

A STUDY OF CERTAIN PROPERTIES OF  
SUPERFLUID HELIUM-4 BASED ON  
MACRO-ORBITAL THEORY



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## SYNOPSIS

Helium-4, a noble element, was first liquified by Kamerlingh Onnes in 1908 [1]. Since then liquid helium-4 (LHE-4) (a system of interacting bosons) has become a subject of extensive investigation. Onnes also observed that LHE-4 exhibits a liquid to liquid transition at  $\sim 2.1 K(T_\lambda)$  [2] which transforms the liquid from its normal phase (*helium-I*) of all properties of a normal liquid to a super-fluid phase (*helium-II*) of very low viscosity and very high thermal conductivity [3, 4]. LHE-4 remains as liquid even at  $0 K$  unless a pressure of about  $25 \text{ atms}$  is applied. Super-fluid phase of LHE-4 possesses some more unique properties such as: (i) it can flow against the gravity along the walls of the container, (ii) its specific heat and related thermodynamic response functions such as expansion coefficient, pressure coefficient *etc.* diverge logarithmically at  $\lambda$ -transition, (iii) it exhibits thermo-mechanical and mechanocaloric effects, (iv) below  $T_\lambda$ , its volume increases with decreasing temperature, (v) for certain velocity (flow/rotational) it sustains quantized circulation, (vi) it sustains four different sound modes, (vii) its dispersion curve at low  $Q$  shows anomalous nature, (viii) it remains calm and still on boiling (no bubble formation), *etc.*

The wealth of experimental and theoretical results has been reviewed by several authors [3, 5, 6]. Several attempts have been made to develop a full microscopic theory but with little success. Most of the microscopic theories so far developed [7–15] start with the basic assumption that the super-fluid phase has macroscopic occupation of  $p = 0$  state. However, the existence of  $p = 0$  condensate has not been confirmed experimentally even after repeated efforts [16]. Recently, in a review article Legget remarks “In the sixty years since London’s original proposal, while there has been almost universal belief that the key to super-fluidity is indeed the onset of BEC at  $T_\lambda$  it has proved very difficult, if not impossible, to verify the existence of the latter phenomenon directly. The main evidence for it comes from high-energy neutron scattering and, very, recently, from the spectrum of atoms evaporated from the liquid surface, and while both are certainly consistent with the existence of a condensate fraction of approximately 10%, neither can be said to establish it beyond all possible doubts.” Consequently, Landau’s two fluid model [17, 18] supplemented by the idea of quantized circulation presented by Onsager [19] and Feynman [20] remains as the only way to understand the properties of LHE-4 despite its several shortcomings [6]. Since a wholly microscopic

theory is not available, LHE-4 continues to be a subject of great interest.

Over the last several years Jain has used a new approach to develop the microscopic understanding of the phenomenon [21–23]; this approach does not assume the existence of either Cooper type pairs in a fermionic system or BEC state (existence of  $p = 0$  condensate) in a bosonic system. This theory (Macro-orbital theory) consistent with microscopic as well as macroscopic uncertainty, and excluded volume principle. It also provides microscopic foundation for the system to behave as a mixture of two fluid as envisage by Landau [17]. Basic objective of the present thesis is to show the agreement of experimentally observed thermodynamic and hydrodynamics properties of LHE-4 with those calculated by using macro-orbital theory and thereby to conclude that the theory has great potential to explain the properties of similar system and to provide a frame work that unifies the physics of bosonic and fermionic system.

This thesis contains seven chapters.

In Chapter-1 we present a brief discussion and review on experimental and theoretical studies of LHE-4.

Chapter-2 describes the salient features of macro-orbital theory. This theory reveals that:

1. Each particle in a fluid can be described by a kind of pair waveform  $\chi = \psi(1, 2)$  representing its wave superposition with another particle. For distinction  $\chi$  is proposed to be known as macro-orbital state [22].
2. Each particle in its macro-orbital has two motion; the  $q$ -motion representing the relative motion of one particle with respect to the other particle decides the quantum size ( $\lambda/2 = \pi/q$ ) of the particle (or the size of the space exclusively occupied by the particle), and the  $K$ -motion (not affected by the inter particle interaction) that participates in defining the quasi-particle excitations of the system. Note that this aspect successfully accounts for the two fluid behavior of a super-fluid [22].
3. All particles in the ground state of a bosonic system have  $q = q_o = \pi/d$  with  $K = 0$ , and they represent an ordered state in the the phase space with  $\Delta\phi = 2\pi$  which accounts for their phase coherence [22]. They also develop a kind of collective binding which explains critical velocities for which superfluidity vanishes [22].

4. Particles in the super-fluid state defines a closed packed arrangement of their equal size wave packets. Consequently, they cease to have relative motion rendering loss of viscosity; they can move in order of their locations on a closed path with  $\Delta\phi = 2n\pi$  ( $n = 1, 2, 3, ..$ ) which accounts for the observed quantum vortices [22].

Chapter-3 presents our theoretical results on the logarithmic divergence of the specific heat at constant pressure ( $C_p$ ), expansion coefficient ( $\alpha_p$ ), isothermal compressibility ( $\kappa_T$ ) and pressure coefficient ( $\beta$ ) of LHE-4 which are found to have very good agreement with experimental results.

Chapter-4 gives a detailed analysis of the excitation spectrum of LHE-4 at low  $Q$ . Following the ordered arrangement of the atoms we estimate the excitation energy at different  $Q$  by assuming different atomic arrangements namely sc, bcc, fcc and hcp arrangements. We also obtain group velocity ( $v_g$ ) as well as phase velocity ( $v_p$ ).

In Chapter-5 we estimate energy gap (the collective binding of atoms in superfluid state) and use it to obtain superfluid density, critical velocity of linear flow and rotational motion, vortex line density, vortex energy *etc.*

In Chapter-6 we make a systematic study of transition temperature, surface tension, bulk potential energy and surface layer thickness. Our results agree closely with their experimental values.

In Chapter-7 we list the important conclusion of our investigation and future course of studies where macro-orbital theory can be used.