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Radiative corrections to the scalar Higgs masses in a non-minimal supersymmetric Standard Model

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Abstract. We calculate the dominant one-loop radiative corrections arising from quark-squark loops to the mass squared matrix of the CP-even Higgs bosons in a non-minimal supersymmetric Standard Model containing two Higgs doublets and a Higgs singlet chiral superfield using one-loop effective potential approximation. We use this result to evaluate upper and lower bounds on the radiatively corrected masses of all the scalar Higgs bosons as a function of the parameters of the model. We find that the one-loop radiative corrections are substantial only for the lightest Higgs boson of the model and can push its mass beyond the reach of LEP. We also calculate an absolute upper bound on the mass of the radiatively corrected lightest Higgs boson and compare it with the corresponding bound in the minimal supersymmetric Standard Model.

1 Introduction

It is well known that supersymmetry is at present the only framework [1] in which the large hierarchy between the Planck (or GUT) scale and the electroweak scale, introduced by the vacuum expectation value of an elementary Higgs scalar [2] in the Standard Model, is natural. The minimal supersymmetric extension of the Standard Model (MSSM) [3] contains two Higgs doublets $H_1=(H_1^0, H_1^-)$ and $H_2=(H_2^+, H_2^0)$, with opposite hypercharge ($Y(H_1)=-1$, $Y(H_2)=+1$), which give masses to down quarks (leptons) and up quarks. After spontaneous symmetry breaking, these give rise to two CP-even neutral Higgs bosons, a CP-odd neutral boson, and two charged Higgs bosons. The lighter CP-even state (h^0) is the lightest of all Higgs bosons in MSSM, its tree level mass being less than that of Z^0 [4], and its couplings to Z^0 being similar to the couplings of Standard Model Higgs boson [5] in the case that the CP-odd neutral scalar is heavier than M_Z.

Recently, it has been pointed out by several authors that the neutral Higgs bosons of the MSSM receive large radiative corrections proportional to $(g^2 m_t^4/m_W^2)$, where m_t and m_W are the top quark and W boson mass, respectively [6–15]. The superpotential for the MSSM [3] is given by

$$W=[h_U QU^c H_2+h_D QD^c H_1+h_E LE^c H_1]+\mu H_1 H_2, \quad (1)$$

where we have suppressed the gauge and generation indices. The three terms in the square brackets are the analogues of the Yukawa couplings of the Standard Model and the last term is the Higgs mixing term which is necessary to give vacuum expectation value to both the Higgs doublets. In realistic models, supersymmetry is softly broken, which ensures that radiative corrections to the mass parameter μ are small, so that $\mu=\mathcal{O}(m_W)$ is technically natural. However, approximate supersymmetry does not explain why μ should be so small in the first place. The simplest method through which a bilinear term in (1) can be generated dynamically is to include an additional singlet Higgs field N [16]. Then, if the superpotential contains a trilinear term $\lambda H_1 H_2 N$, and if N develops a vacuum expectation value $\langle N\rangle=x$, a bilinear term $\lambda x H_1 H_2$ will be generated. When supersymmetry is softly broken, we expect $x=\mathcal{O}(m_W)$, and, since Yukawa couplings being dimensionless are naturally of order 1, $\mu=\lambda x=\mathcal{O}(m_W)$. Such a singlet field appears in grand unified supersymmetric models [17], in the massless mode sector of many superstring models [18], as well as in superstring models based on E_6 [19] and $SU(5)\times U(1)$ gauge groups [20]. A superpotential which is purely trilinear contains only dimensionless couplings and the electroweak scale appears only in the form of soft supersymmetry breaking terms, in the form of scalar masses, trilinear scalar couplings and gaugino masses. Thus, the minimal extension of the supersymmetric Standard Model with an additional Higgs singlet superfield is an appealing alternative to the MSSM, and could be an appropriate model for testing the general assumptions of low energy supersymmetry. However, with the inclusion of the singlet in the model, the tree level upper bound of m_{Z^0} on the mass of the lightest Higgs of the MSSM is lost. It is, therefore, of considerable importance to study the radiative corrections to the Higgs masses in the minimal extension of the minimal supersymmetric standard model in order to devise effective search strategies for the detection of the Higgs bosons in a wide class of phenomenologically viable supersymmetric models.

In this paper we shall study the radiative corrections to the scalar Higgs mass matrix in the minimal extension of the MSSM which includes two Higgs doublets and a Higgs singlet. We shall use the one loop effective potential formalism for the purpose [21]. The superpotential for the model is given by:

$$W = h_U Q U^c H_2 + h_D Q D^c H_1 + h_E L E^c H_1 + \lambda H_1 H_2 N - \tfrac{1}{3} k N^3, \tag{2}$$

where gauge and generation indices are understood [22]. Notice that if $k=0$, then the Lagrangian corresponding to (2) has a global symmetry $N \to N e^{i\theta}$, $H_1 H_2 \to H_1 H_2 e^{-i\theta}$ which is spontaneously broken by the vacuum expectation values of the Higgs fields, giving rise to an unacceptable axion. In order to avoid this, we introduce the additional term proportional to k which explicitly breaks this symmetry. We note that the the superpotential (2) does not contain any bilinear terms and is, therefore, scale independent. This means that if bilinear terms are absent at the tree-level, they cannot be generated by renormalization because of the nonrenormalization properties of supersymmetric theories. Furthermore, there could be problems related to the destablization of the gauge hierarchy* or those related to the domain wall problem [23] in supersymmetric models with Higgs singlet superfields like the one described by the superpotential (2) but these are either model dependent or have very little impact on the low energy phenomenology that we are interested in this paper. In Sect. 2 we shall briefly review the Higgs sector of the minimal extension of the MSSM and set our notation. In Sect. 3 we shall calculate the radiative corrections to the mass squared matrix of the scalar Higgs bosons of this model using one-loop effective potential approximation. Section 4 is devoted to the presentation of the numerical results and their discussion.

2 Tree level masses

We start our discussion with the tree level scalar potential involving only the Higgs fields H_1, H_2 and N [22] which is given by

$$V_0 = V_F + V_D + V_{\text{soft}}, \tag{3}$$

$$V_F = |\lambda|^2 [(|H_1|^2 + |H_2|^2)|N|^2 + |H_1 H_2|^2] + |k|^2 |N|^4 - (\lambda k^* H_1 H_2 N^{*2} + \text{h.c.}), \tag{4}$$

$$V_D = \frac{g^2}{8}(H_2^+ \vec{\sigma} H_2 + H_1^+ \vec{\sigma} H_1)^2 + \frac{g'^2}{8}(|H_2|^2 - |H_1|^2)^2, \tag{5}$$

$$V_{\text{soft}} = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_N^2 |N|^2 - (\lambda A_\lambda H_1 H_2 N + \text{h.c.}) - (\tfrac{1}{3} k A_k N^3 + \text{h.c.}). \tag{6}$$

We have implicitly assumed that squark and slepton fields have vanishing expectation values. Note that there are trilinear terms proportional to λ and k in V_{soft} which are absent in the MSSM. In general λ, k, A_λ, A_k are complex numbers. We can define the overall phases of the fields H_2 and N such that λA_λ and $k A_k$ are real and positive. We shall assume that there is no explicit CP violation in the scalar sector which implies λk^* is real. We can thus choose λ, k, A_λ, A_k to be real. Furthermore, by making an $SU(2)_L \times U(1)_Y$ gauge transformation we can choose

$$v^+ \equiv \langle H^+ \rangle = 0, \quad v_2 \equiv \langle H_2^0 \rangle \in R^+. \tag{7}$$

However, the Higgs potential still depends on the phases of $v_1 \equiv \langle H_1^0 \rangle$ and $x \equiv \langle N \rangle$ which allows for spontaneous CP violation. To forbid this possibility it is sufficient to choose λk to be real and positive. We then find three [22] degenerate minima corresponding to different values for ϕ_1 and ϕ_x, which are phases of v_1 and x:

$$\begin{aligned} &\phi_1 = 0, \quad &&\phi_x = 0, \\ &\phi_1 = \tfrac{2}{3}\pi, \quad &&\phi_x = \tfrac{4}{3}\pi, \\ &\phi_1 = \tfrac{4}{3}\pi, \quad &&\phi_x = \tfrac{2}{3}\pi, \end{aligned} \tag{8}$$

This is a reflection of the underlying Z_3 symmetry of the Lagrangian. We shall henceforth work in the first of these vacua*. The degeneracy between the three vacua can be broken without affecting the low energy physics [23].

Transforming to the unitary gauge, we can write

$$H_1 = \begin{pmatrix} \operatorname{Re} H_1^0 + \frac{i}{\sqrt{2}} \sin\beta\, A^0 \\ \sin\beta\, C^{+*} \end{pmatrix},$$

$$H_2 = \begin{pmatrix} \cos\beta\, C^+ \\ \operatorname{Re} H_2^0 + \frac{i}{\sqrt{2}} \cos\beta\, A^0 \end{pmatrix}, \tag{9}$$

where $\tan\beta = v_2/v_1$, and

$$A^0 \equiv \sqrt{2}[\sin\beta \operatorname{Im} H_1^0 + \cos\beta \operatorname{Im} H_2^0], \tag{10}$$

$$C^+ \equiv \cos\beta\, H^+ + \sin\beta\, H^{-*}. \tag{11}$$

The combinations orthogonal to A^0 and C^+ are the would be neutral Goldstone boson

$$G^0 \equiv \sqrt{2}[\cos\beta \operatorname{Im} H_1^0 - \sin\beta \operatorname{Im} H_2^0]$$

and the charged Goldstone boson

$$G^+ \equiv \cos\beta\, H^{-*} - \sin\beta\, H^+$$

which give masses to $W^\pm$ and Z^0. The physical degrees of freedom of the model are given by three scalars, two pseudoscalars and one charged scalar. Taking into account the constraint $m_W^2 = \frac{1}{2} g^2 (v_1^2 + v_2^2)$, we can express the Higgs masses in terms of six parameters λ, k, A_λ, A_k, x and $\tan\beta$ by using the stationarity constraints in H_1^0, H_2^0 and N to eliminate $m_{H_1}^2$, $m_{H_2}^2$ and m_N^2. We note that the stationarity constraints do not guarantee that (v_1, v_2, x) is

* See [22] for a discussion and references

* Even if λk chosen to be real and negative, it can be shown that the vacuum $\phi_1 = 0$, $\phi_x = 0$ remains a global minimum if we place a mild restriction on a combination of soft SUSY breaking parameters and vacuum expectation values of the scalars [24]

the global minimum of the effective potential (3); for each choice of the parameters we must verify that these lead to the global minimum. In addition we must verify that $v^- \equiv \langle H^- \rangle$ vanishes in the ground state. This is equivalent to the requirement that the squared mass of the charged Higgs boson

$$m_C^2 = m_W^2 - \lambda^2(v_1^2+v_2^2) + \lambda(A_\lambda + kx)\frac{2x}{\sin 2\beta} \tag{12}$$

is positive. The mass of C may be less or greater than m_W^2 depending on the relative sizes of the last two terms.

At tree level the mass squared matrix for the neutral scalar Higgses takes the form (in the basis $\mathrm{Re}\,H_1^0$, $\mathrm{Re}\,H_2^0$, $\mathrm{Re}\,N$):

$$\begin{bmatrix} m_Z^2\cos^2\beta + A_\Sigma\lambda x\tan\beta & -A_\Sigma\lambda x + 2\lambda^2 v_1 v_2 - \frac{m_Z^2}{2}\sin 2\beta & \lambda v_2(2\lambda x\cot\beta - kx - A_\Sigma) \\ -A_\Sigma\lambda x + 2\lambda^2 v_1 v_2 - \frac{m_Z^2}{2}\sin 2\beta & m_Z^2\sin^2\beta + A_\Sigma\lambda x\cot\beta & \lambda v_1(2\lambda x\tan\beta - kx - A_\Sigma) \\ \lambda v_2(2\lambda x\cot\beta - kx - A_\Sigma) & \lambda v_1(2\lambda x\tan\beta - kx - A_\Sigma) & 4k^2x^2 - A_k kx + \frac{A_\lambda \lambda v_1 v_2}{x} \end{bmatrix}, \tag{13}$$

where,

$$A_\Sigma \equiv A_\lambda + kx,$$
$$v^2 \equiv v_1^2 + v_2^2,$$
$$r \equiv x/v. \tag{14}$$

We shall denote the mass eigenstates of the scalar mass matrix by S_1, S_2, S_3, labelled in order of increasing mass. The mass matrix (13) has been discussed in great detail in [22]. Here we note that an upper bound [25] on the tree level mass of the lightest Higgs S_1 can be obtained by using the fact that the smallest eigenvalue of (13) is smaller than the smallest eigenvalue of the upper left 2×2 submatrix:

$$m_{S_1}^2 \le m_Z^2\left[\cos^2 2\beta + \frac{2\lambda^2}{g^2}\cos^2\theta_W \sin^2 2\beta\right]. \tag{15}$$

3 Effective potential and radiative corrections

In this section we compute the radiative corrections to the scalar Higgs boson mass squared matrix (13). For this purpose we shall use the full one-loop effective potential [21], which is quite accurate in estimating the Higgs boson masses. We recall that the effective action is defined as the Legendre transform of the generating functional $W[J]$ through

$$\Gamma[\phi_c] = W[J] - \int d^4x\, J(x)\phi_c(x), \tag{16a}$$

where $\phi(x) \equiv \delta W/\delta J(x)$ is the classical field. The effective action can be expanded in powers of momenta about the zero-momentum point:

$$\Gamma[\phi_c] = \int d^4x\,[-V(\phi_c) + \tfrac{1}{2}(\partial_\mu\phi_c)^2 Z(\phi_c) + \dots\dots\dots]. \tag{16b}$$

$V(\phi_c)$, which is the constant field piece of the effective action, is called the effective potential. One can also expand $\Gamma[\phi]$ in powers of ϕ around $\phi_c = v$, where v is the vacuum expectation value of the field. It can then be shown that the coefficients of such an expansion

$$\Gamma^{(n)} = \left(\frac{\delta^n \Gamma[\phi_c]}{\delta\phi^n}\right)_{\phi_c = v}, \tag{16c}$$

are the proper vertices of the theory. In particular the Fourier transform of $\Gamma^{(2)}$, the inverse propagator, can be written as

$$\tilde\Gamma^{(2)}(p) = p^2 - m^2 + \Sigma(p^2), \tag{16d}$$

where the m^2 is the tree level mass. The physical mass is the value of p^2 at which $\tilde\Gamma^{(2)}$ is zero:

$$m_{\mathrm{phys}}^2 = m^2 - \Sigma(m_{\mathrm{phys}}^2). \tag{17a}$$

From (16b) we get

$$\tilde\Gamma^{(2)}(p=0) = -\left[\frac{\partial^2 V(\phi_c)}{\partial\phi_c^2}\right]_{\phi_c = v} \tag{17b}$$

or

$$\left[\frac{\partial^2 V(\phi_c)}{\partial\phi_c^2}\right]_{\phi_c = v} = m^2 - \Sigma(0) = m_{\mathrm{phys}}^2 + \Sigma(m_{\mathrm{phys}}^2) - \Sigma(0). \tag{17c}$$

In the case of a theory with more than one Higgs field, as is the case with supersymmetric models, the second derivative of V at $\phi_c = v$ is replaced by the second derivative matrix

$$\frac{1}{2}\left[\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right]_{\phi_i = v_i,\, \phi_j = v_j}.$$

From (17c) it is clear that the physical masses of Higgs scalars are given by the eigenvalues of the second derivative matrix evaluated at $\phi_1 = v_1$, $\phi_2 = v_2$ provided that $\Sigma_{ij}(p^2) \approx \Sigma_{ij}(0)$. The effective potential approach to calculate the radiative corrections thus consists in evaluating all self-energies at vanishing external momentum. The most important contributions to the Higgs self-energies $\Sigma_{ij}(p^2)$ at one-loop level come from diagrams with the top quark and stop circulating in the loop, due to large Yukawa coupling of the top quark. The analytic structure of these contributions in the complex p^2 plane is such that $\Sigma_{ij}(p^2) \approx \Sigma_{ij}(0)$ if $p^2 \ll 4M^2$, where M is the mass of the circulating particle in the loop [11]. Therefore, the physical Higgs masses can be approximated by the eigenvalues of the second derivative matrix provided that $m_{\mathrm{phys}}^2 \ll 4\min\{m_t^2, m_{\tilde t}^2\}$. This condition is fulfilled by the light Higgs scalars but may not be satisfied by the heavier scalars. However, the deviation from exact results is generally numerically small in the case of heavier scalars.

Furthermore, for very large values of $\tan\beta$, also the bottom-sbottom contributions in the loops can be nonnegligible. When the external momentum or the Higgs mass approaches or exceeds the threshold of the internal particles (i.e bottom quarks), the full correction can be rather different from the zero-momentum case. However, in that case the corrections themselves are small, either in the absolute sense or relative to the (increased) tree level mass. Thus, for Higgs masses much larger than the bottom quark mass (the phenomenologically relevant case), the generic criterion for the validity of the effective potential approximation is one which is applicable for the top-stop sector.

At tree level the effective potential is nothing but the tree level potential V_0. At one-loop the effective potential can be written as

$$V_1(Q) = V_0(Q) + \Delta V_1(Q), \tag{18a}$$

where $V_0(Q)$ is the level potential evaluated with couplings renormalized at some scale Q, and

$$\Delta V_1(Q) = \frac{1}{64\pi^2}\,\mathrm{Str}\,M^4\left(\ln\frac{M^2}{Q^2} - \frac{3}{2}\right), \tag{18b}$$

where Str denotes the supertrace defined as

$$\mathrm{Str}\,f(M^2) = \sum(-1)^{2J_i}(2J_i+1)f(m_i^2). \tag{18c}$$

In (18c) m_i^2 denotes the field-dependent mass eigenvalue of the ith particle of spin J_i. In (18b), we have used the fact that supersymmetry is softly broken in (3) so that field-independent contributions proportional to $\mathrm{Str}\,M^2$ can be neglected. The logarithmic terms in (18b) are crucial in making $V_1(Q)$ independent of renormalization scale Q, up to higher loop corrections. If we neglect small wave function renormalization effects then the vacuum expectation values are also independent of the renormalization scale.

As discussed above, in the one-loop effective potential approximation, the radiatively corrected Higgs mass-squared matrix can be approximated by the second derivatives of V_1 with respect to the Higgs fields [11]. Since we assume CP to be conserved, the 3×3 matrix of second derivatives of V_1 in the CP-even sector decouples from the CP-odd sector. The dominant contributions to the supertrace (18b) come from the top-stop system. Furthermore, $\mathrm{Str}\,M^2$ is field-independent only when bottom-sbottom contributions are also included. We will, therefore, include both the top-stop and the bottom-sbottom contributions in our calculations. We note that, as in MSSM, the contributions of bottom-sbottom states can be nonnegligible for large values of $\tan\beta$.

Keeping only the dependence on H_1^0, H_2^0 and N the 2×2 $\tilde t_L - \tilde t_R$ and $\tilde b_L - \tilde b_R$ mass squared matrices can be written as:

$$\begin{bmatrix} m_Q^2 + h_t^2|H_2^0|^2 + \left[g^2 - \frac{g'^2}{3}\right]\left[\frac{|H_1]^2 - |H_2|^2}{4}\right] & h_t(A_t H_2^{0*} + \lambda N H_1^0) \\ h_t(A_t H_2^0 + \lambda N^* H_1^{0*}) & m_U^2 + h_t^2|H_2^0|^2 + g'^2\left[\frac{|H_1^0|^2 - |H_2^0|^2}{3}\right] \end{bmatrix}, \tag{19}$$

and

$$\begin{bmatrix} m_Q^2 + h_b^2|H_1^0|^2 - \left[g^2 + \frac{g'^2}{3}\right]\left[\frac{|H_1^0]^2 - |H_2^0|^2}{4}\right] & -h_b(A_b H_1^{0*} + \lambda N H_2^0) \\ -h_b(A_b H_1^0 + \lambda N^* H_2^{0*}) & m_D^2 + h_b^2|H_1^0|^2 - g'^2\left[\frac{|H_1^0|^2 - |H_2^0|^2}{6}\right] \end{bmatrix}, \tag{20}$$

respectively. Here m_Q, m_U, m_D are field independent soft supersymmetry-breaking squark masses. The symbols A_t and A_b are the soft supersymmetry breaking parameters corresponding to the trilinear part of the superpotential, the first two terms in (2),:

$$V_{\mathrm{soft}} = A_t h_t \tilde t_L \tilde t_R^* H_2^0 - A_b h_b \tilde b_L \tilde b_R^* H_1^0 + \text{h.c.} + \ldots \tag{21}$$

We have assumed that A_t and A_b are real. The eigenvalues of the $\tilde t$ and $\tilde b$ mass squared matrices are given by

$$m_{\tilde t_{1,2}}^2 = m_t^2 + \tfrac{1}{2}(m_Q^2 + m_U^2) + \tfrac{1}{4}m_Z^2\cos 2\beta \pm \sqrt{\left[\tfrac{1}{2}(m_Q^2 - m_U^2) + \tfrac{1}{12}(8m_W^2 - 5m_Z^2)\cos 2\beta\right]^2 + m_t^2(A_t + \lambda x\cot\beta)^2}, \tag{22}$$

$$m_{\tilde b_{1,2}}^2 = m_b^2 + \tfrac{1}{2}(m_Q^2 + m_D^2) - \tfrac{1}{4}m_Z^2\cos 2\beta \pm \sqrt{\left[\tfrac{1}{2}(m_Q^2 - m_D^2) - \tfrac{1}{12}(4m_W^2 - m_Z^2)\cos 2\beta\right]^2 + m_b^2(A_b + \lambda x\tan\beta)^2}. \tag{23}$$

The only difference from the minimal supersymmetric Standard Model, as far as the $\tilde t$ and $\tilde b$ masses are concerned, is to replace μ of the MSSM by λx in (22) and (23).

We now come to the question of evaluation of the radiatively corrected mass-squared matrix for the CP-even mass eigenstates. Define $\psi_i \equiv \mathrm{Re}\, H_i^0$ for $i=1, 2$ and $\psi_3 \equiv \mathrm{Re}\, N$. Then we find

$$\boxed{\begin{aligned}\frac{1}{2}\left[\frac{\partial^2 V_1}{\partial\psi_i \partial\psi_j}\right]_{v_1, v_2, x} = &\begin{bmatrix} m_Z^2 \cos^2\beta & -m_Z^2 \frac{\sin 2\beta}{2} + 2\lambda^2 v_1 v_2 & 2\lambda^2 v_1 x - \lambda k v_2 x \\ -m_Z^2 \frac{\sin 2\beta}{2} + 2\lambda^2 v_1 v_2 & m_Z^2 \sin^2\beta & 2\lambda^2 v_2 x - \lambda k v_1 x \\ 2\lambda^2 v_1 x - \lambda k v_2 x & 2\lambda^2 v_2 x - \lambda k v_1 x & 4k^2 x^2 - k A_k x - \lambda k v_1 v_2 \end{bmatrix} \\ &+ \begin{bmatrix} \frac{\lambda x}{2}\tan\beta & \frac{-\lambda x}{2} & \frac{-\lambda v_2}{2} \\ \frac{-\lambda x}{2} & \frac{\lambda x}{2}\cot\beta & \frac{-\lambda v_1}{2} \\ \frac{-\lambda v_2}{2} & \frac{-\lambda v_1}{2} & \frac{\lambda v_1 v_2}{2x} \end{bmatrix} \Delta + \frac{3g^2}{16\pi^2 m_W^2} \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{12} & \Delta_{22} & \Delta_{23} \\ \Delta_{13} & \Delta_{23} & \Delta_{33} \end{bmatrix},\end{aligned}} \tag{24}$$

where

$$\Delta = 2A_\Sigma - \frac{3g^2}{32\pi^2 \sin^2\beta}\frac{m_t^2}{m_W^2}\frac{A_t}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)}(f(m_{\tilde t_1}^2) - f(m_{\tilde t_2}^2)) - \frac{3g^2}{32\pi^2 \cos^2\beta}\frac{m_b^2}{m_W^2}\frac{A_b}{(m_{\tilde b_1}^2 - m_{\tilde b_2}^2)}(f(m_{\tilde b_1}^2) - f(m_{\tilde b_2}^2)), \tag{25}$$

and

$$\Delta_{11} = \frac{m_b^4}{\cos^2\beta}\left[\ln\left(\frac{m_{\tilde b_1}^2 m_{\tilde b_2}^2}{m_b^4}\right) + \frac{2A_b(A_b + \lambda x \tan\beta)}{(m_{\tilde b_1}^2 - m_{\tilde b_2}^2)}\ln\left(\frac{m_{\tilde b_1}^2}{m_{\tilde b_2}^2}\right)\right] + \frac{m_b^4}{\cos^2\beta}\left[\frac{A_b(A_b + \lambda x\tan\beta)}{(m_{\tilde b_1}^2 - m_{\tilde b_2}^2)}\right]^2 g(m_{\tilde b_1}^2, m_{\tilde b_2}^2) + \frac{m_t^4}{\sin^2\beta}\left[\frac{\lambda x(A_t + \lambda x\cot\beta)}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)}\right]^2 g(m_{\tilde t_1}^2, \mathrm{m}_{\tilde t_2}^2), \tag{26}$$

$$\Delta_{22} = \frac{m_t^4}{\sin^2\beta}\left[\ln\left(\frac{m_{\tilde t_1}^2 m_{\tilde t_2}^2}{m_t^4}\right) + \frac{2A_t(A_t + \lambda x \cot\beta)}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)}\ln\left(\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}\right)\right] + \frac{m_t^4}{\sin^2\beta}\left[\frac{A_t(A_t + \lambda x\cot\beta)}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)}\right]^2 g(m_{\tilde t_1}^2, m_{\tilde t_2}^2) + \frac{m_b^4}{\cos^2\beta}\left[\frac{\lambda x(A_b + \lambda x\tan\beta)}{(m_{\tilde b_1}^2 - m_{\tilde b_2}^2)}\right]^2 g(m_{\tilde b_1}^2, m_{\tilde b_2}^2), \tag{27}$$

$$\Delta_{33} = \frac{m_t^4}{\sin^2\beta}\left[\frac{\lambda v_1(A_t + \lambda x\cot\beta)}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)}\right]^2 + \frac{m_b^4}{\cos^2\beta}\left[\frac{\lambda v_2(A_b + \lambda x\tan\beta)}{(m_{\tilde b_1}^2 - m_{\tilde b_2}^2)}\right]^2, \tag{28}$$

$$\Delta_{12} = \frac{m_t^4}{\sin^2\beta}\left[\frac{\lambda x(A_t + \lambda x\cot\beta)}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)}\right]\left[\ln\left(\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}\right) + \frac{A_t(A_t + \lambda x\cot\beta)}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)} g(m_{\tilde t_1}^2, m_{\tilde t_2}^2)\right] + \frac{m_b^4}{\cos^2\beta}\left[\frac{\lambda x(A_b + \lambda x\tan\beta)}{(m_{\tilde b_1}^2 - m_{\tilde b_2}^2)}\right]\left[\ln\left(\frac{m_{\tilde b_1}^2}{m_{\tilde b_2}^2}\right) + \frac{A_b(A_b + \lambda x\tan\beta)}{(m_{\tilde b_1}^2 - m_{\tilde b_2}^2)} g(m_{\tilde b_1}^2, m_{\tilde b_2}^2)\right], \tag{29}$$

$$\Delta_{13} = \frac{m_b^4}{\cos^2\beta}\left[\frac{\lambda v_2(A_b + \lambda x\tan\beta)}{(m_{\tilde b_1}^2 - m_{\tilde b_2}^2)}\right]\left[\ln\left(\frac{m_{\tilde b_1}^2}{m_{\tilde b_2}^2}\right) + \frac{A_b(A_b + \lambda x\tan\beta)}{(m_{\tilde b_1}^2 - m_{\tilde b_2}^2)} g(m_{\tilde b_1}^2, m_{\tilde b_2}^2)\right] + \frac{\lambda^2 v_1 x}{\sin^2\beta}\left\{m_t^4\left[\frac{(A_t + \lambda x\cot\beta)}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)}\right]^2 + \frac{m_t^2}{2}\left[\frac{f(m_{\tilde t_1}^2) - f(m_{\tilde t_2}^2)}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\right]\right\}, \tag{30}$$

$$\Delta_{23} = \frac{m_t^4}{\sin^2\beta}\left[\frac{\lambda v_1(A_t + \lambda x\cot\beta)}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)}\right]\left[\ln\left(\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}\right) + \frac{A_t(A_t + \lambda x\cot\beta)}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)} g(m_{\tilde t_1}^2, m_{\tilde t_2}^2)\right] + \frac{\lambda^2 v_2 x}{\cos^2\beta}\left\{m_b^4\left[\frac{(A_b + \lambda x\tan\beta)}{(m_{\tilde b_1}^2 - m_{\tilde b_2}^2)}\right]^2 + \frac{m_b^2}{2}\left[\frac{f(m_{\tilde b_1}^2) - f(m_{\tilde b_2}^2)}{m_{\tilde b_1}^2 - m_{\tilde b_2}^2}\right]\right\}, \tag{31}$$

Here the functions $f(m^2)$ and $g(m_1^2, m_2^2)$ are defined as in the MSSM [11]:

$$f(m^2) = 2m^2\left(\ln\frac{m^2}{Q^2} - 1\right), \tag{32}$$

and

$$g(m_1^2, m_2^2) = 2 - \left(\frac{m_1^2 + m_2^2}{m_1^2 - m_2^2}\right)\ln\left(\frac{m_1^2}{m_2^2}\right). \tag{33}$$

We note that the explicit Q-dependence in the one-loop expression for the mass-squared matrix through Δ parameter is actually cancelled by the implicit Q-dependence of the parameters A_Σ, λ and $\tan\beta$, so that the physical values of the Higgs masses are independent of the renormalization scale Q. The Q-dependence of Δ_{13} and Δ_{23} through the function $f(m^2)$ can be reabsorbed, upto terms of $\mathcal{O}(g^4)$, by the implicit Q-dependence of $m_W^2=\frac{1}{2}g^2(v_1^2+v_2^2)$. The residual Q-dependence is a two-loop effect that would cancel in a full calculation using two-loop effective potential. Thus the global Q-dependence is numerically very small and a convenient choice for Q is $Q\sim m_W$, so that m_W can be identified with the physical $W^\pm$ mass. Finally, we note that in obtaining (26)–(31) we have neglected the contribution of D-terms in squark masses (22) and (23). Actually D-terms should be neglected everywhere in our formulae since we are including only the quark-squark contributions to ΔV_1. The D-terms contributions to squark masses are, anyway small particularly when the soft SUSY breaking mass parameters are large in comparison to m_W and m_Z, and $\tan\beta$ is not too large compared to unity.

4 Results

The one-loop mass-squared matrix (24) cannot be diagonalized analytically. In this section we shall present numerical results in a physically transparent manner which emerged from a numerical diagonalization of this matrix.

Besides the parameters λ, k, r, $\tan\beta$, A_λ and A_k, which occur in the tree level potential, we have now additional parameters in the one-loop calculation. These include, besides the top quark mass, the soft susy breaking terms in the squark mass matrices and the amount of left-right squark mixing. It is not possible to present an analysis of the full parameter space, and, therefore, we impose some restrictions on parameters in order to discuss our results in a meaningful manner. For simplicity we choose $m_Q=m_U=m_D=\tilde{m}$, as done in the case of MSSM. We shall fix $\lambda=0.87$ and $k=0.63$, the values which are favored by the fixed points of the renormalization group equations [22]. This is done purely for the purposes of illustration. The results derived above can of course be used for any other values of λ and k. Having fixed these parameters, we shall present the results for different values of r ranging between 0.1 to 10.0. We shall study the sensitivity of the results with respect to $\tan\beta$ by considering two different values $\tan\beta=1.5$ and $\tan\beta=4.0$. For fixed λ, k, r and $\tan\beta$, a given value of A_λ fixes the mass of the charged Higgs boson m_C*. The range of values for the parameter A_k will then be determined by imposing [22] the conditions

$$\langle V_{\text{neutral}}(\text{1-loop})\rangle<0, \quad m_{S_1}^2(\text{1-loop})>0, \tag{34}$$

on the one-loop corrected potential in the neutral directions and the one-loop corrected mass of the lightest Higgs boson. In this way we shall be able to explore the allowed range of m_C for which solutions exist. The maximum value of A_k is determined by failure to satisfy one of the constraints in (34). When A_k increase from zero, if $\langle V\rangle>0$ is encountered before $m_{S_1}^2<0$, then there is a lower bound on m_{S_1}, otherwise the lower bound on m_{S_1} is zero.

The results of the above procedure in obtaining the upper and lower bounds on the 1-loop radiatively corrected mass of the Higgs bosons S_1, S_2 and S_3 are shown in Fig. 1 to Fig. 4. We have taken $m_t=150$ GeV, and $A_t=A_b=A=0$ throughout in our calculations. Fig. 1 shows results for small $r=0.1$. From this figure the following features are noteworthy. The lower bound on radiatively corrected m_{S_1} is zero, as at tree level, except in a small range near the upper bound for m_C. In this case the radiative corrections are small for all the Higgs scalar masses. Furthermore, solutions exist only for supersymmetry breaking scalar mass $\tilde{m}\lesssim 200$ GeV. For substantially higher values of $\tilde{m}$ there are no solutions. Figure 2 shows the effect of varying r keeping all other parameters same as in Fig. 1. We notice that the lower bound on m_{S_1} is substantially increased from its tree level value for $\tilde{m}=300$ GeV. Also the upper bound on m_C is significantly reduced. The same is true for the mass of the Higgs scalar S_2. We note that the radiative corrections for m_{S_3} are not appreciable in this case. Further, there are no solutions for $\tilde{m}$ appreciably larger than 300 GeV. Figure 3 shows the results for large $r=10.0$. Here again the lower bound on the lightest Higgs is significantly changed from its tree level value. Unlike the previous cases of small r values, in this case solutions exist for large $\tilde{m}=1$ TeV. We note that the radiative corrections for the heavier Higgs masses are insignificant in this case. For the large values of $\tilde{m}$ that are permitted in this case, the upper bound on m_C is noticeably increased. Finally in Fig. 4 we show the effect of varying $\tan\beta$. There is a large shift in the lower bound on m_{S_1} for $\tilde{m}=300$ GeV. For the same value of $\tilde{m}$ the upper bound on m_C is substantially decreased. Also for large values of $\tan\beta$, the soft susy breaking parameter $\tilde{m}$ is restricted to relatively small values. The lower bound on m_{S_2} is appreciably increased for larger values of m_C, whereas m_{S_3} remains practically unaffected by radiative corrections except that the allowed range of m_C is reduced for larger values of $\tilde{m}$.

We conclude that at low values of $r\simeq 0.1$, the radiative corrections arising from quark-squark contributions are rather small even for $m_t=150$ GeV, but the corrections can be large for large $r\geq 1.0$. Increasing $\tan\beta$ at fixed r decreases the shift in m_{S_1}, leaving m_{S_2} and m_{S_3} unaffected. At low $r\leq 1.0$ the upper bound on m_C is decreased, whereas at larger r the upper bound on m_C increases. At small r only relatively low values of SUSY breaking parameter $\tilde{m}\leq 300$ GeV are permitted irrespective of the values of $\tan\beta$, whereas at larger $r\simeq 10$, large values of $\tilde{m}\simeq 1$ TeV can lead to solutions. These conclusions are not qualitatively changed for nonzero A. From our analysis we find

* The parameter m_C is different from the mass of physical radiatively corrected charged Higgs boson, but this is not relevant for our purpose. Furthermore, radiative corrections to the charged Higgs mass are expected to be relatively small for a reasonable range of parameters as in the case of MSSM [26]. The reason for this is that a global $SU(2)\times SU(2)$ symmetry protects the charged Higgs mass from obtaining large radiative corrections [27]

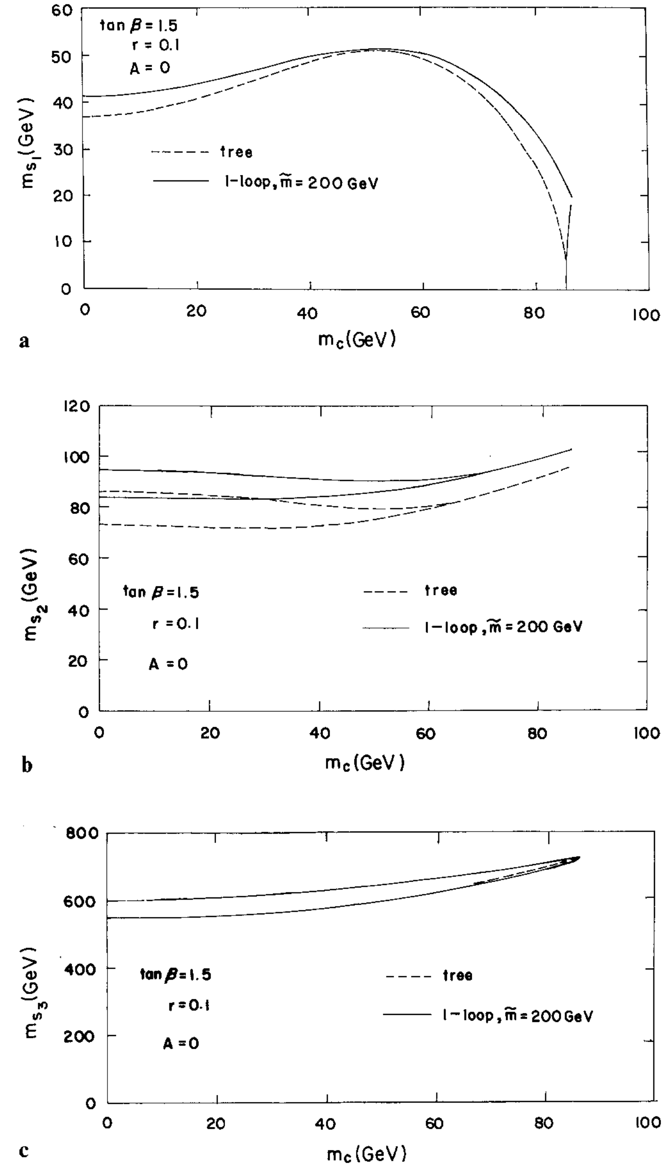

Fig. 1a–c. Curves of maximum and minimum mass for the neutral Higgs boson as a function of m_C. The solid curves are the Higgs masses with radiative corrections and the dashed curves without radiative corrections. The lower bound on m_{s_1} is zero before radiative corrections and is nonzero after radiative corrections only for m_C near its upper bound. For m_C values outside the range plotted there are no solutions which satisfy the criteria (34). We have taken $\lambda=0.87$, $k=0.63$, $\tan\beta=1.5$, $r=0.1$, and $m_t=150$ GeV, $A_t=A_b\equiv A=0$, $\tilde{m}=200$ GeV. Note that there are no solutions for value of $\tilde{m}$ significantly larger than 200 GeV in this case

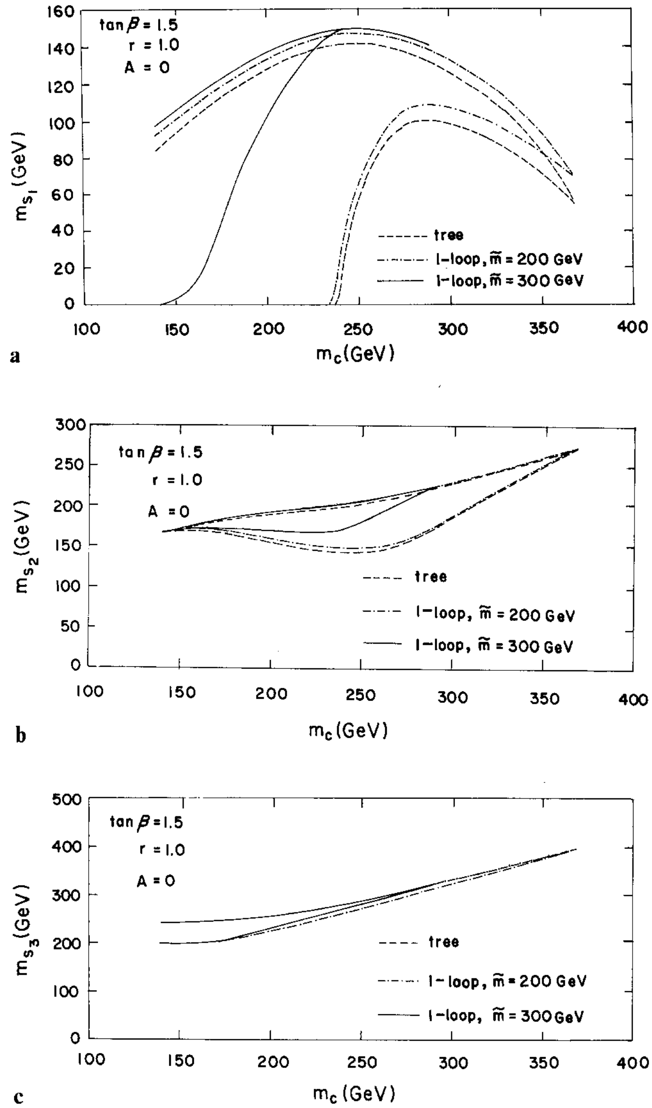

Fig. 2a–c. Same as Fig. 1 except that $r=1.0$, and we have shown curves for two values of $\tilde{m}=200$ GeV (dot dashed) and $\tilde{m}=300$ GeV (solid). Dashed curves are the tree level results. In this case there are no solutions for $\tilde{m}$ much larger than 300 GeV. Note that the upper bound on m_C is considerably reduced compared to the tree level result for $\tilde{m}=300$ GeV. Notice that the 1-loop result in the case of m_{s_3} is not distinguishable from the tree level result for $\tilde{m}=200$ GeV

that the upper bound on m_{S_1} can be as large as 160 GeV for a reasonable range of parameters. As far as the heavier Higgs bosons S_2 and S_3 are concerned, although we have calculated their masses in the effective potential approximation, an approach which may not be valid for larger masses, we expect that, in analogy to the MSSM [28], the differences from an exact diagrammatic approach are numerically small in general.

An absolute upper bound on radiatively corrected m_{S_1} can be obtained by finding the smallest eigenvalue of the upper left 2×2 submatrix of the 3×3 radiatively corrected second derivative matrix (24). We then obtain the upper bound

$$m_{S_1}^2 \le m_Z^2\left[\cos^2 2\beta+\frac{2\lambda^2}{g^2}\cos^2\theta_W\sin^2 2\beta\right] +\frac{3g^2}{16\pi^2 m_W^2}\left[\Delta_{11}\cos^2\beta+\Delta_{22}\sin^2\beta+\Delta_{12}\sin 2\beta\right], \tag{35}$$

to be compared with the tree level bound (15). Notice that this bound is independent of k and Δ. It is easy to see, by making a large x expansion of Δ_{11}, Δ_{12} and Δ_{33} on the right side of (35), that the radiatively corrected upper bound depends only logarithmically on x, the leading powers of x cancelling among the various terms in the large x limit. Thus the upper bound (35) is protected from

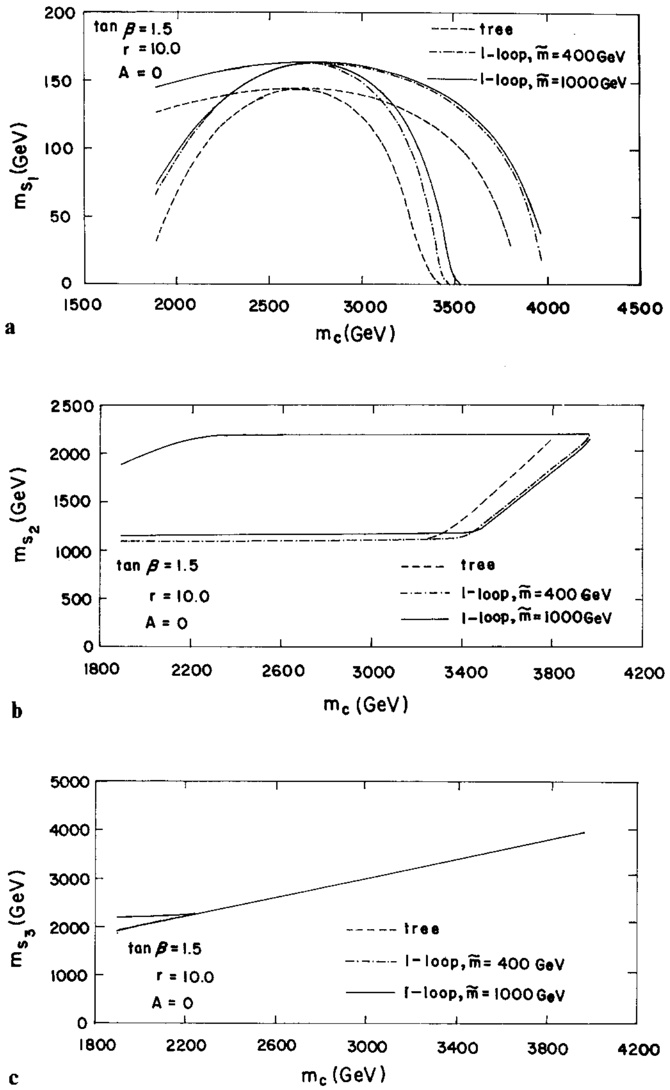

Fig. 3a–c. Same as Fig. 1 but with $r = 10.0$. We have shown results for $\tilde{m} = 400$ GeV (dot dashed) and $\tilde{m} = 1$ TeV (solid), whereas the dashed curve is the tree level result. In this case the allowed range of $\tilde{m}$ is large, upto 1 TeV. The radiative corrections for m_{s_2} and m_{s_3} are not discernible in this case

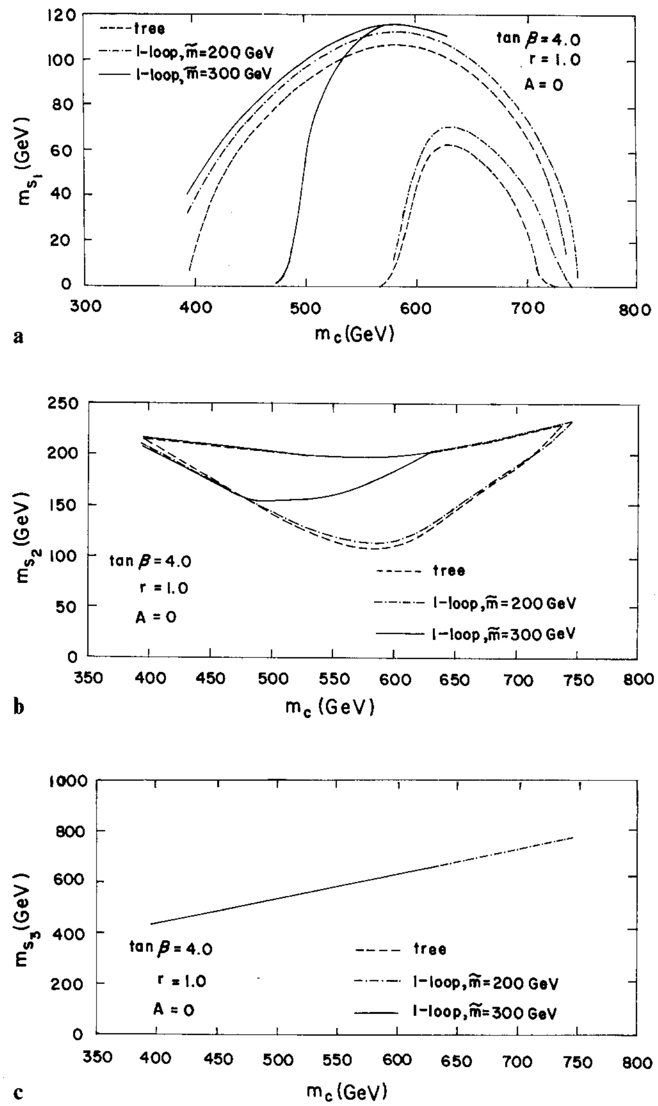

Fig. 4a–c. Same as Fig. 2 except that $\tan\beta = 4.0$. Here also solutions are possible for relatively small values of $\tilde{m} = 200$ GeV (dot dashed) and $\tilde{m} = 300$ GeV (solid). Dashed curves are tree level results. For S_3 the one-loop result for $\tilde{m} = 200$ GeV coincides with the tree level result

large radiative corrections due to quark-squark loops which could, in principle, arise from large x values. In other words the vacuum expectation value x decouples in this limit. We note that x, being the vacuum expectation value of a Higgs singlet, is not constrained by experiment. That the upper bound on the lightest Higgs mass depends only logarithmically on x is a analogous to the logarithmic dependence of electroweak radiative corrections on the Higgs mass in the standard electroweak model with one Higgs doublet [29]. In the present case the phenomenon is a consequence of underlying gauge invariance and supersymmetry of the model.

We have plotted the upper bound (35) in Fig. 5 for $A_t = A_b = A = 0$, $\tilde{m} = \mathrm{TeV}$, $\lambda = 0.87$, $r = 0.1$, and $\tan\beta = 1$ as a function of m_t. The corresponding upper bound for the MSSM is also shown. We see from this graph that the upper bound is the case of nonminimal model lies systematically higher as compared with the minimal model. Varying the parameters for the nonminimal model does not change this conclusion. For $m_t = 150$ GeV, we find the upper bound on m_{S_1} to be $\simeq 165$ GeV as compared with the upper bound of about 115 GeV on the lightest Higgs of the minimal supersymmetric Standard Model.

We conclude that it is possible that the lightest Higgs boson of the nonminimal model, after the radiative corrections due to quark-squark loops are taken into account, may lie outside the range of LEP for some range of parameters. There is an upper bound on its mass which lies systematically above the corresponding upper bound in the case of minimal supersymmetric Standard Model. It is important to delineate the regions of parameter space where the lightest Higgs of the nonminimal model could be accessible experimentally after taking into account the radiative corrections [30]. It is important to discuss the results of Higgs searches in the context of supersymmetric models in as model independent a way as possible.

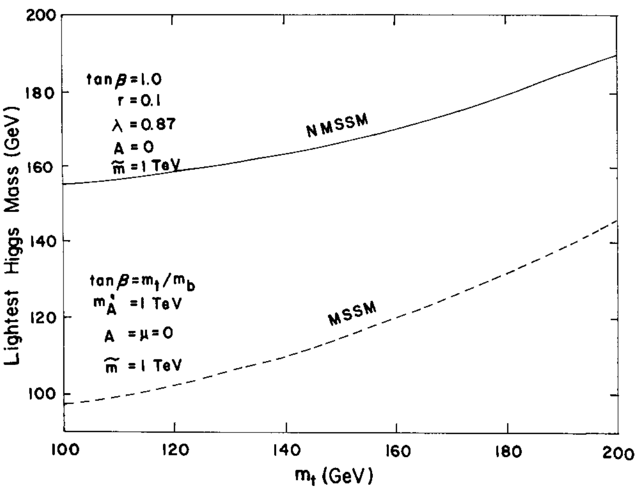

Fig. 5. Upper bound (35) for m_{S_1} (solid curve) compared with the upper bound on the lightest Higgs of the minimal supersymmetric standard model (dashed curve) as a function of m_t for the choice of parameters shown. We have taken $r=0.1$ so that λx ($\simeq 17$) is small. Changing r and $\tan\beta$ for the nonminimal model does not change the results significantly

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