵ_i is the trace of the strain tensor $\hat{\epsilon}_i$ which is given by

$$\hat{\epsilon}_i = \begin{vmatrix} \epsilon_{xxi} & \epsilon_{xyi} & 0 \\ \epsilon_{xyi} & \epsilon_{yyi} & 0 \\ 0 & 0 & \epsilon_{zz} \end{vmatrix}. \quad (2)$$

ϵ_{xxi} , ϵ_{xyi} , ϵ_{yyi} and ϵ_{zz} are the various elements of $\hat{\epsilon}_i$,

$$R_i = [e_i^{-2}(g_i)^2(\epsilon_{xyi})^2 + (3/2)e_i^{-2}c_i^c\epsilon_i(E_{gi} - c_i^c\epsilon_i)],$$

g_i is a constant describing the strain interaction between the conduction and valence bands,

$$A_i^*(E) = (E + G_i^x)(E + H_i)^{-1},$$

$$G_i^x = [E_{gi} - c_i^c\epsilon_i + 3\bar{b}_{0i}\epsilon_{xxi} - \bar{b}_{0i}\epsilon_i(a_{0i} + c_i^c)\epsilon_i],$$

$$a_{0i} = -(x_i + 2y_i)/3, \quad \bar{b}_{0i} = (x_i - y_i)/3,$$

$$d_{0i} = 2Z_i/\sqrt{3}, x_i, y_i$$

and Z_i are the matrix elements of the strain projection operator,

$$H_i = [E_{gi} + c_i^c\epsilon_i], \quad D_i^*(E) = \rho_i(E + H_i)^{-1},$$

$$\rho_i = [d_{0i}\epsilon_{xyi}(3)^{1/2}],$$

$$[b_i^*(E)]^2 = L_i^*(E)[A_i^*(E) - (1/2)D_i^*(E)]^{-1},$$

$$[f_i^*(E)]^2 = L_i^*(E)/C_i^*(E),$$

$$C_i^*(E) = (E + G_i^z)(E + H_i)^{-1}$$

and

$$G_i^z = [E_{gi} - c_i^c\epsilon_i - (a_{0i} + c_i^c)\epsilon_i + 3\bar{b}_{0i}\epsilon_{zzi} - \bar{b}_{0i}\epsilon_i].$$

Therefore, the dispersion law of the electrons in strained layer SLs of non-parabolic semiconductors with graded interfaces can be written by extending the method as given in Ref. [19], as

$$\cos(L_0k) = [\phi(E)/2], \quad (3)$$

where $L_0(= L_1 + L_2)$ is the period length, L_1 and L_2 are the widths of the barrier and well, respectively,

$$\begin{aligned} \phi(E) = & \left\{ 2 \cosh[\beta(E)] \cos[\gamma(E)] \right. \\ & + T(E) \sinh[\beta(E)] \sin[\gamma(E)] \\ & + \Delta_0 \left\{ \left[\frac{K_0^2(E)}{K'(E)} - 3K'(E) \right] \right. \\ & \left. \left. \times \cosh[\beta(E)] \sin[\gamma(E)] \right\} \right\} \end{aligned}$$

$$\begin{aligned}
& + \left(3K_0(E) - \frac{[K'(E)]^2}{K_0(E)} \right) \\
& \times \sinh[\beta(E)] \cos[\gamma(E)] \Big\} \\
& + \Delta_0 \left\{ 2\{K_0(E)\}^2 - 2\{K'(E)\}^2 \right\} \\
& \times \cosh[\beta(E)] \cos[\gamma(E)] \\
& + \frac{1}{12} \left\{ \frac{5K_0^3(E)}{K'(E)} + \frac{5[K'(E)]^3}{K_0(E)} \right. \\
& \left. - 34K'(E)K_0(E) \right\} \\
& \times \sinh[\beta(E)] \sin[\gamma(E)] \Big\} \Big\},
\end{aligned}$$

$$T(E) = \left[\frac{K_0(E)}{K'(E)} - \frac{K'(E)}{K_0(E)} \right],$$

$$\beta(E) = K_0(E)(L_1 - \Delta_0),$$

Δ_0 is the interface width,

$$\gamma(E) = K'(E)(L_2 - \Delta_0),$$

$$\begin{aligned}
K'(E) &= \{(E + G_1^z)^{-1}(E + H_1)(3E^2 + Eq_1 - R_1) \\
& - (E + G_1^z)^{-1}(E + G_1^x + \frac{1}{2}\rho_1)k_x^2 \\
& - (E + G_1^z)^{-1}(E + G_1^x - \frac{1}{2}\rho_1)k_x^2\}^{1/2},
\end{aligned}$$

$$\begin{aligned}
K_0(E) &= \{(V_0 - E - H_2)(E_1 + G_2^z)^{-1} \\
& \times (3E_1^2 + E_1q_2 - R_2) \\
& + (E_1 + G_2^z)^{-1}(E_1G_2^x + \frac{1}{2}\rho_2)k_x^2 \\
& + (E_1 + G_2^z)^{-1}[E_1 + G_2^x - \frac{1}{2}\rho_2]k_y^2\}^{1/2},
\end{aligned}$$

$E_1 = E - V_0$, and $V_0 (= E_{g2} - E_{g1})$ is the potential barrier encountered by the electron. In the presence of a quantizing magnetic field B along the z -direction, the modified dispersion relation of the quantum well SLs in this case can be expressed as

$$(\pi L/d_z)^2 = [\delta(E, n) - 2eB(n + \frac{1}{2})\hbar^{-1}L_0^2]L_0^{-2}, \quad (4)$$

where L and d_z are the size quantum number and the nanothickness along the z -direction respectively, e is the electron charge, B is the quantizing magnetic field along the z -direction, L_0 is the SL

period, $\delta(E, n) = \{\cos^{-1}[\frac{1}{2}\Psi(E, n)]\}^2$, $n = 0, 1, 2, \dots$ is the Landau quantum number,

$$\begin{aligned}
\Psi(E, n) &= \left\{ 2 \cosh[\beta(E, n)] \cos[\gamma(E, n)] + T(E, n) \right. \\
& \times \sinh[\beta(E, n)] \sin[\gamma(E, n)] \\
& + \Delta_0 \left\{ \left[\frac{K_0^2(E, n)}{K'(E, n)} - 3K'(E, n) \right] \right. \\
& \times \cosh[\beta(E, n)] \sin[\gamma(E, n)] \\
& + \left(3K_0(E, n) - \frac{[K'(E, n)]^2}{K_0(E, n)} \right) \\
& \times \sinh[\beta(E, n)] \cos[\gamma(E, n)] \Big\} \\
& + \frac{1}{12} \left(\frac{5K_0^3(E, n)}{K'(E, n)} + \frac{5[K'(E, n)]^3}{K_0(E, n)} \right) \\
& - 34K'(E, n)K_0(E, n) \\
& \left. \times \sinh[\beta(E, n)] \sin[\gamma(E, n)] \right\},
\end{aligned}$$

$$T(E, n) = \left[\frac{K(E, n)}{K'(E, n)} - \frac{K'(E, n)}{K_0(E, n)} \right],$$

$$\beta(E, n) = K_0(E, n)[L_1 - \Delta_0],$$

$$\begin{aligned}
K_0(E, n) &= [(E_1 + G_2^z)^{-1}(V_0 - E - H_2) \\
& \times (3E_1^2 + E_1q_2 - R_2) + (n + \frac{1}{2})\hbar\omega_2(E)]^{1/2},
\end{aligned}$$

$$\hbar = h/2\pi,$$

where h is the Plank's constant,

$$\omega_2(E) = eB/M_2(E),$$

$$\begin{aligned}
M_2(E) &= (\hbar^2/2)(E_1 + G_2^z)[(E_1 + G_2^x + \frac{1}{2}\rho_2) \\
& \times [E_1 + G_2^x - \frac{1}{2}\rho_2]]^{-1/2},
\end{aligned}$$

$$\begin{aligned}
K'(E, n) &= [(E + G_1^z)^{-1}(E + H_1)(3E^2 + Eq_1 - R_1) \\
& - (n + \frac{1}{2})\hbar\omega_1(E)]^{1/2},
\end{aligned}$$

$$\omega_1(E) = eB/M_1(E),$$

where e is the carrier charge and

$$\begin{aligned}
M_1(E) &= (\hbar^2/2)(E + G_1^z)\{(E + G_1^x + \frac{1}{2}\rho_1) \\
& \times [E + G_1^x - \frac{1}{2}\rho_1]\}^{-1/2}.
\end{aligned}$$

Since the lowest mini band is being significantly populated at low temperatures, where the quantum effects become prominent, the electron statistics can be written including the influence of broadening as

$$n_0 = \frac{g_s g_v e B}{h L_0} \sum_{n=0}^{n_{\max}} \sum_{L=1}^{L_{\max}} \frac{A(1 + A \cos \lambda)}{(1 + A^2 + 2A \cos \lambda)} \quad (5)$$

in which g_s is the spin degeneracy, g_v is the valley degeneracy, $A = \exp[(E_F - E')/k_B T]$, E' is the root of Eq. (4), E_F is the Fermi energy in the present case, k_B is Boltzmann constant, T is temperature, $\lambda = \Gamma/k_B T$, $\Gamma = \pi k_B T_D$ and T_D is the Dingle temperature. The carrier contribution to the second- and third-order elastic constants can, respectively, be expressed [16] as

$$\Delta C_{44} = -(G_0^2/9L_0) \frac{\partial n_0}{\partial E_F}, \quad (6)$$

$$\Delta C_{456} = (G_0^3/27L_0) \frac{\partial^2 n_0}{\partial E_F^2}, \quad (7)$$

where G_0 is the deformation potential.

Thus, combining Eqs. (5), (6) and (7), we get

$$\Delta C_{44} = \left(\frac{g_s g_v e B G_0^2}{9 h L_0 k_B T} \right) \sum_{n=0}^{n_{\max}} \sum_{L=1}^{L_{\max}} \times \left[\frac{A(1 - A^2 + 3A \cos \lambda + 3A^2 \cos^2 \lambda)}{(1 + A^2 + 2A \cos \lambda)^2} \right], \quad (8)$$

$$\Delta C_{456} = \left(\frac{g_s g_v e B G_0^3}{27 h L_0 (k_B T)^2} \right) \times \sum_{n=0}^{n_{\max}} \sum_{L=1}^{L_{\max}} [A(1 + A^2 + 2A \cos \lambda)^{-2} \times (1 - A^2 + 3A \cos \lambda + 3A^2 \cos^2 \lambda) - 4A(1 + A^2 + 2A \cos \lambda)^{-3} (A + \cos \lambda) \times (1 - A^2 + 3A \cos \lambda + 3A^2 \cos^2 \lambda) + A(1 + A^2 + 2A \cos \lambda)^{-2} \times (-2A + 3 \cos \lambda + 6A \cos^2 \lambda)]. \quad (9)$$

Eqs. (8) and (9) represent the expressions for ΔC_{44} and ΔC_{456} for the present system. The expression for n_0 for the constituent non-parabolic materials

under magnetic quantization will be given by Eq. (5) where E' has to be determined from

$$\left(n + \frac{1}{2} \right) \hbar \omega_3(E') + \left(\frac{\pi L}{d_z f_i^*(E')} \right)^2 = 1 \quad (10)$$

in which $\omega_3(E') \equiv [2eB/\hbar^2 a^*(E') b^*(E')]$ and L_0 should be replaced by d_z .

In the absence of the magnetic field and considering the bulk specimens of stress free wide-gap semiconductors, the expressions for n_0 , ΔC_{44} and ΔC_{456} can, respectively be written as

$$n_0 = N_c F_{1/2}(\eta), \quad (11)$$

$$\Delta C_{44} = -(G_0^2 N_c / 9 k_B T) F_{-1/2}(\eta), \quad (12)$$

and

$$\Delta C_{456} = (G_0^3 N_c / 27 k_B^2 T^2) F_{-3/2}(\eta), \quad (13)$$

where

$$N_c \equiv 2(2\pi m_i^* k_B T / \hbar^2)^{3/2}, \quad \eta \equiv E_F / k_B T$$

and $F_t(\eta)$ is the one parameter Fermi–Dirac integral of order t [20] which can be defined as

$$F_t(\eta) = (\Gamma(t+1))^{-1} \int_0^\infty x^t [1 + \exp(x - \eta)]^{-1} \quad (14)$$

for $t > -1$

or for all t , analytically continued as a complex contour integral around the negative x -axis as

$$F_t(\eta) = \left(\sqrt{-t} / 2\pi \sqrt{-1} \right) \int_{-\infty}^{+\infty} x^t [1 + e^{-x-\eta}]^{-1} dx \quad (15)$$

It may be noted that Eqs. (11)–(13) are well-known in the literature [15].

2.1. Suggestion for experimental determinations of the electronic contribution to the elastic constants for materials having arbitrary dispersion laws

The magnitude of the thermoelectric power (T_0) in the present case can be written as [21]

$$T_0 = \left(\frac{1}{e T n_0} \right) \int_{-\infty}^{\infty} (E - E_F) R(E) \left[-\frac{\partial f}{\partial E} \right] dE, \quad (16)$$

where $R(E)$ is the total number of states and f is the distribution function. Following Tsidilkovskii [22], Eq. (16) can be written as

$$T_0 = (\pi^2 k_B^2 T / 3en_0) \left(\frac{\partial n_0}{\partial E_F} \right). \quad (17)$$

Using Eqs. (6), (7) and (17), we get

$$\Delta C_{44} = -(G_0^2 e T_0 n_0 / 3\pi^2 k_B^2 T^2), \quad (18)$$

$$\Delta C_{456} = (n_0 e G_0^3 T_0^2 / 3\pi^4 k_B^3 T) \left(1 + \frac{n_0}{T_0} \frac{\partial T_0}{\partial n_0} \right). \quad (19)$$

Thus, we can summarize the whole mathematical background in the following way. From the expression of carrier statistics in strained layer quantum well superlattices of non-parabolic semiconductors with graded structure under the strong magnetic quantization by incorporating all the system parameters, we have formulated the generalized expressions for ΔC_{44} and ΔC_{456} , respectively. The expressions of ΔC_{44} and ΔC_{456} for magneto-quantum size effect form a special case of our analysis, where E' is the only band structure-dependent quantity. In the absence of stress and under the substitution $e_i^2 = 3\hbar^2 E_{gi} / 4m_i^*$, where m_i^* is the effective electron mass at the edge of the conduction band, Eq. (1) assumes the form $E(1 + \alpha.E) = \hbar^2 k^2 / 2m_i^*$. This equation is known in the semiconductor literature as the two-band model of Kane. Besides, this particular equation is being extensively used to study the electronic properties of III–V compounds, ternary and quaternary alloys, and is a special case of Eq. (1). From our generalized formulation, we have obtained the well-known expressions of n_0 , ΔC_{44} and ΔC_{456} in bulk specimens of wide-gap stress-free degenerate materials [15]. This fact is the indirect mathematical test of our generalized analysis. In addition, we have suggested an experimental method for determining ΔC_{44} and ΔC_{456} for materials having arbitrary dispersion laws.

3. Results and discussion

Using Eqs. (5), (8), and (9) for the strained layer InAs/GaSb quantum well SL together with the

parameters [18]

$$m_i^* = 0.023m_0, \quad E_{g1} = 0.41 \text{ eV},$$

$$e_1 = 10 \times 10^{-11} \text{ eVm}, \quad g_1 = 2 \text{ eV}, \quad c_1^c = 20 \text{ eV},$$

$$(S_{44})_1 = 0.3 \times 10^{-3} \text{ kBar}^{-1}, \quad d_{01} = -4.4 \text{ eV},$$

$$(a_{01} + c_1^c) = 8 \text{ eV}, \quad \bar{b}_{01} = -1.8 \text{ eV},$$

$$(S_{11})_1 = 0.09 \times 10^{-3} \text{ kBar}^{-1},$$

$$(S_{12})_1 = 0.48 \times 10^{-3} \text{ kBar}^{-1}, \quad \sigma = 4 \text{ kBar},$$

for InAs and

$$m_2^* = 0.048m_0, \quad E_{g2} = 0.81 \text{ eV},$$

$$e_2 = 14 \times 10^{-11} \text{ eVm}, \quad g_2 = 4 \text{ eV}, \quad c_2^c = 30 \text{ eV},$$

$$d_{02} = -6 \text{ eV}, \quad (S_{44})_2 = 0.6 \times 10^{-3} \text{ kBar}^{-1},$$

$$(a_{02} + c_2^c) = 10 \text{ eV}, \quad \bar{b}_{02} = -4 \text{ eV},$$

$$(S_{11})_2 = 0.71 \times 10^{-3} \text{ kBar}^{-1},$$

$$(S_{12})_2 = 0.46 \times 10^{-3} \text{ kBar}^{-1} \quad \text{for GaSb},$$

$$\Delta_0 = 5 \text{ \AA}, \quad L_0 = 120 \text{ \AA}, \quad L_1 = 60 \text{ \AA},$$

$$d_z = 60 \text{ \AA}, \quad T_D = 9.4 \text{ K},$$

$$T = 4.2 \text{ K}, \quad \text{and} \quad B = 6 \text{ T}.$$

We have plotted in Fig. 1 the normalized ΔC_{44} ($\Delta C_{44}/T_1, T_1 = -(G_0^2 n_0 / 9k_B T)$) and the normalized ΔC_{456} ($\Delta C_{456}/T_2, T_2 = (G_0^3 n_0 / 27k_B^2 T^2)$) versus the electron concentration in stressed quantum well InAs/GaSb SL under strong magnetic field with Δ_0 (the finite width of the interface) $\neq 0$ as shown by curves a and b. The curves c and d of Fig. 1 exhibit the same dependence in the absence of stress. Besides the curves e, f, g, and h show the dependences of ΔC_{44} and ΔC_{456} on n_0 for magneto-size quantized InAs and GaSb, respectively. In Fig. 2, we have plotted the normalized ΔC_{44} and ΔC_{456} for all cases of Fig. 1 as functions of $1/B$ excluding n -InAs for $n_0 = 10^{20} \text{ m}^{-3}$. It appears from Fig. 1 that both ΔC_{44} and ΔC_{456} increase with increasing n_0 in different oscillatory manners for all types of materials as considered

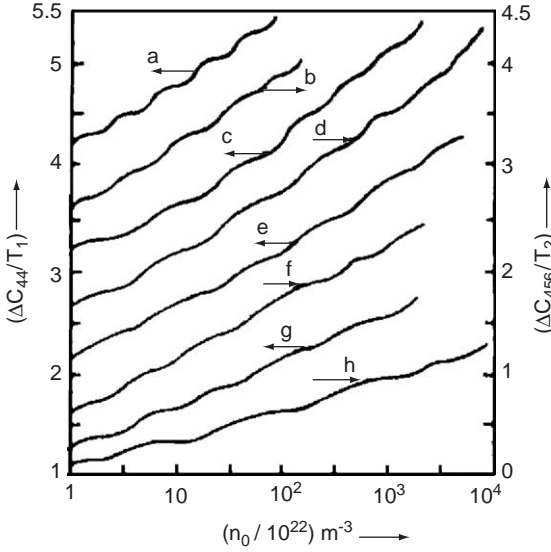


Fig. 1. Plots of the normalized ΔC_{44} and ΔC_{456} versus n_0 in stressed quantum well InAs/GaSb SL with $\Delta \neq 0$ as shown by curves a and b. The curves c and d exhibit the same dependence in absence of stress. The curves e, f, g and h exhibit the dependence of ΔC_{44} and ΔC_{456} on n_0 for magneto-size quantized for InAs/GaSb, respectively ($B = 6$ T).

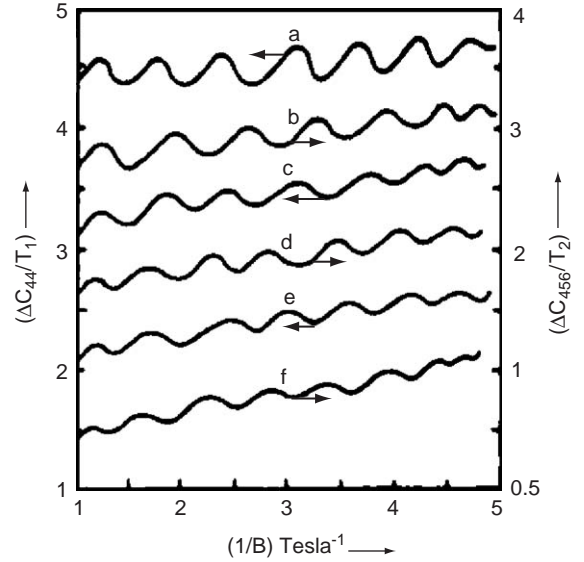


Fig. 2. Plots of the normalized ΔC_{44} and ΔC_{456} . For all cases of Fig. 1 excluding InAs as functions of $1/B$ for $n_0 = 10^{20} \text{ m}^{-3}$.

here. The natures of oscillations are totally determined by the respective band structure. The combined influences of the strain and the finite width of the interface enhance the numerical values of both ΔC_{44} and ΔC_{456} , respectively. The numerical values of ΔC_{44} and ΔC_{456} are the greatest for the present system and the least for magneto-size quantized GaSb, respectively. From Fig. 2, it appears that both ΔC_{44} and ΔC_{456} oscillate with $1/B$ though the nature oscillations are different from that as given in Fig. 1.

It may be noted that our experimental suggestions for the determination of ΔC_{44} and ΔC_{456} are valid for materials having arbitrary dispersion relations. Since the experimental curve of n_0 versus T_0 is not available in the literature to the best of our knowledge for the present system, we cannot compare our theoretical formulation with the proposed experiment although the generalized analysis as presented in this context would be useful and the Eqs. (18) and (19) are important in probing the band structures of different materials.

We wish to note in view of large changes of the elastic constants with n_0 , detailed experimental work on second- and third-order elastic constants as functions of n_0 would be interesting for the present class of quantum confined materials. It may be suggested that the experiments on the velocity of sound involving the shear mode as function of n_0 may exhibit the carrier contribution to the elastic constants for materials having arbitrary carrier energy spectra. It is worth noting that the above statement again suggests the experimental determinations of ΔC_{44} and ΔC_{456} , besides the suggested method of determining them as given by Eqs. (18) and (19), respectively. It may be noted that our study covers different materials having various electron dispersion laws and the formulations of ΔC_{44} and ΔC_{456} are based on the dispersion relations in such compounds.

It is worth remarking that influence of energy band models on ΔC_{44} and ΔC_{456} in various types of materials can also be assessed from our present work. We have not considered other types of compounds or external physical variables for numerical computations in order to keep the presentation brief. With different sets of energy band parameters, we shall get different numerical values of ΔC_{44} and ΔC_{456} although the nature of

variation of the said elastic constants with respect to carrier statistics as shown here would be similar for the other types materials. We have not considered the many body, the hot electron and the allied SL effects in this simplified theoretical formalism due to the absence of proper analytical techniques for including them for the present generalized systems. The inclusion of these effects would increase the accuracy of the results although our suggestion for the experimental determination of ΔC_{44} and ΔC_{456} is independent of incorporating the said effects and the qualitative features of ΔC_{44} and ΔC_{456} would not change in the presence of the aforementioned effects. The formulation as presented in this paper is general and E' is the only band structure-dependent quantity as already stated. Finally, it may be noted that the basic aim of the present paper is not solely to investigate that ΔC_{44} and ΔC_{456} but also to suggest the experimental determination of them for materials having arbitrary band structures which, in turn, is again in dimension independent.

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References

- [1] H. Sasaki, *J. Vac. Sci. Technol.* 19 (1981) 198.
- [2] P.M. Petroff, A.C. Gossard, R.A. Logan, Weigemann, *Appl. Phys. Lett.* 41 (1982) 635.
- [3] L. Esaki, R. Tsu, *IBM J. Res. Dev.* 14 (1970) 61.
- [4] R. Chin, N. Holonyak, G.E. Stillman, J.Y. Tang, K. Hess, *Electron. Lett.* 16 (1980) 467;
F. Capasso, W.T. Tsang, A.L. Hutchinson, G.F. Williams, *Appl. Phys. Lett.* 40 (1982) 38;
J.S. Smith, L.C. Chiu, S. Margalit, A. Yariv, *J. Vac. Sci. Technol. IB* (1983) 376.
- [5] F. Capasso, in: R.W. Willardson, A.C. Beer (Eds.), *Semiconductors and Semimetals*, vol. 22, Part D, Academic Press, New York, 1985, p. 2.
- [6] F. Capasso, R.A. Kiehl, *J. Appl. Phys.* 58 (1985) 1366;
M. Heiblum, D.C. Thomas, C.M. Knoedler, M.I. Nathan, *Appl. Phys. Lett.* 47 (1985) 1105.
- [7] F. Capasso, K. Mohummed, A.Y. Cho, *Appl. Phys. Lett.* 48 (1986) 478.
- [8] M. Kriechbaum, P. Kocevar, H. Pascher, G. Bauer, *IEEE* 24 (1988) 1727.
- [9] K. Ploog, G.H. Dohler, *Adv. Phys.* 32 (1983) 285.
- [10] B.A. Wilson, *IEEE Quant. Electron.* 24 (1988) 1763.
- [11] G.C. Osbourn, *J. Vac. Sci. Technol. A* 3 (1985) 826.
- [12] J.N. Schulman, T.C. McGill, *Appl. Phys. Lett.* 34 (1979) 663.
- [13] K.P. Ghatak, B. De, *Proc. Mater. Res. Soc. USA* 226 (1991) 191.
- [14] A.K. Sreedhar, S.C. Gupta, *Phys. Rev. B* 5 (1972) 3160.
- [15] K.P. Ghatak, *Internat. J. Electron.* 71 (1991) 239;
K.P. Ghatak, B. Nag, *NanoStructured Materials* 10 (1998) 923.
- [16] K.P. Ghatak, S.K. Biswas, D. De, S. Ghosal, S. Chatterjee, *Physica B* 353 (2004) 127.
- [17] R.W. Keyes, *IBM J. Res. Dev.* 5 (1961) 266.
- [18] D.G. Seiler, B.D. Bajaj, A.E. Stephens, *Phys. Rev. B* 16 (1977) 2822.
- [19] H.X. Jiang, J.Y. Lin, *J. Appl. Phys.* 61 (1987) 624.
- [20] S. Chowdhury, L.J. Singh, K.P. Ghatak, *Nanotechnology* 15 (2004) 180.
- [21] B.M. Askerov, N.F. Gashimzade, M.M. Panakhov, *Sov. Phys. Solid State* 29 (1987) 456.
- [22] I.M. Tsidilkovskii, *Band Structure of Semiconductors*, Pergamon Press, London, 1982.