Threshold effects on intermediate mass and proton lifetime predictions in SU(5) with split multiplets

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Including two-loop contributions to the renormalization group equations (RGE's) and threshold modification of the GUT boundary condition for the gauge couplings, we derive analytic formulas for the mass scales and the GUT coupling constant in the SU(5) split-multiplet models recently investigated by Amaldi, de Boer, Frampton, Furstenau, and Liu. The mass scales and the coupling constants are computed using these analytic formulas in conjuction with the iterative convergence procedure adopted for the solutions of RGE's. In each model the threshold effects are evaluated under the assumption that the superheavy masses are degenerate, partially degenerate, or nondegenerate, resulting in a very significant modification of the degenerate intermediate mass of the new particles. The threshold-uncertainty factor is noted to be the least in the partially degenerate case in one of the eight models making the GUT predictions more precise on the intermediate mass. But the corrections are larger both on the proton lifetime and the intermediate mass in the nondegenerate case in all models. For reasonably precise model predictions we conclude that the superheavy masses may be either degenerate, or partially degenerate, or else their mass-splitting coefficient may be small in the nondegenerate case.

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I. INTRODUCTION

Grand unified theories (GUT's) [1–8] are currently being investigated with renewed interest after a more precise determination of the standard model couplings has become available from the CERN e^+e^- collider LEP data [9–14]. The simplest grand unified theory based upon SU(5) [1,2] predicts both the $\sin^2\theta_W$ and the unification mass (M_U) which have been decisively ruled out by experimental data. Although a number of GUT's such as SO(10) and E_6 [8] with one or more intermediate symmetries are being pursued as important theories to describe the basic gauge interactions beyond the standard model, considerable effort is also being made to revive SU(5)-based unification schemes including additional fermionic or bosonic degrees of freedom at the intermediate mass scales [3–6,10,13,14]. Very recently the supersymmetric (SUSY) SU(5) and non-SUSY SU(5) models with split-multiplets [6] have been analyzed in detail in the context of the LEP data by Amaldi, de Boer, Frampton, Furstenau, and Liu (ADFFL) [10]. But the threshold effects due to superheavy masses [15] have been noted to be a major source of modification of different GUT predictions including SO(10) [12–14,16–18] and SUSY SU(5) [19].

One objective of this paper is to derive analytic formulas for the mass scales and the GUT coupling constant such that they can be predicted in terms of the well-known quantities $\sin^2\theta_W(M_Z)$, $\alpha_s(M_Z)$, and $\alpha(M_Z)$ and a reasonable approximation on the two-loop contributions. Another major objective is to estimate the numerical values of threshold effects on the same scales and the GUT coupling constant in the models investigated by ADFFL [10]. In Sec. II we derive analytic expressions. In Sec. III we obtain numerical values of the mass scales following the improved procedure but neglecting threshold effects. The standard-model content of the superheavy masses are identified and expressions for the matching functions and threshold effects are derived in Sec. IV and the Appendix. In Sec. V we compute numerical values of threshold effects and study their implications. The paper is summarized with conclusions in Sec. VI.

II. ANALYTIC FORMULAS FOR MASS SCALES AND GUT-COUPLING CONSTANT

To derive analytical formulas we use renormalization group equations (RGE's) of the standard model for $\mu \le M_I$ (M_I=degenerate intermediate mass of new degrees of freedom):

$$\mu\frac{\partial\alpha_i(\mu)}{\partial\mu}=\frac{a_i}{2\pi}\alpha_i^2(\mu)+\frac{1}{8\pi^2}\sum_{j=1}^{3} b_{ij}\alpha_i^2(\mu)\alpha_j(\mu) , \quad (1)$$

where $i=1,2,3$ refer to $U(1)_Y$, $SU(2)_L$, $SU(3)_C$ gauge groups of the standard group G_{213}. For $\mu > M_I$ we use the same Eq. (1) but with $a_i \to a_i'$ and $b_{ij} \to b_{ij}'$ by including additional contributions due to new degrees of freedom contained in the split-multiplets [10]

$$\begin{aligned} A&=(2,-1,1)+(2,1,1) , \\ B&=(1,2/3,3)+(1,-2/3,3) , \\ C&=(2,1/3,3)+(2,-1/3,3) , \\ D&=(1,-4/3,3)+(1,4/3,3) , \\ E&=(1,2,1)+(1,-2,1) . \end{aligned} \quad (2)$$

With the components of split multiplets given in (2), the eight models I–VIII considered by ADFFL are defined in Table I. The values of a_i and b_{ij} are well known for the standard model. The values of a_i' and b_{ij}' are easily obtained using formulas given in [10] for the

TABLE I. Identification of eight models (I–VIII) considered by ADFFL in Ref. [9].

	Fermions					Scalars				
Model	*A*	*B*	*C*	*D*	*E*	*A*	*B*	*C*	*D*	*E*
I	0	1	1	0	0	1	0	0	0	0
II	0	0	1	0	0	0	0	0	2	0
III	0	0	1	0	0	0	1	0	1	0
IV	0	0	1	0	0	0	1	0	1	1
V	0	1	1	0	0	0	0	1	1	0
VI	0	0	1	1	0	1	0	1	0	0
VII	0	0	1	0	0	0	2	0	0	1
VIII	0	0	1	0	0	0	3	0	0	0

respective models. When μ is near the GUT scale, the threshold modification of the conventional GUT boundary condition $\alpha_i(M_U)=\alpha_G$ is introduced through the matching functions λ_i:

$$\frac{1}{\alpha_i(M_U)}=\frac{1}{\alpha_G}-\frac{\lambda_i}{12\pi}, \quad i=1,2,3 \ , \tag{3}$$

where α_G is the GUT-coupling constant and $\lambda_i(\mu)$ includes the effects of one-loop modification of the light-gauge boson propagators due to superheavy vector bosons, fermions, and scalars [15].

In order to arrive at the analytic expressions for the mass scales, at first we use the two well-known combinations $\alpha^{-1}(M_Z)-\frac{8}{3}\alpha_2^{-1}(M_Z)$ and $\alpha^{-1}(M_Z)-\frac{8}{3}\alpha_3^{-1}(M_Z)$, to obtain two equations involving $\ln(M_U/M_Z)$ and $\ln(M_I/M_Z)$, $\sin^2\theta_W$, $\alpha(M_Z)$, $\alpha_3(M_Z)$, and two-loop and threshold effects. These equations are then solved to express $\ln(M_U/M_Z)$ and $\ln(M_I/M_Z)$. The expression for the inverse GUT-coupling constant α_G^{-1} is obtained using the fine structure constant matching condition at M_Z [21], i.e.,

$$\alpha^{-1}(M_Z)=\tfrac{5}{3}\alpha_1^{-1}(M_Z)+\alpha_2^{-1}(M_Z) \ ,$$

through the evolution equations for $\alpha_1^{-1}(M_Z)$ and $\alpha_2^{-1}(M_Z)$ and the analytic formulas for $\ln(M_U/M_Z)$ and $\ln(M_I/M_Z)$. The analytic formulas for all the models considered in [10] are as given below:

$$\ln\frac{M_U}{M_Z}=\frac{\pi}{\alpha}\left[A_1-B_1\frac{\alpha}{\alpha_S}+C_1\sin^2\theta_W\right]-D_1(P_1^I+P_1^U)$$
$$-E_1(P_2^I+P_2^U)+F_1(P_3^I+P_3^U)+\Delta_U \ ,$$

$$\Delta_U\equiv\Delta\ln\frac{M_U}{M_Z}=\tfrac{1}{3}(D_1\lambda_1+E_1\lambda_2-F_1\lambda_3) \ , \tag{4a}$$

$$\ln\frac{M_I}{M_Z}=\frac{\pi}{\alpha}\left[A_2+B_2\frac{\alpha}{\alpha_s}-C_2\sin^2\theta_W\right]-D_2(P_1^I+P_1^U)$$
$$-E_2(P_2^I+P_2^U)+F_2(P_3^I+P_3^U)+\Delta_I \ ,$$

$$\Delta_I\equiv\Delta\ln\frac{M_I}{M_Z}=\tfrac{1}{3}(D_2\lambda_1+E_2\lambda_2-F_2\lambda_3) \ , \tag{4b}$$

$$\alpha_G^{-1}=\frac{1}{\alpha}\left[A_3+B_3\frac{\alpha}{\alpha_s}-C_3\sin^2\theta_W\right]$$
$$-\frac{1}{\pi}[D_3(P_1^I+P_1^U)+E_3(P_2^I+P_2^U)$$
$$-F_3(P_3^I+P_3^U)]+\Delta_G \ ,$$

$$\Delta_G\equiv\Delta\alpha_G^{-1}=\frac{1}{3\pi}(D_3\lambda_1+E_3\lambda_2-F_3\lambda_3) \ , \tag{4c}$$

where A_i, B_i, C_i, D_i, E_i, and F_i, $i=1,2,3$ are given in Table II for all the eight models [10]. In Eqs. (4a)–(4c), the terms containing $\alpha(M_Z),\alpha_s(M_Z)$ $[\equiv\alpha_3(M_Z)]$, and $\sin^2\theta_W$ are the one-loop terms and those containing the P_i functions defined as

$$P_i^I=\sum_j B_{ij}X_j^I, \quad X_j^I=\ln\frac{\alpha_j(M_I)}{\alpha_j(M_Z)}, \quad B_{ij}=b_{ij}/a_j \ ,$$

$$P_i^U=\sum_j B_{ij}'X_j^U, \quad X_j^U=\ln\frac{\alpha_j(M_U)}{\alpha_j(M_I)}, \quad B_{ij}'=b_{ij}'/a_j' \ ,$$

are the two-loop contributions. The terms Δ_U, Δ_I, and Δ_G containing the λ_i functions are the threshold contributions on M_U, M_I, and α_G^{-1}, respectively.

We note that the formulas are in the identical forms as (4a)–(4c) in the models II and IV, and III and VII. Also in model VI (4a)–(4c) are of the same form as the models II and IV. Certain clear advantages of the present method can be stated as follows. Through these analytic formulas it is possible to estimate the one-loop, two-loop, and threshold contributions separately. Given more improved measurements of $\sin^2\theta_W$, $\alpha_s(M_Z)$ and $\alpha(M_Z)$ in the future, the one-loop term will be known more accurately. The two-loop terms can be estimated using either the one-loop approximation to the gauge coupling constant evolutions or the iterative convergence procedure described in the next section. The experimental uncertainties in the mass scales and α_G^{-1} can also be evaluated analytically from the known uncertainties of $\sin^2\theta_W$, α_s, and α at M_Z by using the one-loop terms in (4a)–(4c).

III. COMPUTATION OF MASS SCALES AND α_G^{-1} WITHOUT THRESHOLD EFFECTS

The mass scales and the GUT coupling have been evaluated by ADFFL [10] using the method of numerical

TABLE II. Coefficients in the analytic formulas for mass scales and unification coupling in (4a)–(4c).

Model	A_1	B_1	C_1	A_2	B_2	C_2	A_3	B_3	C_3
I	$\frac{1}{38}$	$\frac{25}{57}$	$\frac{7}{19}$	$\frac{12}{19}$	$\frac{28}{19}$	$\frac{60}{19}$	$\frac{53}{76}$	$\frac{157}{114}$	$\frac{85}{38}$
II,IV	$\frac{6}{113}$	$\frac{40}{113}$	$\frac{24}{113}$	$\frac{75}{113}$	$\frac{178}{113}$	$\frac{378}{113}$	$\frac{157}{226}$	$\frac{464}{339}$	$\frac{251}{113}$
III,VII	$\frac{4}{83}$	$\frac{92}{249}$	$\frac{20}{83}$	$\frac{50}{83}$	$\frac{344}{249}$	$\frac{248}{83}$	$\frac{157}{249}$	$\frac{871}{747}$	$\frac{460}{249}$
V	0	$\frac{12}{23}$	$\frac{12}{23}$	$\frac{46}{69}$	$\frac{328}{207}$	$\frac{232}{69}$	$\frac{115}{138}$	$\frac{374}{207}$	$\frac{209}{69}$
VI	$\frac{6}{113}$	$\frac{40}{113}$	$\frac{24}{113}$	$\frac{75}{113}$	$\frac{178}{113}$	$\frac{378}{113}$	$\frac{203}{226}$	$\frac{682}{339}$	$\frac{385}{113}$
VIII	$\frac{9}{365}$	$\frac{162}{365}$	$\frac{138}{365}$	$\frac{216}{365}$	$\frac{492}{365}$	$\frac{1068}{365}$	$\frac{177}{292}$	$\frac{159}{146}$	$\frac{249}{146}$
Model	D_1	E_1	F_1	D_2	E_2	F_2	D_3	E_3	F_3
I	$\frac{5}{456}$	$\frac{15}{152}$	$\frac{25}{228}$	$\frac{5}{19}$	$\frac{12}{19}$	$\frac{7}{19}$	$\frac{265}{912}$	$\frac{117}{304}$	$\frac{157}{456}$
II,IV	$\frac{5}{226}$	$\frac{15}{226}$	$\frac{10}{113}$	$\frac{125}{452}$	$\frac{303}{452}$	$\frac{89}{226}$	$\frac{785}{2712}$	$\frac{345}{904}$	$\frac{116}{339}$
III,VII	$\frac{5}{249}$	$\frac{6}{83}$	$\frac{23}{249}$	$\frac{125}{498}$	$\frac{99}{166}$	$\frac{86}{249}$	$\frac{785}{2988}$	$\frac{101}{332}$	$\frac{871}{2988}$
V	0	$\frac{3}{23}$	$\frac{3}{23}$	$\frac{115}{414}$	$\frac{31}{46}$	$\frac{82}{207}$	$\frac{575}{1656}$	$\frac{101}{184}$	$\frac{187}{414}$
VI	$\frac{5}{226}$	$\frac{15}{226}$	$\frac{10}{113}$	$\frac{125}{452}$	$\frac{303}{452}$	$\frac{189}{226}$	$\frac{1015}{2712}$	$\frac{567}{904}$	$\frac{341}{678}$
VIII	$\frac{3}{292}$	$\frac{147}{1460}$	$\frac{81}{730}$	$\frac{18}{73}$	$\frac{213}{365}$	$\frac{123}{365}$	$\frac{295}{1168}$	$\frac{321}{1168}$	$\frac{159}{584}$

solution of the RGE's. We obtain solutions for these quantities by a new method combining the analytic formulas derived in Sec. II with the iterative convergence procedure of solving the RGE's. To describe the method briefly at first we ignore the two-loop and threshold effects due to superheavy fermions and Higgs scalars to start with. In the first step values of M_U^0, M_I^0, and $\alpha_G^{0\,-1}$ are obtained from (4a)–(4c) keeping only one-loop terms. These masses are used in the numerical solutions of RGE's as the limiting values for the two different ranges: $M_Z \le \mu \le M_I^0$:

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_Z)} - \frac{1}{2\pi} a_i \ln\frac{\mu}{M_Z} - \frac{1}{4\pi}\sum_j B_{ij} \ln\frac{\alpha_j(\mu)}{\alpha_j(M_Z)} \; . \tag{5a}$$

$M_I^0 \le \mu \le M_U^0$:

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_I)} - \frac{1}{2\pi} a_i' \ln\frac{\mu}{M_I} - \frac{1}{4\pi}\sum_j B_{ij}' \ln\frac{\alpha_j(\mu)}{\alpha_j(M_I)} \; . \tag{5b}$$

The one-loop values of $\alpha_i(\mu)$ are obtained ignoring the third term in the right-hand side (RHS) of (5) and are used to estimate these terms in the next approximation to obtain better values of $\alpha_i(\mu)$. The process is iterated until the values of $\alpha_i(\mu)$ obtained for each value of μ converge. The values of $\alpha_i(M_I^0)$ and $\alpha_i(M_U^0)$ obtained in this manner are used to evaluate the P_i^I and P_i^U functions and the two-loop correction terms in the RHS of (4a)–(4c) yielding new values of M_U^0, M_I^0, and α_G^0 to begin the

TABLE III. Model predictions for mass scales and GUT-coupling constant without threshold effects for $\alpha_s = 0.113$.

Models	M_I^0 (GeV)	M_U^0 (GeV)	$\alpha_G^{0\,-1}$
I	$10^{3.045\pm1.3}$	$10^{15.975\pm0.32}$	35.11 ± 1.02
II	$10^{2.151\pm1.35}$	$10^{15.386\pm0.25}$	35.00 ± 1.01
III	$10^{3.623\pm1.2}$	$10^{15.39\pm0.26}$	36.31 ± 0.9
IV	$10^{2.25\pm1.35}$	$10^{15.389\pm0.25}$	35.07 ± 1.01
V	$10^{3.606\pm1.36}$	$10^{16.543\pm0.4}$	33.25 ± 1.32
VI	$10^{2.347\pm1.35}$	$10^{15.5\pm0.25}$	31.80 ± 1.2
VII	$10^{3.697\pm1.2}$	$10^{15.466\pm0.26}$	36.24 ± 0.9
VIII	$10^{3.904\pm1.16}$	$10^{15.988\pm0.32}$	36.6 ± 0.8

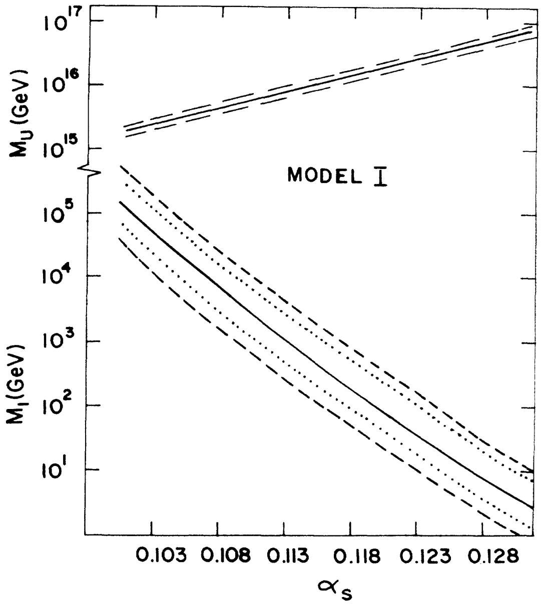

FIG. 1. Variation of mass scales as a function of strong interaction coupling α_s. The solid lines are the predictions in model I without threshold effects. The dotted (dashed) line represents threshold corrections for $\beta=5(10)$ for partially degenerate case as described in the text.

second step. The new values of mass scales obtained in this manner are again used as the limiting values of the two ranges in (5a) and (5b) where the iteration procedure is repeated to obtain improved values of $\alpha_i(\mu)$ including $\alpha_i(M_I^0)$, $\alpha_i(M_U^0)$, P_i^I and P_i^U at the end of the second step. The third step starts with improved values of M_U^0, M_I^0, and α_G^0. In fact the values of mass scales and α_G^{-1} obtained in this manner are found to converge after the fifth or the sixth step in all the eight models. As a cross-check

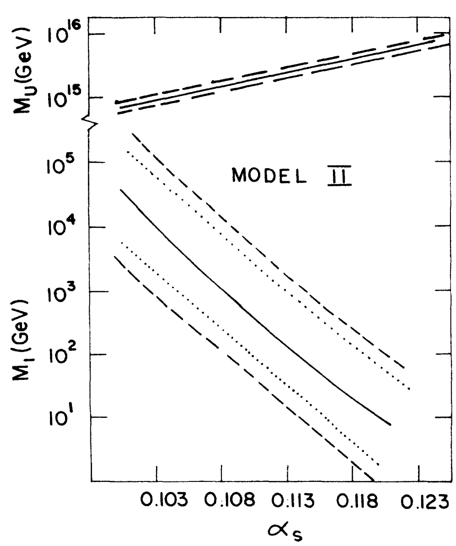

FIG. 2. Same as Fig. 1 but for model II in the degenerate case.

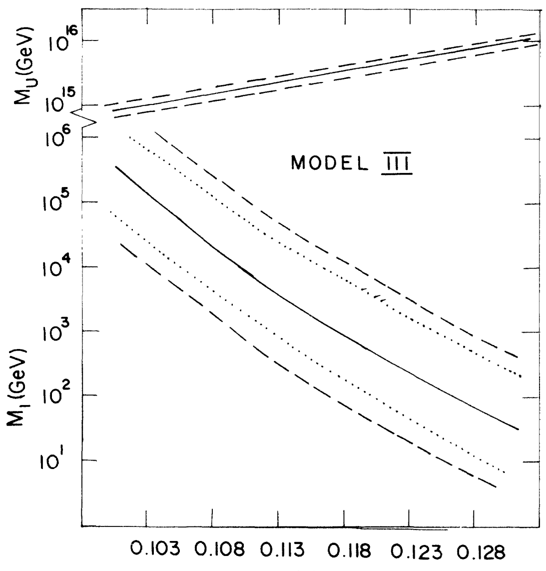

FIG. 3. Same as Fig. 1 but for model III in the degenerate case.

of the validity of the procedure, the gauge couplings $\alpha_i^{-1}(\mu)$ obtained in the last step are plotted against μ using (5) to verify the change of slope at the intermediate scale $\mu=M_I$, and the values of M_U and α_G^{-1} at the meeting point $\mu=M_U$. All the relevant quantities obtained ignoring the small logarithms in the λ_i function are denoted as M_I^0, M_U^0, and α_G^0 as presented in Table III. We have used the same input parameters as ADFFL [10], $\sin^2\theta_W=0.2333\pm0.0008$, $\alpha_s=0.113\pm0.005$, and α^{-1}

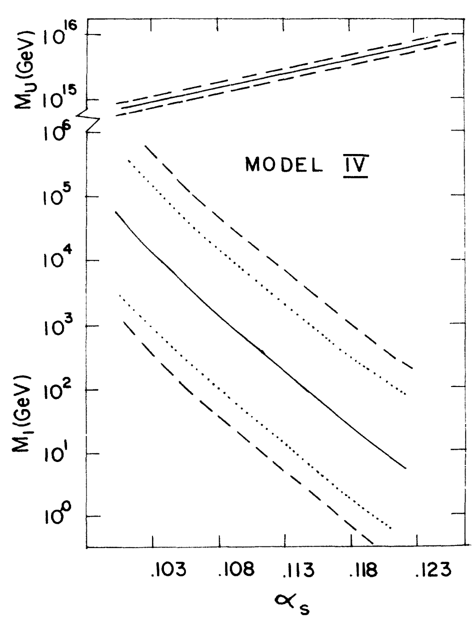

FIG. 4. Same as Fig. 1 but for model IV in the degenerate case.

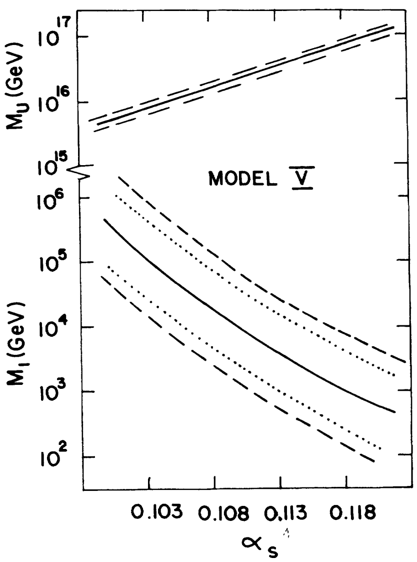

FIG. 5. Same as Fig. 1 but for model V in the degenerate case.

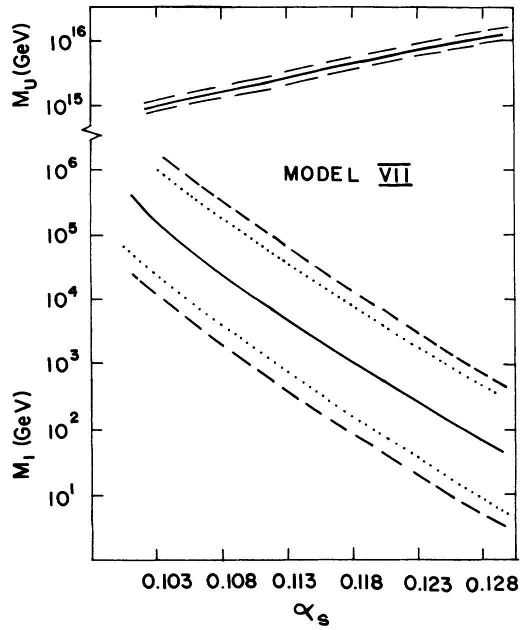

FIG. 7. Same as Fig. 1 but for model VII in the degenerate case.

$=127.9\pm0.2$ at the Z mass. We find that the unification masses are very close to those obtained in [10] having negligible differences in most of the models. Although there are no drastic differences on M_I^0, our values do possess quantitative differences in models V and VIII.

We have analyzed variations of the mass scales with α_s as shown in Figs. 1–8 by the solid lines for all the eight models [10]. These curves, in addition to expressing anticorrelation between M_U and M_I, also indicate the range of values of α_s within which the model predictions are possible.

IV. DEPENDENCE OF THRESHOLD EFFECTS ON SUPERHEAVY MASSES

In Table IV we present the G_{213} content of the superheavy masses of fermions (M_i^F) and scalars (M_i^s), $i=A,B,C,D,E$, having the transformation properties given in (2), while choosing the Higgs scalar extended

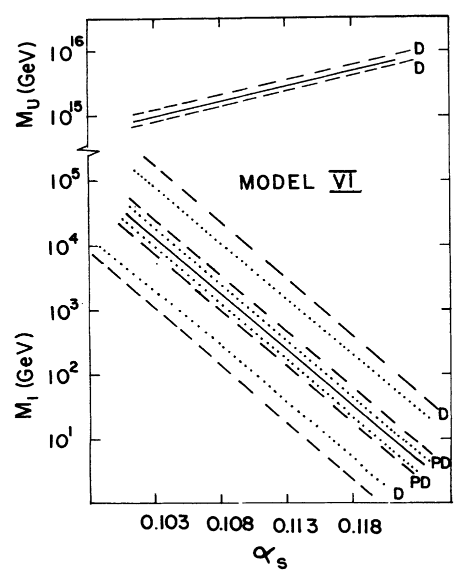

FIG. 6. Same as Fig. 1 but for both degenerate (D) and partially degenerate (PD) cases.

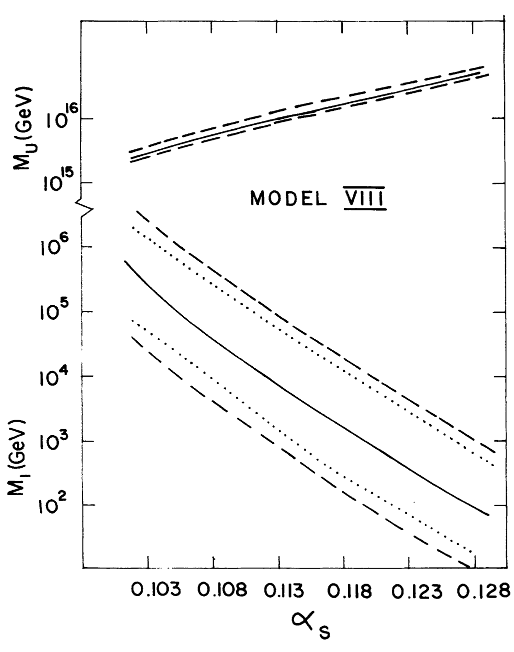

FIG. 8. Same as Fig. 1 but for model VIII in the degenerate case.

TABLE IV. Superheavy masses near the unification scale contributing to threshold effects in models I–VIII where subscripts $A,B,C,D,$ and E refer to the components of split-multiplets.

Model	Masses of fermions	Masses of scalars
I	$M_A^F.M_D^F,M_E^F$	M_B^S
II	M_A^F,M_B^F,M_D^F,M_E^F	$2M_C^2,M_E^S$
III	M_A^F,M_B^F,M_D^F,M_E^F	M_A^S,M_C^S,M_E^S
IV	M_A^F,M_B^F,M_D^F,M_E^F	M_A^S,M_C^S
V	M_A^F,M_D^F,M_E^F	M_E^S
VI	M_A^F,M_B^F,M_E^F	M_B^S,M_D^S,M_E^S
VII	M_A^F,M_B^F,M_D^F,M_E^F	$2M_A^S,M_C^S,M_D^S$
VIII	M_A^F,M_B^F,M_D^F,M_E^F	$3M_A^S$

survival hypothesis has been used [21]. In every model the superheavy Higgs scalar masses near M_U in **24** and **5** used for spontaneous symmetry breaking (SSB) are $5\supset M^{(5)}(1,2/3,3)$; $\mathbf{24}\supset M_1^{(24)}(3,0,1)+M_2^{(24)}(1,0,8)$. Using the notation $\eta_j^F=\ln(M_j^F/M_U)$, $\eta_j^s=\ln(M_j^s/M_U)$, $j=A,B,C,D,E$, $\eta^{(5)}=\ln(M^{(5)}/M_U)$, and $\eta_i^{(24)}=\ln(M_i^{(24)}/M_U)$, $i=1,2$, the matching functions are derived in the Appendix. Using (4a)–(4c) we express Δ_U, Δ_I, and Δ_G in terms of η_i's and consider three different cases, degenerate (D), partially degenerate (PD) and nondegenerate (ND), for each model and extremize Δ_U in each case leading to constraints on η_i. These constraints are then used to compute Δ_I and Δ_G.

To illustrate the method, if the masses are nondegenerate (ND) the extremization of Δ_U in model I leads to $\eta_A^F=\eta_E^F=\eta_1^{(24)}=\eta^{(+)}=\pm\ln\beta$; $\eta_D^F=\eta_B^S=\eta_2^{(24)}=\eta^{(5)}=\eta^{(-)}=\mp\ln\beta$; $\Delta_U=5/114\pm\frac{10}{19}\ln\beta$; $\Delta_I=\frac{1}{19}(1\mp45\ln\beta)$ and $\Delta_G=1/228\pi(75\mp411\ \ln\beta)$. In the partially degenerate (PD) case we assume all fermions and Higgs scalars belonging to the split multiplets possess separate degenerate masses M^F and M^S, and the two superheavy components in **24** possess the same masses $M^{(24)}$ but different from $M^{(5)}$: $M_i^F=M^F$, $M_i^S=M^S$, $i=A,B,C,D,E$; $M_i^{(24)}=M^{(24)}$, $i=1,2$, $\eta_i^F=\eta^F$, $\eta_i^S=\eta^S$, $\eta_i^{(24)}=\eta^{(24)}$. Then extremization of Δ_U leads to $\eta^F=\eta^{(+)}=\pm\ln\beta$, $\eta^S=\eta^{(5)}=\eta^{(24)}=\eta^{(-)}=\mp\ln\beta$ with $\Delta_U=1/114(5\pm17\ln\beta)$, $\Delta_I=\frac{1}{19}(1\pm11\ln\beta)$ and $\Delta_G=1/228\pi(75\pm198\ln\beta)$. Formulas for the degenerate (D) case are obtained most easily by taking $\eta_i^F=\eta_i^S=\eta^{(5)}=\eta^{(24)}=\eta=\pm\ln\beta$ yielding $\Delta_U=1/114(5\mp9\ln\beta)$, $\Delta_I=\frac{1}{19}(1\pm21\ln\beta)$ and $\Delta_G=1/228\pi(75\pm378\ln\beta)$. The corrections for all other models can be derived using (4a)–(4c) and the expressions obtained in the Appendix and can be summarized as

$$\Delta_U=A_U+B_U\ln\beta\ ,$$
$$\Delta_I=A_I+B_I\ln\beta\ , \tag{6}$$
$$\Delta_G=A_G+B_G\ln\beta\ .$$

For all models $A_U\simeq0.044$, but $A_I\simeq0.0526$, 0.0530, and 0.0522 for models I–III, IV–VI, and VI, VIII, respectively. The values of other parameters are given in Table V.

TABLE V. Numerical values of coefficients in the threshold formula (6) for models I–VIII when the superheavy masses are degenerate (D), partially degenerate (PD), and nondegenerate (ND) as defined in the text.

Model	A_G	Type	B_U	B_I	B_G
		(D)	∓0.079	±1.108	±0.5270
I	0.1047	(PD)	∓0.149	±0.580	±0.2764
		(ND)	±0.526	∓2.368	∓0.5737
		(D)	∓0.0700	±1.115	±0.6600
II	0.1047	(PD)	∓0.1415	±2.220	±0.6835
		(ND)	±0.5130	∓3.854	∓0.7272
		(D)	∓0.0736	±1.096	±0.6000
III	0.1047	(PD)	±0.1686	∓1.890	∓0.6240
		(ND)	±0.5528	∓3.176	∓0.6216
		(D)	∓0.0265	±1.785	±0.565
IV	0.0975	(PD)	±0.1415	±1.560	±0.686
		(ND)	±0.5575	∓3.584	∓0.780
		(D)	±0.0870	±1.116	±0.5940
V	0.1053	(PD)	∓0.0869	±0.884	±0.4000
		(ND)	±1.0430	∓2.478	∓0.7623
		(D)	∓0.071	±1.115	±0.627
VI	0.1056	(PD)	±0.141	∓0.230	∓0.033
		(ND)	±0.460	∓2.247	∓0.758
		(D)	∓0.0700	±1.100	±0.625
VII	0.1044	(PD)	±0.1680	±1.890	±0.595
		(ND)	±0.5943	∓3.570	∓0.732
		(D)	∓0.0820	±1.027	±0.5500
VIII	0.1044	(PD)	±0.2958	±1.898	±0.6289
		(ND)	±0.7500	∓3.050	∓0.5930

V. COMPUTATION OF THRESHOLD EFFECTS AND IMPLICATIONS ON MODEL PREDICTIONS

In the absence of any accurate information on the value of β, we follow the standard-model Higgs boson mass as a guide and take $\beta \le 10$. Using (6) and the values of coefficients given in Table V we have computed numerical values of threshold corrections on the unification mass, intermediate scale and the GUT-coupling constant. Using $\beta=5$ and 10 the ratios M_U/M_U^0, M_I/M_I^0, and α_G^{-1} are given in Table VI for models I–VIII for the degenerate, nondegenerate, and partially degenerate cases where M_U^0, M_I^0, and α_G^0 are the two-loop computed values neglecting threshold effects as explained in Sec. III. The corrected inverse coupling is $\alpha_G^{-1}=\alpha_G^{0\,-1}+\Delta\alpha_G^{-1}$. The proton lifetime ($\tau_p$) for the $p \to e^+\pi^0$ mode is computed using the formula

$$\tau_p=\frac{M_U^4}{\alpha_G^2 M_p^5}\,, \tag{7}$$

where M_p = proton mass, and compared with the experimental limit for the $p\to e^+\pi^0$ mode, $\tau_p \ge 3\times 10^{32}$ yr. We define τ_p^0 with the quantities on the RHS of (7) having the superscript.

In Figs. 1–8 we have presented a variation of M_U^0, M_I^0, M_U, and M_I as a function of α_s including threshold effects with $\beta=5$ and 10 in the case where the correction appeared to be the least on the intermediate mass. It is quite clear that the corrections on the unification mass and proton lifetime are small and remain stable against threshold effects in all models for the degenerate and partially degenerate cases shown in Figs. 1–8 with the extremal value of $M_U/M_U^0 \simeq 10^{\pm 0.2}$ and $\tau_p/\tau_p^0 \simeq 10^{\pm 1}$ for $\beta=10$.

In Figs. 9–12 we have plotted $(\tau_p)_{\max}$ and $(\tau_p)_{\min}$ as a

TABLE VI. Numerical values of threshold effects on intermediate mass, unification scale and the GUT-coupling constant in models I–VIII in three different cases.

Model	β	Degenerate M_I/M_I^0	Degenerate M_U/M_U^0	Degenerate $\Delta\alpha_G^{-1}$	Partially degenerate M_I/M_I^0	Partially degenerate M_U/M_U^0	Partially degenerate $\Delta\alpha_G^{-1}$	Nondegenerate M_I/M_I^0	Nondegenerate M_U/M_U^0	Nondegenerate $\Delta\alpha_G^{-1}$
I	5	$10^{\substack{+0.80\\-0.75}}$	$10^{\substack{-0.04\\+0.08}}$	${}^{+0.95}_{-0.74}$	$10^{\substack{+0.43\\-0.38}}$	$10^{\substack{-0.08\\+0.12}}$	${}^{+0.55}_{-0.34}$	$10^{\substack{-1.63\\+1.68}}$	$10^{\substack{+0.39\\-0.35}}$	${}^{-0.82}_{+1.03}$
	10	$10^{\substack{+1.13\\-1.08}}$	$10^{\substack{-0.06\\+0.10}}$	${}^{+1.32}_{-1.11}$	$10^{\substack{+0.60\\-0.56}}$	$10^{\substack{-0.13\\+0.17}}$	${}^{+0.74}_{-0.53}$	$10^{\substack{-2.34\\+2.39}}$	$10^{\substack{+0.55\\-0.51}}$	${}^{-1.22}_{+1.43}$
II	5	$10^{\substack{+0.80\\-0.76}}$	$10^{\substack{-0.03\\+0.07}}$	${}^{+1.17}_{-0.96}$	$10^{\substack{+1.57\\-1.53}}$	$10^{\substack{-0.08\\+0.12}}$	${}^{+1.20}_{-0.99}$	$10^{\substack{-2.48\\+2.52}}$	$10^{\substack{+0.38\\-0.34}}$	${}^{-1.07}_{+1.28}$
	10	$10^{\substack{+1.14\\-1.09}}$	$10^{\substack{-0.05\\+0.09}}$	${}^{+1.62}_{-1.41}$	$10^{\substack{+2.24\\-2.20}}$	$10^{\substack{-0.12\\+0.16}}$	${}^{+1.68}_{-1.47}$	$10^{\substack{-3.56\\+3.60}}$	$10^{\substack{+0.59\\-0.49}}$	${}^{-1.57}_{+1.78}$
III	5	$10^{\substack{+0.79\\-0.74}}$	$10^{\substack{-0.03\\+0.07}}$	${}^{+1.07}_{-0.86}$	$10^{\substack{-1.30\\+1.34}}$	$10^{\substack{+0.14\\-0.10}}$	${}^{-0.90}_{+1.11}$	$10^{\substack{-2.20\\+2.24}}$	$10^{\substack{+0.41\\-0.37}}$	${}^{-0.90}_{+1.10}$
	10	$10^{\substack{+1.11\\-1.07}}$	$10^{\substack{-0.05\\+0.09}}$	${}^{+1.49}_{-1.28}$	$10^{\substack{-1.87\\+1.91}}$	$10^{\substack{+0.19\\-0.15}}$	${}^{-1.33}_{+1.54}$	$10^{\substack{-3.15\\+3.20}}$	$10^{\substack{+0.57\\-0.53}}$	${}^{-1.33}_{+1.54}$
IV	5	$10^{\substack{+1.27\\-1.22}}$	$10^{\substack{-0.00\\+0.04}}$	${}^{+1.01}_{-0.81}$	$10^{\substack{+1.11\\-1.07}}$	$10^{\substack{+0.12\\-0.08}}$	${}^{+1.20}_{-1.10}$	$10^{\substack{-2.48\\+2.59}}$	$10^{\substack{+0.41\\-0.37}}$	${}^{-1.16}_{+1.35}$
	10	$10^{\substack{+1.81\\-1.76}}$	$10^{\substack{-0.01\\+0.05}}$	${}^{+1.40}_{-1.20}$	$10^{\substack{+1.58\\-1.54}}$	$10^{\substack{+0.16\\-0.12}}$	${}^{+1.68}_{-1.48}$	$10^{\substack{-3.56\\+3.61}}$	$10^{\substack{+0.58\\-0.54}}$	${}^{-1.70}_{+1.89}$
V	5	$10^{\substack{+0.80\\-0.76}}$	$10^{\substack{+0.08\\-0.04}}$	${}^{+1.06}_{-0.85}$	$10^{\substack{+0.64\\-0.59}}$	$10^{\substack{-0.04\\+0.08}}$	${}^{+0.75}_{-0.54}$	$10^{\substack{-1.71\\+1.75}}$	$10^{\substack{+0.75\\-0.71}}$	${}^{-1.12}_{+1.99}$
	10	$10^{\substack{+1.14\\-1.09}}$	$10^{\substack{+0.11\\-0.07}}$	${}^{+1.47}_{-1.26}$	$1-{}^{+0.91}_{-0.86}$	$10^{\substack{-0.07\\+0.11}}$	${}^{+1.03}_{-0.82}$	$10^{\substack{-2.45\\+2.50}}$	$10^{\substack{+1.06\\-1.02}}$	${}^{-1.65}_{+1.86}$
VI	5	$10^{\substack{+0.80\\-0.76}}$	$10^{\substack{-0.03\\+0.07}}$	${}^{+1.14}_{-0.90}$	$10^{\substack{+0.18\\-0.14}}$	$10^{\substack{+0.12\\-0.08}}$	${}^{+0.16}_{-0.05}$	$10^{\substack{-1.55\\+1.59}}$	$10^{\substack{+0.34\\-0.30}}$	${}^{-1.11}_{+1.92}$
	10	$10^{\substack{+1.14\\-1.09}}$	$10^{\substack{-0.05\\+0.09}}$	${}^{+1.55}_{-1.94}$	$10^{\substack{+0.25\\-0.21}}$	$10^{\substack{+01.6\\-0.12}}$	${}^{+0.18}_{-0.02}$	$10^{\substack{-2.22\\+2.26}}$	$10^{\substack{+0.48\\-0.44}}$	${}^{-1.64}_{+1.85}$
VII	5	$10^{\substack{+0.79\\-0.75}}$	$10^{\substack{-0.03\\+0.07}}$	${}^{+1.11}_{-0.90}$	$10^{\substack{+1.34\\-1.30}}$	$10^{\substack{+0.14\\-0.10}}$	${}^{+1.06}_{-0.85}$	$10^{\substack{-2.47\\+2.52}}$	$10^{\substack{+0.43\\-0.40}}$	${}^{-1.07}_{+1.28}$
	10	$10^{\substack{+1.12\\-1.08}}$	$10^{\substack{-0.05\\+0.09}}$	${}^{+1.54}_{-1.33}$	$10^{\substack{+1.91\\-1.87}}$	$10^{\substack{+0.19\\-0.15}}$	${}^{+1.47}_{-1.27}$	$10^{\substack{-3.55\\+3.59}}$	$10^{\substack{+0.61\\-0.58}}$	${}^{-1.58}_{+1.79}$
VIII	5	$10^{\substack{+0.74\\-0.70}}$	$10^{\substack{-0.04\\+0.08}}$	${}^{+0.99}_{-0.78}$	$10^{\substack{+1.35\\-1.30}}$	$10^{\substack{+0.23\\-0.19}}$	${}^{+1.12}_{-0.91}$	$10^{\substack{-2.11\\+2.15}}$	$10^{\substack{+0.54\\-0.50}}$	${}^{-0.85}_{+1.06}$
	10	$10^{\substack{+1.05\\-1.00}}$	$10^{\substack{-0.06\\+0.10}}$	${}^{+1.37}_{-1.16}$	$10^{\substack{+1.92\\-1.87}}$	$10^{\substack{+0.31\\-0.27}}$	${}^{+1.55}_{-1.34}$	$10^{\substack{-3.03\\+3.07}}$	$10^{\substack{+0.77\\-0.73}}$	${}^{-1.26}_{+1.47}$

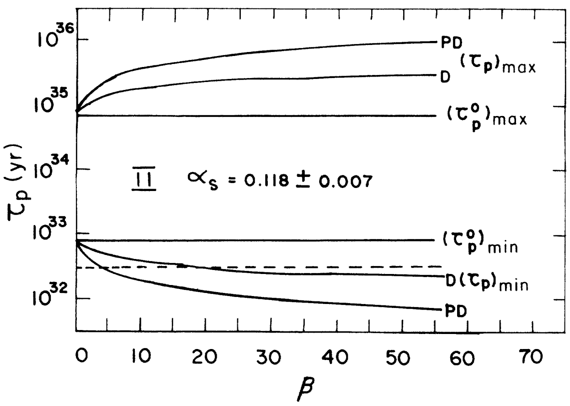

FIG. 9. Predictions on maximal $[(\tau_p)_{max}]$ and minimal $[(\tau_p)_{min}]$ values of proton lifetime in model II as a function of the superheavy mass splitting coefficient β in the degenerate (D) and partially degenerate (PD) cases, τ_p^0 represents proton lifetime for the $p \rightarrow e^+\pi^0$ mode without threshold effects. The dashed horizontal line represents the experimental lower limit on τ_p.

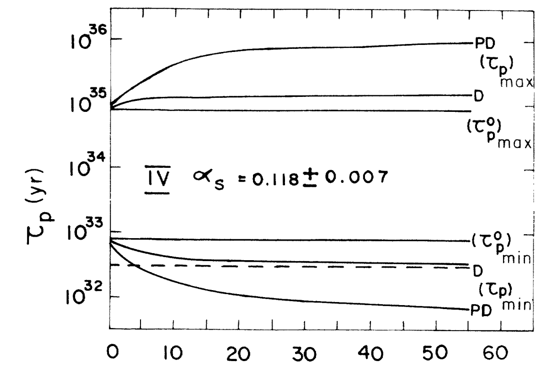

FIG. 11. Same as Fig. 9 but for model IV.

function of β with fixed $\alpha_s = 0.118 \pm 0.0007$ [22] in four different lowest unification mass models: II, III, IV, and VII. It is clear that even for $\beta = 5-10$ in the degenerate or partially degenerate case $(\tau_p)_{max}$ exceeds the Super-Kamiokande limit by a factor of 10–40. Thus, in order to verify or rule out the model by proton decay experiments through the $p \rightarrow e^+\pi^0$ mode, improvements on the accuracy of planned experiments are essential. But in the nondegenerate case the corrections to M_U and τ_p are much larger with the observed extremal modifications $M_U/M_U^0 \simeq 10^{\pm 1}$ and $\tau_p/\tau_p^0 \simeq 10^{\pm 4.2}$ in model V for the same value of β.

The corrections are also found to be larger for the intermediate masses in every model even for the degenerate and partially degenerate cases as shown in Figs. 1–8 by the dashed (dotted) lines for $\beta = 10(5)$. Taking $\alpha_s = 0.118 \pm 0.007$ and $\beta \simeq 10$ including experimental and threshold uncertainties we find that for verifying the model predictions at accelerator energies the search limits on the new fermions and scalars should extend by a factor of nearly 100 compared to the uncorrected predictions in all models, except VI where the threshold correction factor is only $10^{\pm 0.25}$ on M_I in the partially degenerate case. This uncertainty is very reasonable, making the predictions in model VI more precise compared to others. With the improvement of experimental precision in the future, the threshold uncertainty would be the main factor controlling the experimental search for new particles. In this sense model VI is singled out if the superheavy masses are partially degenerate. For the nondegenerate case the threshold effects are found to introduce much larger uncertainties.

VI. SUMMARY AND CONCLUSION

We have derived analytic formulas for mass scales, and the GUT-coupling constant up to two-loop order modified by matching function corrections. When threshold effects are excluded, these formulas are capable of predicting successfully the corresponding values in terms of $\sin^2\theta_W$, α_s, α, and two-loop contributions. Another advantage of the analytic formulas has been to evaluate the experimental uncertainties of relevant quantities in terms of the uncertainties of $\sin^2\theta_W$, α_s, and α. The mass scales and α_G have been computed using a novel procedure that uses these formulas and iterative con-

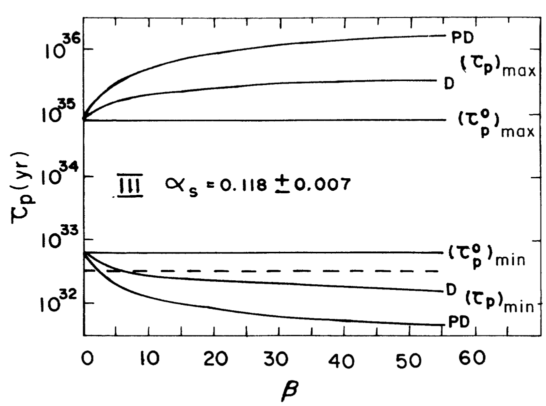

FIG. 10. Same as Fig. 9 but for model III.

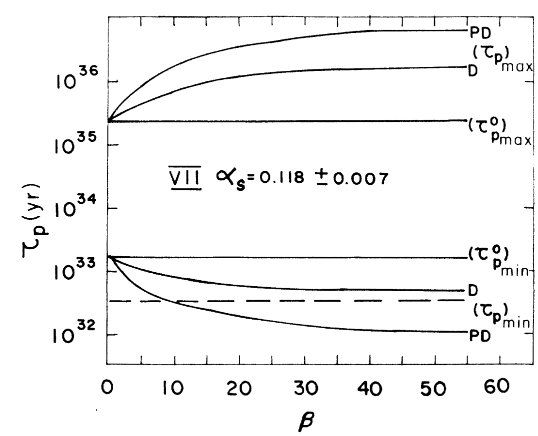

FIG. 12. Same as Fig. 9 but for model VII.

vergence method of solutions of RGE's. Formulas for extremal values of threshold effects have also been obtained in all the eight models considered by ADFFL [10] such that the corresponding numerical values can be readily obtained for any value of the heavy-mass-splitting coefficient in the degenerate, partially degenerate, or nondegenerate cases.

Although the numerical values of threshold corrections are found to be small on the unification mass and proton lifetime in the degenerate and partially degenerate cases, they are quite substantial and are found to change the intermediate mass by nearly one order for $\beta \simeq 10$ in almost all models except model VI. In all such models including experimental uncertainties and threshold effects we find that the search limits for the new fermions and scalars at future accelerator energies should be increased by a factor 100 over the uncorrected predictions. But in model VI in the partially degenerate case the threshold correction factor on M_I is only $10^{\pm 0.25}$, making it more precise compared to others. In the nondegenerate case the corrections are much larger and found to change the proton lifetime and the intermediate masses by more than one order even for a smaller value of the splitting coefficient $\beta \simeq 5$. The threshold corrections are larger in the split-multiplet models mainly because of additional heavy fermions near the GUT scale. Since the corrections are quite large in the nondegenerate case threatening to upset precise model predictions for $\beta \simeq 10$, we conclude that the superheavy masses might be either degenerate or partially degenerate if the model predictions are to be reasonably precise. Alternatively the mass-splitting coefficient must be restricted to smaller values with $\beta \leq 5$ in the nondegenerate case.

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APPENDIX

In this appendix we derive matching functions and threshold effects in all the eight models I–VIII considered by ADFFL. Using the notation

$$f_1 = 5 + \tfrac{12}{5}\eta_A^F + \tfrac{32}{5}\eta_D^F + \tfrac{24}{5}\eta_E^F + \tfrac{2}{5}\eta^{(5)} ,$$
$$f_2 = 3 + 4\eta_A^F + 2\eta_1^{(24)} ,$$
$$f_3 = 2 + 4\eta_D^F + \eta^{(5)} + 3\eta_2^{(24)} ,$$

the matching functions in models I–VIII are stated below.

Model I: $\lambda_1 = f_1 + \tfrac{2}{5}\eta_B^S ,$
$\lambda_2 = f_2$
$\lambda_3 = f_3 + \eta_B^S .$

Model II: $\lambda_1 = f_1 + \tfrac{8}{5}\eta_B^S + \tfrac{2}{5}\eta_C^S + \tfrac{12}{5}\eta_E^S ,$
$\lambda_2 = f_2 + 6\eta_C^F ,$
$\lambda_3 = f_3 + 4\eta_B^F + 4\eta_C^S .$

Model III: $\lambda_1 = f_1 + \tfrac{8}{5}\eta_B^S + \tfrac{3}{5}\eta_A^S + \tfrac{1}{5}\eta_C^S + \tfrac{6}{5}\eta_E^S ,$
$\lambda_2 = f_2 + \eta_A^S + 3\eta_C^S ,$
$\lambda_3 = f_3 + 4\eta_B^F + 2\eta_C^S .$

Model IV: $\lambda_1 = f_1 + \tfrac{8}{5}\eta_B^F + \tfrac{3}{5}\eta_A^S + \tfrac{1}{5}\eta_C^S ,$
$\lambda_2 = f_2 + \eta_A^S + 3\eta_C^S .$
$\lambda_3 = f_3 + 4\eta_B^S + 2\eta_C^S .$

Model V: $\lambda_1 = f_1 + \tfrac{6}{5}\eta_E^S + \tfrac{2}{5}\eta^{(5)} ,$
$\lambda_2 = f_2 ,$
$\lambda_3 = f_3 .$

Model VI: $\lambda_1 = f_1 + \tfrac{8}{5}\eta_B^F - \tfrac{32}{5}\eta_D^F + \tfrac{2}{5}\eta_B^S + \tfrac{8}{5}\eta_D^S + \tfrac{6}{5}\eta_E^S ,$
$\lambda_2 = f_2 ,$
$\lambda_3 = f_3 - 4\eta_A^F + 4\eta_B^F + \eta_B^S + \eta_D^S .$

Model VII: $\lambda_1 = f_1 + \tfrac{8}{5}\eta_B^S + \tfrac{6}{5}\eta_A^S + \tfrac{1}{5}\eta_C^S + \tfrac{8}{5}\eta_D^S ,$
$\lambda_2 = f_2 + 2\eta_A^S + 3\eta_C^S ,$
$\lambda_3 = f_3 + 4\eta_B^S + 2\eta_C^S + \eta_D^s .$

Model VIII: $\lambda_1 = f_1 + \tfrac{8}{5}\eta_B^S + \tfrac{9}{5}\eta_E^S ,$
$\lambda_2 = f_2 + 3\eta_A^S ,$
$\lambda_3 = f_3 + 4\eta_B^S .$

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