INVESTIGATIONS
ON
SOME GAUGE MODELS
(Relating to Particle Physics)

by

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to
my beloved parents

CERTIFICATE

I certify that the dissertation entitled **"Investigation on some gauge models"** submitted by Supriya Bhattacharjee, in partial fulfilment for the requirement of the Degree of Master of Philosophy incorporates the Student's bonafide researches under my supervision.

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CONTENTS

CHAPTER I

INTRODUCTION

One of the great triumphs of theoretical physics has been the development of the standard model[1-5] of elementary particles and their interactions. The standard model combines the $SU(3)_C$-colour gauge theory of the strong interaction with $SU(2)_L X U(1)_Y$ model of weak and electromagnetic interactions. This model has been spectacularly successful in describing and predicting an enormous range of phenomena within a theoretically satisfactory framework. With the discovery of the standard model, based upon $SU(3)_C X SU(2)_L X U(1)_Y$, the phenomena of strong and electroweak interactions could be described with similar technique as the quantum electrodynamics — a long cherished goal of theoretical physicists. The strong interaction is now described by $SU(3)_C$ at the elementary level of coloured quarks and massless gluons. Besides, the eight gluons that carry the colour forces, the electroweak gauge group, $SU(2)_L X U(1)_Y$, and its spontaneous symmetry breaking to $U(1)_{em}$, predicts three massive weak vector-bosons $W_L^{\pm}, Z_L$, and one massless gauge boson which is the photon. The charged weak gauge bosons, $W_L^{\pm}$, mediate charged-current weak interaction between left-handed charged fermions, whereas the neutral Z°_L bosons mediates the weak neutral current interactions. At low energies, the electroweak theory reproduces the wellknown V-A structure of the weak interactions[6].

The existence of neutral-current weak interactions have been confirmed within present experimental accuracies and agrees with the standard model predictions in an excellent fashion[7]. The carriers of electro-weak interactions, such as $W^{\pm}_{L}$ and Z°_{L} bosons have been experimentally detected at the CERN $\bar{p}p$ collider[8,9]. The observed experimental values of their masses are completely compatible with the predictions of the $SU(2)_L \times U(1)_Y$ model, thus supporting the gauge hypothesis of a partially unified electroweak interactions. These and a large number of other experimental observations have vindicated the electroweak unification idea beyond any doubt. The existence of scaling of structure functions in deep inelastic lepton-hadron scattering, scaling violations, jet structure in $e^+e^- \longrightarrow$ hadrons and at the $\bar{p}p$ collider, the QCD (Quantum chromodynamics) running coupling constant, and numerous other processes point out to $SU(3)_C$ as the gauge theory of strong interactions[5]. Thus, the standard model based upon the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ has been established unequivocally. Inspite of its crowning success, the standard model has some theoretical and conceptual difficulties : The basic Lagrangian based upon $SU(3)_C \times SU(2)_L \times U(1)_Y$ possesses three different coupling constants; therefore, it is not a true unification of forces. What has been achieved is only a partial unification of weak and electromagnetic interactions. The strong interaction, based upon $SU(3)_C$, still remains as a separate entity, unaffected by electroweak unification. Even in the electroweak model only the left-handed fermions play the dominant role in contributing to the charged and neutral current processes. In the standard model, neutrinos are massless. Experimental measurements on β-decay, neutrino-oscillations and attractive theoretical models for the solution of solar-neutrino puzzle, suggest that neutrinos might be having a small mass. The standard model also does not explain why parity (P) is violated in weak interaction, neither does it explain the origin of the CP violation in strong and weak interactions, although, through Kobayashi-Moskawa[10] ansatz, the weak CP-violating parametes are put in by hand. The standard model does not answer why quarks are different from leptons. Also it fails to explain why electric charge is quantized.

In order to solve some of these difficulties, an attractive attempt was made to construct a theory in which both left-handed and right-handed fermions couple to charged and neutral gauge bosons and it, therefore, involves both V-A as well as V+A charged currents. In the models with V±A structure, it is possible to generate neutrino masses over a wide range in a very natural way. At first proposed by Mahapatra and Pati,[11] and Pati andSalam,[11] these models are based upon the electroweak gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and are known as the left-right symmetric (LRS) model. Besides explaining the parity violation in weak interactions, the LRS model ascribes a spontaneous origin to weak CP-violations. Pati and Salam also advanced the exciting idea that, above a certain scale, the quarks might be treated in a similar footing as leptons by means of $SU(4)_C$ unification.[11]

The purpose of this dissertation is to review and learn the theory and predictions of the electroweak model based upon $SU(2)_L \times U(1)_Y$ and study its successes. Our next goal is to review some aspects of the LR model and its variants.[12] Lastly we observe how the weak interaction phenomenology based upon LR model and its variants[12] should be modified in view of the extended survival hypothesis or the minimal fine tuning hypothesis.

The dissertation is organized in the following manner. In chapter 2, we review the Weinberg-Salam-Glashow electroweak theory based upon $SU(2)_L \times U(1)_Y$. In chapter 3, the predicted values of masses of gauge bosons and neutral currents are computed and compared with the experimental results. Chapter 4 deals with the theories of LR models. For the generation of gauge boson masses in LR models, both *Higgs doublets and triplets are considered here*. Constraints arising from potential minimization in these models are discussed. The generation of neutrino masses and the see-saw mechanism that operates in LR model are also described in this chapter. In chapter 5, predictions of the neutral current parameters obtained for LRS model are compared with the $SU(2)_L XU(1)_Y$ model parameters and the experimental results. Here we also discuss the light and heavy gauge boson masses in the absence of left handed Higgs triplet and for no W_L-W_R mixing in the case of $SU(2)_L\ XU(1)_R\ XU(1)_{B-L}$ model. We also discuss here how neutrino masses get modified in $SU(2)_L\ XU(1)_R\ XU(1)_{B-L}$ model. In chapter 6, we observe how the gauge boson masses and neutral current parameters are modified in view of the constraint imposed by minimal fine tuning hypothesis. Here we also suggest how limits on $W^{\pm}_R$ and and Z_R gauge boson masses could be obtained from ν-e scattering and $e^+e^- \longrightarrow \mu^+\mu^-, \tau^+\tau^-$. Finally we summarize our review work and observation in chapter 7.

CHAPTER 2

THE WEINBERG-SALAM-GLASHOW MODEL

The simplest unification model of the weak and electromagnetic forces is the model which was independently developed by Glashow[1], Weinberg[3] and Salam[2]. The model assumes the $SU(2)_L XU(1)_Y$ group as the fundamental gauge group of electroweak interactions. The Weinberg-Salam model is reviewed in this chapter.

In section 2.1 the electroweak Lagrangian containing the gauge boson and the leptonic terms, is written down. In sec 2.2 the mechanism of spontaneous symmetry breaking by Higgs scalar[13] is described. We obtain gauge boson masses, and the effective Lagrangian for neutral current and charged current interactions in sec. 2.3. In sec. 2.4 extension of Weinberg-Salam model to the hadronic sector by GIM mechanism[15] is reviewed. A brief summary of this chapter is provided in section 2.5.

2.1 The electroweak Lagrangian

Considering the model in its purely leptonic form and starting with the fermions, such as electron and its neutrino which form a left-handed weak isospin doublet

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad (2.1.1)$$

where
$$\left.\begin{aligned} \nu_L &= \frac{1}{2}(1 - r_5)\nu \\ e_L &= \frac{1}{2}(1 - r_5)e \end{aligned}\right\} \qquad (2.1.2)$$

Since the neutrino is apparently massless,

$$\nu_R = \frac{1}{2}(1 + r_5)\nu = 0 \qquad (2.1.3)$$

So, we designate only one right-handed fermion

$$R \equiv e_R = \frac{1}{2}(1 + r_5)e \qquad (2.1.4)$$

which is an isospin singlet invariant under SU(2) group.

To incorporate electromagnetism we define a weak hypercharge Y. Requiring that the Gell-Mann-Nishijima relation

$$Q = I_3 + \frac{Y}{2} \qquad (2.1.5)$$

be satisfied, leads to the assignments

$$\left.\begin{aligned} Y_L &= -1 \\ Y_R &= -2 \end{aligned}\right\} \qquad (2.1.6)$$

By construction, the weak isospin projection I_3 and the weak hypercharge Y are commuting observables

$$[I_3, Y] = 0 \qquad (2.1.7)$$

We now take the group of transformation generated by I and Y to be the gauge group $SU(2)_L \times U(1)_Y$ of the theory. Introducing the gauge bosons

$$\left.\begin{aligned} &b^1_\mu,\ b^2_\mu,\ b^3_\mu \quad \text{for} \quad SU(2) \\ &\tilde{A}_\mu \quad \text{for} \quad U(1) \end{aligned}\right\} \qquad (2.1.8)$$

the Lagrangian is written as

$$\alpha = \alpha_{gauge} + \alpha_{leptons} \tag{2.1.9}$$

where, $\alpha_{gauge} = -\frac{1}{4} F^{1}_{\mu\nu} F^{1\mu\nu} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}$ (2.1.10)

The field strength tensor are

$$F^{l}_{\mu\nu} = \partial_\nu b^{l}_{\mu} - \partial_\mu b^{l}_{\nu} + g\epsilon_{jkl} b^{j}_{\mu} b^{k}_{\nu} \tag{2.1.11}$$

and for U(1) gauge field, it is.

$$f_{\mu\nu} = \partial_\nu \tilde{A}_\mu - \partial_\mu \tilde{A}_\nu \tag{2.1.12}$$

and,

$$\begin{aligned} \alpha_{lepton} &= \bar{R} i\gamma^\mu (\partial_\mu + \frac{ig'}{2} \tilde{A}_\mu Y)R \\ &+ \bar{L} i\gamma^\mu (\partial_\mu + \frac{ig'}{2} \tilde{A}_\mu Y + \frac{ig}{2} \bar{\tau}.\bar{b}_\mu)L \end{aligned} \tag{2.1.13}$$

where g is the coupling constant for weak isospin group $SU(2)_L$ and g'/2 is the coupling constant for the weak hypercharge group $U(1)_Y$.

2.2 Spontaneous symmetry breaking by Higgs mechanism

The theory is not a satisfactory one, for two reasons:

(i) It contains four massless weak gauge bosons (b^1_μ, b^2_μ, b^3_μ, $\tilde{A}$) whereas nature has only one, the photon.

(ii) In addition, the local $SU(2)_L$ invariance forbids gauge boson mass terms but the Lagrangian is renormalised. By massless gauge bosons, the range of weak interaction becomes infinite as against the experimental fact where the weak interaction has a range of the order of a fermi which implies the mediating vector boson masses must be of the order of 100 GeV.

Therefore the theory has to be modified so that there will remain only one conserve quantity (the electric charge) and one massless gauge boson (the photon) and the other gauge bosons acquire masses.

This way to get massive gauge bosons out of a Lagrangian involving massless gauge bosons was found out by Higgs-Kibble mechanism.[13] To accomplish this, we introduce a complex doublet of scalar field

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix} \tag{2.2.1}$$

which transforms like an SU(2) doublet and must have weak hypercharge

$$Y_\phi = +1 \tag{2.2.2}$$

by virtue of Gell-Mann-Nishijima relation (2.1.5).

We add a term to the Lagrangian

$$\mathcal{L}_{scalar} = (D^\mu\phi)^+ (D_\mu\phi) - V(\phi^+\phi) \tag{2.2.3}$$

where $$D^\mu = \partial_\mu + \frac{ig'}{2}\tilde{A}_\mu Y + \frac{ig}{2} \bar{\tau}.\bar{b}_\mu \tag{2.2.4}$$

is the co-variant derivative and as usual the potential is

$$V(\phi^+\phi) = \mu^2(\phi^+\phi) + |\lambda| (\phi^+\phi)^2 \tag{2.2.5}$$

Besides this, we also add an interaction term which involves Yukawa couplings of the scalars to the fermions

$$\alpha_{Yukawa} = - G_e [\bar{R}(\phi^+L) + (\bar{L}\phi)R] \tag{2.2.6}$$

which is invariant under $SU(2)_L$ X $U(1)_Y$ transformation and is a Lorentz scalar. Therefore the total Lagrangian becomes,

$$\alpha = \alpha_{gauge} + \alpha_{leptons} + \alpha_{scalars} + \alpha_{Yukawa} \tag{2.2.7}$$

$$\begin{aligned} \alpha = & - \frac{1}{4} F^1_{\mu\nu} F^{\mu\nu 1} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} \\ & + \bar{L}i\gamma^\mu(\partial_\mu + \frac{ig'}{2} \tilde{A}_\mu Y + \frac{ig}{2} \bar{\tau}.\bar{b}_\mu)L \\ & + \bar{R}i\gamma^\mu(\partial_\mu + \frac{ig'}{2} \tilde{A}_\mu Y)R + (D_\mu\phi)^+(D^\mu\phi) \\ & - \mu^2\phi^+\phi - \lambda(\phi^+\phi)^2 - G_e (\bar{L}\phi R + \bar{R}\phi^+L) \end{aligned} \tag{2.2.8}$$

Now considering the case of spontaneous symmetry breaking where $\mu^2<0$ and from the potential (2.2.5)

$$V(\phi^+\phi) = \mu^2 (\phi^+\phi) + |\lambda| (\phi^+\phi)^2$$

The vacuum expectation value of the scalar field

$$<\phi>_o = \begin{pmatrix} 0 \\ V/\sqrt{2} \end{pmatrix} \tag{2.2.9}$$

where $$V = \sqrt{\frac{-\mu^2}{|\lambda|}} \tag{2.2.10}$$

This breaks $SU(2)_L X U(1)_Y$ symmetries but preserves the invariance under the $U(1)_{em}$ symmetry, generated by the electric charge operator.

A would-be-Goldstone boson [14] is associated with every generator of the gauge group that does not leave the vacuum invariant. The vacuum is left invariant by a generator G, if,

$$e^{i\alpha G} < \phi >_o = < \phi >_o \tag{2.2.11}$$

for infinitesimal transformation

$$(1 + i\alpha G) < \phi >_o = < \phi >_o$$

$$\text{or,} \qquad i\alpha G < \phi >_o = 0$$

$$\text{or,} \qquad G < \phi >_o = 0 \tag{2.2.12}$$

For the generators of $SU(2)_L X U(1)_Y$ we find

$$\tau_1 <\phi>_o = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ V \end{pmatrix} = \begin{pmatrix} V \\ 0 \end{pmatrix} \neq 0$$

$$\tau_2 <\phi>_o = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} o \\ V \end{pmatrix} = \begin{pmatrix} -iV \\ 0 \end{pmatrix} \neq 0 \tag{2.2.13}$$

$$\tau_3 <\phi>_o = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ V \end{pmatrix} = \begin{pmatrix} 0 \\ V \end{pmatrix} \neq 0$$

$$Y <\phi>_o = 1 \begin{pmatrix} 0 \\ V \end{pmatrix} \neq 0 \tag{2.2.14}$$

$$\text{But} \quad Q <\phi>_o = \frac{1}{2} (\tau_3 + Y) <\phi>_o = 0 \tag{2.2.15}$$

Three of the four generators are broken but the linear combination corresponding to electric charge is not. The photon will therefore remain massless.

Expanding the Lagrangian about the minimum of V by writing

$$\phi = \exp\left\{\frac{-i\xi.\tau}{2V}\right\} \phi = \begin{pmatrix} 0 \\ V + \eta \end{pmatrix} \tag{2.2.16}$$

and transforming to U gauge

$$\phi = \phi' = \exp\left\{\frac{i\xi.\tau}{2}\right\} \phi = \begin{pmatrix} 0 \\ V + \eta \end{pmatrix} \tag{2.2.17}$$

$$\tau.\tilde{A}_\mu \longrightarrow \tau.\tilde{A}_\mu'$$

$$B_\mu \longrightarrow B_\mu$$

$$R \longrightarrow R$$

$$L \longrightarrow L' = \exp\left\{\frac{i\xi.\tau}{2V}\right\} L$$

By spontaneous symmetry breaking, the Yukawa term, in the Lagrangian, becomes in terms of U gauge fields

$$\begin{aligned} \alpha_{Yukawa} &= -G_e\,(\bar{R}\,\phi^+ L + \bar{L}\,\phi\,R) \\ &= -G_e\,[\bar{R}\,(\phi'^+ L') + (\bar{L}'\phi')R] \\ &= -G_e\,[\bar{R}\,(\phi^+ L) + (\bar{L}\phi)\,R] \\ &= -G_e\,\left[\bar{e}_R \left(0 \quad \frac{V+\eta}{\sqrt{2}}\right) \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} + \left(\bar{\nu}_e \quad \bar{e}_L\right) \begin{pmatrix} 0 \\ \frac{V+\eta}{\sqrt{2}} \end{pmatrix} e_R \right] \\ &= -G_e\left(\frac{V+\eta}{\sqrt{2}}\right) (\bar{e}_R e_L + \bar{e}_L e_R) \\ &= \frac{G_e V}{\sqrt{2}}\,\bar{e}e - \frac{G_e\eta}{\sqrt{2}}\,e\,\bar{e} \end{aligned}$$

This expression gives the electron mass

$$m_e = \frac{G_e V}{\sqrt{2}} \tag{2.2.18}$$

The scalar term in the Lagrangian becomes

$$
\begin{aligned}
\alpha_{scalar} &= (D^{\mu}\phi)^{+}(D_{\mu}\phi) - \mu^{2}(\phi^{+}\phi) - |\lambda|\,(\phi^{+}\phi)^{2} \\
&= (D^{\mu}\phi')^{+}\,(D_{\mu}\phi') - \mu^{2}(\phi'^{+}\phi') - |\lambda|(\phi'^{+}\phi')^{2} \\
&= (\partial^{\mu} - \frac{ig'}{2}\tilde{A}_{\mu}Y - \frac{ig}{2}\,\bar{\tau}.\bar{b}_{\mu})\,\begin{pmatrix}0 & \frac{V+\eta}{\sqrt{2}}\end{pmatrix} \\
&\quad (\partial^{\mu} + \frac{ig'}{2}\tilde{A}_{\mu}Y + \frac{ig}{2}\,\bar{\tau}.\bar{b}_{\mu})\begin{pmatrix}0 \\ \frac{V+\eta}{\sqrt{2}}\end{pmatrix} \\
&\quad -\mu^{2}\begin{pmatrix}0 & \frac{V+\eta}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}0 \\ \frac{V+\eta}{\sqrt{2}}\end{pmatrix} - |\lambda|\left[\begin{pmatrix}0 & \frac{V+\eta}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}0 \\ \frac{V+\eta}{\sqrt{2}}\end{pmatrix}\right]^{2} \\
&= \frac{1}{2}(\partial^{\mu}\eta)(\partial_{\mu}\eta) + \frac{V^{2}}{8}\,[g^{2}|b_{\mu}^{1} - ib_{\mu}^{2}|^{2} + (g'\tilde{A}_{\mu} - gb_{\mu}^{3})^{2}] \\
&\quad - \mu^{2}\,\frac{(V+\eta)^{2}}{2} - |\lambda|\,\frac{(V+\eta)^{4}}{4} \\
&= \frac{1}{2}(\partial^{\mu}\eta)(\partial_{\mu}\eta) - \mu^{2}\eta^{2} + \frac{V^{2}}{8}\,[g^{2}|b_{\mu}^{1} - ib_{\mu}^{2}|^{2} \\
&\quad + (g'\tilde{A}_{\mu} - gb_{\mu}^{3})^{2}] + \ldots \text{ Interaction terms}
\end{aligned}
\qquad (2.2.19)
$$

It is seen from the Lagrangian that η field has acquired a mass.

$M_H^2 = -2\mu^2 > 0$ which is the physical Higgs boson.

2.3 The gauge boson masses, the electroweak angle and the charged and neutral currents

Defining charged gauge field as

$$W_{\mu}^{\pm} = \frac{b_{\mu}^{1} \mp ib_{\mu}^{2}}{2} \qquad (2.3.1)$$

the term proportional to g^2V^2 is recognizable as a mass term for the charged vector boson.

From the eq.(2.2.19) we see that it contains a term

$$\frac{g^2V^2}{8} \left(|W^+_\mu|^2 + | W^-_\mu |^2 \right) \tag{2.3.2}$$

Thus the masses of the charged intermediate bosons are

$$M_W = \frac{|gV|}{2} \tag{2.3.3}$$

Finally defining the orthogonal combinations

$$Z_\mu = - \frac{g'\tilde{A}_\mu + gb^3_\mu}{\sqrt{g^2 + g'^2}} \tag{2.3.4}$$

and,

$$A_\mu = \frac{g\tilde{A}_\mu + g'b^3_\mu}{\sqrt{g^2 + g'^2}} \tag{2.3.5}$$

Introducing a weak mixing angle

$$\theta_W \quad \text{as} \quad g'/g = \tan \theta_W \tag{2.3.6}$$

The eqs. (2.3.4) and (2.3.5) can be written as

$$Z_\mu = -\tilde{A}_\mu \, \text{Sin}\, \theta_W + b^3_\mu \, \text{Cos}\, \theta_W \tag{2.3.7}$$

and,

$$A_\mu = \tilde{A}_\mu \, \text{Cos}\, \theta_W + b^3_\mu \, \text{Sin}\, \theta_W \tag{2.3.8}$$

which may be inverted as

$$\tilde{A}_\mu = A_\mu \, \text{Cos}\, \theta_W - Z_\mu \, \text{Sin}\, \theta_W \tag{2.3.9}$$

$$b^3_\mu = A_\mu \, \text{Sin}\, \theta_W + Z_\mu \, \text{Cos}\, \theta_W \tag{2.3.10}$$

From eq. (2.2.19)

$$\alpha = \frac{1}{2}\{(\partial^{\mu}\eta)(\partial_{\mu}\eta) - M_H^2\eta^2\}$$
$$+ \frac{1}{2}\frac{g^2V^2}{4}W_{\mu}^{+}W_{\mu}^{-} + \frac{1}{2}\frac{V^2}{4}(g^2+g'^2)\left(\frac{-g'\tilde{A}_{\mu} + gb_{\mu}^{3}}{\sqrt{g^2+g'^2}}\right)$$
$$= \frac{1}{2}\{(\partial^{\mu}\eta)(\partial_{\mu}\eta) - M_H\eta^2\} + \frac{1}{2}\frac{g^2V^2}{4}W_{\mu}^{+}W_{\mu}^{-}$$
$$+ \frac{1}{2}\frac{V^2}{4}(g^2+g'^2)Z_{\mu}Z^{\mu} + (0)A_{\mu}A^{\mu} \tag{2.3.11}$$

Therefore the mass of Z_{μ}

(2.3.12)

can be written as

$$M_{Z_{\mu}} = \frac{gV}{2}\sqrt{1+\frac{g'^2}{g^2}} \tag{2.3.13}$$

$$\text{or } M_Z = M_W\sqrt{1+\frac{g'^2}{g^2}} \tag{2.3.14}$$

Thus the Higgs mechanism has also generated masses for W and Z bosons and the field A_{μ} is massless, as there is no term proportional to A_{μ}^2 in the eq. (2.3.11). We would like to identify it with the electromagnetic field.

The interaction among the gauge boson and leptons may be followed from α_{lepton}. After spontaneous symmetry breaking

$$\alpha = \bar{L}i\gamma^{\mu}(ig'A_{\mu}.Y + \frac{ig}{2}\bar{\tau}.b_{\mu})L$$

1024 23

$$= -Y\frac{g'}{2}\,\bar{L}\gamma^{\mu}L\tilde{A}_{\mu} - \frac{g}{2}\,\bar{L}\gamma^{\mu}\begin{pmatrix} b^3_{\mu} & b^1_{\mu} - ib^2_{\mu} \\ b^1_{\mu}+ib^2_{\mu} & -b^3_{\mu}\end{pmatrix} L$$

$$= -Y\frac{g'}{2}\,L\gamma^{\mu}L\tilde{A}_{\mu} - \frac{g}{2}\,(\bar{\nu}_L\ \bar{e}_L)\gamma^{\mu}\begin{pmatrix} b^3_{\mu} & \sqrt{2}W^+_{\mu} \\ \sqrt{2}\,W^-_{\mu} & -b^3_{\mu}\end{pmatrix}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}$$

$$= -Y\frac{g'}{2}\,\bar{L}\gamma^{\mu}L\tilde{A}_{\mu} - \frac{g}{\sqrt{2}}\,[\bar{\nu}_L\gamma^{\mu}e_L\ W^+_{\mu} + \bar{e}_L\gamma^{\mu}\nu_L\ W^-_{\mu}]$$
$$- \frac{g}{2}\,[\,\bar{\nu}_L\gamma^{\mu}\nu_L\ b^3_{\mu} + \bar{e}_L\gamma^{\mu}e_L\ b^3_{\mu}\,] \qquad (2.3.15)$$

For the charged gauge bosons we have

$$\alpha_{W-L} = -\frac{g}{\sqrt{2}}\,(\bar{\nu}_L\gamma^{\mu}e_L\ W^+_{\mu} + \bar{e}_L\gamma^{\mu}\nu_L\ W^-_{\mu}) \qquad (2.3.16)$$

Substituting the value of

$$\bar{\nu}_L\gamma^{\mu}e_L = \frac{1}{2}\,\bar{\nu}\gamma^{\mu}\,(1-\gamma_5)\,e$$

and

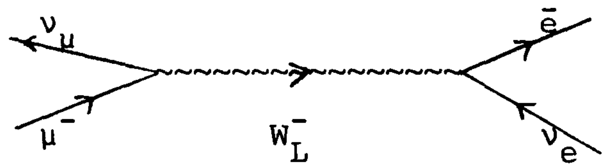
$$\bar{e}_L\gamma^{\mu}\nu_L = \frac{1}{2}\,\bar{e}\,\gamma^{\mu}\,(1-\gamma_5)\nu \qquad (2.3.17)$$

$$\alpha_{W-L} = -\frac{g}{2\sqrt{2}}\,[\,\bar{\nu}\gamma^{\mu}(1-\gamma_5)\,e\,W^+_{\mu} + \bar{e}\gamma^{\mu}(1-\gamma_5)\nu W^-_{\mu}] \qquad (2.3.18)$$

Considering Muon decay process in the Weinberg-Salam model

$$\mu^- \longrightarrow \bar{e} + \nu_{\mu} + \bar{\nu}_e \qquad (2.3.19)$$

The Feynman diagram for this decay process is



The Lagrangian for one vertex is

$$\alpha = \frac{-g}{2\sqrt{2}}\ \bar{e}\,\gamma_{\sigma}\,(1-\gamma_5)\,\nu_e\,W^{\sigma -} \qquad (2.3.20)$$

and for another vertex is,

$$\alpha = -\frac{g}{2\sqrt{2}} \bar{\nu}_\mu \gamma^\rho (1 - \gamma_5) \mu W_\rho^+ \qquad (2.3.21)$$

So the effective Lagrangian is

$$\alpha_{eff} = \frac{g^2}{8} [\bar{\nu}_\mu \gamma^\rho (1-\gamma_5) \mu \bar{e} \gamma_\sigma (1-\gamma_5) \nu_e W_\rho^+ W^{\sigma -}] \qquad (2.3.22)$$

and in V-A form the four fermion interaction, the effective Lagrangian for moun-decay is

$$\alpha_{eff} = -\frac{G_F}{\sqrt{2}} [\bar{\nu}_\mu \gamma_\lambda (1-\gamma_5) \mu \bar{e} \gamma^\lambda (1-\gamma_5) \nu_e] \qquad (2.3.23)$$

Where G_F is the Fermi coupling constant. Introducing the vacuum expectation value (VEV) in eq. (2.3.22) we get,

$$\alpha_{eff} = \frac{g^2}{8} [\bar{\nu}_\mu \gamma^\rho (1-\gamma_5) \mu \bar{e} \gamma_\sigma (1-\gamma_5) \nu_e] <0 | W_\rho^+ W^{\sigma -} | o> \qquad (2.3.24)$$

where the factor $< 0| W_\rho^+ W^{\sigma -} | 0>$ is called the W-boson propagator and is given by

$$<0|W_\rho^+ W^{\sigma -} | 0 > = \frac{\delta_\rho^\sigma + q_\rho q^\sigma / M_W}{(q^2 - M_W^2)} \qquad (2.3.25)$$

In the energy limit $q^2 \longrightarrow 0$ and $q^2 << M_W^2$

$$< 0| W_\rho^+ W^{\sigma -} |0> = -\delta_\rho^\sigma / M_W^2 \qquad (2.3.26)$$

Therefore,

$$\alpha_{eff} = \frac{-g^2}{8M_W^2} \delta_\rho^\sigma [\bar{\nu}_\mu \gamma^\rho (1-\gamma_5) \mu \bar{e} \gamma_\sigma (1-\gamma_5) \nu_e]$$

$$= -\frac{g^2}{8M_W^2} [\bar{\nu}_\mu \gamma_\lambda (1-\gamma_5) \mu \bar{e} \gamma^\lambda (1-\gamma_5) \nu_e] \qquad (2.3.27)$$

Now comparing eqs (2.3.23) and (2.3.27) we get,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \qquad (2.3.28)$$

This relation implies that the expression (2.3.18) reproduces low energy phenomenology of intermediate boson model.

Substituting the value of $M_W^2 = \frac{g^2 v^2}{2}$ in the equation (2.3.28) we get,

$$\frac{g^2}{8}\left(\frac{4}{g^2 v^2}\right) = \frac{G_F}{\sqrt{2}} \qquad (2.3.29)$$

$$\text{or,} \quad V = (G_F/\sqrt{2})^{-\frac{1}{2}} \simeq 246 \text{ Gev.} \qquad (2.3.30)$$

and the vacuum expectation value of the scalar field is

$$<\phi>_o = \frac{V}{2} = (G_F\sqrt{8})^{-\frac{1}{2}} \simeq 174 \text{ GeV} \qquad (2.3.31)$$

The coupling constants of the $SU(2)_L$ and $U(1)_Y$ gauge groups may be written as

$$g = \frac{e}{\sin\theta_W} \geq e \qquad (2.3.32)$$

$$g' = \frac{e}{\cos\theta_W} \geq e \qquad (2.3.33)$$

$$\text{provided} \quad \frac{gg'}{\sqrt{g^2+g'^2}} = e \qquad (2.3.34)$$

Substituting the value of g in eq. (2.3.28) and using eq.(2.3.6)

$$M_W^2 = \frac{g^2}{4\sqrt{2}}\, G_F = \frac{e^2}{4\sqrt{2}}\, G_F \sin^2\theta_W$$

$$= \pi\alpha/\sqrt{2}\ G_F \sin^2\theta_W = \frac{37.4 \text{ GeV}/C^2)}{\sin\theta_W} \qquad (2.3.35)$$

and

$$M_Z^2 = M_W^2 / \cos^2\theta_W \geq M_W^2 \qquad (2.3.36)$$

From the relation (2.3.32) and (2.3.33) the strength of weak and electromagnetic interaction are related by a single parameter. The feebleness of the weak interactions at low energy is laid to the large mass of the intermediate bosons and not to the small coupling constant.

The dimensionless couplingconstant that endowed the electron with mass in eq. (2.2.18) is small

$$G_e = \frac{m_e\sqrt{2}}{V} = m_e 2^{\frac{3}{4}} \; G_F^{\frac{1}{2}}$$
$$\simeq 3 \times 10^{-6} \qquad (2.3.37)$$

The neutral gauge bosons couplings to leptons are given by,

$$\alpha_{o-1} = \frac{gg'}{\sqrt{g^2+g'^2}} \; \bar{e}\,\gamma^\mu e A_\mu - \frac{\sqrt{g^2+g'^2}}{2} \; \bar{\nu}_L \gamma^\mu \nu_L Z_\mu$$

$$+ \frac{Z_\mu}{\sqrt{g^2+g'^2}} \; [-g'^2 \bar{e}_R \gamma^\mu e_R + \frac{(g^2-g'^2)}{2} \; \bar{e}_L \gamma^\mu e_L] \qquad (2.3.38)$$

Here we may identify A_μ as the photon provided

$$\frac{gg'}{\sqrt{g^2+g'^2}} = e \qquad (2.3.39)$$

This interaction Lagrangian in terms of the weak mixing angle θ_W becomes,

$$\alpha_{o-1} = e\bar{e}\gamma^\mu e A_\mu - \frac{g}{2 \operatorname{Cos} \theta_W} \; \bar{\nu}_L \gamma^\mu \nu_L \; Z_\mu$$

$$- \frac{\sqrt{g^2+g'^2}}{2} \; [2.\; g'^2 \; \frac{\bar{e}_R \gamma^\mu e_R}{g^2+g'^2} \; Z_\mu + \frac{g'^2-g^2}{g^2+g'^2} \; \bar{e}_L \gamma^\mu e_L Z_\mu] \qquad (2.3.40)$$

$$= e\bar{e}\gamma^\mu e A_\mu - \frac{g}{2 \operatorname{Cos} \theta_W} \; \bar{\nu}_L \gamma^\mu \nu_L Z_\mu$$

$$- \frac{g}{2 \operatorname{Cos} \theta_W} \; [2 \operatorname{Sin}^2\theta_W \; \bar{e}_R \gamma^\mu e_R Z_\mu + (2 \operatorname{Sin}^2\theta_W - 1)\bar{e}_L \gamma^\mu e_L Z_\mu]$$

$$\alpha = e\bar{e}\gamma^{\mu}eA_{\mu} - \frac{1}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{\nu}\gamma^{\mu}(1-\gamma_5)\,\nu\, Z_{\mu}$$

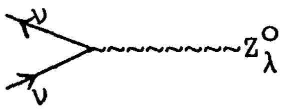
$$- \frac{1}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{\frac{1}{2}} [2\,\mathrm{Sin}^2\theta_W\, \bar{e}\gamma^{\mu}(1+\gamma_5)\, e\, Z_{\mu}$$

$$+ (2\,\mathrm{Sin}^2\theta_W - 1)\, \bar{e}\gamma^{\mu}(1-\gamma_5)\, e\, Z_{\mu}] \qquad (2.3.41)$$

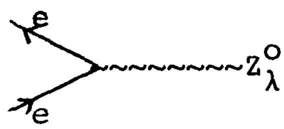
From this Lagrangian the Feynman rules for vertices are deduced,

$$-ie\bar{e}\gamma_{\lambda}e$$

$$-i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{\nu}\gamma_{\lambda}(1-\gamma_5)e$$



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{\nu}\gamma_{\lambda}(1-\gamma_5)\nu$$



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{e}\gamma_{\lambda}$$

$$[2\,\mathrm{Sin}^2\theta(1+\gamma_5)+(2\,\mathrm{Sin}^2\theta_W - 1)(1-\gamma_5)]$$

In the $SU(2)_L \times U(1)_Y$ model, the properties of the gauge bosons are co-related with those of the neutral current interactions by means of the weak mixing angle θ_W. From the eq. (2.3.41) the weak neutral current is given by

$$J_{\lambda}^{o} = J_{\lambda}^{(3)} - 2\,\mathrm{Sin}^2\,\theta_W \quad J_{\lambda}^{(em)} \qquad (2.3.42)$$

Where $J_{\lambda}^{(3)}$ is the weak isospin current and $J_{\lambda}^{(em)}$ is the electromagnetic current,

$$J_{\lambda}^{(3)} = \frac{1}{2} \sum_{i} \bar{\Psi}_i \, \tau_3 \gamma_{\lambda} \, (1-\gamma_5)\psi_i \tag{2.3.43}$$

$$= \frac{1}{2} \{\bar{\nu} e \gamma_{\lambda}(1-\gamma_5)\nu - \bar{e}\gamma_{\lambda}(1-\gamma_5)e + (e \to \mu) + (e \to \tau)\}$$

and

$$J_{\lambda}^{(em)} = -\bar{e}\gamma_{\lambda} e - \bar{\mu}\gamma_{\lambda}\mu - \bar{\tau}\gamma_{\lambda}\tau \tag{2.3.44}$$

$$= -\frac{1}{2} [\bar{e}\gamma_{\lambda}(1-\gamma_5) + \bar{e}\gamma_{\lambda}(1+\gamma_5)e + (e \to \mu) + (e \to \tau)$$

Therefore, $J_{\lambda}^{(o)} = \frac{1}{2} \{\bar{\nu}_e \gamma_{\lambda}(1-\gamma_5)\nu_e - \bar{e}\gamma_{\lambda}(1-\gamma_5)e$

$$+(e \to \mu) + (e \to \tau)\} - 2 \sin^2\theta_W \{-\frac{1}{2} [\bar{e}\gamma_{\lambda}(1-\gamma_5) e$$

$$+\bar{e}\gamma_{\lambda}(1+\gamma_5)e + (e \to \mu) + (e \to \tau)]\}$$

$$= \frac{1}{2} \{\bar{\nu}_e \gamma_{\lambda}(1-\gamma_5)\nu_e\} + \frac{1}{2} [2 \sin^2\theta_W \, \bar{e}\gamma_{\lambda}(1+\gamma_5)e$$

$$+ (2 \sin^2\theta_W - 1) \, \bar{e}\gamma_{\lambda}(1-\gamma_5)e + (e \to \mu) + (e \to \tau)] \tag{2.3.45}$$

Now comparing eqs. (2.3.41) and (2.3.45) we obtain

$$\alpha_{o-1} = -J_{\mu}^{em} A_{\mu} - \frac{1}{\sqrt{2}} \left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{\frac{1}{2}} Z_{\mu} \, 2J_{\mu}^{(o)}$$

$$= - J_{\mu}^{em} A_{\mu} - \sqrt{2} \left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{\frac{1}{2}} J_{\mu}^{(o)} \, Z_{\mu} \tag{2.3.46}$$

which gives the neutral current in Weinberg-Salam and Glashow model.

2.4 The GIM Mechanism

The extension of the Weinberg-Salam model to the hadronic sector is accomplished through the medium of the quark model. Due to the similarity of quarks to the leptons, construction of the theory at the quark level is relatively straight forward. If each weak isospin doublet of leptons is acompanied by a colour triplet of weak isospin doublet of quarks, an anomaly free, hence, renormalisable theory could be established.

Similar to the left-handed parts of ν_e and e, the left-handed parts of the u and d quarks of a definite colour also form a doublet under $SU(2)_L$ while their right-handed parts are singlets, that is, left-handed weak isospin doublets of quark is

$$L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \longrightarrow L_u = \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \qquad (2.4.1)$$

with weak hypercharge $Y(q_L) = \frac{1}{3}$

where $\quad d_\theta = d \cos \theta_c + S \sin \theta_c \qquad (2.4.2)$

and θ_c is the Cabibbo angle. The right-handed weak isospin singlets are represented as

$$\begin{aligned} R_u &\equiv u_R = \frac{1}{2}(1+\gamma_5)u \\ R_d &\equiv d_R = \frac{1}{2}(1+\gamma_5)d \end{aligned} \qquad (2.4.3)$$

with weak hypercharge

$$Y(u_R) = \frac{4}{3} \quad \text{and} \quad Y(d_R) = -\frac{1}{3},$$

The charge raising weak current is,

$$\begin{aligned} J_\mu^{(+)} &= \bar{u}\gamma_\mu (1-\gamma_5)d \,.\, \cos\theta_c \\ &+ \bar{u}\gamma_\mu(1-\gamma_5)S \,.\, \sin\theta_c \end{aligned} \qquad (2.4.4)$$

and the neutral current becomes

$$J_\mu^{(o)} = J_\mu^{(3)} - \sin^2\theta_W J_\mu^{em}$$

$$= \frac{1}{2} \{\bar{u}\gamma_\mu(1-\gamma_5)u - \bar{d}\gamma_\mu(1-\gamma_5)d \cos^2\theta_c$$

$$- \bar{S}\gamma_\mu(1-\gamma_5)S \sin^2\theta_c - \bar{S}\gamma_\mu(1-\gamma_5)d \sin\theta_c \cos\theta_c$$

$$- \bar{d}\gamma_\mu (1-\gamma_5)S \sin\theta_c \cos\theta_c - \sin\theta_W\{\tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s\}. \quad (2.4.5)$$

This hadronic neutral current contains flavour-changing ($d \leftrightarrow S$) terms

Preserving Cabbibo rotation and using the $SU(2)_L \times U(1)_Y$ model, Glashow, Illiopulos and Maini (GIM)[15] suggested a method to get rid of the undesirable flavour changing neutral currents by introducing an extra quark degrees of freedom namely the charm quark as early as 1970.

According to them, the second left-handed weak isospin doublet is represented by

$$\begin{pmatrix} C \\ S_\theta \end{pmatrix}_L$$

involving the charm quark where

$$S_\theta = S \cos\theta_c - d \sin\theta_c \quad (2.4.6)$$

and C_R and S_R are the right handed singlets.

The hadronic neutral current becomes,

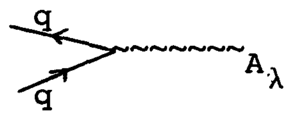
$$J_\mu^{(o)} = \frac{1}{2} [(\bar{u}\gamma_\mu(1-\gamma_5)u + \bar{C}\gamma_\mu(1-\gamma_5)C$$

$$- \bar{d}\gamma_\mu(1-\gamma_5)d - \bar{S}\gamma_\mu(1-\gamma_5)S]$$

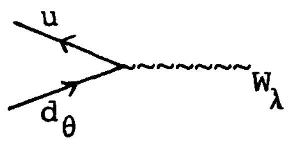
$$- 2 \sin^2\theta_W J_\mu^{em} \quad (2.4.7)$$

This is flavour conserving neutral current.

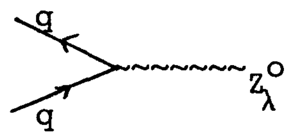
The Feynman rules for interactions between gauge bosons and quarks are represented in the following fig.



$$-iQ_q \; \bar{q}\gamma_\lambda q$$



$$-i\left(\frac{G_F M_W^{\,2}}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{u}\gamma_\lambda(1-\gamma_5)d_\theta$$



$$-i\left(\frac{G_F M_Z^{\,2}}{\sqrt{2}}\right) \bar{q}\gamma_\lambda[R_q(1+\gamma_5)+L_q(1-\gamma_5)]q$$

Where $\quad R_q = -\,Q_q \,\mathrm{Sin}^2\theta_W$

$$L_q = \tau_3 - 2Q_q \,\mathrm{Sin}^2\theta_W$$

The complete gauge invariant Lagrangian for electroweak interactions including both leptons and quarks takes the form

$$\alpha = -\frac{1}{4} F^1_{\mu\nu} F^{1\mu\nu} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}$$

$$+ i \sum_{n=e,\mu,\tau} \bar{L}^{(n)}\gamma^\mu(\partial_\mu + \frac{ig'}{2} \tilde{A}_\mu Y + \frac{ig}{2} \vec{\tau}.\vec{b}_\mu)L^{(n)}$$

$$+ i \sum_{n=e,\mu,\tau} \bar{R}^{(n)}\gamma^\mu(\partial_\mu + \frac{ig'}{2} \tilde{A}_\mu Y)R^{(n)}$$

$$+ i \sum_{q=u,d,c,s,t,b} \bar{L}_q\gamma^\mu[\partial_\mu + \frac{ig'}{2} \tilde{A}_\mu Y(q_L) + \frac{ig}{2} \bar{\tau}.\bar{b}_\mu]L_q$$

$$+ i \sum_{q=u,d,c,s,t,b} \bar{R}_q \gamma^\mu [\partial_\mu + \frac{ig'}{2} \tilde{A}_\mu Y(q_R)] R_q$$

$$+ (D^\mu \phi)^+ (D_\mu \phi) - V(\phi^+ \phi)$$

$$- i \sum_{j=e,\mu,\tau} G_j [\bar{R}_j (\phi^+ L_j) + (\bar{L}_j \phi) R_j]$$

$$- i \sum_{K=u,d,c,s,t,b} G_K [\bar{R}_K (\phi^+ L_K) + (L_K \phi) R_K] \quad (2.4.8)$$

2.5 Summary

A renormalizable gauge invariant description of leptons and hadrons may be obtained by postulating an isospin triplet vector boson $\vec{b}_\mu$, corresponding to $SU(2)_L$ an isosinglet vector boson $\tilde{A}_\mu$, corresponding to $U(1)_Y$. The Lagrangian respecting $SU(2)_L X U(1)_Y$ gauge symmetry forbids mass terms for the gauge bosons. On the otherhand, short range nature of weak interactions requires massive gauge bosons. This dilemma is overcome by Higgs mechanism. The addition of a potential energy term due to a Higgs doublet results in the spontaneous symmetry breaking,

$$SU(2)_L X U(1)_Y \xrightarrow{M_W} U(1)_{em},$$

when the neutral component acquires vacuum expectation value. This in turn gives masses to the weak-gauge bosons, quarks and leptons, while maintaining the photon massless and generating the Higgs mass term.

At low energies the model yields charged and neutral current interactions consistent with the V-A theory. The quarks carrying colour and flavour quantum numbers are included in the Lagrangian by the elegant GIM mechanism, that avoids flavour changing neutral currents. The masses of the $W^{\pm}$ and Z° boson, the existence of the neutral current and neutral current parameters, and the electroweak mixing angle, predicted by the Weinberg-Salam model have been experimentally verified. In the next chapter, we review experimental tests of the model.

EXPERIMENTAL TESTS OF THE WEINBERG-SALAM-GLASHOW MODEL

The $SU(2)_L XU(1)_Y$ model makes precise predictions about the existence of $W^{\pm}$, Z gauge bosons and, especially, about their mass values. The existence of neutral current is the other important prediction of the model. In this chapter, the predicted values of both gauge boson masses and neutral current parameters are computed and compared with their experimental values. In Sec. 3.1 of this chapter derivation of gauge boson masses is shown considering radiative correction , following the procedure of Marciano and Sirlin[16]. In Sec. 3.2 experimental values of different parameters obtained by UA 1 and UA 2 collaborations are reported . Comparison of experimental and predicted values of gauge boson masses is made in Sec. 3.3. In Sec. 3.4, neutral current parameters are obtained for neutrino-hadron, neutrino-electron, electron-hadron, electron-muon reactions, and the neutral current experiments are analysed in sec 3.5. In sec. 3.6, we compare the predicted values and the experimental values of different neutral current parameters. The world average of $\sin^2\theta_W$ obtained in this manner is discussed in the sec.3.7. Finally we summarize the chapter briefly in sec.3.8.

3.1 Measurement of $W^{\pm}$, Z gauge boson masses

The predicted values of masses of $W^{\pm}$ and Z bosons are 83.0±2.9 and 93.8±2.5 GeV respectively. From various experiments the UA 1 and UA 2 (after the underground area at (CERN) collaborations have collected several hundred $W^{\pm} \longrightarrow e^{\pm}\nu$ events and about 10% as many $Z \longrightarrow e^{+}e^{-}$.

From those data they have extracted properties of the $W^{\pm}$ and Z bosons that can be compared with the predicted values. Calculating gauge boson masses, m_W and m_Z, from the theoretical expression (considering radiative correction)[16]

$$m_W = \left[\frac{\pi\alpha}{\sqrt{2}\ G_F \ \mathrm{Sin}^2\theta_W \ (1-\Delta r)} \right]^{\frac{1}{2}} \qquad (3.1.1)$$

and

$$m_Z = \frac{m_W}{\mathrm{Cos}\theta_W} \qquad (3.1.2)$$

where Δr denotes the $O(\alpha)$ radiative correction.

α is the fine structure constant

$= \dfrac{1}{137.035963}$

and G_F is the Fermi coupling constant

$= 1.16634 \pm 0.000002 \times 10^{-5}\ \mathrm{GeV}^{-2}$

Following Marciano and Sirlin,[16] theoretical results of the masses are compared with the experimental results.

From (3.1.1) $m_W = \dfrac{A}{\mathrm{Sin}\ \theta_W}$ (3.1.3) where $$A = \left[\frac{\pi\alpha}{\sqrt{2}\, G_F} \right]^{\frac{1}{2}} \left[\frac{1}{1-\Delta r} \right]^{\frac{1}{2}} \tag{3.1.4}$$

$$= \frac{37.2810 \pm 0.0003}{(1-\Delta r)^{\frac{1}{2}}} \text{ GeV} \tag{3.1.5}$$

and $$m_Z = \frac{m_W}{\cos\theta_W} = \frac{A}{\sin\theta_W \cos\theta_W} = \frac{2A}{\sin 2\theta_W} \tag{3.1.6}$$

From these expressions m_W and m_Z interdependence can be expressed as follows.

From equation (3.1.3) $m_W = \dfrac{A}{\cos\theta_W}$

$$\text{or } \sin\theta_W = \frac{A}{m_W} \tag{3.1.7}$$

$$\text{or } \cos\theta_W = \sqrt{1 - \frac{A^2}{m_W^2}} \tag{3.1.8}$$

Substituting these values of $\cos\theta_W$ in equation (3.1.2) we have,

$$m_Z = \frac{m_W}{\sqrt{1 - \frac{A^2}{m_W^2}}} \tag{3.1.9}$$

from eq (3.1.6)

$$\sin 2\theta_W = \frac{2A}{m_Z} \tag{3.1.10}$$

and $$\cos 2\theta_W = \sqrt{1 - \frac{4A^2}{m_Z^2}}$$

or $$\cos\theta_W = \left[\frac{1}{2} \left(1 + \sqrt{1 - \frac{4A^2}{m_Z^2}}\right) \right]^{\frac{1}{2}} \tag{3.1.11}$$

Substituting the value of $\cos\theta_W$ in eq. (3.1.2) we get,

$$m_Z = \frac{m_W}{\left[\frac{1}{2} \left(1 + \sqrt{1 - \frac{4A^2}{m_Z^2}}\right) \right]^{\frac{1}{2}}} \tag{3.1.12}$$

or, $$m_W = m_Z \left[\frac{1}{2}\left(1 + \sqrt{1 - \frac{4A^2}{m_Z^2}}\right)\right]^{\frac{1}{2}} \qquad (3.1.13)$$

from (3.1.9)

$$m_Z - m_W = \frac{m_W}{\left(1 - \frac{A^2}{m_W^2}\right)^{\frac{1}{2}}} - m_W = m_W \left[\frac{1}{\left(1 - \frac{A^2}{m_W^2}\right)^{\frac{1}{2}}} - 1\right] \qquad (3.1.14)$$

and from (3.1.13)

$$m_Z - m_W = m_Z - m_Z \left[\frac{1}{2}\left(1 + \left(1 - \frac{4A^2}{m_Z^2}\right)^{\frac{1}{2}}\right]$$

$$= m_Z \left[1 - \left\{\frac{1}{2}\left(1 + \left(1 - \frac{4A^2}{m_Z^2}\right)^{\frac{1}{2}}\right\}^{\frac{1}{2}}\right] \qquad (3.1.15)$$

from (3.1.3)

$$m_W = \frac{37.281 \pm 0.003}{\sin\theta_W (1 - \Delta\gamma)^{\frac{1}{2}}}$$

or $$\Delta r = 1 - \frac{(37.281 \pm 0.0003)^2}{m_W^2 \sin^2\theta_W}$$

$$\Delta r = 1 - \frac{(37.281 \pm 0.0003)^2}{m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right)} \quad \text{using } (3.1.2) \qquad (3.1.16)$$

where $$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} \qquad (3.1.17)$$

The $O(\alpha)$ radiative correction $\Delta\gamma$ computed by Sirlin and Marciano are very large.

$\Delta r = 0.07 \pm 0.013$

(assuming[8] $m_Z = 93$ GeV/C^2, $m_t = 3.5$ GeV/C^2)

$M_{Higgs} = 100$ GeV/C^2

A numerical evaluation of $\Delta\gamma$ employing $\sin^2\theta_W = 0.217$ (the central value from deep inelastic ν_μ scattering)[17]

$m_H \simeq m_Z$, $\quad m_C = 1.5$ GeV

$m_b = 4.5$ GeV, $\quad m_t = 36$ GeV

Wetzel's[18] analysis leads to

$$\Delta r = 0.0696 \pm 0.0020 \text{ for } \quad (3.1.18) \quad \begin{cases} \sin^2\theta_W = 0.217 \\ m_H = m_Z \\ m_t = 36 \text{ GeV.} \end{cases}$$

where an estimate of uncertainties in the hadronic contribution has been included. This numerical value is not sensitive to smallshifts in $\sin^2\theta_W$, m_H or m_t. For this reason, we will take the value

$\Delta r = 0.0696 \pm 0.020$ as the standard value and we will take it to be constant throughout.

Combining (3.1.5) and (3.1.18) leads to

$$A = 38.65 \pm 0.04 \text{ GeV} \tag{3.1.19}$$

Prediction of m_W and m_Z are possible from a separate determination by substituting the value of A from (3.1.19) to (3.1.3) and (3.1.6) or inverting those equations, one can determine $\sin^2\theta_W$ by measuring m_W or m_Z. But the generally quoted predictions for m_W and m_Z are obtained by using the world average value,

$$\sin^2\theta_W = 0.217 \pm 0.014 \tag{3.1.20}$$

(from deep-inelastic ν_μ scattering and the eD asymmetry) which leads to

$$M_W = 83.0 \begin{matrix} + 2.9 \\ - 2.7 \end{matrix} \text{ GeV} \tag{3.1.21}$$

$$M_Z = 93.8 \begin{matrix} + 2.4 \\ - 2.4 \end{matrix} \text{ GeV} \tag{3.1.22}$$

The errors are of course corrected. The predicted mass difference is

$$m_Z - m_W = 10.8 \pm 0.5 \text{ GeV} \qquad (3.1.23)$$

3.2 Experimental values obtained by UA1 and UA2 collaborations

Experimentally, the $W^{\pm}$, and Z mas values obtained at CERN[8,9] in the year 1987 are analysed below.

$$\left.\begin{aligned} m_W &= 82.7 \pm 1.0 \pm 2.7 \text{ GeV} \\ m_Z &= 93.1 \pm 1.0 \pm 3.1 \text{ GeV} \end{aligned}\right\} \text{UA1 collaborations} \qquad (3.2.1)$$

and

$$\left.\begin{aligned} m_W &= 80.2 \pm 0.6 \pm 0.5 \pm 1.3 \\ m_Z &= 91.5 \pm 1.2 \pm 1.7 \end{aligned}\right\} \text{UA2 collaborations} \qquad (3.2.2)$$

Both experiments agree with one another as well as with the standard model prediction.

The mass difference

$$m_Z - m_W = 10.4 \pm 1.4 \pm 0.8 \text{ GeV From UA1} \qquad (3.2.3)$$

and, $$m_Z - m_W = 11.3 \pm 1.3 \pm 0.5 \pm 0.8 \text{ from UA2} \qquad (3.2.4)$$

provides a nice test of the theory.

To examine these results, we first determine,

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}$$

which does not require the knowledge of Δr in the use of measurement of both m_W and m_Z

The equation has the advantage that many systematic and calibration errors cancel in the ratio m_W/m_Z when both masses are measured with the same detector. That procedure yields,

$$\mathrm{Sin}^2\theta_W = 0.211 \pm 0.025 \text{ from UA1} \tag{3.2.5}$$

$$\mathrm{Sin}^2\theta_W = 0.232 \pm 0.025 \pm 0.008 \text{ from UA2} \tag{3.2.6}$$

One can test the standard model at the level of its quantum corrections and probe for new physics by experimentally measuring Δr. Using the value of $\mathrm{Sin}^2\theta_W$ in equation (3.1.16), Δr values are found out.

$$\Delta r = 0.038 \pm 0.10 \pm 0.067 \text{ from UA1} \tag{3.2.7}$$

$$\Delta r = 0.068 \pm 0.087 \pm 0.030 \text{ from UA2} \tag{3.2.8}$$

Another way to test the standard model is to extract $\mathrm{Sin}^2\theta_W$ from eq (3.1.7) from $\Delta r = 0.0696$

$$\mathrm{Sin}^2\hat{\theta}_W = \frac{A^2}{m_W^2} = \frac{(38.65\ \mathrm{GeV})^2}{m_W^2} \tag{3.2.9}$$

$$\mathrm{Sin}^2 2\hat{\theta}_W = \frac{4A^2}{m_Z^2} = \frac{(77.30\ \mathrm{GeV})^2}{m_Z^2} \tag{3.2.10}$$

which gives,

$$\mathrm{Sin}^2\hat{\theta}_W = 0.218 \pm 0.005 \pm 0.014 \text{ from UA1} \tag{3.2.11}$$

$$\mathrm{Sin}^2\hat{\theta}_W = 0.232 \pm 0.003 \pm 0.008 \text{ from UA2} \tag{3.2.12}$$

The results are in excellent agreement with previously published UA1 and UA2 results.[16,19]

Another useful quantity for testing the standard model is the ρ parameter which should be 1(one) in the Higgs doublet scenario.

For $\Delta r = 0.696$, writing $\rho = \frac{m_W^2}{m_Z^2 \cos\theta_W}$

where $m_W = \frac{A}{\sin\theta_W}$

Therefore ρ can be expressed as

$$\rho = \frac{m_W^2}{m_Z^2 (1 - \frac{A^2}{m_W^2})} = \frac{m_W^2}{m_Z^2 (1 - \frac{38.65 \text{ GeV}}{m_W})} \quad (3.2.13)$$

Now, using the information of the Z° mass, one can determine the parameter ρ, related immediately to the isospin of the Higgs particle.

$$\rho = 1.009 \pm 0.028 \pm 0.02 \text{ from UA 1} \quad (3.2.14)$$

$$\rho = 1.001 \pm 0.028 \pm 0.006 \text{ from UA2} \quad (3.2.15)$$

UA2 value of ρ is in perfect agreement with the prediction of $\rho=1$ for a Higgs doublet and its deviation from 1 in the UA1 results reflects the difference in the $\sin^2\theta_W$ values of eqs (3.2.5) and (3.2.11) because the value of ρ depends on $\sin^2\theta_W$ values. The ρ parameter provides a particularly good test of standard model at the tree level.

3.3 Comparison of predicted value with the experimental results

A first test of the standard model is given by the comparison of the measured mass difference $m_Z - m_W$ with the predicted value as a function of m_Z and Δr eliminating $\sin^2\theta_W$ in relation (3.1.3) and (3.1.6). The two measurements of $\sin^2\theta_W$ are in good agreement with the most accurate estimates from low energy experiments. All these results are summarized in fig. 3.1.

Table 3.1 Measurement of standard model parameters.

Parameter	UA1 Electron channel	UA1 muon channel	Low energy experiments
m_W(GeV)	82.7±1.0(Stat.)±2.7(Syst.)	$81.8^{+6.0}_{-5.3}$(Stat)±2.6(Syst.)	
m_Z (GeV)	93.1±1.0(Stat.)±3.1(Syst.)	$90.7^{+5.2}_{-4.8}$(Stat.)±3.2(Syst.)	
m_Z-m_W(GeV)	10.4±1.4(Stat.)±0.8(Syst.)	$8.9^{+7.4}_{-7.7}$(Stat.)±1.9(Syst.)	
$\sin^2\theta_W$	0.211±0.025(Stat.)	0.187±0.148(Stat)±0.033	0.232±0.004
$\sin^2\hat{\theta}_W$	0.218±0.005(Stat)±0.014 (Syst.)	$0.233^{+0.033}_{-0.029}$(Stat)±0.014 (Syst.)	±0.003
			Minimal Standard model prediction
ρ	1.009±0.028±0.020	1.05±0.16±0.05	1
Δr_1	0.038±0.100±0.067		0.0711± 0.0013
Δr_2	0.125±0.021±0.057		

Now the values of m_W, m_Z reported from UA1 and UA2 collaborations (taken in the year 1984, 1986, 1987) are presented in the table 3.2.

We conclude that within errors, the experimental results are in excellent agreement with the theory and thus the values are compatible with the $SU(2)_L XU(1)_Y$ model supporting the hypothesis of a unified electroweak interactions.

All UA1 and UA2 results published and the results obtained in low energy neutrino experiments[21-23] which average to

$$\sin^2\theta_W = 0.232 \pm 0.004 \pm 0.003.$$

3.4 Measurement of neutral current parameters

The neutral current phenomena involving neutrino-hadron, neutrino electron, electron-hadron, and electron-muon reactions can be characterised by thirteen independent parameters. The fundamental goal of neutral current physics is therefore the complete determination of all these parameters. Once the goal is accomplished we can then compare these parameters to any chosen gauge model.

Defining the effective Larangian for low energy neutral current interactions as

$$\alpha = \alpha^{\nu H} + \alpha^{\nu e} + \alpha^{eH} + \alpha^{e\mu} + \ldots \qquad (3.4.1)$$

Where $\alpha^{\nu H}$ contains the terms relevant for neutrino-hadron scattering etc.

Table 3.2 Comparison of UA 1 and UA2 results with standard model prediction.

Quantity	Years	References	Values from UA1 and UA2 collaborations		Standard model $\sin^2\theta_W=0.23$
			UA1	UA2	
m_W (GeV)	1984	[16]	$80.9\pm1.5\pm2.4$	$81.0\pm2.5\pm1.3$	$83.0^{+2.9}_{-2.7}$
	1986	[19]	$83.5^{+1.1}_{-1.0}\pm2.8$	$81.2\pm1.1\pm1.3$	
	1987	[8],[9]	$82.7\pm1.0\pm2.7$	$80.2\pm0.6\pm0.5\pm1.3$	
m_Z (GeV)	1984	[16]	$95.6\pm1.5\pm2.9$	$91.9\pm1.3\pm1.4$	$93.8^{+2.4}_{-2.4}$
	1986	[19]	$93.0\pm1.4\pm3.2$	$92.5\pm1.3\pm1.5$	
	1987	[8],[9]	$93.1\pm1.0\pm3.1$	$91.5\pm1.2\pm1.7$	
(m_Z-m_W) (GeV)	1984	[16]	$14.7\pm2.1\pm0.4$	$10.9\pm2.8\pm0.2$	10.8 ± 0.5
	1986	[19]	$9.5^{+1.8}_{-1.7}\pm0.5$	$11.3\pm1.7\pm0.2$	
	1987	[8],[9]	$10.4\pm1.4\pm0.8$	$11.3\pm1.3\pm0.5\pm0.8$	
Δr	1984	[16]	$0.252\pm0.072\pm0.045$	$0.051\pm0.173\pm0.030$	0.0696 ± 0.002
	1986	[19]	0.028 ± 0.12	0.08 ± 0.10	
	1987	[8],[9]	$0.038\pm0.100\pm0.067$	$0.068\pm0.087\pm0.030$	
ρ	1984	[16]	$0.928\pm0.038\pm0.016$	$1.006\pm0.052\pm0.010$	1
	1986	[19]	$1.028\pm0.037\pm0.019$	$0.996\pm0.033\pm0.009$	
	1987	[8],[9]	$1.009\pm0.028\pm0.02$	$1.001\pm0.028\pm0.006$	

Table contd..

Table 3.2 contd.

Quantity	Years	References	Values from UA 1 and UA 2 collaborations		Standard model $\sin^2\theta_W=0.23$
			UA1	UA2	
$\sin^2\hat{\theta}_W = \left(\frac{38.65\,GeV}{m_W}\right)^2$	1984	[16]	0.228±0.008±0.014	0.228±0.014±0.007	
	1986	[19]	0.214±0.015	0.227±0.010	0.217±0.014
	1987	[8],[9]	0.218±0.005	0.232±0.003±0.008	
$\sin^2\theta_W = 1-\frac{m_W^2}{m_Z^2}$	1984	[16]	0.284±0.035	0.223±0.053	
	1986	[19]	0.194±0.032	0.229±0.030	0.217±0.014
	1987	[8],[9]	0.211±0.025	0.232±0.025±0.010	

(i) **Neutrino-hadron interaction:**

To the extent that hadrons are made up of valence u and d quarks the effective Lagrangian describing νH interactions contains four independent parameters α, β, γ and δ and is given by,

$$\alpha = \frac{-G_F}{\sqrt{2}} \bar{\nu}\gamma_\lambda(1-\gamma_5)\nu \{ \frac{1}{2} [\bar{u}\gamma_\lambda(\alpha+\beta\gamma_5) u - \bar{d}\gamma_\lambda(\alpha + \beta\gamma_5)d] + \frac{1}{2} [\bar{u}\gamma_\lambda(\gamma +\delta\gamma) u + \bar{d}\gamma_\lambda (\gamma+\delta\gamma_5)d]\} \qquad (3.4.1)$$

The parameters α, β, γ and δ have the following meaning

$\alpha \equiv$ isovector vector, $\beta \equiv$ isovector axial vector,

$\gamma \equiv$ isoscalar vector and $\delta \equiv$ isoscalar axial vector coefficients.

Here isovector and isoscalar mean $\frac{1}{2}(\bar{u}u - \bar{d}d)$ and $\frac{1}{2}(\bar{u}u + \bar{d}d)$ respectively. The νH interaction can also be parameterised by four "chiral coupling constants" $\varepsilon_L(u)$, $\varepsilon_R(u)$, $\varepsilon_L(d)$ and $\varepsilon_L(d)$ as [20]

$$\alpha^{\nu H} = - \frac{G_F}{\sqrt{2}} \bar{\nu}\gamma^\mu(1- \gamma_5)\nu J_\mu{}^H \qquad (3.4.2)$$

when the hadronic neutral current is given by

$$J_\mu^H = \sum_i [\varepsilon_L(i)\bar{q}_i \gamma_\mu(1-\gamma_5)q_i] + [\varepsilon_R(i)\bar{q}_i \gamma_\mu(1+\gamma_5)q_i]\}$$
$$= \sum_i \bar{q}_i\gamma_\mu(g_V^i + g_A^i\gamma_5)q_i \qquad (3.4.3)$$

The vector and the axial vector couplings $g^{i}_{V,A}$ are related to the chiral coupling $\varepsilon^{(i)}_{L,R}$ by

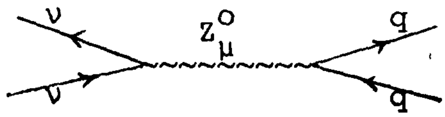
$$g^{i}_{V} = \varepsilon_{L}(i) + \varepsilon_{R}(i) \tag{3.4.4}$$

$$g^{i}_{A} = \varepsilon_{R}(i) - \varepsilon_{L}(i) \tag{3.4.5}$$

The two sets of coupling are related by,

$$\begin{aligned} \varepsilon_{L}(u) &= \frac{1}{4}\ (\alpha+\beta+\gamma+\delta) \\ \varepsilon_{R}(u) &= \frac{1}{4}\ (\alpha-\beta+\gamma-\delta) \\ \varepsilon_{L}(d) &= \frac{1}{4}\ (-\alpha-\beta+\gamma+\delta) \\ \varepsilon_{R}(d) &= \frac{1}{4}\ (-\alpha+\beta+\gamma-\delta) \end{aligned} \tag{3.4.6}$$

The Feynman graph for ν-H scattering from WS - GIM model is drawn as follows:



The effective Lagrangian can be written from this interaction as

$$\alpha_{eff} = -\frac{1}{\sqrt{2}}\left[\frac{G_F M_Z{}^2}{\sqrt{2}}\right]^{\frac{1}{2}} \bar{\nu}\gamma_{\mu}(1-\gamma_5)\nu\ Z^{o}_{\mu}$$

$$\Big\{-\frac{1}{\sqrt{2}}\left[\frac{G_F M_Z{}^2}{\sqrt{2}}\right]^{\frac{1}{2}} \{\bar{u}\gamma_{\nu}[-2Q_u \mathrm{Sin}^2\theta_W(1+\gamma_5)$$

$$+ (\tau_3(u)-2Q_u \mathrm{Sin}^2\theta)(1-\gamma^5)]u$$

$$+ \bar{d}\gamma_{\nu}[-2Q_d \mathrm{Sin}^2\theta_W(1+\gamma_5)+(\tau_3(d)-2Q_d \mathrm{Sin}^2\theta_W(1-\gamma_5)]d$$

$$+ (u \to C) + (u \to S)\}Z^{o}_{\nu}\Big\} \tag{3.4.7}$$

Substituting $Q_u = \frac{2}{3}$, $Q_d = -\frac{1}{3}$

$\tau_3(u) = +1$, $\tau_3(d) = -1$, $\sin^2\theta_W = x_W$,

and $Z^o_\mu \, Z^o_\nu$ by its propagator, for, $q^2 << M_Z^2$,

$$\langle 0 | Z_\mu \, Z_\nu | 0 \rangle \approx - \frac{\delta_{\mu\nu}}{M_Z^2}$$

the eq.(3.4.7) becomes,

$$\begin{aligned} \alpha_{eff} = & -\frac{1}{2} \frac{G_F}{\sqrt{2}} \; \bar{\nu}\gamma^\mu (1+\gamma_5)\nu \{ \bar{u}\gamma_\nu \; [- \frac{4x_W}{3} (1+\gamma_5) \\ & +(1- \frac{4x_W}{3}) (1-\gamma^5)] u + \bar{d}\gamma_\nu \; [\frac{2x_W}{3}(1+\gamma_5) \\ & +(-1+ \frac{2x_W}{3}) (1+\gamma_5)] d + \ldots\ldots \} \end{aligned} \qquad (3.4.8)$$

Comparing eqs.(3.4.2) and (3.4.8) one gets,

$$\begin{aligned} \varepsilon_L(u) &= \frac{1}{2} - \frac{2}{3} x_W \\ \varepsilon_R(u) &= - \frac{2}{3} x_W \\ \varepsilon_L(d) &= -\frac{1}{2} + \frac{2}{3} x_W \\ \varepsilon_R(d) &= \frac{x_W}{3} \end{aligned} \qquad (3.4.9)$$

Then using eq. (3.4.6) we obtain

$$\begin{aligned} \alpha &= 1 - 2 x_w \\ \beta &= 1 \\ \gamma &= - \frac{2x_w}{3} \\ \delta &= 0 \end{aligned} \qquad (3.4.10)$$

(ii) Neutrino electron reaction

The Lagrangian,

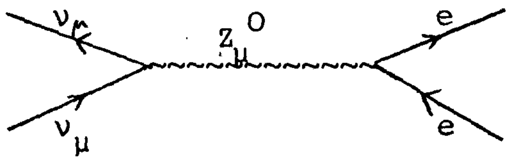
$$\alpha^{\nu e} = \frac{G_F}{\sqrt{2}} \; \bar{\nu}\gamma_\mu \; (1-\gamma_5)\nu \; J^e_\mu \qquad (3.4.11)$$

Where $J_\mu^e = \varepsilon_L(e)\ \bar{e}\gamma_\mu(1-\gamma_5)e+\varepsilon_R(e)\bar{e}\gamma_\mu(1+\gamma_5)e$

$$= \bar{e}\gamma_\mu(g_V{}^e+g_A^e\ \gamma_5)e \qquad (3.4.12)$$

and where $g^e_{V,A}=\varepsilon_R(e) \pm \varepsilon_L(e)$ (3.4.13)

The Feynman diagram of ν-e scattering can be represented as



The Lagrangian,

$$\alpha_{eff} = -\frac{1}{\sqrt{2}}\left(\frac{G_F M_Z{}^2}{\sqrt{2}}\right)^{\frac{1}{2}}\ \bar{\nu}\gamma_\mu\ (1-\gamma_5)\nu Z_\mu^\circ$$

$$\left\{-\frac{1}{\sqrt{2}}\left(\frac{G_F M_Z{}^2}{\sqrt{2}}\right)^{\frac{1}{2}}\ \bar{e}\gamma_\nu[(2\mathrm{Sin}^2\theta_W(1+\gamma_5)+(2\mathrm{Sin}^2\theta_W-1)(1-\gamma_5)]\ eZ_\nu^\circ\right\} \qquad (3.4.14)$$

Substituting $Z_\mu^\circ Z_\nu^\circ$ by its propagator, for $q^2 << M_Z{}^2$ and

$\mathrm{Sin}^2\theta_W = x_w$,

eq. (3.4.14) becomes,

$$\alpha_{eff} = -\frac{1}{2}\ \frac{G_F}{\sqrt{2}}\ \bar{\nu}\gamma_\mu\ (1-\gamma_5)\nu$$

$$\bar{e}\gamma_\mu[(4x_w-1) + \gamma_5]e \qquad (3.4.15)$$

Comparing eq.(3.4.15) and (3.4.11)

$$g_V^e = \frac{1}{2}\ (4x_w-1)$$

and $$g_A^e = -\frac{1}{2} \qquad (3.4.16)$$

(iii) **Electron-hadron scattering**

This class of relation includes the SLAC $e_{L,R}$ D and atomic physics experiments. The parity conserving part of the neutral current interactions between electrons and nucleons is completely overwhelmed by the much stronger electromagnetic interactions. We therefore concentrate on the parity non-conserving effective Lagrangian which is described by four independent parameters $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ and $\tilde{\delta}$ in analogy with νH case. The parameters have meaning similar to α, β, γ and δ. We have,

$$\begin{aligned}\alpha = -\frac{G_F}{\sqrt{2}}\{&\bar{e}\gamma_\lambda\gamma_5 e\ [\frac{\tilde{\alpha}}{2}(\bar{u}\gamma_\lambda u - \bar{d}\gamma_\lambda d)\\ &+\frac{\tilde{\gamma}}{2}(\bar{u}\gamma_\lambda u + \bar{d}\gamma_\lambda d)] + \bar{e}\gamma_\lambda e[\frac{\tilde{\beta}}{2}(\bar{u}\gamma_\lambda\gamma_5 u - \bar{d}\gamma_\lambda\gamma_5 d)\\ &+\frac{\tilde{\delta}}{2}(\bar{u}\gamma_\lambda\gamma_5 u + \bar{d}\gamma_\lambda\gamma_5 d)]\}\end{aligned} \qquad (3.4.17)$$

It can also be written as

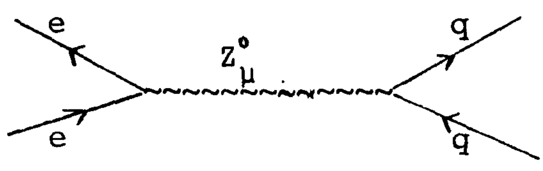
$$\alpha^{eH} = -\frac{G_F}{\sqrt{2}}\sum_i (C_{1i}\bar{e}\gamma_\mu\gamma_5 e\bar{q}_i\gamma_\mu q_i + C_{2i}\bar{e}\gamma_\mu e\bar{q}_i\gamma_\mu\gamma_5 q_i) \qquad (3.4.18)$$

where $q_i = u, d, \ldots.$

The two sets of coupling are related by

$$\begin{aligned}C_{1u} &= \tfrac{1}{2}(\tilde{\alpha}+\tilde{\gamma})\\ C_{2u} &= \tfrac{1}{2}(\tilde{\beta}+\tilde{\delta})\\ C_{1d} &= \tfrac{1}{2}(-\tilde{\alpha}+\tilde{\gamma})\\ C_{2d} &= \tfrac{1}{2}(-\tilde{\beta}+\tilde{\delta})\end{aligned} \qquad (3.4.19)$$

The effective Lagrangian for electron-hadron scattering can be written from the Feynman diagram as



$$\alpha_{eff} = -\frac{1}{\sqrt{2}}\left(\frac{G_F M_Z{}^2}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{e}\gamma^\mu[(2\mathrm{Sin}^2\theta_W - 1)(1-\gamma_5) + 2\mathrm{Sin}^2\theta_W(1+\gamma_5)]e\, Z^\circ_\mu$$
$$x - \frac{1}{\sqrt{2}}\left(\frac{G_F M_Z{}^2}{\sqrt{2}}\right)^{\frac{1}{2}} \{\bar{q}_i\gamma_\nu[\tau_3(q_i) - 2Q_{qi}\mathrm{Sin}^2\theta_W(1-\gamma_5)]$$
$$+(-2Q_{qi}\,\mathrm{Sin}^2\theta_W)(1+\gamma_5)]q_i\}Z^\circ_\nu\} \qquad (3.4.20)$$

Substituting $q_i = u, d, \ldots$

$$\tau_3(u) = +1, \quad \tau_3(d) = -1$$
$$Q_u = 2/3, \quad Q_d = -\frac{1}{3}$$

and $Z^\circ_\mu Z^\circ_\nu$ by its propagator

$$\sim \frac{\delta_{\mu\nu}}{M_Z{}^2}$$

the eq. (3.4.20) becomes,

$$\alpha_{eff} = -\frac{1}{2}\frac{G_F}{\sqrt{2}}\,\bar{e}\gamma^\mu[(2x_w - 1)(1-\gamma_5) + 2x_w(1+\gamma_5)]e$$
$$\{\bar{u}\gamma_\nu[(1 - \frac{4x_w}{3})(1-\gamma_5) + (-\frac{4}{3}x_w)(1+\gamma_5)]u$$
$$+ \bar{d}\gamma_\nu[(-1 + \frac{2}{3}x_w)(1-\gamma_5) + (\frac{2}{3}x_w)(1+\gamma_5)]d\} + \ldots$$
$$= -\frac{G_F}{\sqrt{2}}\,\frac{1}{2}\,\bar{e}\gamma^\mu[(4x_w - 1) - \gamma_5]e$$
$$\{\bar{u}\gamma_\nu[(1 - \frac{8}{3}x_w) + \gamma_5]u + \bar{d}\gamma_\nu[(-1 + \frac{4}{3}x_w) - \gamma_5]d\}$$

$$\alpha_{eff} = -\frac{G_F}{\sqrt{2}}\,\frac{1}{2}\,\{[\bar{e}\gamma^\mu(4x_w - 1)e\;\bar{u}\gamma_\nu\gamma_5 u\;\bar{e}$$
$$-\bar{e}\gamma^\mu\gamma^5 e\;\bar{u}\gamma_\nu(1 - \frac{8}{3}x_w)u]$$
$$[-\bar{e}\gamma^\mu(4x_w - 1)\;e\bar{d}\gamma_\nu\gamma_5 d \qquad (3.4.21)$$
$$-\bar{e}\gamma^\mu\gamma_5 e\;\bar{d}\gamma_\nu(-1 + \frac{4x_w}{3})d]\}$$

C_{1u} and C_{2u} are the coefficients of $\bar{e}\gamma_\mu\gamma_5 e\bar{u}\gamma_\mu u$ and $\bar{e}\gamma_\mu e\bar{u}\gamma_\mu\gamma_5 u$ respectively. Similarly C_{1d} and C_{2d} are the coefficients of $\bar{e}\gamma_\mu\gamma_5 e\bar{d}\gamma_\mu d$ and $\bar{e}\gamma_\mu e\, \bar{d}\gamma_\mu\gamma_5 d$ respectively.

Comparing eqs. (3.4.21) and (3.4.18) where $\sin^2\theta_W = x_W$

$$
\begin{aligned}
C_{1u} &= \tfrac{1}{2}\left(-1 + \tfrac{8}{3}x_w\right) = -\tfrac{1}{2} + \tfrac{4}{3}x_w \\
C_{2u} &= \tfrac{1}{2}\left(4x_w - 1\right) = 2x_w - \tfrac{1}{2} \\
C_{1d} &= -\tfrac{1}{2}\left(-1 + \tfrac{4}{3}x_w\right) = \tfrac{1}{2} - \tfrac{2}{3}x_w \\
C_{2d} &= -\tfrac{1}{2}\left(4x_w - 1\right) = \tfrac{1}{2} - 2x_w
\end{aligned}
\tag{3.4.22}
$$

and

$$
\begin{aligned}
\tilde{\alpha} &= -(1 - 2x_w) \\
\tilde{\beta} &= -(1 - 4x_w) \\
\tilde{\gamma} &= \tfrac{2}{3}x_w \\
\text{and} \quad \tilde{\delta} &= 0
\end{aligned}
\tag{3.4.23}
$$

(iv) Electron-position annihilation into muon pairs

Writing the most general weak neutral current effective Lagrangian for $e^+e^- \rightarrow \mu^+\mu^-$ involving V and couplings as

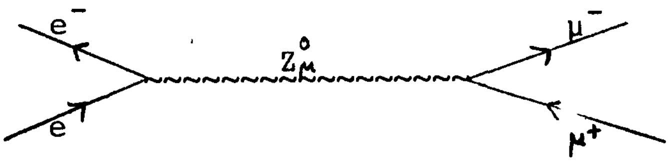
$$
\begin{aligned}
\alpha = -\frac{G_F}{\sqrt{2}} \{ & h_{vv}(\bar{e}\gamma_\mu e + \bar{\mu}\gamma_\mu\mu)(\bar{e}\gamma^\mu e + \bar{\mu}\gamma^\mu\mu) \\
& + 2h_{VA}(\bar{e}\gamma_\mu e + \bar{\mu}\gamma_\mu\mu)(\bar{e}\gamma^\mu\gamma_5 e + \bar{\mu}\gamma^\mu\gamma_5\mu) \\
& + h_{AA}(\bar{e}\gamma_\mu\gamma_5 e + \bar{\mu}\gamma_\mu\gamma_5\mu)(\bar{e}\gamma^\mu\gamma_5 e + \bar{\mu}\gamma^\mu\gamma_5\mu)\}
\end{aligned}
\tag{3.4.24}
$$

(3.4.24)

where μ-e universality is assumed. These constants h_{VV}, h_{AA}, h_{VA} could be measured by (i) studying the magnitude and

and energy dependence of the cross-section, (ii) the forward backward asymmetry in the angular distribution of the final muon, (iii) and the muon longitudinal polarization respectively.

The Feynman diagram for this reaction is represented as



The effective Lagrangian is given by,

$$\alpha_{eff} = -\frac{1}{\sqrt{2}} \left[\frac{G_F\ M_Z{}^2}{\sqrt{2}}\right]^{\frac{1}{2}} \bar{e}\gamma_\mu\ [(2\mathrm{Sin}^2\theta_W-1)(1-\gamma_5) + 2\mathrm{Sin}^2\theta_W(1+\gamma_5)]\ e\ Z^\circ_\mu$$

$$\times\left\{(-\frac{1}{\sqrt{2}})\left[\frac{G_F M_Z{}^2}{\sqrt{2}}\right]^{\frac{1}{2}} \bar{\mu}\gamma_\nu\ [(2\ \mathrm{Sin}^2\theta_W-1)(1-\gamma_5)\right.$$

$$\left. + 2\mathrm{Sin}^2\theta_W(1+\gamma_5)]\ \mu\ Z^\circ_\nu\right\} \qquad (3.4.25)$$

Substituting $Z^\circ_\mu\ Z^\circ_\nu$ by its propagator for $q^2 << M_Z{}^2$ and $\mathrm{Sin}^2\theta_W = x$, the eq. (3.4.22) becomes

$$\alpha_{eff} = \frac{1}{2}\ \frac{G_F M_Z{}^2}{\sqrt{2}}\ \left(\frac{\delta_{\mu\nu}}{-M_Z{}^2}\right) \bar{e}\gamma_\mu[(4x_w-1)-\gamma_5]\ e$$

$$\bar{\mu}\gamma_\nu[(4x_w-1)-\gamma_5]\mu$$

$$= -\frac{1}{2}\ \frac{G_F}{\sqrt{2}}\ [(4x_w-1)^2\ \bar{e}\gamma_\mu e\ \bar{\mu}\gamma_\mu\mu$$

$$-(4x_w-1)\ \bar{e}\gamma_\mu e\ \bar{\mu}\gamma_\mu\gamma_5\ \mu$$

$$-(4x_w-1)\ \bar{e}\gamma_\mu\gamma_5 e\ \bar{\mu}\gamma_\mu\mu \qquad (3.4.26)$$

$$+ \bar{e}\gamma_\mu\gamma_5\ e\bar{\mu}\gamma_\mu\gamma_5\ \mu\ ]$$

Comparing eqs (3.4.26) and (3.4.24) the following values can be found out.

$$h_{VA} = \frac{1}{4}(1 - 4x_W)$$

$$h_{VV} = \frac{(1 - 4x_W)^2}{4} \qquad (3.4.27)$$

$$h_{AA} = \frac{1}{4}$$

We have thus seen that low energy neutral current experiments can be described by thirteen parameters $\alpha, \beta, \gamma, \delta$, g_V, g_A, $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, $\tilde{\delta}$, h_{VV}, h_{AA}, and h_{VA}. All these parameters can be shown in a neutral current coupling pyramid[24,25] as shown below.

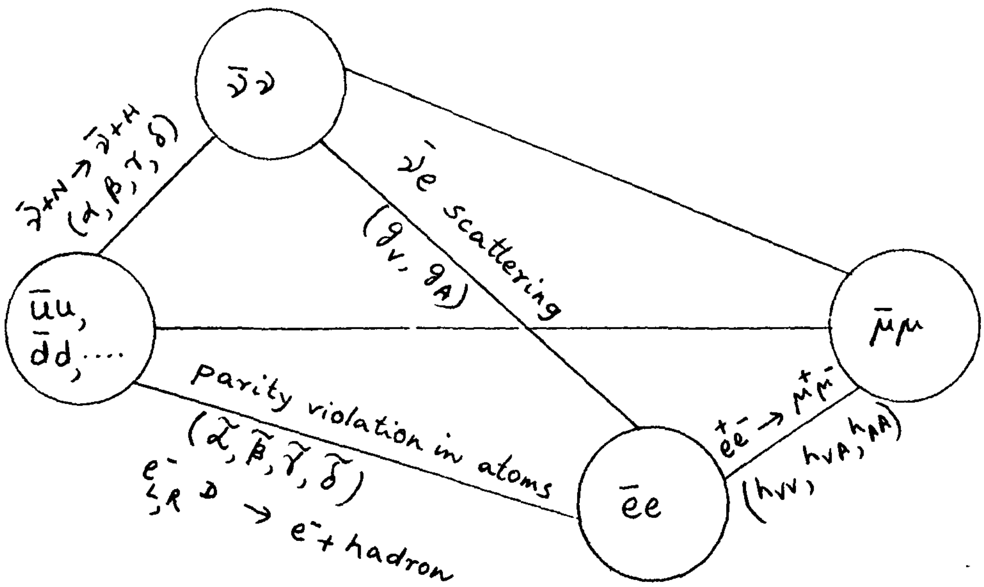


Fig. 3.2 Neutral-current coupling pyramid.

The neutral current parameters can be arranged in a table shown below.

Table 3.3 Predicted values for neutral current parameters.

Process	Parameters	WS-GIM prediction $\sin^2\theta_W = x_w$
	$\varepsilon_L(u)$	$\frac{1}{2} - \frac{2}{3}x_w$
	$\varepsilon_R(u)$	$-\frac{2}{3}x_w$
	$\varepsilon_L(d)$	$-\frac{1}{3} + \frac{1}{3}x_w$
	$\varepsilon_R(d)$	$\frac{1}{3}x_w$
νH reaction	α	$1 - 2x_w$
	β	1
	γ	$-\frac{2}{3}x_w$
	δ	0
νe reaction	g_V^e	$-\frac{1}{2} + 2x_w$
	g_A^e	$-\frac{1}{2}$
	C_{1u}	$-\frac{1}{2} + \frac{4}{3}x_w$
	C_{2u}	$-\frac{1}{2} + 2x_w$
	C_{1d}	$\frac{1}{2} - 2x_w/3$
	C_{2d}	$\frac{1}{2} - 2x_w$
eH reaction	$\tilde{\alpha}$	$-(1-2x_w)$
	$\tilde{\beta}$	$-(1-4x_w)$
	$\tilde{\gamma}$	$\frac{2}{3}x_w$
	$\tilde{\delta}$	0
$e^+e^- \to \mu^+\mu^-$	h_{AA}	$\frac{1}{4}$
	h_{VA}	$\frac{1-4x_w}{4}$
	h_{VV}	$(2x_w - \frac{1}{2})^2$

3.5 Neutral-current experiments

As discussed in Sec. (3.4), experiments can be carried out for various processes at low energies, in order to determine various neutral current parameters occuring in the effective Lagrangian. The experimental values of these parameters would then yield the values of $\sin^2\theta$ through the low-energy limit of the electroweak theory. Experimentally, these have been already measured. In fact, eight years before the gauge bosons were detected at the $\bar{p}p$ collider, neutral current experiments suggested that the electroweak theory might be correct. We describe below some of the neutral current processes where the experimental values of the relevant parameters have been measured.

(a) Neutrino-hadron reactions

Experimental data on neutrino-hadron reactions are much more in number compared to other processes. These data have been analysed to determine the space-time structure of neutral currents. As discussed in Sec. 3.5, the experimental values of neutral current parameters obtained from neutrino-hadron processes can be used to determine $\sin^2\theta_W$ very accurately.

(i) Deep Inelastic scattering from iso-scalar target

The primary neutral current quantities are measured for isoscalar targets as the neutral current to charged current ratio. For neutrino and antineutrino these are

$$R_\nu = \frac{\nu N \rightarrow \nu X}{\nu N \rightarrow \mu^- X} \qquad (3.5.1)$$

$$R_{\bar\nu} = \frac{\bar\nu N \rightarrow \bar\nu X}{\nu N \rightarrow \mu^+ X} \qquad (3.5.2)$$

where ν represents neutrino and X represents hadrons.

The differential cross-sections with respect to the hadron energy are

$$\frac{d\sigma}{dy}(\nu N \rightarrow \nu X) \quad \text{and} \quad \frac{d\sigma}{dy}(\bar\nu N \rightarrow \bar\nu X) \quad ,$$

where $Y = \frac{E_h}{E_\nu}$, E_h is the hadron energy and E_ν is the incident ν energy and the cross-section for both charged and neutral current interactions are given (in units of $G^2 M_W E_\nu/\pi$) and with $N = \frac{1}{2}(n+p)$ by,

$$\frac{d\sigma^{cc}}{dy}(\nu N) = Q + \bar Q\,(1-y)^2 \qquad (3.5.3)$$

$$\frac{d\sigma^{cc}}{dy}(\bar\nu N) = \bar Q + Q\,(1-y)^2 \qquad (3.5.4)$$

$$\frac{d\sigma^{Nc}}{dy}(\nu N) = [\,|\varepsilon_L(u)|^2+|\varepsilon_L(d)|^2]\,[Q+\bar Q(1-y)^2]$$
$$+[\,|\varepsilon_R(u)|^2+|\varepsilon_R(d)|^2]\,[\bar Q+Q(1-y)^2] \qquad (3.5.5)$$

$$\frac{d\sigma^{NC}}{dy}(\bar\nu N) = [\,|\varepsilon_L(u)|^2+|\varepsilon_L(d)|^2]\,[\bar Q+Q(1-y)^2]$$
$$+[\,|\varepsilon_R(u)|^2+|\varepsilon_R(d)|^2]\,[Q+\bar Q(1-y)^2] \qquad (3.5.6)$$

Where $Q = \int x\,[u(x) + d(x)]\,dx$

$\bar Q = \int x\,[\bar u(x) + \bar d(x)]\,dx$

and $x = Q^2/2\,P.Q.$ (the Bjorken scaling variables) and $\overset{(-)}{u}(x)$ and $\overset{(-)}{d}(x)$ being the up and down quark (antiquark) distribution functions.

From eqs (3.5.3) and (3.5.4) two more equations (3.5.7) and (3.5.8) are derived.[25]

$$\frac{\sigma_{NC}(\nu N)+\sigma_{NC}(\bar{\nu}N)}{\sigma_{CC}(\nu N)+\sigma_{CC}(\bar{\nu}N)} = \frac{1}{4}(\alpha^2+\beta^2+\gamma^2+\delta^2)$$

$$= |\varepsilon_L(u)|^2+|\varepsilon_L(d)|^2+|\varepsilon_R(u)|^2+|\varepsilon_R(d)|^2 \qquad (3.5.7)$$

and,

$$\frac{\sigma_{NC}(\nu N)-\sigma_{NC}(\bar{\nu}N)}{\sigma_{CC}(\nu N)-\sigma_{CC}(\bar{\nu}N)} = \frac{1}{2}(\alpha\beta+\gamma\delta)$$

$$= |\varepsilon_L(u)|^2+|\varepsilon_L(d)|^2-|\varepsilon_R(u)|^2-|\varepsilon_R(d)|^2 \qquad (3.5.8)$$

The accurate data on deep-inelastic scattering on isoscalar targets are from CDHS[26] and CHARM[27] groups which reports

$$R_\nu \equiv \left(\frac{NC}{CC}\right)_\nu = 0.307 \pm 0.008$$

$$R_{\bar{\nu}} \equiv \left(\frac{NC}{CC}\right)_{\bar{\nu}} = 0.373 \pm 0.025 \qquad (3.5.9)$$

for CDHS, and,

$$R_\nu \equiv 0.320 \pm 0.021$$

$$R_{\bar{\nu}} \equiv 0.39 \pm 0.024 \qquad (3.5.10)$$

for CHARM.

From the formulae (3.5.7) and (3.5.8) we can derive,

$$|\varepsilon_L(u)|^2+|\varepsilon_L(d)|^2 = \frac{R_\nu - \gamma^2 R_{\bar{\nu}}}{1-\gamma^2} + \text{corrections} \qquad (3.5.11)$$

and $|\varepsilon_R(u)|^2+|\varepsilon_R(d)|^2 = \dfrac{\gamma^2(R_\nu - R_{\bar{\nu}})}{1-\gamma^2}$ + correction (3.5.12)

where γ is measured, as,

$$\gamma \equiv \frac{\sigma^{CC}_{\bar{\nu}}}{\sigma^{CC}_{\nu}} = 0.49 \pm 0.019 \qquad (3.5.13)$$

The corrections arise from the theoretical uncertainties in the strange quark effects are small.

Using the CDHS data and taking into account corrections for neutron excess in an Fe target, one obtains

$$\begin{aligned} |\varepsilon_L(u)|^2+|\varepsilon_L(d)|^2 &= 0.300 \pm 0.015 \\ |\varepsilon_R(u)|^2+|\varepsilon_R(d)|^2 &= 0.024 \pm 0.008 \end{aligned} \qquad (3.5.14)$$

where errors include the corrections. Similar numbers can be obtained using the data of other groups: Harvard-Pensylvania, Wisconsin-Fermilab (HPWF), BEBC etc.

(ii) <u>Deep inelastic scattering on proton and neutron targets</u>

There are several measurements of deep-inelastic $\nu(\bar{\nu})$ scattering from proton and neutron targets. The data are usually presented in terms of the ratio of neutral current to charged current cross-sections.

$$R_\nu^P = \frac{\nu P \to \nu X}{\nu p \to \mu^- X}\ , \quad R_{\bar{\nu}}^P = \frac{\bar{\nu}_p \to \bar{\nu} X}{\bar{\nu}_p \to \mu^+ X} \qquad (3.5.15)$$

The measured report from BEBC and other experiments are mentioned below:[28-33]

$R_\nu^P = 0.52 \pm 0.04$ (BEBC)[28]

$R_\nu^P = 0.48 \pm 0.17$ (FNAL, Harris et al 1977)[29]

$R_{\bar{\nu}}^P = 0.42 \pm 0.13$ (FNAL, Derric et al, 1978)[30]

$R_\nu^{n/p} = 1.22 \pm 0.35$ (Marriner 1977)[31]

$$\begin{cases} R_{\bar{\nu}}^{n/p} = 0.64 \pm 0.18 \text{ (FNAL, Roe, 1979)}^{32} \\ R_{\bar{\nu}}^{n/p} = 1.64 \pm 0.20 \text{ (FIIM, Bell et al, 1979)}^{33} \end{cases} \quad (3.5.16)$$

The measured value of R_{ν}^{P} from BEBC group can be expressed from the following calculated expression

$$R_{\nu}^{P} = 2.1|\varepsilon_L(u)|^2 + 1.0|\varepsilon_L(d)|^2$$
$$+ 0.70|\varepsilon_R(u)|^2 + 0.36|\varepsilon_R(d)|^2 \quad (3.5.17)$$

Similarly the values of R_{ν}^{P}, $R_{\bar{\nu}}^{P}$, $R_{\nu}^{n/p}$ and $R_{\bar{\nu}}^{n/p}$ from other (FNAL, FIIM) experiments can be measured in terms of $\varepsilon_L(u)^2$ $\varepsilon_L(d)^2$, $\varepsilon_R(u)^2$ and $\varepsilon_R(d)^2$.

Semi inclusive pion production

The reaction involved in the semi-inclusive neutral-current pion production on isoscalar targets are of the following type

$$\bar{\nu} + N \rightarrow \bar{\nu} + \pi^{\pm} + X$$

Where we make use of quark fragmentation model. Knowledge of the quark fragmentation function i.e., the probability that a given type of quark u or d, produces a π^+ or π^- allows one to extract information on the relative strength of the couplings to u and d quarks from the above measurements.

From charged current reactions that . in νN ($\bar{\nu}N$) collisions. the probability for a u(d) quark to fragment into a $\pi^{+(-)}$ in the forward direction is much larger than the probability for a u(d) quark to fragment into a $\pi^{-(+)}$ i.e.,

$$\left(\frac{\pi^+}{\pi^-}\right)_{\nu\to\mu^-} = \frac{D_u^{\pi+}}{D_u^{\pi-}} \simeq 3 \qquad (3.5.18)$$

$$\left(\frac{\pi^+}{\pi^-}\right)_{\nu\to\mu^+} = \frac{D_d^{\pi+}}{D_d^{\pi-}} = \frac{D_u^{\pi-}}{D_u^{\pi+}} \simeq \frac{1}{3} \qquad (3.5.19)$$

where $D^{\pi}_{u(d)}$ is the probability amplitude.

In neutral current reactions on isoscalar targets where there are as many u quarks as d quarks, an asymmetry between π^+ and π^- would give information on the difference in the magnitude of neutral current interaction between u and d quarks. Quantitatively we have following expressions,[34-36]

$$\left(\frac{\pi^+}{\pi^-}\right)_{\nu\to\nu} = \frac{[|\varepsilon_L(u)|^2+\frac{1}{3}|\varepsilon_R(u)|^2]D_u^{\pi+}+[|\varepsilon_L(d)|^2+\frac{1}{3}|\varepsilon_R(d)|^2]D_u^{\pi-}}{[|\varepsilon_L(u)|^2+\frac{1}{3}|\varepsilon_R(u)|^2]D_u^{\pi-}+[|\varepsilon_L(d)|^2+\frac{1}{3}|\varepsilon_R(d)|^2]D_u^{\pi+}} \qquad (3.5.20)$$

$$\left(\frac{\pi^+}{\pi^-}\right)_{\bar\nu\to\bar\nu} = \begin{cases}\varepsilon_L(u) \leftrightarrow \varepsilon_R(u)\\ \varepsilon_L(d) \leftrightarrow \varepsilon_R(d)\end{cases} \qquad (3.5.21)$$

where $D_d^{\pi+} = D_u^{\pi-}$ and $D_d^{\pi-} = D_u^{\pi+}$

The earliest results obtained in the Gargamelle experiment at low energy ($E_\nu \simeq$ 1-5 GeV) have been supplemented by the measurement of Fermilab IHEP-ITEP-Michigan (FIIM) group using high energy antineutrino ($E_{\bar\nu}$ > 20 GeV)

$$R_{\nu}^{+/-} \equiv \left(\frac{\pi^+}{\pi^-}\right) = 0.77 \pm 0.14 \text{ GGM} \quad (\text{Kluttig et al})^{37} \tag{3.5.22}$$

$$R_{\bar{\nu}}^{+/-} = \left(\frac{\pi^+}{\pi^-}\right) = 1.65 \pm 0.33 \text{ GGM}^{37} \tag{3.5.23}$$

$$= 1.27 \begin{smallmatrix} +0.36 \\ -0.27 \end{smallmatrix} \text{ FIIM}^{32} \tag{3.5.24}$$

Using experimental values from eqs. (3.5.22) and (3.5.23), Sehgal obtained[35] from the equation (3.5.20) and (3.5.21)

$$\begin{aligned} |\epsilon_L(u)|^2 &= 0.11 \pm 0.03 \\ |\epsilon_L(d)|^2 &= 0.19 \pm 0.03 \\ |\epsilon_R(u)|^2 &= 0.03 \pm 0.015 \\ |\epsilon_R(d)|^2 &= 0.00 \pm 0.0\,15 \end{aligned} \tag{3.5.25}$$

Sehgal's analysis was subsequently confirmed by semi inclusive data at higher energies coming from BEBC and FMMS collaborations and by deep-inelastic scattering data on protons.

Elastic scattering: The elastic scattering of neutrinos and antineutrinos from protons are the simplest semi leptonic neutral current interactions

$$\begin{aligned} &\nu + p \rightarrow \nu + p \\ \text{and} \quad &\bar{\nu} + p \rightarrow \bar{\nu} + p \end{aligned} \tag{3.5.26}$$

The data obtained from these reactions are combined with those from inelastic scattering from an isoscalar target to yield important isospin information. The measurable quantities are,

$$
\left.\begin{aligned}
R^{el}_{\nu} &= \frac{\nu p \rightarrow \nu p}{\nu n \rightarrow \mu^- p} \\
\text{and} \quad R^{el}_{\bar{\nu}} &= \frac{\bar{\nu} p \rightarrow \bar{\nu} p}{\nu p \rightarrow \mu^+ n}
\end{aligned}\right\} \tag{3.5.27}
$$

The experimental values are

$$
\begin{aligned}
R^{el}_{\nu} &= 0.11 \pm 0.02 \quad \text{HPB (Entenberg et al)}^{39} \\
&= 0.20 \pm 0.06 \quad \text{CIR (Lee et al)}^{40} \\
&= 0.10 \pm 0.03 \quad \text{AP (Faissner et al)}^{41} \\
&= 0.12 \pm 0.06 \quad \text{GGM (Pohl et al)}^{42} \\
R^{el}_{\bar{\nu}} &= 0.19 \pm 0.05 \quad \text{HPB (Entenberg et al)}^{39}
\end{aligned}
$$

A most ambitious fit to all neutrino hadron data has been performed by Langacker et al[43], and reviewed by Kim et al[7] .They obtained a unique solution that is essentially the same as solution of Hung and Sakurai[24]. The results are represented in a tabular form.

Table 3.3 Neutrino-hadron data

Fit to data	WS model ($\sin^2\theta_W$=0.23)
$\epsilon_L(u) = 0.340 \pm 0.033$	0.347
$\epsilon_L(d) = -0.424 \pm 0.026$	-0.423
$\epsilon_R(u) = -0.179 \pm 0.019$	-0.153
$\epsilon_R(d) = -0.017 \pm 0.058$	0.077
$\alpha = 0.589 \pm 0.067$	0.540
$\beta = 0.937 \pm 0.062$	1.000
$\gamma = -0.273 \pm 0.081$	-0.153
$\delta = 0.101 \pm 0.093$	-0.000

(b) **Neutrino-electron reaction**

The neutrinos involving reactions $\nu_\mu e$, $\bar{\nu}_\mu e$ and $\bar{\nu}_e e$ are discussed because of their simplicity.

The cross-section for these reactions can be computed from the effective Lagrangian (eq (3.4.10)) as

$$\frac{d\sigma}{dy}(\nu_\mu e) = \frac{2G^2 m_e E_\nu}{\pi} [g_L^2 + g_R^2(1-y)^2 - \frac{m_e}{E_\nu} y\, g_L g_R] \qquad (3.5.28)$$

$$\frac{d\sigma}{dy}(\bar{\nu}_\mu e) = \frac{2G^2 m_e E_\nu}{\pi} [g_R^2 + g_L^2(1-y)^2 - \frac{m_e}{E_\nu} y\, g_L g_R] \qquad (3.5.29)$$

$$\frac{d\sigma}{dy}(\bar{\nu}_e e) = \frac{2G^2 m_e E_\nu}{\pi} [G_R^2 + G_L(1-y)^2 - \frac{m_e}{E_\nu} y G_L G_R] \qquad (3.5.30)$$

where $y = \frac{E_e}{E_\nu}$ and E_e is the K.E of electron in the lab.

$$g_{L,R} = \frac{1}{2}(g_V \pm g_A) \qquad (3.5.31)$$

$$G_{L,R} = \frac{1}{2}(G_V \pm G_A) = \begin{Bmatrix} 1 + g_L \\ g_R \end{Bmatrix} \qquad (3.5.32)$$

The total cross-sections can be obtained by integrating eq.(3.5.28) to (3.5.30). It is then customary to present the experimental results on a $g_V - g_A$ plane by ellipses, as can be seen in fig.3.3 . In the Fig. 3.3, the $\bar{\nu}_e e$ curve intersects with the $\nu_\mu e$ and $\bar{\nu}_\mu e$ ellipses at two regions leaving the vector-axial-vector ambiguity inact. The best fit[38] to the data gives

$$\left.\begin{aligned} g_A &= -0.52 \pm 0.06 \\ g_V &= 0.06 \pm 0.08 \end{aligned}\right\} \text{(axial dominant)}$$

With the VA ambiguity $g_V \leftrightarrow g_A$. The axial-dominant solution is in excellent agreement with the standard model, which, for $\sin^2\theta_W = 0.23$ gives

$$g_A = -0.5$$
$$g_V = -0.04$$

(c) Electron-hadron interaction:

Starting with a beam of longitudinally polarized electron which may be scattered inelastically by a nucleus, parity nonconservation due to weak neutral current interactions can be detected by studying how the observed cross-section depends on the incident electron helicity. If it is found that $\sigma(\lambda = \frac{1}{2}) \neq \sigma(\lambda = -\frac{1}{2})$ where λ stands for the incident electron helicity in

$$e^-_{\text{polarized}} + \text{nucleus} \rightarrow e^- + \text{any}.$$

We have unambiguous proof for parity violation, independent of any model.

Defining parity violating asymmetry A as follows:[44]

$$A = \frac{d\sigma(\lambda=\frac{1}{2}) - d\sigma(\lambda=-\frac{1}{2})}{d\sigma(\lambda=\frac{1}{2}) + d\sigma(\lambda=-\frac{1}{2})} \tag{3.5.33}$$

In parton model, the interference between Z boson exchange and electromagnetism leads to an asymmetry[45]

$$\frac{A(x,y)}{Q^2} = \frac{G_F}{2\sqrt{2}\pi\alpha} \frac{\sum_i x q_i(x) Q_i (C_{1i} \pm F(y) C_{2i})}{\sum_i x\, q_i(x)\, Q_i^2} \tag{3.5.34}$$

Where $Q^2 = -q^2 > 0$ is the momentum transfer, $xq_i(x)$ is the distribution function for parton i of momentum fraction x, eQ_i is the charge of parton i, and,

$$F(y) = \frac{1-(1-y)^2}{1+(1-y)^2} \tag{3.5.35}$$

The upper and lower signs refer to quarks and antiquarks respectively.

Neglecting antiquarks and heavy quark and considering deuteron as the target, eq. (3.5.34) can be written as,

$$\frac{A_D}{Q^2} = \frac{3G_F}{5\sqrt{2}\pi\alpha}\left[\left(C_{1u}-\tfrac{1}{2}C_{1d}\right)+F(y)\left(C_{2u}-\tfrac{1}{2}C_{2d}\right)\right] \tag{3.5.36}$$

But from the WS-GIM model,

$$C_{1u}-\frac{1}{2}C_{1d} = -\frac{3}{4}+\frac{5}{3}\sin^2\theta_W \tag{3.5.37}$$

and

$$C_{2u}-\frac{1}{2}C_{2d} = 3\left(\sin^2\theta_W-\frac{1}{4}\right) \tag{3.5.38}$$

From SLAC polarized electron experiment,

$$\frac{A_D}{Q^2} = [(-9.7\pm 2.6)+(4.9\pm 8.1)F(y)]\times 10^{-5} \tag{3.5.39}$$

where

$$C_{1u}-\frac{1}{2}C_{1d} = -0.45 \pm 0.12 \tag{3.5.40}$$

$$C_{2u}-\frac{1}{2}C_{2d} = 0.23 \pm 0.38 \tag{3.5.41}$$

The asymmetry on a proton target can be written as,

$$\frac{A_P}{Q^2} \simeq \frac{2G_F}{3\sqrt{2}\pi\alpha}\left[\left(C_{1u}-\tfrac{1}{4}C_{1d}\right)+F(y)\left(C_{2u}-\tfrac{1}{4}C_{2d}\right)\right] \tag{3.5.42}$$

Which is considerably less reliable than the expression for $\frac{A_D}{Q^2}$,

Parity Violation in Atoms

The existing data on atomic parity violation is confusing, with different groups reporting conflicting experimental results[46-49] and these have large experimental errors, and the atomic physics calculations are difficult and uncertain. Now, the theoretical expression for the weak charge for heavy atom is given by

$$Q_W(N,Z) = -2[C_{1u}(2Z+N)+C_{1d}(Z+2N)] \tag{3.5.43}$$

Where N, Z are total number of nucleons, protons of the atom.

For Bi(Z = 126, N=83) and Thalium (Z=123, N=81) predictions are

$$Q_W^{Bi} = Q_W(126,\ 83) = -584\ C_{1u} - 670\ C_{1d} \tag{3.5.44}$$

and

$$Q_W^{Th} = Q_W(123,\ 81) = -570\ C_{1u} - 654\ C_{1d} \tag{3.5.45}$$

The Novosibirsk[47] and Berkeley[46,49] experiments determine

$$Q_W^{Bi} = -\ 140 \pm 40 \text{ (Novosibirsk)} \tag{3.5.46}$$

$$Q_W^{Th} = -280 \pm 140 \quad \text{(Berkeley)} \tag{3.5.47}$$

Using (3.5.44) and (3.5.45)

$$C_{1u} + 1.15\ C_{1d} = 0.24 \pm 0.068 \tag{3.5.48}$$

$$C_{1u} + 1.15\ C_{1d} = 0.49 \pm 0.41 \tag{3.5.49}$$

and comparing the SLAC polarized electron results

$$C_{1u} - \frac{1}{2} C_{1d} = -0.45 \pm 0.12 \qquad (3.5.50)$$

$$C_{2u} - \frac{1}{2} C_{2d} = 0.23 \pm 0.38 \qquad (3.5.51)$$

are found experimentally.

The regions in C_{1u}-C_{1d} plane allowed by eqs (3.5.48), (3.5.49) and (3.5.50) are shown in fig. 3.4. This allowed region is determined in two different ways. (i) fit to the eD and atomic physics results or (ii) fit simultaneously to the eD, ν-hadron, and ν-e data.

In fig (ii) the regions allowed by these two procedures together with Weinberg-Salam model is shown.

(d) **Weak interaction prediction for muon pair production**

The basic process of electron positron annihilation into muon pairs is

$$e^+e^- \longrightarrow \mu^+ + \mu^-$$

The JADE collaboration has extracted h_{VV} and h_{AA} by analysing the muon pair data[50]

$$h_{VV} = 0.01 \pm 0.08 \ (0.0016 \text{ for } \sin^2\theta_W = 0.23)$$

$$h_{AA} = 0.18 \pm 0.16 \ (0.25)$$

where the parentheses are the WS model predictions. The allowed regions in an h_{VV} - h_{AA} plane is indicated in fig. 3.5. The errors are still large, but it is amusing that the "origin" h_{VV} - h_{AA} = 0 (no neutral current interactions) is not favoured by the data. If we imagine a phenomenological model in which the low energy predictions for the neutral current interactions coincide exactly with those of the WS-model with $\sin^2\theta_W = 0.23$ and yet Weinberg's Z mass prediction may fail .

3.6 Comparison of predicted and experimental values of the neutral current parameters

Here we compare the experimentally determined coupling parameters with the predicted values. In this respect we first discuss the factorization constraints that must be satisfied by any model in which the neutral current interactions are mediated by a single Z-boson. With the single Z-boson hypothesis the interactions appearing in the neutral current pyramid of fig 3.2 can be completely determined by specifying seven ($\epsilon_L(u)$, $\epsilon_L(d)$, $\epsilon_R(u)$, $\epsilon_R(d)$, g_V^e, g_A^e and ρ) parameters the coupling of Z to $\bar{\nu}_L\nu_L$ and to $\bar{u}u$, $\bar{d}d$, and $\bar{e}e$ (vector and axial vector for each) where we have assumed μe universality.

On the otherhand, there are as many as thirteen phenomenological parameters in the neutral current pyramid. We therefore expect that in a single Z model there must be six independent "factorization" relation among the thirteen parameters.[24,25]

The relations are,

$$C_{1i} = 2g_A^e \, g_V^i \, / \, \rho \qquad (3.6.1)$$

$$C_{2i} = 2g_V^e \, g_A^i \, / \, \rho \qquad (3.6.2)$$

as well as,

$$h_{VV} = g_V^{e^2} / \rho \qquad (3.6.3)$$

$$h_{AA} = g_A^{e^2} / \rho \qquad (3.6.4)$$

$$h_{VA} = g_V^e \, g_A^e / \rho \qquad (3.6.5)$$

from (3.6.1) and (3.6.2)

$$\frac{C_{1d}}{C_{1u}} = \frac{g_V^d}{g_V^u} \quad \text{and} \quad \frac{C_{2d}}{C_{2u}} = \frac{g_A^d}{g_A^u} \qquad (3.6.6)$$

The isospin structure of the hadronic current is the same in νH and eH interactions. One fits g_V^d/g_V^u and $d_A^d \, / \, g_A^u$ directly to the νH data and obtains

$$g_V^d/g_V^u \quad = -2.76 \; {}^{+1.12}_{-2.08} \quad \text{(90\% confidence level)}$$

$$g_A^d/g_A^u \quad = 0.78 \pm 0.26 \qquad \text{"}$$

The allowed regions in C_{1u}-C_{1d} and C_{2u}-C_{2d} plane are shown in fig. 3.4 and 3.6.

The sixth factorization relation allows the measurements of eD scattering and ν-H data to put constraints on g_V^e/g_A^e. The result is obtained by fitting to the eD and νH data directly in terms of g_V^e/g_A^e.

A simultaneous fit to all the data with $\varepsilon_L(u)$, $\varepsilon_L(d)$, $\varepsilon_R(u)$, $\varepsilon_R(d)$, g_V^e, g_A^e and ρ as the free parameter yield the best fit values of the parameters both for ρ as a free parameter and $\rho= 1$ presented in table 3.4.

Table 3.4 Determination of $\varepsilon_L(u)$, $\varepsilon_L(d)$, $\varepsilon_R(u)$, $\varepsilon_R(d)$, g_V^e, g_A^e and ρ for (i) factorization assumed, ρ a free parameter, (ii) factorization assumed and $\rho = 1.0$. The predictions of the WS-GIM model for $\mathrm{Sin}^2\theta_W=0.23$

Parameters	Factorization assumed	Factorization plus $\rho = 1.0$	WS-GIM $\mathrm{Sin}^2\theta_W=0.23$
$\varepsilon_L(u)$	0.339±0.033	0.339±0.030	0.345
$\varepsilon_L(d)$	-0.424±0.026	-0.425±0.025	-0.423
$\varepsilon_R(u)$	-0.179±0.019	-0.179±0.018	-0.155
$\varepsilon_R(d)$	-0.016±0.058	-0.016±0.052	+0.077
g_V^e	0.043±0.063	0.043±0.056	-0.036
g_A^e	-0.545±0.056	-0.545±0.044	-0.50
ρ	1.001 ±0.21		

In table 3.5 the experimental and the predicted values of neutral current parameters are presented. The experimental values are in good agreement with $SU(2)_L \times U(1)_Y$ model parameters with $\mathrm{Sin}^2\theta_W$ set equal to 0.23.

Table 3.5 Comparison of the theoretical prediction of the neutral current parameters of the WS-model with those of experimental results.

Neutral-current parameters	Physical process	WS-GIM model prediction in terms of x_W	WS-GIM prediction with $x_W = 0.23$	Experimental values
1	2	3	4	5
$\varepsilon_L(u)$	Neutrino-hadon scattering	$\frac{1}{2} - \frac{2}{3} x_W$	0.347	0.340±0.033
$\varepsilon_L(d)$		$-\frac{1}{2} + \frac{1}{3} x_W$	-0.423	-0.424±0.026
$\varepsilon_R(u)$		$-2/3\ x_W$	-0.153	-0.179±0.019
$\varepsilon_R(d)$		$1/3\ x_W$	0.077	-0.017±0.058
α		$(1-2x)$	0.54	0.589±0.067
β		1	1	0.937±0.062
γ		$-2/3\ x$	-0.153	-0.273±0.081
δ		0	0	0.101±0.093
g_V^e	Neutrino-electron scattering	$-1/2\ (1-4x)$	-0.04	0.043±0.063
g_A^e		$-1/2$	-0.5	-0.545±0.056
C_{1u}	Electron-hadron scattering	$-1/2 + 4/3\ x_W$	-0.193	-
C_{1d}		$1/2 - 2/3\ x_W$	0.347	-
C_{2u}		$-1/2 + 2x_W$	-0.040	-
C_{2d}		$1/2 - 2x_W$	0.040	-
$\tilde{\alpha}$		$-(1-2x_W)$	-0.54	-0.68 ± 0.19
$\tilde{\beta}$		$-(1-4x_W)$	-0.08	0.06 ± 0.21
$\tilde{\gamma}$		$2/3\ x_W$	0.153	0.24 ± 0.10
$\tilde{\delta}$		0	0	0.00 ± 0.20

Table contd..

Table 3.5 contd..

1	2	3	4	5
h_{VV}	$e^+e^- \longrightarrow \mu^+\mu^-$	$1/4(1-4x)^2$	0.0016	0.02±0.04
h_{AA}		1/4	0.25	0.35±0.11
h_{VA}		1/4(1-4x)	0.02	-0.27 ±0.04
$C_{1u} - .5C_{1d}$	eD asymmetry	$3/4 + 5/3\, x_w$	-0.367	-0.45±0.12
$C_{2u} - .5C_{2d}$		$3\,(x_w - 1/4)$	-0.060	+0.23±0.38
$C_{1u}+1.15C_{1d}$ $C_{1u} + 1.15C_{1d}$	Parity violation in atoms	$\frac{3}{40} + 0.566x_w$	0.205	(i)0.24±0.068(Novosibirsk) (ii)0.49±0.41 (Berkeley) (iii)0.20±0.30 (Seattle)

3.7 Comparison of the value of $\mathrm{Sin}^2\theta_W$ from different experiments: Deep inelastic $\nu_\mu N$ scattering -

Determination of $\mathrm{Sin}^2\theta_w$ in deep inelastic $\nu_\mu N$ scattering generally measure the neutral to charged current cross-section ratio R_ν as

$$R_\nu \equiv \frac{\sigma(\nu_\mu N \longrightarrow \nu_\mu X)}{\sigma(\nu_\mu N \longrightarrow \mu^- X)} \tag{3.7.1}$$

Inclusion of radiative corrections tend to reduce the uncorrected values of $\mathrm{Sin}^2\theta_W^{\nu N}$ determined from R_ν by $\simeq 4\%$

$$\text{i.e.,}\quad \mathrm{Sin}^2\theta_W^{\nu N} - \mathrm{Sin}^2\theta_W \equiv \Delta S^2 \simeq 0.01 \tag{3.7.2}$$

This sizable shift is mainly due to charged current cross-section correction.

In the table we give values of $\mathrm{Sin}^2\theta_W$ obtained (after including radiative corrections) by a number of collaborations. The results are all consistent, although the two Fermilab experiments (CCFRR and FMM) tend to have higher central values. So, the final values of

$$\mathrm{Sin}^2\theta_W \text{ is } = 0.222 \pm 0.007 \tag{3.7.3}$$

or without radiative correction

$$\mathrm{Sin}^2\theta_W = 0.232 \pm 0.007 \tag{3.7.4}$$

Table 3.6 Determination of $\mathrm{Sin}^2\theta_W$ by a number of different deep inelastic $\nu_\mu N$ scattering experiments [26,27,51,52]

Year	Group	$\mathrm{Sin}^2\theta_W$
1979	CDHS	0.218 ± 0.013
1981	CHARM	0.212 ± 0.015
1985	CCFRR	0.242 ± 0.012 ± 0.005
1985	FMM	0.247 ± 0.012 ± 0.013
1985	CDHS	0.219 ± 0.007 ± 0.006
1985	CHARM	0.215 ± 0.010

<u>Neutrino-electron scattering</u>:

The cross-section for all four reactions

$$\overset{(-)}{\nu}_\mu e \longrightarrow \overset{(-)}{\nu}_\mu e$$

$$\overset{(-)}{\nu}_e e \longrightarrow \overset{(-)}{\nu}_e e$$

have been measured and found to agree with WS model prediction. A $\nu_e e$ experiment at LOS Alamos found

$$\sin^2\theta_W = 0.21 \begin{matrix} +0.09+0.05 \\ -0.13-0.07 \end{matrix}$$

which is not precise.

Better precision has been achieved in $\overset{(-)}{\nu}_\mu e$ scattering experiments that measure the ratio

$$R \equiv \frac{\sigma(\nu_\mu e \longrightarrow \nu_\mu e)}{\sigma(\bar{\nu}_\mu e \longrightarrow \bar{\nu}_\mu e)}$$

$$= 1.265 \begin{matrix} +\ 0.72 \\ -\ 0.42 \end{matrix} \quad (\text{CHARM})$$ [53]

$$= 1.38 \begin{matrix} +\ 0.40 \\ -\ 0.31 \end{matrix} + 0.17 \ (\text{BNL})$$ [54]

Taken together they yield

$$\sin^2\theta_W = 0.212 \pm 0.023 \ (\overset{(-)}{\nu}_\mu e \ \text{average})$$

Elastic νp Scattering

A BNL experiment[55] has reported the value of $\sin^2\theta_W$ from measurement of

$$\nu_\mu p \longrightarrow \nu_\mu p$$

and, $$\bar{\nu}_\mu p \longrightarrow \bar{\nu}_\mu p$$

$$\sin^2\theta_W = 0.220 \pm 0.016 \begin{matrix} +0.023 \\ -0.031 \end{matrix}$$

Most precise value of $\sin^2\theta_W$ can be obtained from the following experiments.

ν-N (isoscalar)		ν-nucleon	
1986, CHARM [56]	0.236±0.005	BEBC [59] (H_2) 1986	0.225±0.03
1986, CDHS [57]	0.225±0.005	BBKOPS [60] (ν)	0.220±0.031
1985-86 CCFRR [51]	0.239±0.010		
1985 FMMF [52]	0.244±0.016	ν-electron	
1985 CDHS [58]	0.227±0.012	BBKOPS [61]	0.209±0.032
		CHARM [62]	0.215±0.034
Average		<all νe>	0.223±0.018
0.232±0.003±0.003+0.013			

The best values for $\sin^2\theta_W$ from the two most precise experiments scattering neutrinos off isoscalar targets is 0.231±0.004 (experimental) (0.232±0.003)±0.005 (theor).The groups analysing the worlds' neutral current data for νN(ISOSC) using structure functions find.

0.233±0.003±0.005

and 0.230±0.003±0.004

Thus we have seen that the prediction of the W-S model are in very good accord with low and high energy phenomenology.

A thorough analysis of all experiments contributing to the neutral current sector was recently carried out by U. Amaldi and collaboraters.[63] Measurements of the weak angle in the minimal model (ρ=1) and with ρ free are summarized in the table below.

Table 3.7 Measurement of the weak angle

Reaction	$\sin^2\theta_W(\rho=1)$	$\sin^2\theta_W$	ρ
$\nu_\mu A \longrightarrow \mu A$	0.233±0.003±0.005	0.232±0.014±0.008	
$\nu_\mu p \longrightarrow \nu_\mu p$	0.210±0.033	0.205±0.041	
$\nu_\mu e \longrightarrow \nu_\mu e$	0.233±0.018±0.002	0.221±0.021±0.003	
W/Z	0.228±0.007±0.002	0.228±0.008±0.003	
Parity violation in atoms	0.209±0.018±0.014		
Polarized e on deuterium	0.221±0.015±0.013		
μc DIS	0.25±0.08		
Average	0.230±0.005	0.229±0.006	0.998±0.009

An overall excellent agreement with the minimal standard model is obtained, with the result $\sin^2\theta_W = 0.230 \pm 0.005$.

3.8 Summary

The predicted values of $W^{\pm}$ andZ gauge boson masses in the WS-GIM model are in good agreement with the experimental results. For comparison with the model, an average between the UA1 and UA2 masses is established. The error in the mass difference is smaller than that in their separate masses because of energy-scale uncertainties which cancel in the difference. The neutral current experiments also provide a spectacular confirmation of the low energy form of the WS-GIM model formulated by Glashow, Weinberg and Salam.

Analysis of the νH experiments leads to a unique determination of the hadronic coupling constants $\varepsilon_L(u)$, $\varepsilon_L(d)$, $\varepsilon_R(u)$ and $\varepsilon_R(d)$. Theoretical uncertainties lead to variation in the coupling constants, which are in all cases, smaller than the statistical errors. Analyses of the leptonic reactions $\nu_\mu e \to \nu_\mu e, \bar{\nu}_\mu e \to \bar{\nu}_\mu e$ and $\bar{\nu}_e e \to \bar{\nu}_e e$ lead to two solutions, one of which is dominantly vector, the other is axial vector. The results obtained from the asymmetry in polarized eD scattering are axial vector dominant solutions.

Due to the inconsistencies in the results reported on parity violation in atomic physics, Kim et al combined the data from Novosibirsk (Bi) and Berkeley (Tl) experiments with eD asymmetry to determine C_{1u}, C_{2u}, C_{1d}, C_{2d} in model-independent way. The results are in good agreement with WS-GIM model, and with the region allowed by all other neutral current data assuming factorization. The value of $\sin^2\theta_W$ and ρ are in excellent agreement with the predicted values. The reported average value of $\sin^2\theta_W$ from UA1 and UA2 collaborations is

$$0.232 \pm 0.004 \pm 0.003$$

while that from neutral current experiments is 0.232 ± 0.005. We thus conclude that, within errors, the observed experimental values are completely compatible with the $SU(2)_L X U(1)_Y$ model.

LEFT-RIGHT-GAUGE MODELS

In the standard theory the V-A character of weak interactions is obtained as the low energy limit of spontaneously broken $SU(2)_L XU(1)_Y$ gauge interactions. By allowing only the left-handed chiral fermions to transform nontrivially under $SU(2)_L XU(1)_Y$, the V-A structure is built into the theory in a certain sense. On the other hand, it is natural to conjecture that both V-A and V+A type of charged and neutral currents are present, with equal strengths at a larger scale $M_R >> M_W$, so that as we come down to the W-boson mass and lower energies the V+A structure is damped out. The weak CP violation manifested in K° decay does not have a spontaneous origin in the standard model. On the other hand, the model which embodies both V-A and V+A structures can ascribe the weak CP violation to have a spontaneous origin. The neutrino in the standard model is massless. At present there are reasons to believe, both experimentally and cosmologically[64], that the neutrino might have a small mass. In the left-right symmetric models[65], since both left and right handed helicities of the neutrino are included, the neutrino can naturally have a Dirac mass[66]. On the other hand, if neutrinos are Majorana particles, there could be suitable Higgs representations which can also generate

Majorana neutrino masses[67,68]. The gauge models based upon the electroweak gauge group[65,67] $SU(2)_L X SU(2)_R X U(1)_{B-L}$, or $SU(2)_L X SU(2)_R X SU(4)_C$, proposed by Mahapatra and Pati, and Pati and Salam, do possess these desirable and attractive features. We concentrate upon the left-right model, based upon the gauge group $SU(2)_L X SU(2)_R X U(1)_{B-L}$ in this chapter. In Sec. 4.1, introduction of the model is provided.

In sec 4.2, we mention fermion representation in L-R symmetric model. In Sec 4.3, Higgs sectors without generating neutrino masses are discussed. The gauge boson masses in LR model are derived in sec 4.4. The constraints arising from minimization of potential are discussed in Sec. 4.5. In section 4.6, we review how neutrino masses are generated in L-R models. The see-saw mechanism[69] which explains about the small neutrino masses is also discussed. We discuss in sec. 4.7 about the neutrinoless double-β decay which is the most interesting prediction of the model. A brief summary of the chapter is given in sec 4.9.

4.2 Fermion representation in L-R models

Considering the gauge theory based on the group $SU(2)_L X SU(2)_R X U(1)_{B-L}$ under which the fermions transform in the left-right symmetric manner. We denote the $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$ coupling constants by g_L, g_R and g'

respectively. The immediate consequence of the left-right symmetry which we will impose on the Lagrangian is the left and right gauge couplings of fermions with gauge bosons are the same i.e. $g_L=g_R=g$.

The quarks and lepton of three generations are placed in $SU(2)_L$ and $SU(2)_R$ doublets as given below.

$$Q_{1L}=\begin{pmatrix}u\\d\end{pmatrix}_L^{r,y,b}, \quad Q_{2L}=\begin{pmatrix}c\\s\end{pmatrix}_L^{r,y,b}, \quad Q_{3L}=\begin{pmatrix}t\\b\end{pmatrix}_L^{r,y,b}$$

$$Q_{1R}=\begin{pmatrix}u\\d\end{pmatrix}_R^{r,y,b}, \quad Q_{2R}=\begin{pmatrix}c\\s\end{pmatrix}_R^{r,y,b} \quad Q_{3R}=\begin{pmatrix}t\\b\end{pmatrix}_R^{r,y,b} \tag{4.2.1}$$

$$\psi_{1L}=\begin{pmatrix}\nu_e\\e\end{pmatrix}_L \quad \psi_{2L}=\begin{pmatrix}\nu_\mu\\\mu\end{pmatrix}_L, \quad \psi_{3L}=\begin{pmatrix}\nu_\tau\\\tau\end{pmatrix}_L$$

$$\psi_{1R}=\begin{pmatrix}\nu_e\\e\end{pmatrix}_R, \quad \psi_{2R}=\begin{pmatrix}\nu_\mu\\\mu\end{pmatrix}_R, \quad \psi_{3R}=\begin{pmatrix}\nu_\tau\\\tau\end{pmatrix}_R \tag{4.2.2}$$

The representation content of fermionic multiplet is

$$\begin{aligned} Q_{iL}&=(\tfrac{1}{2},\ 0,\ \tfrac{1}{3}), & Q_{iR}&=(0,\ \tfrac{1}{2},\ \tfrac{1}{3}) \\ \psi_{iL}&=(\tfrac{1}{2},\ 0,\ -1), & \psi_{iR}&=(0,\ \tfrac{1}{2},\ -1) \end{aligned} \tag{4.2.3}$$

where i = 1,2,3 and the symbol $\frac{1}{2}(0)$ implies a doublet (singlet) under the gauge group.

The electric charge operator is defined as

$$Q_{el} = T_{3L} + T_{3R} + \frac{B - L}{2} \tag{4.2.4}$$

Where T_L, T_R and B-L are the generators of the $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ respectively.

In the standard model, fermions appeared as left-handed doublets and right handed singlets. But in L-R symmetric models, they appear in both as doublets.

In the standard model, ν_{eR}, $\nu_{\mu R}$, $\nu_{\tau R}$ are absent which causes any neutrino to be massless. But in the LR models, the presence of these right-handed neutrinos allows Dirac mass of the neutrino to exist like quark and charged lepton masses.

4.3 Description of Higgs sector in left-right models

The original idea of Pati and Mahapatra is based upon left right symmetric (LRS) gauge theory. In this case both $SU(2)_L$ and $SU(2)_R$ possess the same coupling constant ($g_L=g_R=g$) with fermions at a mass scale when $SU(2)_L XSU(2)_R XU(1)_{B-L}$ is a good symmetry. There could be left-right gauge groups with unequal gauge coupling constants ($g_L \neq g_R$) which are known as left-right asymmetric (LRA) models. In this case the gauge group is also based upon $SU(2)_L X SU(2)_R X U(1)_{B-L}$. In the left-right symmetric or asymmetric models based upon the gauge group $SU(2)_L X SU(2)_R X U(1)_{B-L}$, the symmetry is broken spontaneously to the Weinberg-Salam model, $SU(2)_L X U(1)_Y$ by a Higgs field with and without generating neutrino masses. The sets of Higgs fields in the two cases are different as we describe them below.

Higgs sector without generating Majorana neutrino masses

In this case there is a left(right) handed doublets, $\chi_L(\chi_R)$ carrying B-L=1 which are needed to break $SU(2)_R X U(1)_{B-L} \longrightarrow U(1)_Y$. Simultaneously maintaining L-R symmetry. Besides, the Weinberg-Salam doublet is contained in the scalar field ϕ which transform as a doublet both under $SU(2)_L$ and $SU(2)_R$.

$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^\circ \end{pmatrix} = (\tfrac{1}{2},\ 0,\ 1), \qquad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^\circ \end{pmatrix} = (0,\ \tfrac{1}{2},\ 1)$$

$$\phi = \begin{pmatrix} \phi_1^\circ & \phi_1^+ \\ \phi_2^- & \phi_2^\circ \end{pmatrix} = (\tfrac{1}{2},\ \tfrac{1}{2},\ 0)$$

$$\tilde{\phi} = \tau_2\ \phi^*\ \tau_2 = \begin{pmatrix} \phi_2^{*\circ} & -\phi_2^+ \\ -\phi_1^- & \phi_1^{*\circ} \end{pmatrix} \tag{4·3·1}$$

The symmetry breaking is realized by giving vacuum expectation value (VEV)

$$\langle\chi_L\rangle = \begin{pmatrix} 0 \\ \upsilon_L \end{pmatrix} \qquad \langle\chi_R\rangle = \begin{pmatrix} 0 \\ \upsilon_R \end{pmatrix} \tag{4.3.2}$$

and

$$\langle\phi_1\rangle = \langle\phi\rangle = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} \quad \langle\phi_2\rangle = \langle\tilde{\phi}\rangle = \begin{pmatrix} k' & 0 \\ 0 & k \end{pmatrix}$$

where k' could be zero.

4.4 Gauge boson masses in the left-right models

In this section we discuss derivation of gauge boson masses in the left-right asymmetric (LRA) model, where the $SU(2)_L$ and $SU(2)_R$ have unequal coupling constants ($g_L \neq g_R$). We show how the formulas reduce to the LRS case when we use $g_L = g_R = g$. We will discuss the cases of Higgs sectors without and with Majorana neutrino masses in each case.

(a) LRA model without Majorana neutrino masses

In this case the Higgs sectors are represented by

$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^\circ \end{pmatrix} \quad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^\circ \end{pmatrix} \qquad \phi = \begin{pmatrix} \phi_1^\circ & \phi_1^+ \\ \phi_2^- & \phi_2^\circ \end{pmatrix}$$

and

$$\langle\chi_L\rangle = \begin{pmatrix} 0 \\ \upsilon_L \end{pmatrix} \quad \langle\chi_R\rangle = \begin{pmatrix} 0 \\ \upsilon_R \end{pmatrix} \quad \langle\phi\rangle = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} \tag{4.4.1}$$

with $\upsilon_R > k >> k', \upsilon_L$

The gauge boson masses are obtained from the scalar contribution to the Lagrangian

$$\alpha_{scalar} = (D_\mu\chi_L)^+(D^\mu\chi_L) + (D_\mu\chi_R)^+(D^\mu\chi_R)$$

$$+ \mathrm{Tr}(D_\mu\phi^+)(D^\mu\phi) - V(\chi_L, \chi_R, \phi) \qquad (4.4.2)$$

Where,

$$D^\mu\chi_L = \partial^\mu\chi_R - \frac{1}{2} ig_L \vec{\tau}.\vec{W}^\mu_L\chi_L - \frac{ig'}{2} B^\mu\chi_L$$

$$D^\mu\chi_R = \partial^\mu\chi_R - \frac{1}{2} ig_R \vec{\tau}.\vec{W}^\mu_R\chi_R - \frac{ig'}{2} B^\mu\chi_R$$

$$D^\mu\phi = \partial^\mu\phi - \frac{1}{2} ig_L \vec{\tau}.\vec{W}^\mu_L \phi + \frac{1}{2} ig_R \phi \vec{\tau}.\vec{W}^\mu_R \qquad (4.4.3)$$

Using the vacuum expectation values from (4.4.1) in (4.4.2) then gives charged gauge boson mass matrix

$$M^2 = \begin{array}{c c} & \begin{array}{cc} W_L & W_R \end{array} \\ \begin{array}{c} W_L \\ W_R \end{array} & \begin{pmatrix} \frac{1}{2} g_L^2(k^2+k'^2+\upsilon_L^2) & -g_Lg_Rkk' \\ -g_Lg_Rkk' & \frac{1}{2} g_R^2(k^2+k'^2+\upsilon_R^2) \end{pmatrix} \end{array} \qquad (4.4.4)$$

Defining the mass eigen states as

$$W_1^+ = W_L^+ \cos\xi + W_R^+ \sin\xi$$
$$W_2^+ = -W_L^+ \sin\xi + W_R^+ \cos\xi \qquad (4.4.5)$$

The eigen values of (4.4.4) are given by,

$$M_W^{\pm 2} = \frac{1}{4}[g_L^2(v_L^2+k^2+k'^2) + g_R^2(v_R^2+k^2+k'^2)$$
$$\pm\{[g_L^2(k^2+k'^2+v_L^2)-g_R^2(v_R^2+k^2+k'^2)]^2+g_L^2g_R^2k^2k'^2\}^{\frac{1}{2}}] \qquad (4.4.6)$$

Where, the left-right mixing angles ξ, is given by the relation

$$\tan 2\xi = \frac{4g_L\, g_R k\, k'}{g_R^2(v_R^2+k^2+k'^2) - g_L^2\,(v_L^2+k^2+k'^2)} \qquad (4.4.7)$$

It is noted that second term inside the radical in (4.4.6) is proportional to $\tan^2 2\xi$ and vanishes in the limit $\xi \longrightarrow 0$. In the limit of vanishing L-R mixing, the left-handed ($W_L^{\pm}$) and right-handed ($W_R^{\pm}$) charged gauge boson masses computed using minus and plus signs, respectively in (4.4.6)

$$M^2_{W_R^{\pm}} \simeq \frac{1}{2} g_R^2 \, (v_R^2 + k^2 + k'^2) \tag{4.4.8}$$

$$M^2_{W_L^{\pm}} \simeq \frac{1}{2} g_L^2 \, (v_L^2 + k^2 + k'^2) \tag{4.4.9}$$

This can be directly verified from (4.4.4) neglecting off-diagonal elements. If we use the conditions $v_R^2 >> k^2 >> k'^2, v_L^2$ eqs (4.4.8) and (4.4.9) yield

$$\begin{aligned} M^2_{W_R} &\simeq \frac{1}{2} g_R^2 \, v_R^2 \\ M^2_{W_L} &\simeq \frac{1}{2} g_L^2 \, \phi^2 \end{aligned} \tag{4.4.10}$$

satisfying $M_{W_R} >> M_{W_L}$. In such a limit the L-R mixing angle is

$$\tan 2\xi \simeq \frac{4 g_L k k'}{g_R v_R^2} \tag{4.4.11}$$

$W_L^{\pm}$ are charged W-bosons of the $SU(2)_L X U(1)_Y$ model, after the symmetry is spontaneously broken down to $U(1)_{em}$. These have predominantly V-A structure of weak currents. $W_R^{\pm}$ are the charged and heavy right-handed bosons of the $SU(2)_R X U(1)_{B-L}$, after the symmetry is broken down to $U(1)_Y$. They have predominantly V+A couplings with quarks and leptons.

Neutral gauge boson masses - The neutral gauge boson mass matrix for the spontaneous breaking,

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{\langle\chi_R\rangle\neq 0} SU(2)_L \times U(1)_Y \xrightarrow[\langle\chi_L\rangle\neq 0]{\langle\phi\rangle\neq 0} U(1)_{em}$$

can be similarly written as

$$M_Z^2 = \begin{array}{c} \\ W_L^3 \\ W_R^3 \\ B \end{array} \begin{array}{c} \begin{array}{ccc} W_L^3 & W_R^3 & B \end{array} \\ \begin{bmatrix} \frac{1}{2}g_L^2(v_L^2+k^2+k'^2) & -\frac{1}{2}g_L g_R(k^2+k'^2) & -g' g_L v_L^2 \\ -\frac{1}{2}g_L g_R(k^2+k'^2) & \frac{1}{2}g_R^2(v_R^2+k^2+k'^2) & -g' g_R v_R^2 \\ -g' g_L v_L^2 & -g' g_R v_R^2 & g'^2(v_L^2+v_R^2) \end{bmatrix} \end{array} \tag{4.4.12}$$

Where W_L^3, W_R^3 and B are the neutral components of gauge bosons in $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$, respectively.

Using the definitions

$$x_\omega = \sin^2\theta_W = \frac{e^2(M_W)}{g_L^2(M_W)}, \quad \frac{1}{e^2 M_R)} = \frac{1}{g_L^2(M_R)} + \frac{1}{g_R^2(M_R)} + \frac{1}{g'^2(M_R)}$$

$$R^2 = g_R^2/g_L^2$$

$$\eta_R = \frac{k^2+k'^2}{v_R^2}, \quad \eta_L = \frac{v_L^2}{k^2+k'^2}, \quad z = \frac{2k'^2}{k^2+k'^2} \tag{4.4.13}$$

The mass matrix can be written as

$$M_Z^2 = \frac{1}{2}g_L^2 v_R^2 \begin{bmatrix} \eta_R(1+\eta_L) & -R\eta_L & -2\epsilon\eta_R\eta_L \\ -R\eta_R & R^2(2+\eta_R) & -2R\epsilon \\ 2\epsilon\eta_R\eta_L & -2R\epsilon & 2\epsilon^2(1+\eta_R\eta_L) \end{bmatrix} \tag{4.4.14}$$

This yields for the left and right handed neutral gauge bosons (Z_1, Z_2)

$$M^2_{Z_{1,2}} = \frac{1}{2} g_L^2 v_R^2 \frac{(R^2-x_w)(1+\eta_R\eta_L)}{R^2-(R^2+1)x_w} \Big[1+ \frac{(R^2-(R^2+1)x_w)(R^2-1+\eta_R\frac{(R^2+1)}{2}}{(R^2-x_w)(1+\eta_R\eta_L)}$$

$$\pm \Big\{ 1+ \frac{R^2-(R^2+1)x_w}{(R^2-x_w)^2(1+\eta_R\eta_L)^2} [(R^2-(R^2+1)x_w)(R^2-1+\eta_R \frac{(R^2+1)}{2})^2$$

$$-2x_w(1+\eta_R\eta_L)(R^2-1+\eta_R \frac{(R^2+1)}{2}$$

$$-4R^4\eta_R\eta_L+2R^2(1+\eta_R\eta_L)(R^2-1+\eta_R \frac{(1-R^2)}{2})]\}^{\frac{1}{2}}] \qquad (4.4.15)$$

In (4.4.15), the lighter (heavier) of the two masses corresponds to $Z_1(Z_2)$-neutral boson of the $SU(2)_L X U(1)_Y$ $(SU(2)_R X U(1)_{B-L})$ theory.

(b) **Gauge boson masses in the LRS model**

In this case the $SU(2)_L$ and $SU(2)_R$ gauge groups have the same coupling constants at $\mu = M_{W_R^\pm} = M_R$, i.e. $g_L(M_R)=g_R(M_R)=g$.

Then in (4.5.15), $R^2=1$, $$\frac{1}{e^2(M_R)} = \frac{2}{g^2} + \frac{1}{g'^2} \qquad (4.4.16)$$

The left-right $W_L^\pm - W_R^\pm$ mixing angle for $v_R^2 >> k^2 >> k'^2$, v_L^2 is given by

$$\tan 2\xi = \frac{4kk'}{v_R^2} \qquad . \qquad (4.4.17)$$

In the vanishing mixing limit $\xi \to o$ $(k' \to o,\ Z \to o)$, the charged and neutral gauge boson masses are given by,

$$M^2_{W_L} \simeq \frac{1}{2} g^2 \ (k^2+k'^2+v_L^2)$$
$$M^2_{W_R} \simeq \frac{1}{2} g^2 \ (k^2+k'^2+v_R^2) \qquad (4.4.18)$$

$$M^2_{Z_1} \simeq \frac{1}{2} g^2 \left(\frac{g^2 + 2g'^2}{g^2+g'^2}\right) \ (k^2+k'^2+v_L^2)$$
$$M^2_{Z_2} \simeq \frac{1}{2} g^2 \ \left(\frac{g^2+g'^2}{g'^2}\right) \ (v_R^2+k^2+k'^2) \qquad (4.4.19)$$

(c) **Gauge boson masses in the LRA and LRS models with Higgs sector generating Majorana neutrino masses**

The Higgs doublets χ_L and χ_R used in sec (4.4a) and (4.4b) do not possess B-L= 2. The LRS doublet $\phi(\frac{1}{2}, \frac{1}{2}, 0)$ has vanishing B-L charge. As we have seen in previous sections, if neutrino is a Majorana particle a coupling of the type $\Delta_L \nu^c \nu$ or $\Delta_R N^c N$ is possible if the Higgs particles Δ_L and Δ_R carry B-L=2. As described earlier, the choice of the Higgs sector for generating Majorana neutrino masses consists of the following:

$$\phi \equiv (\tfrac{1}{2}, \tfrac{1}{2}, 0), \quad \tilde{\phi} = \tau_2 \phi^* \tau_2 \equiv (\tfrac{1}{2}, \tfrac{1}{2}, 0)$$
$$\Delta_L \equiv (1,0,2), \quad \Delta_R \equiv (0,1,2) \qquad (4.4.20)$$

Where Δ_L (Δ_R) is the left(right) handed triplet carrying B-L = 2. The neutral components of Δ_L and Δ_R are assigned VEV's of the most general form

$$\langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ V_L & 0 \end{pmatrix} \qquad \Delta_R = \begin{pmatrix} 0 & 0 \\ V_R & 0 \end{pmatrix}$$

$$\langle\phi\rangle = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} \qquad \langle\tilde{\phi}\rangle = \begin{pmatrix} k' & 0 \\ o & k \end{pmatrix} \qquad (4.4.21)$$

with these VEV's the charged $W_L^{\pm}$ and $W_R^{\pm}$ gauge boson-masses and also the neutral gauge boson masses assume the same form as expressed in eqs (4.4.4) - (4.4.19).

Before closing this section we write down the structure of the neutral gauge bosons including the massless photon (A_μ) in terms of $W_{L\mu}^3$, $W_{R\mu}^3$ and B_μ and the electroweak mixing angle θ_w.

$$A_\mu = \mathrm{Sin}\theta_w(W_{L\mu}^3+W_{R\mu}^3) + (\mathrm{Cos}\ 2\theta_W)^{\frac{1}{2}}\ B_\mu$$

$$Z_{1\mu} \equiv Z_{L\mu} = \cos\theta_W\ W_{L\mu}^3 - \mathrm{Sin}\theta_w \tan\theta_w\ W_{R\mu}^3 - \tan\theta_w(\cos 2\theta_w)^{\frac{1}{2}}\ B_\mu \quad (4.4.22)$$

$$Z_{2\mu} \equiv Z_{R\mu} = \frac{(\cos\ 2\theta_w)^{\frac{1}{2}}}{\cos\theta_w}\ W_{R\mu}^3 - \tan\theta_w B_\mu$$

where

$$\tan\theta_w = \frac{g'}{(g^2+g'^2)^{\frac{1}{2}}} \qquad (4.4.23)$$

In the LRS model,

$$M_{ZL} = \frac{M_{WL}}{\cos\theta_w} \qquad (4.4.24)$$

as in the standard model, but M_{ZR} is related to the corresponding charged gauge boson mass by a different relation

$$M_{ZR} = \frac{M_{W_R} \cos\theta_w}{\sqrt{\cos 2\theta_w}} \qquad (4.4.25)$$

These relations can be verified from eqs (4.4.18) and (4.4.19).

4.5 Minimisation of Higgs potential in LRS model

Higgs potential can be written as

$$\begin{aligned} V = & -\sum_{i,j=1}^{2} \mu_{ij} \,\mathrm{tr}\, \phi_i^+\phi_j + \sum_{i,j,k,l} \mathrm{tr}\,(\phi_i^+\phi_j)\mathrm{tr}(\phi_k^+\phi_l) \\ & + \sum_{ij,k,l=1}^{2} \lambda'_{ijk,l} \,\mathrm{tr}\, \phi_i^+\phi_j\, \phi_k^+\phi_j - \mu^2 (\chi_L^+\chi_L + \chi_R^+\chi_R) \\ & + \rho_1[(\chi_L^+\chi_L)^2 + (\chi_R^+\chi_R)^2] + \rho_2(\chi_L^+\chi_L\chi_L^+\chi_L + \chi_R^+\chi_R\chi_R^+\chi_R) \\ & + \rho_3\chi_L^+\chi_L\chi_R^+\chi_R + \sum_{i,j=1}^{2} \alpha_{ij} \,\mathrm{tr}\, \phi_i^+\phi_j(\chi_L^+\chi_L + \chi_R^+\chi_R) \\ & + \sum_{i,j=1}^{2} \beta_{ij}(\mathrm{tr}\, \chi_L^+\chi_L\phi_i\phi_j^+ + \mathrm{tr}\, \chi_R^+\chi_R\phi_i^+\phi_j) \\ & + \sum_{i,j=1}^{2} \gamma_{ij} \,\mathrm{tr}\, \chi_L^+\phi_i\chi_R\phi_j^+ \end{aligned} \qquad (4.5.1)$$

Left-right symmetry under which the fields ϕ_i and ϕ_2' transform as $\phi_i \longleftrightarrow \phi_i^+$ $(i = 1,2)$ dictates $\mu_{ij}=\mu_{ji}$, $\lambda_{1212}=\lambda_{2121}$,

$$\lambda_{iijk}=\lambda_{iikj},\ \lambda_{ijkk}=\lambda_{jikk},\ \lambda'_{lijk}=\lambda_{klij}=\lambda'_{jkli}=\lambda'_{ijkl} \qquad (4.5.2)$$

and the potential becomes

$$
\begin{aligned}
& V(\chi_L, \chi_R, k, k') \\
&= [-(\mu_{11}^2+\mu_{22}^2) + (\lambda_{1111}+\lambda_{1122}+\lambda_{2211}+\lambda_{2222}) \\
&\times (k^2+k'^2)](k^2+k'^2)+4(\lambda_{1221}+\lambda_{2112}+2\lambda'_{1221})k^2k'^2 \\
&+ (\lambda'_{1111}+\lambda'_{2222})(k^4+k'^4) \\
&+ [-\mu_{12}^2+4(\lambda_{1112}+\lambda_{1211}+\lambda_{2221}+\lambda_{2122} \\
&+ (\lambda'_{1112}+\lambda'_{2221}) (k^2+k'^2)]kk' \\
&+ 4(2\lambda_{1212}+\lambda'_{1212})k^2k'^2 \\
&- \mu^2(\upsilon_L^2+\upsilon_R^2) + \rho_1(\upsilon_L^4+\upsilon_R^4)+\rho_2(\dot{v}_L^4 + v_R^4) \\
&+ \rho_3(\upsilon_L^2\ \upsilon_R^2) + (\upsilon_L^2+\upsilon_R^2)[(\alpha_{11}+ \alpha_{22}+\beta_{11})k^2 \\
&+ (\alpha_{11}+ \alpha_{22}+ \beta_{22})k'^2+(4\alpha_{12}+2\beta_{12})kk'] \\
&+ 2\upsilon_L\upsilon_R[(\gamma_{11}+\gamma_{22})kk'+\gamma_{12}(k^2+k'^2)]
\end{aligned}
\qquad (4.5.3)
$$

From the extremizing conditions

$$\frac{\partial V}{\partial \upsilon_L} = 0 = \frac{\partial V}{\partial \upsilon_R} \qquad (4.5.4)$$

We obtain

$$
\begin{aligned}
& -2\mu^2\upsilon_L+4\rho_1\upsilon_L^3 +4\rho_2\upsilon_L^3 + 2\rho_3\upsilon_L\upsilon_R^2 \\
& +2\upsilon_L(\alpha_{11}+\alpha_{22}+\beta_{11})k^2+2\upsilon_L(\alpha_{11}+\alpha_{22}+\beta_{22})k'^2 \\
& 2\upsilon_R(\gamma_{11}+\gamma_{22})kk'+2\upsilon_R\gamma_{12}(k^2+k'^2)=0
\end{aligned}
\qquad (4.5.5)
$$

or

$$
\begin{aligned}
& \mu^2\upsilon_L=2\upsilon_L^3(\rho_1+\rho_2)+\rho_3\upsilon_L\upsilon_R^2+\upsilon_L(\alpha_{11}+\alpha_{22}+\beta_{11})k^2 \\
& +\upsilon_L(\alpha_{11}+\alpha_{22}+\beta_{22})k'^2+\upsilon_R(\gamma_{11}+\gamma_{22})kk'+\upsilon_R\gamma_{12}(k^2+k'^2)
\end{aligned}
\qquad (4.5.6)
$$

and $\mu^2\upsilon_R = 2\upsilon_R^3(\rho_1+\rho_2)+\rho_3\upsilon_L\upsilon_R$

$+ \upsilon_R(\alpha_{11}+\alpha_{22}+\beta_{11})k^2+\upsilon_R(\alpha_{11}+\alpha_{22}+\beta_{22})k'^2$

$$+ \upsilon_L(\gamma_{11}+\gamma_{22})\ kk'+\upsilon_L\gamma_{12}(k^2+k'^2) \qquad (4.5.7)$$

Multiplying equation (4.5.6) by υ_R and equation (4.5.7) by υ_L and subtracting one obtains

$[\upsilon_L\upsilon_R\{\ 2(\rho_1+\rho_2)-\rho_3\}-\gamma_{12}(k^2+k'^2)$

$$- (\gamma_{11}+\gamma_{22})kk']\ (\upsilon_L^2-\upsilon_R^2) = 0 \qquad (4.5.8)$$

The possible solutions are

(i) $\upsilon_L^2 = \upsilon_R^2$, in this case, LR symmetry is not broken,

$$\text{(ii)}\quad \upsilon_L\upsilon_R = \frac{\gamma_{12}(k^2+k'^2) + (\gamma_{11}+\gamma_{22})kk'}{2(\rho_1+\rho_2)-\rho_3} \qquad (4.5.9)$$

for $\upsilon_L^2 \neq \upsilon_R^2$

In the approximation $k' \ll k$ and $\upsilon_L = 0$ from eq (4.5.3)

$V(\chi_R, k) = [-(\mu_{11}^2+\mu_{22}^2)+(\lambda_{1111}+\lambda_{1122}+\lambda_{2211}+\lambda_{2222})k^2]k^2$

$+ (\lambda'_{1111}+\lambda'_{2222})k^4$

$$-\mu^2\upsilon_R^2+(\rho_1+\rho_2)\upsilon_4^4+\upsilon_R^2\ (\alpha_{11}+\alpha_{22}+\beta_{11})k^2 \qquad (4.5.10)$$

Again from the extremizing condition $\frac{\partial V}{\partial \upsilon_R} = 0$ we obtain

$$-2\mu^2\upsilon_R+4(\rho_1+\rho_2)\upsilon_R^3+2\upsilon_R(\alpha_{11}+\alpha_{22}+\beta_{11})k^2 = 0$$

or $-\mu^2+2(\rho_1+\rho_2)\upsilon_R^2+2(\alpha_{11}+\alpha_{22}+\beta_{11})k^2=0$

$$\text{or}\quad \upsilon_R^2 = \frac{\mu_2^2-2(\alpha_{11}+\alpha_{22}+\beta_{11})k^2}{2(\rho_1+\rho_2)} \qquad (4.5.11)$$

which is the minimum solution and the parity is spontaneously broken. From eq.(4.5.9)

$$\upsilon_L \upsilon_R = \frac{\beta}{\rho-\rho'}, \quad k^2$$

in the limit $k >> k'$ and $\beta = \gamma_{12}$

$$\rho-\rho' = 2(\rho_1+\rho_2)-\rho_3$$

$$\upsilon_L = \frac{\gamma k^2}{\upsilon_R} \quad \text{with} \quad \gamma = \frac{\beta}{\rho-\rho'}$$

The fermion mass in the present case is generated by the Lagrangian

$$\alpha_{mass} = h_1 \bar{\Psi}_L \phi \psi_R + h_2 \bar{\Psi}_L \tilde{\phi} \psi_R$$

$$+ h_3 \bar{Q}_L \phi Q_R + h_4 \bar{Q} \tilde{\phi} Q_R \qquad (4.5.12)$$

There is no coupling of the fermions to the χ' fields, and the neutrino gets the Dirac mass in a manner similar to quarks and leptons due to the VEV's of ϕ and $\tilde{\phi}$

$$m_{\nu e} = h_1 k + h_2 k'$$

$$m_e = h_1 k' + h_2 k \qquad (4.5.13)$$

$$m_u = h_3 k + h_4 k'$$

$$m_d = h_3 k' + h_4 k$$

But such large neutrino masses are not favoured by experiments.

4.6 Generation of neutrino masses by Higgs triplets:

In this case left-handed and right handed triplets of Higgs scalars Δ_L and Δ_R are used to break left-right symmetry. Assuming neutrino to be a majorana particle[67,68], due to the coupling of Δ' fields to the farmions, neutrino acquires a majorana mass.

The symmetry breaking can be realized by two L-R conjugate triplets $\Delta_L(1,0,2)$, $\Delta_R(0.1,2)$ and

$$\phi\ (1/2,\ 1/2^*,\ 0),\quad \tilde{\phi} \equiv \tau_2\phi^*\tau_2(1/2,\ 1/2^*,\ 0) \qquad (4.6.1)$$

With the numbers in the brackets denoting $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ quantum numbers respectively. Writing in the matrix form,

$$\Delta_{L,R} = \begin{bmatrix} \frac{1}{\sqrt{2}}\delta^+ & \delta^{++} \\ \delta^o & -\frac{1}{\sqrt{2}}\delta^+ \end{bmatrix} \qquad (4.6.2)$$

$$\phi = \begin{bmatrix} \phi_1^o & \phi_1^+ \\ \phi_2^- & \phi_2^o \end{bmatrix} \qquad \tilde{\phi} = \begin{bmatrix} \phi_2^{o*} & -\phi_2^{o*} \\ \phi_1^- & \phi_1^{o*} \end{bmatrix}$$

in the 2X2 representation.

The most general form of the vacuum expectation value (VEV) consistent with $U(1)_{em}$ is

$$<\phi> = <\phi_1> = \begin{bmatrix} k & 0 \\ 0 & k' \end{bmatrix},\ <\phi_2> = <\tilde{\phi}> = \begin{bmatrix} k' & 0 \\ 0 & k \end{bmatrix}$$

$$\text{and}\quad <\Delta_{L,R}> = \begin{pmatrix} 0 & 0 \\ V & 0 \end{pmatrix}_{L,R} \qquad (4.6.3)$$

and the Higgs potential satisfying gauge, left-right and discrete symmetries ($\Delta_L \to \Delta_L$, $\Delta_R \longrightarrow \Delta_R$, $\phi \longrightarrow e^{i\pi/2}\phi$) can be written as[70]

$$V= -\sum_{i,j=1}^{2} \mu_{ij}^{2} \text{ tr } \phi_i^+\phi_j + \sum_{i,j,k,l=1}^{2} \lambda_{ijkl} \text{ tr}(\phi_i^+\phi_j) \text{ tr}(\phi_k^+\phi_l)$$

$$+ \sum_{i,j,k,l=1}^{2} \lambda_i' \text{ tr } \phi_i^+\phi_j\phi_k^+\phi_l - \mu^2\text{tr } (\Delta_L^+\Delta_L+\Delta_R^+\Delta_R)$$

$$+ \rho_1[(\text{tr } \Delta_L^+\Delta_L)^2 + (\text{tr } \Delta_R^+\Delta_R)^2]$$

$$+ \rho_2(\text{tr } \Delta_L^+\Delta_L\Delta_L^+\Delta_L + \text{tr}\Delta_R^+\Delta_R\Delta_R^+ \Delta_R) + \rho_3\text{tr } \Delta_L^+\Delta_L\Delta_R^+\Delta_R$$

$$+ \sum_{i,j=1}^{2} \alpha_{ij} \text{ tr } \phi_i^+\phi_j(\text{tr } \Delta_L^+\Delta_L + \text{ tr } \Delta_R^+\Delta_R)$$

$$+ \sum_{i,j=1}^{2} \beta_{ij}(\text{tr } \Delta_L^+\Delta_L\phi_i\phi_j^+ + \text{ tr } \Delta_R^+\Delta_R\phi_i^+\phi_j)$$

$$+ \sum_{i,j=1}^{2} \gamma_{ij} \text{ tr } \Delta_L^+\phi_i\Delta_R\phi_j^+ \tag{4.6.4}$$

which gives

$$V(\Delta_L, \Delta_R, k_1, k_2) = -\mu^2(V_L^2+V_R^2) + \frac{\rho}{4} (V_L^4+V_R^4)$$

$$+ \frac{\rho'}{2} V_L^2V_R^2 + (V_L^2+V_R^2) [(\alpha_{11}+\alpha_{22}+\beta_{11})k^2$$

$$+ (\alpha_{11}+ \alpha_{22}+ \beta_{22}) k'^2 + (4\alpha_{12}+ 2\beta_{12})kk']$$

$$+ 2V_LV_R [(\gamma_{11}+\gamma_{22}) kk' + \gamma_{12}(k^2+k'^2)] \tag{4.6.5}$$

+ terms which depend on k, k' only.

where $\rho = 4(\rho_1+\rho_2)$, $\rho' =2\rho_3$ (4.6.6) as previously, in the approximation

$$k' << k,$$

$$V(\Delta_L, \Delta_R, k) = -\mu^2 (V_L^2 + V_R^2) + \frac{\rho}{4}(V_L^4 + V_R^4)$$

$$+ \frac{\rho'}{2} V_L^2 V_R^2 + \frac{\alpha}{2}(V_L^2 + V_R^2) k^2 + \beta V_L V_R k^2 \quad (4.6.7)$$

with $\alpha = 2(\alpha_{11} + \alpha_{22} + \beta_{11})$ (4.6.8)

$\beta = 2\gamma_{12}$

Again from the extremizing condition

$$\frac{\partial V}{\partial V_L} = \frac{\partial V}{\partial V_R} = 0, \text{ one obtains },$$

$$\mu^2 V_L = \rho V_L^3 + \rho' V_L V_R^2 + \alpha k^2 V_L + \beta k^2 V_R \quad (4.6.9)$$

$$\mu^2 V_R = \rho V_R^3 + \rho' V_R V_L^2 + \alpha k^2 V_R + \beta k^2 V_L \quad (4.6.10)$$

and

$$[(\rho - \rho') V_L V_R - \beta k^2] (V_L^2 - V_R^2) = 0 \quad (4.6.11)$$

The possible solutions are

(a) $V_L^2 = V_R^2$

for which L - R symmetry is not broken.

(b) $V_L \neq V_R$ in which case,

$$V_L V_R = \frac{\beta'}{\rho - \rho'} k^2 \quad (4.6.12)$$

writing $\gamma = \frac{\beta}{\rho - \rho'}$,

$$V_L = \frac{\gamma k^2}{V_R} \quad (4.6.13)$$

Unlike the χ' fields, if neutrino is a majorana particle, there is a coupling with the Δ' fields which generates majorana mass terms for the neutrino. The fermion mass is generated by the Lagrangian,

$$\alpha = h_1 \bar{\Psi}_L \phi \psi_R + h_2 \psi_L \tilde{\phi} \psi_R + h_3 \bar{Q}_L \phi Q_R$$

$$+ h_4 \bar{Q}_L \tilde{\phi} Q_R + i h_5 (\psi_L^T C \tau_2 \Delta_L \psi_L + \psi_R^T C \tau_2 \Delta_R \psi_R) + \text{H.C.} \qquad (4.6.14)$$

where $\tilde{\phi} \equiv \tau_2 \phi^* \tau_2$ and C is the Dirac change-conjugation matrix. Considering only one generation of fermions, the masses for charged fermions are

$$\begin{aligned} m_e &= h_1 k' + h_2 k \\ m_u &= h_3 k + h_4 k' \\ m_d &= h_3 k' + h_4 k \end{aligned} \qquad (4.6.15)$$

where m_e, m_u, m_d are the masses of electron, up and down quark respectively. The Lagrangian for ν_L, ν_R sector becomes[70],

$$\alpha^{\nu}_{mass} = h_5 [V_L (\nu_L^T C \nu_L + \nu_L^{\dagger} C^{\dagger} \nu_L^*) + V_R (\nu_R^T C \nu_R + \nu_R^{\dagger} C^{\dagger} \nu_R^*)]$$

$$+ (h_1 k + h_3 k') (\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) \qquad (4.6.16)$$

which is a mixture of Majorana and Dirac mass terms. This Lagrangian can also be written as

$$\alpha^{\nu}_{mass} = h_5 (V_L \nu^T C \nu - V_R N^T C N)$$

$$+ (h_1 k + h_2 k') \nu^T C N + \text{H.C} \qquad (4.6.17)$$

where $\nu \equiv \nu_L$ and $N \equiv C (\bar{\nu}_R)^T$

and using the properties of charge conjugation matrix

$$C^T = -C, \ C^2 = -1 \ \text{ and } C \gamma_{\mu} C^T = -\gamma_{\mu}^T$$

$$\nu_R^{\dagger} C^{\dagger} \nu_R^* = -N^T C N \ \text{ and } \ \bar{\nu}_R \nu_L = N^T C \nu = \nu^T C N.$$

In the matrix form, the above Lagrangian takes the form

$$\alpha_{mass} = (\nu^T N^T) \, M C \begin{pmatrix} \nu \\ N \end{pmatrix} + \text{H.C.} \qquad (4.6.18)$$

where $$M = \begin{pmatrix} a & c \\ c & b \end{pmatrix} \qquad (4.6.19)$$

a,b denote the Majorana mass and C denotes the Dirac mass term. and

$$a = h_5 V_L,\ b = -h_5 V_R,\ C = \frac{1}{2}\,(h_1 k + h_2 k') \qquad (4.6.20)$$

The eigen states of this mass matrix is given by,

$$\begin{aligned} \nu &= \nu_e \cos\ \zeta + N_e \sin \zeta \\ N &= -\nu_e \sin \zeta + N_e \cos \zeta \end{aligned} \qquad (4.6.21)$$

$$\text{with} \quad \tan 2\,\zeta = = \frac{2c}{b-a} \simeq \frac{2c}{b} \qquad (4.6.22)$$

Studying the eigen values of (4.6.20) assuming $k' \ll k$, Mahapatra and Senjanovic obtain light and heavy Majorana neutrino masses as[70]

$$\begin{aligned} m_{\nu e} &\simeq a - \frac{c^2}{b} \\ m_{Ne} &= b. \end{aligned} \qquad (4.6.23)$$

Using eqs. (4.6.13) and (4.6.20), masses can be written as,

$$\begin{aligned} m_{\nu e} &\simeq (h_5 \gamma + \frac{1}{4}\ \frac{h_1^2}{h_5})\ \frac{k^2}{V_R} \\ m_{Ne} &= -h_5 V_R \end{aligned} \qquad (4.6.24)$$

For simplicity we put $k' = 0$. If all Yukawa couplings are equal, $h_1 = h_2 = h_5 = h$, then it is easy to see that,

$$m_{Ne} \simeq \frac{h}{g}\ m_{WR}$$

and

$$m_{\nu e} \simeq \frac{g}{h}\ \frac{m_e^2}{m_{WR}} \qquad (4.6.25)$$

Where g is the gauge coupling constant in the LRS model, and, as will be demonstrated subsequently in this dissertation. $M_{WR} \sim g V_R$, where M_{WR} stands for the $W_R^{\pm}$ gauge boson mass.

If $h/g \simeq 1$,

$$m_{\nu e} \simeq \frac{m_e^2}{m_{WR}} \tag{4.6.26}$$

Thus, as $m_{WR} \longrightarrow \infty$, $m_{\nu e} \longrightarrow 0$.
i.e. in the V-A limit of charged and neutral currents, the neutrino has zero mass as in the standard model. But for any finite value of m_{WR} the neutrino acquires a mass much smaller compared to the Dirac mass of the quark or charged lepton in the same family. The formula (4.6.26) holds for every generation and can be generalised as

$$m_{\nu\alpha} \simeq \frac{m_\alpha^2}{m_{WR}} \tag{4.6.27}$$

$\alpha = e, \mu, \tau$.

We calculate the neutrino masses for assumed value of m_{WR} in the range 1 TeV - 10^{12}GeV. as presented in Table 4.1.

Table 4.1 Neutrino masses for three generations as a function of $W_R^{\pm}$ boson masses - using see-saw mechanism and left-right symmetric model.

m_{WR} (GeV)	$m_{\nu e}$(eV) m_e=.511MeV	$m_{\nu\mu}$ (keV) m_μ = 105.6MeV	$m_{\nu\tau}$ (MeV) m_τ = 1870 MeV
10^3	0.261	11.2	3.5
10^4	0.026	1.12	0.35
10^5	2.6×10^{-3}	0.11	0.04
10^{12}	2.6×10^{-10}	1.12×10^{-8}	3.5×10^{-9}

4.7 Neutrinoless double β-decay

In order to confirm the finite neutrino mass, there are various experiments such as the β-decays of ^{3}H, ^{35}S, and ^{63}Ni, the neutrino oscillation, the electron capture in ^{163}Ho and so on. Concerning the V+A charged current, the precise measurements on the β-and μ-decays have been performed. In addition to these problems, the question whether neutrinos are Dirac or Majorana particle can be tested .he neutrinoless double -decay directly.

The process can take place in the second order in Fermi coupling if neutrinos are Majorana particles (shown in the fig. 4.1)

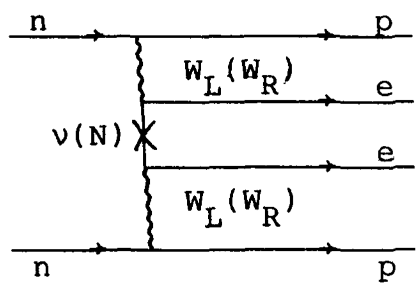


Fig.4.1. The diagram showing neutrino less double - β- decay through exchange of W_L and ν_i (i = e,μ,τ) or W_R and N_i.

The neutrinoless double β-decay $(\beta\beta)_0$ has been analysed in terms of the amplitude η, the lepton number nonconserving parameter or the admixture of left and right handed neutrino electron currents, where η appears in the leptonic current as

$$\bar{e}\gamma_\mu \left[\frac{(1-\gamma_5)}{2} + \eta \frac{(1+\gamma_5)}{2}\right] \nu \qquad (4.7.1)$$

through the finite mass of the electron neutrino. The value obtained for the contribution involving W_L-W_R and ν-N mixing is

$$\eta \leq 10^{-2} \frac{m_e}{m_N} \leq 10^{-7} \qquad (4.7.2)$$

for $m_N \geq 100$ GeV. The values of η obtained for $m_N = 10^2$-10^4 are listed in the table below.

Table 4.2 The values of η for $m_N = 10^2$-10^4 GeV are obtained from the relation (4.7.2)

m_N (GeV)	10^2	10^3	10^4
η	5×10^{-8}	5×10^{-9}	5×10^{-10}

We see the relation (4.7.2) is satisfied for the values of

$$m_N = 10^2 - 10^4 \text{ i.e. } m_N \geq 10^2.$$

The limit on η consistent with $(\beta\beta)_o$ decay rates of $^{48}Ca \rightarrow {}^{48}Ti$, $^{76}Ge \rightarrow {}^{76}Se$, and $^{82}Se \rightarrow {}^{82}Kr$. is,[71]

$$|\eta| \leq 5\times10^{-4}$$

The value of η corresponds to $m_{\nu e} \leq 1$ KeV for the light neutrino and implies a half-life for $(\beta\beta)_o$ decay of ^{82}Se, eg,

$$T_{1/2}^{(\beta\beta)_o} \simeq \frac{1.1 \times 10^{14\pm2}}{\eta^2} \quad Yr \qquad (4.7.3)$$

$$\geq 40 \times 20^{20\pm2} \quad Yr$$

Again due to the tiny neutrino masses, the exchange of heavy right handed leptons $N_i (i = e,\mu,\tau)$ will obviously dominate and η is given by

$$\eta \leq \left(\frac{m_{WL}}{m_{WR}}\right)^4 \frac{1}{m_N} f_{nuc}. \qquad (4.7.4)$$

Where f_{nuc} is the nuclear structure estimated by Halprin et al[71] to be about 0.35 GeV.

For $m_{Ne} \geq 100$ GeV, and $\left(\frac{m_{WL}}{m_{WR}}\right)^2 \leq \frac{1}{10}$

η becomes $\leq 3.5 \times 10^{-5}$

For the values of η in eq (4.7.4) $(\beta\beta)_o$ would require a half-life measurement of order $8\times10^{22\pm2}$ yr. The values of η are found out for $m_N = m_{WR} = 10^2 - 10^4$ GeV.

Table 4.3 The values of η obtained for $m_N = m_{WR} = 10^2\text{-}10^4$GeV from the relation (4.7.4)

m_N (GeV)	10^2	10^3	10^4
η	3.5×10^{-5}	3.5×10^{-6}	3.5×10^{-7}

A measurement of a half-life for $(\beta\beta)_o$ decay in the range 10^{20} to

10^{24} Yr would place a limit on m_N for the left-right symmetric electroweak model if the limit on $\left(\frac{m_{WL}}{m_{WR}}\right)^2$ is known from other considerations. Neutrinoless double β-decay is a lepton number violating process which provides constraints on neutrino masses and couplings. The existence of Majorana neutrinos is one of the simplest ways to account for masses much smaller than those of charged leptons (or of quarks). The effective neutrino mass $\langle m_\nu \rangle$ can be derived from the life time and from the knowledge of the nuclear matrix element. Unfortunately, strong discrepancies still exist between different theoretical estimates of this last quantity, as an example, upper limit on $\langle m_\nu \rangle$ derived from the UCS-LBL result on ^{76}Ge, are given by 1eV, 2eV and 19eV.[72-74]

We now briefly comment on some other muon and electron number changing processes. The interesting possible decay such as $\mu \to e\gamma$ and $\mu \to 3e$ may sometimes impose severe constraints on models with neutrino masses. The processes $\mu \to e\gamma$ receives its two dominant contributions from the $\nu_i^H W_R W_R$ and $\nu_i^H W_L W_L$ triangle loop diagrams (Fig.4.2). These contributions to the branching ratio may be estimated respectively.

$$B_L = \frac{\Gamma(\mu \to e + \gamma)}{\Gamma_\mu \text{ total}}$$

$$= \frac{3\alpha}{32\pi} \left(\sin\theta \quad \cos\theta \quad \frac{m_{\nu_2}^2 - m_{\nu_1}^2}{m_{W_L}^2}\right)^2 \tag{4.7.5}$$

$$B_R = \frac{3\alpha}{32\pi} \left(\frac{m_{W_L}^2}{m_{W_R}^2}\right)^2 \left(\sin\theta' \quad \cos\theta' \quad \frac{m_{W_2}^2 - m_{N_1}^2}{m_{W_L}^2}\right)^2 \tag{4.7.6}$$

For $m_W \sim 80$ GeV, and the limiting values $m_{\nu e} \leq 30$ ev, $m_{\nu\mu} \sim 0.6$ MeV, $B_L < 4.5 \times 10^{-26}$ which is too small. Eq. (4.7.6) holds for L-R symmetric model where neutrinos are predominantly left-handed and light while the N's are predominantly right handed and heavy, but the masses smaller than but of the same order as m_{WR}. In this case, eq (4.7.6) for $\frac{m_{WL}^2}{m_{WR}^2} \sim \frac{1}{10}$, $m_{W_2}^2 - m_{N_1}^2 \sim 10^4$ GeV.

$$B_R \approx 4 \times 10^{-8} (\sin\theta' \cos\theta')^2 \qquad (4.7.7)$$

From cosmological limits on the

ν_e, ν_μ masses, $\theta' \leq \left(\frac{m_e}{m_\mu}\right)^{\frac{1}{2}}$

$$\text{and} \quad B_R \leq 2 \times 10^{-10} \qquad (4.7.8)$$

When compared with $\mu \to e\gamma$ process, the branching ratio becomes

$$B(\mu \to ee\bar{e}) = \Gamma(\mu \to ee\bar{e})/\Gamma(\mu \to e\nu_\mu\bar{\nu}_e) \approx \frac{\alpha}{\sin^2\theta_W} B(\mu \to e\gamma)$$

For $\sin^2\theta_W \approx 0.22$,

$$B(\mu \to ee\bar{e}) / B(\mu \to e\gamma) \approx (1-10)\% \qquad (4.7.9)$$

Other possible muon and lepton number changing process are $e\bar{\mu} \to \mu\bar{e}$,

$$\mu^- + A(Z) \longrightarrow e^- + A(Z), \qquad (4.7.10)$$

$$\mu^- + A(Z) \longrightarrow e^+ + A(Z-2), \text{ etc.} \qquad (4.7.11)$$

For the process (4.7.10)

$$B \leq 1.3 \times 10^{-10} \sin^2\theta' \cos^2\theta' \qquad (4.7.12)$$

Choosing $\theta' \sim \left(\frac{m_e}{m_\mu}\right)^{\frac{1}{2}}$

$$B \leq 6 \times 10^{-13} \qquad (4.7.13)$$

and for the process (4.7.11)

$$B \leq 4 \times 10^{-10} \sin^2\theta' \cos^2\theta' \left(\frac{m_2 - m_1}{m_A}\right)^2 \qquad (4.7.14)$$

Where m_1, m_2 are the masses of Majorana neutrinos and m_A is the target mass.

Detection of both conversion processes with comparable branching ratios provides clear cut evidence for the existence of heavy Majorana neutrinos and for the L-R symmetric electo-weak model.

4.8 Summary

Unified electroweak gauge theories based on the gauge group $SU(2)_L x SU(2)_R x U(1)_{B-L}$ in which breakdown of parity invariance is spontaneous, lead most naturally to a massive neutrino. Dirac mass is generated by Yukawa interaction of fermions with Higgs doublets carrying B-L = 0 and Y = +1. But when Higgs triplets with B-L = ±2 are used, neutrino also gets Majorana mass. The small mass of the physical neutrino is then generated by a see-saw mechanism. Mahapatra and Senjanovic have shown that smallness of neutrino mass can be understood as a result of the observed maximality of parity violation in low energy weak interactions. In particular, in the limit $m_{WR} \longrightarrow \infty$, $m_{\nu e} \longrightarrow 0$, the weak interaction becomes pure V-A type. In left-right symmetric model, the V-A limit of charged and neutral currents corresponds to the vanishing of neutrino mass. The neutrinoless double β-decay is expected to provide strong evidence for a heavy neutrino in the mass range 100 GeV - few TeV.

In the next section we discuss gauge model based upon $SU(2)_L X U(1)_R X U(1)_{B-L}$ where it is possible to generate nonvanishing Majorana neutrinomass even if $M_{W_R} \longrightarrow \infty$. Here it has been found[75] that vanishing neutrino mass is a consequence of V-A limit of neutral currents only. Neutrino masses from various experiments are also summarised in the next chapter.

CHAPTER 5

GAUGE BOSON MASSES AND NEUTRAL CURRENT PHENOMENOLOGY IN LEFT-RIGHT MODELS AND MAJORANA MASS GENERATION IN TWO-STEP BREAKING

In this chapter, we discuss how far gauge boson masses, discussed in chapter 4, are modified in the two-step spontaneous breaking of $SU(2)_L XSU(2)_R XU(1)_{B-L}$ models. We also discuss derivation of neutral current parameters in left-right models. Certain new aspects of Majorana mass generation for neutrinos are discussed and experimental results on neutrino masses summarized.

In Sec 5.1 of this chapter, masses of charged and neutral gauge bosons in the L-R symmetric model are rewritten by using suitable parameters. In sec 5.2, neutral current parameters of LRsmodel are compared with that of $SU(2)_L XU(1)_Y$ model and with experimental results. In sec 5.3, we discuss how the light and heavy gauge boson masses get modified in $SU(2)_L XU(1)_R XU(1)_{B-L}$ model, descending from left-right gauge models. As noted in the previous chapter, neutrino masses vanish in $SU(2)_L XSU(2)_R XU(1)_{B-L}$ model for $m_{WR} \longrightarrow \infty$. In sec 5.4 we discuss how, according to an alternative proposal, neutrino masses get completely decoupled from $m^{\pm}_{WR}$, but depends on m_{ZR}. In sec.5.5, results obtained from different experiments on neutrino masses are compared. A brief summary of the chapter is given in sec 5.6.

5.1 Reparametrization of masses of chargedand neutral gauge bosons in LR symmetric model.

Starting with a phenomenological Lagrangian implied by the gauge group $SU(2)_L XSU(2)_R XU(1)_{B-L}$ the Higgs representation

$$\Delta_L = (1,0,2), \quad \Delta_R = (0,1,2), \quad \phi = (1/2,\ 1/2,\ 0).$$

are used with their VEV's

$$\langle\Delta_L\rangle = \begin{pmatrix} 0 & V_L \\ 0 & 0 \end{pmatrix} \quad \langle\Delta_R\rangle = \begin{pmatrix} 0 & V_R \\ 0 & 0 \end{pmatrix} \quad \langle\phi\rangle = \begin{pmatrix} k' & 0 \\ 0 & k \end{pmatrix} \qquad (5.1.1)$$

Following Rizzo and Senjanovic[76], the following parameters are used to explain the weak interaction phenomenology

$$\eta_R = \frac{k^2+k'^2}{V_R^2} \qquad \eta_L = \frac{V_L^2}{k^2+k'^2} \qquad Z = \frac{2k'^2}{k^2+k'^2} \qquad (5.1.2)$$

where η_R measures the amount of breaking of $SU(2)_R$, η_L measures the presence of left-handed Higgs triplet and Z tests the amount of W_L-W_R mixing. Using these parameters the charged gauge boson mass matrix (4.4.4) becomes,

$$M_W^2 = \frac{1}{2} g^2 V_R^2 \begin{pmatrix} \eta_R(1+\eta_L) & -\eta_R[1-(1-Z)^2]^{\frac{1}{2}} \\ -\eta_R[1-(1-Z)^2]^{\frac{1}{2}} & (\eta_R+1) \end{pmatrix} \qquad (5.1.3)$$

and the neutral gauge boson mass matrix becomes,

$$M_Z^2 = \frac{1}{2} g^2 V_R^2 \begin{pmatrix} \eta_R(1+2\eta_L) & -\eta_R & -2\varepsilon\eta_R\eta_L \\ -\eta_R & 2+\eta_R & -2\varepsilon \\ -2\varepsilon\eta_L\eta_R & -2\varepsilon & 2\varepsilon^2(1+\eta_L\eta_R) \end{pmatrix} \qquad (5.1.4)$$

where $\varepsilon = g'/g$ (5.1.5)

The charged current constraint from μ-decay becomes

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8}\left[\frac{\cos^2\xi}{M_{W_1}^2} + \frac{\sin^2\xi}{M_{W_2}^2}\right] \tag{5.1.6}$$

Where M_{W_i}, i=1,2 are the masses of the charged W-boson. The left and right handed gauge bosons get mixed and the mixing angle can be expressed in terms of the above parameters as

$$\tan 2\xi = \frac{2\eta_R\,[1-(1-Z)^2]^{\frac{1}{2}}}{1-\eta_R\eta_L} \tag{5.1.7}$$

On diagonalizing the mass matrices, in the usual way, one obtains the four mass eigen values $M_{W_1}, M_{W_2}, M_{Z_1}$ and M_{Z_2}. In obtaining the masses, the relation

$$\sin^2\theta_W = x_w = \frac{e^2}{g^2}$$

and $\dfrac{1}{e^2} = \dfrac{2}{g^2} + \dfrac{1}{g'^2}$ are used.

The eigen values of the masses for charged gauge bosons are, -

$$M_W^2 = \frac{1}{4}\,g^2V_R^2\,(1+2\eta_R+\eta_R\eta_L) \pm\{(1-\eta_L\eta_R)^2+4\eta_R^2\,[1-(1-Z)]\}^{\frac{1}{2}} \tag{5.1.8}$$

Where,

$$M_{W_1}^2 = \frac{1}{4} g^2V_R^2(1+2\eta_R+\eta_R\eta_L)-\{(1-\eta_L\eta_R)^2+4\eta_R^2[1-(1-Z)]\}^{\frac{1}{2}}$$

is the lighter mass, and,

$$M_{W_2}^2 = \frac{1}{4} g^2V_R^2 \ (1+2\eta_R+\eta_R\eta_L)+\{(1-\eta_L\eta_R)^2+4\eta_R^2 \ [1-(1-Z)^2]\}^{\frac{1}{2}}$$

is the heavier mass.

For neutral gauge bosons

$$M_Z^2 = \frac{1}{2} g^2V_R^2(1+\eta_L\eta_R) \ \frac{1-x_W}{1-2x_W} \ \{1+ \frac{\eta_R}{1+ \eta_L\eta_R} \ \frac{1-2x_W}{1-x_W}$$

$$\pm \ [1+ \ \frac{(1-2x_W)\eta_R^2(1-2x_W)-2\eta_R(1+\eta_L\eta_R)x_W-4\eta_L\eta_R}{(1+\eta_L\eta_R)^2(1-x_W)^2}\Big]^{\frac{1}{2}}\Big\}$$

Where,

$$M_{Z_1} = \frac{1}{2} g^2V_R^2 \ (1+\eta_L\eta_R) \ \frac{1-X_W}{1-2X_W} \ \{1+ \frac{\eta_R}{1+\eta_L\eta_R} \ \frac{1-2X_W}{1-x_W}$$

$$-[1+ \ \frac{(1-2X_W)\eta_R^2 \ (1-2x_W)-2\eta_R(1+\eta_L\eta_R)X_W-4\eta_L\eta_R}{(1+\eta_L\eta_R)^2(1-X_W)^2} \ ]^{1/2}\}$$

is the lighter gauge boson mass and,

$$M_{Z_2} = \frac{1}{2} g^2V_R^2(1+\eta_L\eta_R) \ \frac{1-X_W}{1-2X_W} \ \{1+ \ \frac{\eta_R}{1+\eta_L\eta_R} \ \frac{1-2X_W}{1-X_W}$$

$$+[1+ \frac{(1-2X_W)\eta_R^2(1-2X_W)-2\eta_R(1+\eta_L\eta_R)X_W+4\eta_L\eta_R}{(1+\eta_L\eta_R)^2(1-X_W)^2} \ ]^{1/2}\}$$

is the heavier gauge boson mass.

We discuss in the sec. 5.3, how the masses get modified in the absence of left-handed Higgs triplet and for no W_L-W_R mixing.

5.2 Neutral current parameters in LRS model.

To discuss about the neutral current data one needs to define the Fermiconstant G_F in terms of the model parameters.

$$\frac{G_F}{\sqrt{2}} = \frac{1}{4\eta_R V_R^2} \quad \frac{1+\eta_R}{1+\eta_L+\eta_L\eta_R+\eta_R(1-Z)^2} \tag{5.2.1}$$

One further introduces the parameters

$$\alpha \equiv -\frac{\eta_R}{1+\eta_R} \quad [1-(1-Z)^2]^{\frac{1}{2}} \tag{5.2.2}$$

which is the coefficient of the mixing term in the general charged current Hamiltonian and

$$\beta \equiv \frac{\eta_R(1+\eta_L)}{1+\eta_R} \tag{5.2.3}$$

Where β is the coefficient of right-handed currents. Turning into the various neutral processes, the effective Hamiltonian for both the SLAC asymmetry[44] experiment and atomic parity violation can be expressed as

$$H_{PV} = H_{PV}^S \; AB$$

where
$$A = \frac{1+\eta_R(1-Z)^2+\eta_L(1+\eta_R)}{1+\eta_L(2+\eta_R)} \tag{5.2.4}$$

and
$$B = \frac{1-\eta_L\eta_R}{1+\eta_R} \tag{5.2.5}$$

and H_{PV}^S is same as in the standard model.

(i) Neutrino-Hadron Scattering

For neutrino-hadron scattering, Hamiltonian can be expressed as, -

$$H^{\nu H} = \frac{G_F}{\sqrt{2}} \bar{\nu}\gamma_\mu (1-\gamma_5)\nu[\varepsilon_L(q)\bar{q}\gamma_\mu(1-\gamma_5)q + \varepsilon_R(q)\bar{q}\gamma_\mu(1+\gamma_5)q]$$

$$= \frac{G_F}{\sqrt{2}} \bar{\nu}\gamma_\mu (1-\gamma_5)\nu[(\varepsilon_L(u)+\varepsilon_R(u))\bar{u}\gamma_\mu 1$$

$$+ (\varepsilon_R(u)-\varepsilon_L(u))\bar{u}\gamma_\mu\gamma_5 u+(\varepsilon_L(d)+\varepsilon_R(d))\bar{d}\gamma_\mu d$$

$$+ (\varepsilon_R(d)-\varepsilon_L(d))\bar{d}\gamma_\mu\gamma_5 d] \qquad (5.2.6)$$

where the parameters are given by

$$\varepsilon_L(u) = \frac{A}{2}\left[\frac{1}{2}\frac{\eta_R+2}{\eta_R+1} - \frac{4X_W}{3}\right]$$

$$\varepsilon_R(u) = \frac{A}{2}\left[\frac{1}{2}\frac{\eta_R}{\eta_R+1} - \frac{4X_W}{3}\right]$$

$$\varepsilon_L(d) = \frac{A}{2}\left[-\frac{1}{2}\frac{\eta_R+2}{\eta_R+1} + \frac{2X_W}{3}\right] \qquad (5.2.7)$$

$$\varepsilon_R(d) = \frac{A}{2}\left[-\frac{1}{2}\frac{\eta_R}{\eta_R+1} + \frac{2X_W}{3}\right]$$

(ii) Neutrino-electron scattering

For neutrino electron interaction, the Hamiltonian can be written as,

$$H^{\nu e} = \frac{G_F}{\sqrt{2}} \bar{\nu}\gamma_\mu(1-\gamma_5)\nu[\varepsilon_L(e)\bar{e}\gamma_\mu(1-\gamma_5)e+\varepsilon_R(e)\bar{e}\gamma_\mu(1+\gamma_5)e] \qquad (5.2.8)$$

$$= \frac{G_F}{\sqrt{2}} \bar{\nu}\gamma_\mu (1-\gamma_5)\nu\ \bar{e}\gamma_\mu(g_V+g_A\gamma_5)e$$

Where, $g^e_{V,A} = \varepsilon_R(e) \pm \varepsilon_L(e)$

and the parameters are,

$$g_V = -\frac{A}{2}[-1-4x_W]$$
$$g_A = -\frac{A}{2(\eta_R+1)} \qquad (5.2.9)$$

(iii) <u>Parity violation in atomsandasymmetry in e-d scattering</u>

TheHamiltonian for electron-hadron scattering can be written as,

$$H^{eH} = \frac{G_F}{\sqrt{2}} \sum_i (C_{1i}\bar{e}\gamma_\mu\gamma_5 e\bar{q}_i\gamma_\mu q_i + C_{2i}\bar{e}\gamma_\mu q_i q_5 q_i) \qquad (5.2.10)$$

where q_i = u, d, ...

and the parameters are expressed as,

$$C_{1u} = \frac{AB}{2}[-1+\frac{8X_W}{3}]$$
$$C_{1d} = \frac{AB}{2}[1-\frac{4X_W}{3}] \qquad (5.2.11)$$
$$C_{2u} = -C_{2d} = \frac{AB}{2}[-1+4X_W]$$

(iv) <u>Asymmetries in $e^+e^- \longrightarrow \mu^+\mu^-$</u>

The Hamiltonian for $e^+e^- \longrightarrow \mu^+\mu^-$ involving V and A couplings can be written as,

$$H = \frac{G_F}{\sqrt{2}}[h_{VV}(\bar{e}\gamma_\mu e+\bar{\mu}\gamma_\mu\mu)(\bar{e}\gamma^\mu e+\mu\gamma^\mu\mu)$$
$$+ 2h_{VA}(\bar{e}\gamma_\mu e+\bar{\mu}\gamma_5\mu)(\bar{e}\gamma^\mu\gamma_5 e+\bar{\mu}\gamma^\mu\gamma_5\mu)$$
$$+h_{AA}(\bar{e}\gamma_\mu\gamma_5 e+\bar{\mu}\gamma_\mu\gamma_5\mu)(\bar{e}\gamma^\mu\gamma_5 e+\bar{\mu}\gamma^\mu\gamma_5\mu) \qquad (5.2.12)$$

Where the parameters are

$$h_{VA} = \frac{AB}{4} [1-4X_W]$$

$$h_{AA} = \frac{A}{4} [1+\eta_R\eta_L]/(\eta_R+1) \qquad (5.2.13)$$

$$h_{VV} = \frac{A}{4} [\frac{1+2\eta_R+\eta_L\eta_R}{(1+\eta_R)} \qquad (1-4X_W)^2$$

Comparison of the neutral current parameters obtained from the LRS model with that of WS model

In the following table 5.1, neutral current parametes are written down in terms of η_R, η_L, X_W, A and B. It is apparent that in the absence of η_L (left-handed triplet) and η_R (right-handed charged boson is infinitely heavier) all neutral current parameters obtained in LRS model reduce to those in the $SU(2)_L \times U(1)_Y$ model. From the table 5.1, we calculate the neutral current parameters and compare them with experimental data. For simplicity, we choose the case, when,

$$\eta_R = \frac{k^2}{V_R^2} \quad \text{as} \quad k' = 0 \qquad (5.2.14)$$

$$\eta_L = 0 \quad \text{and} \quad Z = 0$$

Thus, the expression for A, and B can be written from relations (5.2.4) and (5.2.5) as,

$$A = 1+\eta_R \quad \text{and} \quad B = \frac{1}{1+\eta_R} \qquad (5.2.15)$$

Table 5.1 - Neutral current Parameters of LRS and WS model

$\epsilon_L(u)$	$A/2\,[\frac{1}{2}\,\frac{\eta_R+2}{\eta_R+1} - \frac{4X_W}{3}]$	$1/2 - 2/3\ X_W$
$\epsilon_R(u)$	$A/2\,[1/2\,\frac{\eta_R}{\eta_R+1} - \frac{4X_W}{3}]$	$-2/3\ X_W$
$\epsilon_L(d)$	$A/2[-1/2\,\frac{\eta_R+2}{\eta_R+1} + \frac{2X_W}{3}]$	$-1/2 + \frac{X_W}{3}$
$\epsilon_R(d)$	$A/2\,[-1/2\,\frac{\eta_R}{(\eta_R+1)} + \frac{2X_W}{3}]$	$1/3\ X_W$
g_V	$A/2\,[-1 + 4X_W]$	$-1/2 + 2X_W$
g_A	$-A/2\,[\frac{1}{(\eta_R+1)}]$	$-1/2$
C_{1u}	$AB/2\,[-1+\frac{8X_W}{3}]$	$-1/2 + \frac{4X_W}{3}$
C_{1d}	$AB/2\,[1- 4X_W/3]$	$1/2 - 2X_W/3$
$C_{2u}=-C_{2d}$	$AB/2\,[-1+4X_W]$	$-1/2 + 2X_W$
h_{VA}	$AB/4\,[1-4X_W]$	$1/4\,(1-4X_W)$
h_{AA}	$A/4\,\frac{1+\eta_R\eta_L}{(\eta_R+1)}$	$1/4$
h_{VV}	$A/4\,\frac{1+2\eta_R+\eta_L\eta_R}{(1+\eta_R)}\,(1-4X_W)^2$	$1/4\,(1 - 4X_W)^2$

In the table 5.2, neutral current parameters are found out for a given value of η_R and compared with experimental data.

Table 5.2 - Calculated and experimental values of neutral current parameters

Parameters	Parameters with $X_W = 0.23$			Experimental results of the parameters
	Standard model	LRs Model		
		$\eta_R = 0.2$	$\eta_R = 0.4$	
$\varepsilon_L(u)$	0.347	0.366	0.385	0.340 ± 0.033
$\varepsilon_R(u)$	-0.153	-0.134	-0.115	-0.179 ± 0.019
$\varepsilon_L(d)$	-0.423	-0.458	-0.493	-0.424 ± 0.026
$\varepsilon_R(d)$	0.077	0.042	0.0073	- 0.017 ± 0.058
g_V	-0.04	-0.048	-0.056	0.043 ± 0.063
g_A	-0.5	-0.05	-0.05	-0.545 ± 0.056
C_{1u}	-0.193	-0.193	-0.193	-
C_{1d}	0.347	-0.347	0.347	-
C_{2u}	-0.04	-0.04	-0.04	-
C_{2d}	0.04	0.04	0.04	-
h_{VV}	0.0016	0.0022	0.0029	0.02 ± 0.04
h_{AA}	0.25	0.25	0.25	0.35 ± 0.11
h_{VA}	0.02	0.02	0.02	-0.27 ± 0.04

It is seen that the values of $\varepsilon_L(u)$ and $\varepsilon_L(d)$ are increasing while that of $\varepsilon_R(u)$ and $\varepsilon_R(d)$ are decreasing as η_R increases for the condions (5.2.14) and for a fixed value of $\mathrm{Sin}^2\theta_W=0.23$.

But the value of g_A remains constant for the above condition while g_V increases with η_R. The values of C_{1u}, C_{1d}, C_{2u}, and C_{2d} remain same as they are in the standard model because these values do not depend on η_R.

The accurate measurement of e-d asymmetry yields[44]

$$C_{1u} - .5C_{1d} = -0.45 \pm 0.12$$
$$C_{2u} - .5\, C_{2d} = 0.23 \pm 0.38 \qquad (5.2.16)$$

while the theoretical values obtained by satisfying certain conditions

$$C_{1u} - .5\, C_{1d} = -0.367$$
$$C_{2u} - .5\, C_{2d} = -0.06 \qquad (5.2.17)$$

The theoretical values of $C_{2u} - .5C_{2d}$ is too less from the experimental value.

The atomic parity violation measurements on heavy atoms yield[44,45]

$$\begin{aligned} C_{1u}+1.15C_{1d} &= 0.24 \pm 0.068 \text{ (Novosibirsk)} \\ &= 0.49 \pm 0.41 \text{ (Berkeley)} \\ &= 0.20 \pm 0.30 \text{ (Seattle)} \\ &= 0.15 \pm 0.035 \end{aligned} \qquad (5.2.18)$$

while the theoretical values are,

$$C_{1u} + 1.15C_{1d} = 0.205 \qquad (5.2.19)$$

which agrees more with the values obtained from Novosibirsk and Seattle than the values obtained from Berkeley experiment.

5.3 Light and heavy gauge boson masses in $SU(2)_L X U(1)_R X U(1)_{B-L}$ model arising from left-right models

In this section, we describe how the masses of light and heavy gauge bosons get modified due to the two step break ing of $SU(2)_R$.

Considering the LRS breaking pattern as

$$SU(2)_L X SU(2)_R X U(1)_{B-L}$$

$$\xrightarrow[\langle\chi_R\rangle \neq 0]{m_{W_R} > m_{W_L}} SU(2)_L X U(1)_R X U(1)_{B-L}$$

$$\xrightarrow[\langle\Delta_R\rangle \neq 0]{m_{Z_R} > m_{W_L}} SU(2)_L X U(1)_Y \tag{5.3.1}$$

$$\xrightarrow[\substack{\langle\Delta_L\rangle \neq 0 \neq \langle\phi\rangle \\ \langle\chi_L\rangle \neq 0}]{m_{W_L}} U(1)_{em}.$$

where the Higgs representations are already defined in Chapter 4.

In the absence of left-handed Higgs triplet ($\eta_L=0$) and for no W_L-W_R mixing ($Z=0$), the mass matrix for charged gauge boson becomes (using the parameters from (5.1.2)).

$$M_W^2 = \frac{1}{2} g^2 V_R^2 \begin{bmatrix} \eta_R & 0 \\ 0 & 1+\eta_R \end{bmatrix} \tag{5.3.2}$$

where $\eta_R = \dfrac{k^2+k'^2}{V_R^2} \approx \dfrac{k^2}{V_R^2}$ (5.3.3) assuming $k^2 >> k'^2$, for simplicity. In the first step, the symmetry breaking takes place at mass scales

$m_{WR} >> m_{WL}$, with $\Delta I_{3R} \simeq 0$. This is achieved by a Higgs triplet $\varepsilon_R \equiv (0,1,0)$. Only $SU(2)_R$ is broken, but local B-L symmetry remains unbroken. From charge conservation condition

$$\Delta I_{3R} \simeq - \frac{1}{2} \Delta (B-L) \tag{5.3.4}$$

which is satisfied since

$$I_{3R} = B-L = 0 \qquad \text{for} \quad \varepsilon_R$$

In the second step, for $m_{ZR} >> m_{WL}$, thecondition $\Delta I_{3L} \simeq 0$ is also satisfied and the charge conservation is satisfied since $I_{3R} = \pm 1$ and $B-L = -2$ for Δ_R.

Eq. (5.3.4) relates the breaking of $U(1)_R$ and $U(1)_{B-L}$ local symmetries.

For the two step breaking of $SU(2)_R$, ε_R plays an essential role. As $I_{3R} = 0$ for ε_R, it contributes only to the mass of right-handed charged gauge boson but does not effect the neutral gauge boson mass matrix.

Writing VEV of ε_R as

$$\langle \varepsilon_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varepsilon_X \\ 0 \end{pmatrix} . \tag{5.3.5}$$

The charged gauge boson mass matrix is modified as ,

$$M_W^2 = \frac{1}{2} g^2 V_R^2 \begin{bmatrix} \eta_R(1+\eta_L) & -\eta_R[1-(1-Z)^2]^{\frac{1}{2}} \\ -\eta_R[1-(1-Z)^2]^{\frac{1}{2}} & (1+\eta_R+\eta_R^X) \end{bmatrix} \quad (5.3.6)$$

where $\eta_R^X = \dfrac{\epsilon_X}{V_R^2}$ (5.3.7)

and the mixing angle ξ can be expressed as

$$\tan 2\xi = \frac{2\eta_R[1-(1-Z)^2]^{\frac{1}{2}}}{[(1+\eta_R+\eta_R^X)-\eta_R(1+\eta_L)]} \quad (5.3.8)$$

The eigen values of charged gauge bosons are modified as,

$$M_W^2 = \frac{1}{4} g^2 V_R^2 \, [\eta_R(1+\eta_L)+(1+\eta_R+\eta_R^X) \pm \{[\eta_R(1+\eta_L)-(1+\eta_R+\eta_R^X)]^2+4\eta_R^2[1-(1-Z)^2]\}^{\frac{1}{2}}] \quad (5.3.9)$$

In the absence of left-handed Higgs triplet (η_L=0) and for no W_L-W_R mixing (Z=0), the mass-matrix for charged gauge boson can be written as,

$$M_W^2 = \frac{1}{2} g^2 V_R^2 \begin{bmatrix} \eta_R & 0 \\ 0 & (1+\eta_R+\eta_R^X) \end{bmatrix} \quad (5.3.10)$$

Where $\eta_R = \dfrac{k^2}{V_R^2}$ assuming $k^2 >> k'^2$ for simplicity

and $\eta_R^X = \dfrac{\epsilon_X}{V_R^2}$

For no mixing case, $\xi = 0$, $\tan 2\xi = 0$ (5.3.11)

The eigen values of charged gauge bosons are also further modified

$$M_W^2 = \frac{1}{4}\, g^2 V_R^2\, [\eta_R + (1+\eta_R+\eta_R^\chi\ ) \pm \{[\eta_R - (1+\eta_R+\eta_R^\chi)]^2\}^{\frac{1}{2}}] \qquad (5.3.12)$$

where the light and heavy charged gauge boson masses take the form, -

$$M_{W_1} = [\frac{1}{4}\, g^2 V_R^2\, \{(\eta_R + 1 + \eta_R + \eta_R^\chi) - (\eta_R^\chi + 1)\}]^{\frac{1}{2}}$$
$$= [\frac{1}{2}\, g^2 V_R^2\, \eta_R]^{\frac{1}{2}} \qquad (5.3.13)$$

and

$$M_{W_2} = [\frac{1}{4}\, g^2 V_R^2\, \{(\eta_R + 1 + \eta_R + \eta_R^\chi) + (\eta_R^\chi + 1)\}]^{\frac{1}{2}}$$
$$= [\frac{1}{2}\, g^2 V_R^2\, (1+\eta_R+\eta_R^\chi)]^{\frac{1}{2}} \qquad (5.3.14)$$

respectively.

But in the presence of left-handedtriplet and for no mixing case, the lighter (left-handed) charge gauge boson mass becomes,

$$M_{W_1} = [\frac{1}{2}\, g^2 V_R^2\, \eta_R (1+\eta_L)]^{\frac{1}{2}} \qquad (5.3.15)$$

Thus, it is seen that the mass M_{W_1} shown in eq.(5.3.13) gets reduced, as compared to (5.3.15), by a quantity $gV_R(\frac{\eta_R\eta_L}{2})^{\frac{1}{2}}$ in the absence of left-handed Higgs triplet. It is also evident from eqs.(5.3.9) and (5.3.14) that whether the left-handed Higgs triplet is included or not, right-handed heavier charged gauge boson mass M_{W_2}, does not change for no mixing case. M_{W_2} can be made large by making η_R^χ large to break

$$SU(2)_R \xrightarrow{M_{W_2} = m_{W_R}} U(1)_R .$$

In no mixing case, Z=0, and ξ=0.both in eqs (5.1.7) and (5.3.8) and the muon decay constraints coming from eq (5.1.6) and the two steps symmetry breaking case are the same, namely,

$$\frac{G_F}{\sqrt{2}} = \frac{1}{8}\,\frac{g^2}{M^2_{W_1}} = \frac{1}{4k^2(1+\eta_L)} \quad \text{when } k'^2=0 . \qquad (5.3.16)$$

But in the absence of η_L eq (5.3.16) becomes,

$$\frac{G_F}{\sqrt{2}} = \frac{1}{4k^2} \qquad (5.3.17)$$

The relation (5.3.17) can also be obtained from eq.(5.2.1).

The introduction of Higgs triplet χ_R does not affect the neutral mass matrix, the masses of the neutral gauge bosons remains unaffected irrespective of the amount of mixing between the charged left and right handed bosons.

5.4 Neutrino mass in $SU(2)_L XU(1)_R XU(1)_{B-L}$ model

It is shown in the last chapter that the left and right-handed Majorana neutrinos can acquire the small and large masses separately, due to the spontaneous symmetry breaking in $SU(2)_L XSU(2)_R XU(1)_{B-L}$ model with minimal Higgs representations. The left and right-handed Majorana neutrinos take the form

$$m_{\nu e} \sim \frac{m_e^2}{m_{W_R}} \quad \text{and } m_N \sim m_{WR} \qquad (5.4.1)$$

respectively, where m_e is the charged lepton mass and m_{WR} is the charged right-handed gauge boson mass.

From eq.(5.4.1) when,

$$m_{WR} \longrightarrow \infty, \qquad m_{\nu e} \longrightarrow 0.$$

which is already discussed in the last chapter.

The Higgs representations used for the two step symmetry breaking of $SU(2)_R$ of $SU(2)_L X SU(2)_R X U(1)_{B-L}$ are,

$\chi_L(1,0,0)$, $\chi_R(0,1,0)$, $\Delta_L(1,0,2)$,

$\Delta_R(0,1,2)$ and $\phi(1/2, 1/2, 0)$ with their vacuum expectation values as,

$$\langle\chi_L\rangle = \begin{pmatrix} 0 & V_L^{\chi} \\ 0 & 0 \end{pmatrix} \qquad \langle\chi_R\rangle = \begin{pmatrix} 0 & V_R^{\chi} \\ 0 & 0 \end{pmatrix}$$

$$\langle\Delta_L\rangle = \begin{pmatrix} 0 & V_L \\ 0 & 0 \end{pmatrix} \qquad \langle\Delta_R\rangle = \begin{pmatrix} 0 & V_R \\ 0 & 0 \end{pmatrix} \qquad \langle\phi\rangle = \begin{pmatrix} k' & 0 \\ 0 & k \end{pmatrix} \tag{5.4.2}$$

Where χ_L and χ_R are the left and right-handed Higgs triplet- with B-L=0 and Δ_L and Δ_R are Higgs triplet with nonzero B-L values. The values of χ_R, χ_L, Δ_R, Δ_L and ϕ all contribute to the $W_R^{\pm}$ and $W_L^{\pm}$ masses and Δ_R, Δ_L and ϕ to the Z_R and Z_L masses. The symmetry breaking can be achieved by choosing,

$$V_R^{\chi 2} >> V_R^2 >> k^2 >> V_L^{\chi 2} \approx V_L^2 \tag{5.4.3}$$

and $k'^2 << k^2$

Then one has,

$$M_{WR} \sim gV_R^{\chi}\ , \quad m_{ZR} \sim gV_R, \quad m_{WL} \sim m_{ZL} \sim g\mathbf{k} \tag{5.4.4}$$

The neutrino Majorana mass does not arise from the contribution of the triplets χ_L and χ_R (B-L = 0), but arises from the contribution of Δ_R and Δ_L, since for them $B - L \neq 0$. It is shown in chapter 4, that Majorana neutrino masses can be written as,[70]

$$m_{\nu e} \sim (h_5\gamma + \frac{1}{4}\ \frac{h_1^2}{h_5}\ )\ \frac{\mathbf{k}^2}{V_R} \tag{5.4.5}$$

and $m_{Ne} = h_5 V_R$

where γ is a ratio of Higgs self coupling,

With the reasonable assumption as,

$$h_1 \approx h_5 \approx h \tag{5.4.6}$$

where h s' are Yukawa couplings and using the conditions (5.4.4), (5.4.5) and (5.4.6) one gets, -

$$m_{\nu e} \sim \frac{h}{g}\ \frac{m_{WL}}{m_{ZR}} \sim \frac{g}{h}\ \frac{m_e^2}{m_{ZR}} \tag{5.4.7}$$

and

$$m_N \sim \frac{h}{g}\ m_{ZR} \tag{5.4.8}$$

If $h/g \approx 1$, eqs (5.4.7) and (5.4.8) become,

$$m_{\nu e} \sim \frac{m_e^2}{m_{ZR}} \tag{5.4.9}$$

$$\text{and} \quad m_N \sim m_{ZR} \tag{5.4.10}$$

Thus it is seen that neutrino can have a finite non-vanishing mass even if m_{WR} is very large as long as m_{ZR} is not too large. That is, the vanishing of neutrino mass in this case is a result of the V-A limit of neutral currents. Now choosing

$$V_L{\chi}^2 \approx V_L^2 \approx 0 \quad \text{and} \quad V_R{\chi}^2 >> V_R^2$$

and $\quad m_{ZR} = 500 \text{ GeV} - 10^5 \text{ GeV},$

we calculate the neutrino masses for the three generations as a function of right handed neutral gauge boson mass m_{ZR}.

Table 5.3 Neutrino masses for three generations for m_{ZR} = 500 GeV-10^5GeV

m_{ZR} (GeV)	$m_{\nu e}$(eV) m_e=.511MeV	$m_{\nu\mu}$(keV) m_μ = 105.6 MeV	$m_{\nu\tau}$(MeV) m_τ = 1870 MeV
5×10^2	0.52	22.3	6.99
10^3	0.26	11.15	3.50
10^4	0.026	1.12	0.35
10^5	2.6×10^{-3}	0.11	0.04

5.5 Results of neutrino masses from different experiments

The experimental limits on the neutrino masses from the mass spectra of leptons (in MeV) obtained[77] are

$$\begin{aligned} m_{\nu e} &\leq 10^{-5} \\ m_{\nu\mu} &< 0.5 \\ m_{\nu\tau} &< 250 \end{aligned} \qquad (5.5.1)$$

But according to Particle data group[78] the following bounds on neutrino masses aregiven

$$m_{\bar{\nu}e} < 46 \ (60 \ eV)^{79}$$

$$m_{\nu\mu} < 500 \ KeV \tag{5.5.2}$$

$$m_{\nu\mu} < 160 \ MeV \ (\ 25 \ MeV)^{80}$$

From the analysis of the electron energy spectrum of the tritium β-decay ITEP group[81] in Moscow reported the finite neutrino mass

$$17 \ eV < m_{\bar{\nu}e} < 40 \ eV \tag{5.5.3}$$

at 95% confidence level.

Out of the other recent results of other groups we mention only the 95% confidence level limit of the Zurich group[64] who implemented ^3H in a carbon matrix

$$m_{\bar{\nu}e} < 18 \ eV \tag{5.5.4}$$

Absence of neutrinoless double β-decay in ^{76}Ge, leads to a limit $m_{\nu(Majorana)} < 2eV$. and $m_{\nu\mu} < 0.25$ MeV (90% confidence level)

$$m_{\nu\tau}{}^{82} < 85 \ MeV \ (95\% \ confidence \ level)$$

$$\text{also} \quad m_{\nu\tau}{}^{83} < 76 \ MeV \ (95\% \ confidence \ level) \tag{5.5.5}$$

From the cosmological arguments first advanced by Cowsik and Mc Cllelund[84], then upper limit on the mass (summed over all flavour) of stable "light" neutrinos and antineutrinos as being quoted recently as

$$\Sigma\, m_{\nu,\bar{\nu}} \leq 40 \text{ eV} \qquad (5.5.6)$$

A summary of the situation then is that the mass of ν_e (which has the most stringent limit) should be less than about 20 eV. From an analysis of the neutrino burst from the Supernova 1987a it is possible to improve this limit[85] to 12 eV.

From the resolution of the solar neutrino puzzle[86], mass of neutrino is found out to be $\sim m_{\nu\mu} \sim 0.008$ eV while following Bethe if one uses the highly speculative but very interesting see-saw mechanism one expects

$$m_{\nu e} \sim 10^{-7} \text{ eV} \qquad (5.5.7)$$

To satisfy this condition, the value of $m_{WR} \sim m_{ZR}$ should be equal to $\sim 2.6 \times 10^{18}$ eV, We summarize four experiments published in the year 1986 and 1987.

Table 5.4 Summary of antineutrino mass experiments published in 1986,1987

Experiment	References	$m_{\bar{\nu}e}$ (eV)
Zurich	M. Fritshi et al,[64]	<18
Tokyo	H. Kawakami et al [87]	<32±2
LAMPF	J.F. Wilkerson et al [88]	<27
Moscow	S. Boris et al [81]	30±2
SN 1987 A	M. Roos [89]	≤20

The claim by the Moscow group to have found a non zero $\bar{\nu}_e$ mass is not supported by the other experiments, which are compatible with a vanishing mass. The Zurich measurement yields the most stringent limit $\leq$ 18 eV.

5.6 Summary

It is shown that the two steps breaking of $SU(2)_R$ in left-right symmetric $SU(2)_L X SU(2)_R X U(1)_{B-L}$ model yields a Majorana neutrino mass independent of the heavy m_{WR} but inversely proportional to the ligher m_{ZR}. Since M_{ZR} decouples from M_{WR}, $m_\nu \longrightarrow 0$ only if $M_{ZR} \longrightarrow \infty$. In the case when $M_{WR} \longrightarrow \infty$, M_{ZR} could be finite, yielding nonvanishing Majorana neutrino mass. In $SU(2)_L X U(1)_R X U(1)_{B-L}$ model, when W_L-W_R do not mix, left-handed charged gauge boson mass gets modified while right handed charged gauge boson mass remains unchanged in the absence of left-handed Higgs triplet. The neutrino mass ($m_{\nu e}$) from SN 1987A [89], which in its most conservative form, reads

$$m_{\nu e} \leq 20 \text{ eV} \qquad (5.6.1)$$

is comparable to those already obtained in the laboratory. The laboratory limits have not been improved since the Berkeley Conference. Experimental data from various laboratories were summarized as

$$\begin{aligned} m_{\nu e} &< 18 \text{ eV} && \text{Zurich data,} \\ m_{\nu e} &< 27 \text{ eV} && \text{Los Alamos data} \qquad (5.6.2) \\ m_{\nu e} &< 32 \text{ eV} && \text{INS (Japan) data} \end{aligned}$$

The ITEP claim of observing a non-vanishing ν_e mass

$$17 \text{ eV} < m_{\nu e} < 40 \text{ eV}$$

$$m_{\nu e} = 30.3 {}^{+2}_{-8} \text{ eV} \qquad (5.6.3)$$

The only new laboratory result on neutrino masses has to do with τ neutrino. According to the ARGUS[90] data,

$$m_{\nu\tau} \leq 50 \text{ MeV} \qquad (5.6.4)$$

from the measured 5π-invariant mass in the decay $\tau \longrightarrow 5\pi\nu$.

These laboratory limits (5.6.1) — (5.6.4) can be made to be consistent with the left right models, or the $SU(2)_L X U(1)_R X U(1)_{B-L}$ model with $M^{\pm}_{WR}$ (or M_{ZR}) between 500 GeV - few TeV.

However, solution of the solar-neutrino puzzle requires a very small neutrino mass, $m_{\nu e} \simeq 10^{-7}$. for which we need $M_{W_R} (\sim M_{ZR}) > 10^{10}$ GeV. This implies that if the conventional treatment of these models are to explain small neutrino mass compatible with solutions to the solar neutrino puzzle, it is extremely difficult to verify left-right models by low energy experiments.

CHAPTER 6

MINIMAL FINE TUNING, GAUGE BOSON MASSES AND NEUTRAL CURRENT PARAMETERS IN THE LEFT-RIGHT MODELS

In this chapter, we emphasize upon the idea of minimal fine tuning of parameters and specify the Higgs sector and VEV's neededfor spontaneous symmetry breaking. The minimal fine tuning constraint leads to a modification of gauge boson masses and neutral current parameters. Its implication on neutral current parameters are also discussed.

In sec 6.1, we specify the Higgs sector and the VEV's needed for the LRA model based upon $SU(2)_L XSU(2)_R XU(1)_{B-L}$ ($g_L \neq g_R$). In sec 6.2 we note down modified formulae for charged and neutral gauge boson masses. The corresponding formulae for the neutral current parameters are also described in this section. Implications on neutrino masses are discussed in sec 6.3. A summary of the chapter is provided in sec 6.4.

6.1 Minimal fine tuning and the Higgs sector

The hypothesis of minimal fine tuning of parameters can be stated in the following manner[91]. Only those particles (scalars and fermions) remain light which are needed to be light. Others acquire heavy or superheavy masses characteristic of next higher scale(s) of spontaneous symmetry breaking of the gauge symmetry. An example of this hypothesis can be described in the context of LRS model breaking down to $U(1)_{em}$ through $SU(2)_L X U(1)_Y$:

$$SU(2)_L X SU(2)_R X U(1)_{B-L} \xrightarrow[M_{WR}]{\langle\Delta_R\rangle \neq 0} SU(2)_L X U(1)_Y \xrightarrow[\langle\Delta_L\rangle \neq 0]{\langle\phi\rangle \neq 0} U(1)_{em}$$

$$(g_L = g_R = g) \qquad\qquad M_{W_L} \qquad (6.1.1)$$

In this case $\langle\Delta_R\rangle \neq 0$ is necessary for spontaneous symmetry breaking at the first stage and $\langle\phi\rangle \neq 0$ is necessary at the second stage. Since $\langle\phi\rangle \neq 0$ executes the breaking of

$SU(2)_L X U(1)_Y \xrightarrow[\langle\phi\rangle \neq 0]{M_W} U(1)_{em}$ it is not necessary to attribute nonzero VEV's to Δ_L. Similarly the VEV $k \neq 0$ occuring in

$$\langle\phi\rangle = \begin{pmatrix} k & o \\ o & k' \end{pmatrix}, \quad \langle\tilde{\phi}\rangle = \begin{pmatrix} k' & o \\ o & k \end{pmatrix} \qquad (6.1.2)$$

is sufficient to break $SU(2)_L X U(1)_Y \longrightarrow U(1)_{em}$ using $k' \neq o$ is not necessary.

Since $\langle\Delta_L\rangle \neq 0$ is not needed, the left-handed Higgs triplet does not remain light, but acquires mass $\sim M_{W_R}$ in the LRS model. Similarly with $k'=0$, the left-right mixing angle vanishes.

$$\tan 2\xi \longrightarrow 0.$$

In the LRA model the two coupling constants are unequal at $\mu \geq M_{W_R}$. In the LRS model the equality between coupling constants $(g_L = g_R = g)$ is maintained at $\mu \geq M_{WR}$ by choosing left-right symmetric Higgs sectors. For every left-handed doublet (triplet) there must exist a corresponding right-handed doublet (triplet) of scalars. In the LRA model maintaining such a symmetry is not compulsory. Therefore from the point of minimal fine tuning of parameters, the following VEV's are the minimal requirements

(A) LRS model ($g_L=g_R=g$)

$$\langle\phi\rangle = \begin{pmatrix} k & o \\ o & o \end{pmatrix}, \quad \langle\tilde{\phi}\rangle = \begin{pmatrix} o & o \\ o & k \end{pmatrix}$$

$$\langle\Delta_L\rangle = 0 \;, \quad \langle\Delta_R\rangle = \begin{pmatrix} o & o \\ V_R & o \end{pmatrix} \qquad (6.1.3)$$

(B) LRA model ($g_L \neq g_R$)

$$\langle\phi\rangle = \begin{pmatrix} k & o \\ o & o \end{pmatrix}, \quad \langle\tilde{\phi}\rangle = \begin{pmatrix} o & o \\ o & k \end{pmatrix} \langle\Delta_R\rangle = \begin{pmatrix} o & o \\ V_R & o \end{pmatrix} \qquad (6.1.4$$

It is noted that Δ_L is not present in the Lagrangian respecting LRA gauge symmetry (Case B). Similarly if we take doublets instead of triplets, the counterpart of the right handed doublet i.e., $\chi_L(\frac{1}{2}, 0, 1)$ need not be present in the Lagrangian.

6.2 Gauge boson masses and neutral current parameters from minimal fine-tuning constraint

(A) LRS model: Using formulae discussed in the previous sections,

We have ($V_L=0$, $k'=0$, $V_R^2 \gg k^2 \neq 0$)

$$\eta_L = \frac{V_L^2}{k^2+k'^2} = 0, \quad Z = \frac{2k'^2}{k^2+k'^2} \doteq 0$$

$$\eta_R = \frac{k^2+k'^2}{V_R^2} = \frac{k^2}{V_R^2}$$

$$\tan 2\xi = 2\eta_R \frac{[1-(1-Z)^2]^{\frac{1}{2}}}{(1-\eta_R\eta_L)} = 0 \qquad (6.2.1)$$

The mass matrices are

$$M_W^2 = \tfrac{1}{2} g^2 V_R^2 \begin{bmatrix} \eta_R & 0 \\ 0 & 1+\eta_R \end{bmatrix}$$

$$(6.2.2)$$

$$M_Z^2 = \tfrac{1}{2} g^2 V_R^2 \begin{bmatrix} \eta_R & -\eta_R & 0 \\ -\eta_R & 2+\eta_R & -2\varepsilon \\ 0 & -2\varepsilon & 2\varepsilon^2 \end{bmatrix}$$

Where $\varepsilon = g'/g$. These formulae

$$M_{W_L}^2 = \frac{1}{2} g^2 k^2 \qquad (6.2.3)$$

$$M_{W_R}^2 = \frac{1}{2} g^2 (V_R^2 + k^2)$$

$$M_{ZL} = \frac{M_{W_L}}{\cos\theta_W} \qquad (6.2.4)$$

$$M_{ZR} = \frac{M_{W_R} \cos\theta_W}{\sqrt{\cos\theta_W}}$$

The neutral-current parameters take the form

for (i) atomic parity violation and asymmetry in e-d scattering:-

$$C_{1u} = -\frac{1}{2} + \frac{4X_W}{3}$$

$$C_{1d} = \frac{1}{2} - \frac{2X_W}{3} \qquad (6.2.5)$$

$$C_{2u} = -C_{2d} = -\frac{1}{2} + 2X_W$$

(ii) Neutrino-hadron scattering :-

$$\varepsilon_L(u) = \frac{1+\eta_R}{2}\left[\frac{1}{2}\,\frac{\eta_R+2}{\eta_R+1} - \frac{4X_W}{3}\right]$$

$$\varepsilon_R(u) = \frac{1+\eta_R}{2}\left[\frac{1}{2}\,\frac{\eta_R}{\eta_R+1} - \frac{4X_W}{3}\right] \tag{6.2.6}$$

$$\varepsilon_L(d) = \frac{1+\eta_R}{2}\left[-\frac{1}{2}\,\frac{\eta_R+2}{\eta_R+1} + \frac{2X_W}{3}\right]$$

$$\varepsilon_R(d) = \frac{1+\eta_R}{2}\left[-\frac{1}{2}\,\frac{\eta_R}{\eta_R+1} + \frac{2X_W}{3}\right]$$

(iii) Neutrino-electron scattering

$$g_V = -\frac{(1+\eta_R)}{2}\,[-1+4X_W]$$

$$g_A = -\frac{1}{2} \tag{6.2.7}$$

(iv) Asymmetries in $e^+e^- \longrightarrow \mu^+\mu^-$

$$h_{VA} = \frac{1}{4}\,[\,1-4X_W]$$

$$h_{AA} = \frac{1}{4} \tag{6.2.8}$$

$$h_{VV} = \frac{1}{4}\,(1+2\eta_R)(1-4X_W)^2$$

where $X = \sin^2\theta_W = \dfrac{e^2\,(M_W)}{g^2(M_W)}$. It is well known that if $x_W \simeq 0.23$, $M_{W_R} \geq$ few TeV, $M_{Z_R} \simeq 500\text{-}1000$ GeV, this model agrees well with existing low-energy data.

It is noted that C_{1u}, C_{1d}, C_{2u} and C_{2d} in this case are the same as in the standard model, so that the atomic parity violation experiments cannot distinguish between left-right symmetric models from the standard model. Similarly g_A, h_{VA} and h_{AA} have the same expression as the standard model, but g_V receives a correction to the standard model formula by

$$\delta g_V = \frac{k^2}{V_R^2}(g_V)_{std} \simeq \frac{M_{W_L}^2}{M_{W_R}^2}(g_V)_{std} \tag{6.2.9}$$

where $(g_V)_{std} = -\frac{1}{2} + 2x_w$ denotes the value obtained in the standard model. Accurate measurements of g_V from ν_μ-e and ν_e-e scattering experiments should then set a limit on the right-handed gauge boson masses. Similarly h_{VV} receives a correction

$$\delta h_{VV} = \frac{2k^2}{V_R^2}(h_{VV})_{std} \simeq \frac{2M_{W_L}^2}{M_{W_R}^2}(h_{VV})_{std} . \tag{6.2.10}$$

where $(h_{VV})_{std} = \frac{1}{4}(1-4x_w)^2$ denotes the standard model value. Thus, accurate measurements on h_{VV} from $e^+e^- \longrightarrow \mu^+\mu^-, \tau^+\tau^-$ experiments would set a limit on $W_R^\pm$ gauge boson masses, although h_{AA} and h_{VA} cannot distinguish between the two models. In Figs.6.1 and 6.2 we have plotted g_V and h_{VV} as a function of M_{W_R}/M_{W_L}. It is clear that if $M_{W_R}^\pm \simeq (6-10)M_{W_L}^\pm$, LRS model predictions are similar to the standard model.

(b) <u>LRA model</u> $(g_L \neq g_R)$

In this case, eqs (6.2.1) hold, but in addition $R^2 = g_L^2 / g_R^2 \neq 1$. The appropriate formulas for gauge boson masses are obtained from the formulae given in chapter 5 using $k' = V_L = 0$. The neutral current parameters C_{1u}, C_{1d}, C_{2u} and C_{2d}

remain the same as in the standard model, and the parameters $\varepsilon_L(u)$, $\varepsilon_R(u)$, $\varepsilon_L(d)$ and $\varepsilon_R(d)$ remain the same as in the LRS models. The parameters h_{VA} and h_{AA} are also same as in the LRS or the standard model. But g_V and h_{VV} both differ from the LRS model expressions

$$g_V = \frac{(1+\eta_R)}{2} \quad [-1- 4x_w F] \tag{6.2.11}$$

where the parameter

$$F = \frac{R^2(2+\eta_R)+\eta_R}{2R^2(1+\eta_R)}$$

$$\begin{aligned} h_{VV} = \frac{1}{(R^2+1)^2} \{ & \frac{R^2-1}{4} [8x_w(R^2+1)-3R^2-1] \\ & + [1- \frac{2(R^2+1)}{R^2} x_w]^2 \; [R^4+ \frac{\eta_R}{2} (R^2+1)^2]\} \end{aligned} \tag{6.2.12}$$

In this case also accurate measurements on g_V would set a limit on the $W_R^{\pm}$ gauge boson masses. Similarly h_{VV} clearly distinguishes the model from the standard and LRS models.

If the $W_R^{\pm}$ gauge boson masses $\geq$ few TeVand M_{ZR}~1 TeV this model agrees well with the neutral current and CP-violating parameters.

6.3 Formula for neutrino masses by see-saw mechanism

In this section we note that the see-saw mechanism becomes more natural leading to a simple formula for neutrino masses when minimal fine tuning constraint restricts the VEV's. With $\upsilon_L = k' = 0$, the scalar potential responsible for

$SU(2)_R X U(1)_{B-L}$ breaking is of the form

$$V(\upsilon_R,k)=-\mu^2\upsilon_R^2 + \frac{\rho}{4}\ \upsilon_R^4 + \alpha\upsilon_R^2 k^2 + \text{terms involving k only.} \qquad (6.3.1)$$

Here ρ,and α are of order unity. Using extremizing condition

$$\frac{\partial V}{\partial \upsilon_R} = 0 \ \text{ yields, } \ -\mu^2 + \rho\upsilon_R^2 + 2\alpha k^2 = 0$$
$$\text{or } \ \upsilon_R^2 = (\mu^2 - 2\alpha k^2)/\rho \qquad (6.3.2)$$

Thus the hierarchy $\upsilon_R^2 >> k^2$, can be maintained with $\mu^2 >> k^2$, In the $\nu_L - \nu_R$ sector, the mass term in the Lagrangian is

$$\alpha^{\nu}_{mass} = h_S[\upsilon_R(\nu_R^T C \nu_R + \nu_R^+ C^+ \nu_R^*)]$$
$$+h_1 k\ (\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L) \qquad (6.3.3)$$

Denoting the two components as $\nu \equiv \nu_L$ and $N = C(\bar{\nu}_R)^T$, where C is the Dirac chargeconjugation matrix, the mass term can be written in the form

$$\alpha_{mass} = (\nu^T\ N^T)\ MC\ \binom{\nu}{N} + H.C. \qquad (6.3.4)$$

where the neutrino mass matrix is

$$M = \begin{pmatrix} 0 & \frac{1}{2}h_1 k \\ \frac{1}{2}h_1 k & -h_5\upsilon_R \end{pmatrix}. \qquad (6.3.5)$$

The physical eigen states can be written as

$$\nu_e = \nu\cos\xi + N\,\mathrm{Sin}\xi$$
$$N_e = -\nu\sin\xi + N\cos\xi \qquad (6.3.6)$$

where the left and the right handed neutrino mixing parameter is written as,

$$\tan 2\xi = - \frac{h_1 k}{h_5 \upsilon_R} \tag{6.3.7}$$

Diagonalization of the mass matrix then yields the eigen values, when

$$\upsilon_R^2 >> k^2$$

$$m_{\nu e} = \frac{h_1^2 k^2}{4 h_5 \upsilon_R} \tag{6.3.8}$$

$$m_{Ne} = - h_5 \upsilon_R \tag{6.3.9}$$

Using $M_{W_R} = \frac{1}{\sqrt{2}} g\upsilon_R$, $m_e = h_1 k$ and assuming $|h_5| \simeq |h_1| = h$ yields

$$m_{Ne} \simeq \frac{h}{g} M_{W_R} \tag{6.3.10}$$

$$m_{\nu e} \simeq \frac{h}{g} \frac{m_e^2}{M_{W_R}} \tag{6.3.11}$$

Note that with the assumed hierarchy $\upsilon_R^2 >> k^2$, the formulae (6.3.8) - (6.3.11) are obtained with more exactness as compared to the corresponding ones discussed in chapter 5. Here although the final formula is the same for the Majorana neutrino masses, the intermediate steps involve less parameters and approximations.

6.4 SUMMARY

With the imposition of constraint arising out of minimal fine tuning, the VEV's are restricted both in LRS and LRA models with $V_L=k'=0$. In the LRA model, the presence of left-handed doublets (χ_L) or triplets (Δ_L) are needed, since the $\phi(2,2,0)$ executes the spontaneous symmetry breaking of $SU(2)_L X U(1)_Y$. The gauge boson masses and the neutral current parameters assume simpler forms. In both the LRS and LRA cases, the atomic parity violating parameters C_{1u}, C_{1d}, C_{2u} and C_{2d} yield identical expressions as the standard model. Similarly two of the parameters in $e^+e^- \longrightarrow \mu^+\mu^-$ or $\tau^+\tau^-$, namely h_{AA} and h_{VA}, have identical forms as the standard model. Also the parameter g_A in ν_μ-e and ν_e-e scattering remains unaffected. But the neutral current parameters g_V and h_{VV} are significantly affected compared to the standard model, such that accurate measurements of these parameters might provide a lower limit on the $W_R^\pm$ and Z_R gauge boson masses. The parameters $\varepsilon_L(u)$, $\varepsilon_R(u)$, $\varepsilon_L(d)$, $\varepsilon_R(d)$ occuring in ν-hadron processes are modified in the same manner in both LRS and LRAmodels with the minimal fine-tuning constraint.

The expression for the neutral current parameters discussed in this and other chapters have been derived at the tree level. Radiative corrections might change them to an extent by which our conclusions and statements, drawn in this chapter are expected to be valid within good approximation.

CHAPTER 7

DISCUSSION AND CONCLUSION

The $SU(2)_L X U(1)_Y$ electroweak gauge model, discovered by Glashow, Weinberg and Salam is broken spontaneously to $U(1)_{em}$ at the W_L-boson scale by a standard doublet carrying $Y= +1$. Predictions of the broken gauge symmetry on the neutral current parameters, $\sin^2\theta_W$, $W_L^{\pm}$ and Z_L boson masses and CP-violation in weak decays agrees remarkably well with experiments. Although, experimentally, there is no compelling reason at present to search for alternatives to the standard model based upon $SU(3)_C X SU(2)_L X U(1)_Y$, there are many aesthetic reasons which point out that the standard model might not be the ultimate gauge theory of basic forces in nature. One such motivation is the neutrino mass; indications in favour of neutrino mass seem to be positive in various experiments. Further, solution of the solar neutrino puzzle requires a tiny neutrino mass. Another such motivation is the origin of parity violation. In the standard model, the fermions transform differently under left and right gauge transformations and parity violation seem to be intrinsic. Similar to the spontaneous breaking of gauge symmetry, the breaking of parity might have also a spontaneous origin only if it is restored at a scale much larger than the W_L-boson mass. A gauge model with spontaneous parity violation might provide a spontaneous origin to weak CP violation also. In the years

1973-1975, Pati, Salam, and Mahapatra, proposed models based upon left-right symmetric gauge theory, which, during the course of its development, agreed with all available experimental data. Besides they fulfilled the objectives of establishing, theoretically, that parity (P) and weak CP symmetries, present in LRS theories could break down spontaneously. In the Pati-Salam model, a completely new additional possibility called quark-lepton unification was also achieved through the gauge group $SU(2)_L X SU(2)_R X SU(4)_c (g_L=g_R)$. Developing the idea of P and CP-violation proposed by Mahapatra and Pati, in the years 1974-75, Mahapatra and Senjanovic showed how Majorana neutrino masses can be generated in left-right symmetric gauge models.

In this dissertation, we have reviewed the electroweak gauge model based upon $SU(2)_L X U(1)_Y$ and compared its predictions with experiments. We have reviewed how spontaneous symmetry breaking of gauge models based upon $SU(2)_L X SU(2)_R X U(1)_{B-L}$ with both $g_L=g_R$(LRS) and $g_L \neq g_R$(LRA), either directly to the standard group, or through the intermediate symmetry, $SU(2)_L X U(1)_R X U(1)_{B-L}$, takes place with appropriate choice of Higgs sector, capable of generating Majorana neutrino masses over a wide range. The phenomenology of computing the neutral current parameters in terms of the VEV's and electroweak mixing angles has been discussed. It is well known by now from various phenomenological analyses that if $\sin^2\theta_W \simeq 0.23$, $M_{W_R}^{\pm} \geq 1$ TeV-few TeV, or larger, and $M_{Z_R} \geq 500$ GeV, predictions of LRS and LRA models on charged and neutral current parameters and also on CP-violating parameters agree well with the existing data at low energies.

In the LRS or LRA models, breaking spontaneously directly to the Weinberg-Salam model, the Majorana neutrino mass is governed by the relation

$$m_{\nu_i} \simeq \frac{m_i^2}{M_{WR}} , \; m_{N_i} \simeq M_{WR}, \; i = e, \mu, \tau \qquad (7.1.1)$$

where m_i stands for the charged lepton mass of the i-th family (generation) and m_{Ni} is the corresponding right-handed Majorana neutrino mass. In these cases $m_{\nu i} \longrightarrow 0$ $M_{WR} \longrightarrow \infty$, since M_{ZR} and M_{WR} are decoupled from each other. Thus in this case vanishing neutrino mass is a consequence of V-A limit of neutral currents only. The Majorana mass ranges for widely varying values of $M_{WR}^{\pm}$ and M_{ZR} have been computed and compared with the existing experimental data.

Finally we make some new observations on LRS and LRA models, in view of the constraint imposed by minimal fine tuning of parameters in gauge theories. It is noted under such a constraint that, for the spontaneous breaking of LRA gauge symmetry, or for $SU(2)_L XU(1)_R XU(1)_{B-L}$, the left-handed doublet or triplet (under $SU(2)_R$), are not necessary. Both for LRA and LRS models, the VEV's should be such that V_L=k'=0. In these cases the atomic parity violating neutral current parameters, C_{1u}, C_{1d}, C_{2u}, C_{2d} and the parameters g_A, h_{VA} and h_{AA} occuring in ν-e and $e^+e^- \longrightarrow \mu^+\mu^-$ have identical expressions as the standard model. Accurate experimental measurements of the parameters g_V and h_{VV} might set a lower limit for $W_R^{\pm}$ or Z_R gauge boson masses.

In particular, in the LRS model we derive that the changes in g_V and h_{VV}, compared to the Weinberg-Salam model, can be expressed as

$$\delta g_V \simeq \frac{M^2_{W_L}}{M^2_{WR}} (g_V)_{std.} \tag{7.1.3}$$

$$\delta h_{VV} \simeq \frac{2\, M^2_{W_L}}{M^2_{WR}} (h_{VV})_{std.} \tag{7.1.4}$$

where $(g_V)_{std.}$ and $(h_{VV})_{std.}$ denote the corresponding standard model predictions.

However, these statements are valid at the tree-level only and radiative corrections might change them. Since radiative corrections are usually small, our conclusions and statements are expected to be valid, at least, approximately.

REFERENCES

REFERENCES

1. S.L. Glashow, Nucl. Phys. **22**, 579 (1961).
2. A. Salam, in Elementary Particle Theory ed. by N. Svartholm, 367 (1969).
3. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).
4. E.S. Abers, and B.W. Lee, Phys. Rev. **9c** (1973).
5. C. Quigg, Gauge theories of strong weak and electromagnetic interaction (Cambridge Univ. Press, London, 1976).
6. E.C.G. Sudarshan and R.E. Marshak, Phys. Rev. **109**, 1860 (1958); R.P. Feynman and M. Gell-Mann, Phys. Rev. **109**; 193 (1958).
7. J.E. Kim, P. Langacker, M. Levine and R. Williams, Rev. Mod. Phys. **53**, 211 (1981).
8. E. Locci, UA1 collaborations, Proc. of the international Euro Physics Conf. on HEP, Vol. **1**, 11 (1987).
9. The UA2 collaborations, R. Ansari et al, Phys. Lett. B. **186**, 440 (1987).
10. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
11. J.C. Pati, and A. Salam, Phys. Rev. D**10**, 275 (1974); R.N. Mahapatra and J.C. Pati, Phys. Rev. **11**, 566, **11**, 2558 (1975); G. Senjanovic and R.N. Mahapatra, Phys. Rev. **12**, 1502 (1975).
12. M.K. Parida, A. Roychoudhuri, Phys. Rev. D**26**, 2364 (1982).
13. P.W. Higgs, Phys. Lett. **12**, 132 (1964), Phys. Rev. Lett. **13**, 508 (1964); Phys. Rev. **145**, 1156 (1966); T.W.B. Kibble, Phys. Rev. **155**, 1554 (1967).
14. J. Goldstone, Nuovo Cimento, **19**, 154 (1961).
15. S.L. Glashow, J. Illiopoulous, and L. Maiani, Phys. Rev. D**2**, 1285 (1970).
16. W.J. Marciano, and A. Sirlin. Phys. Rev. D**29**, 945 (1984).
17. A. Sirlin and W.J. Marciano,Nucl. Phys. B**189**, 442 (1981); C. Clewellyin Smith and J. Wheater, Phys. Rev. Lett. **105**B, 486 (1981).
18. W. Wetzel, Z. Physics C**11**, 117 (1981).

19. W.J. Marciano and Z. Parsa, -Ann. Rev. of Nucl. and Part. Sci, Vol. **36**, (171-205) ed. J.D. Jackson, H.E. Gove and R.F. Schwitters (1986).

20. M.M. Nagels et al, Nucl. Phys. B 147, 189 (1979).

21. CHARM Collab, J.V. Allaby et al, Phys. Lett. B **177**, (1986).

22. CC FR Collab, F. Merrit, presented Proc. 12th Intern. Conf. on Neutrino physics and astrophysics (1986).

23. FMMF collab, R. Brook, presented Proc. 12th Intern. Conf. on Neutrino Physics and astrophysics, 1986.

24. P.Q. Hung and J.J. Sakurai, Phys. Lett. **63B**, 295 (1976); Phys. Lett. **69B**, 323 (1977).

25. G. Rajasekaran and K.V.L. Sharma, Pramana **2**, 62 (1974); M.K. Parida and G. Rajasekaran Pramana, **13**, 183 (1979), Hazra and M.K. Parida, Pramana **19**, 73 (1982).

26. C. Geweniger in Proc. Neutrino, -'79, Bergen (1979), ed . A. Haatuft, and C. Jarlskog, vol. 2, 392 (1979).

27. M. Jonker et al, Phys. Lett. **99B**, 265 (1981).

28. J. Blietschau et al, Phy. Lett B**88**, 381 (1979).

29. F.A. Harris et al, Phys. Rev. Lett. **39**, 437 (1977).

30. M. Derric et al, Phys. Rev. D**18**, 7 (1978).

31. J. Marriner, Lawrence, Berkeley, Lab. Rep. LBL-6438 (1977).

32. B.P. Roe, Proc. Neutrino '79, Bergen, ed. by A. Haatuft, and C. Jarlskog, Vol. 2, 592 (1979).

33. J. Bell, et al, Phys. Rev. Lett. (1979).

34. P.Q. Hung, Phys. Lett **69B**, 217 (1977).

35. L.M. Sehgal, Phys. Lett. **71B**, 99 (1977).

36. J. Okada, and S. Pakvasa, Nucl. Phys. **B112**, 400, (1976).

37. H. Kluttig, J.G. Morfin and W. Van Doninck, Phys. Lett. **71B**, 446 (1977).

38. F. Dydak, 1979. Proc. 1979 EPS Intern. Conf. on H.E.P. 25, CERN.

39. A. Entenberg, et al, Phys. Rev. Lett 42, 1198 (1979).

40. W. Lee et al, Phys. Rev. Lett **38**, 202 (1977).

41. H. Faissner et al, Phys. Rev. D**21**, 555 (1980).

42. M.L. Pohl et al, Phys. Lett. B**72**, 489 (1978).

43. P. Langacker et al, in Proc. Neutrino-79, ed. A. Haatuft, C. Jarlskog (Astvedt Industrier, A/S Bergen) vol. 1, 276 (1979).

44. C.Y. Prescott et al, Phys. Lett. B**77**, 347 (1978), Phys. Lett. B84, 524 (1979).

45. R.N. Cahn and F.J. Gilman, Phys. Rev. D**17**, 1313 (1978).

46. G.E.P. Baird, Phys Rev. Lett 39, 798 (1977), Lewis, Phys. Rev. Lett. **39**. 795 (1977).

47. L.M. Barkov and M.S. Zolotov, 1978a, Zh. Eksp. Theor. Fiz.Pis'ma Red **27**, 379 [JETP Lett **27**, 357 (1978)], in Neutrino '78, ed. by E.C. Fowler 423 (1978).

48. R. Conti et al, Phys. Rev. Lett. **42**, 343 (1979).

49. E.D. Commins, in Neutrino '79. ed. by A. Haaluft and C. Jarlskog, (Astvedt Industrier A/S, Bergen) Vol 1, 69 (1979).

50. W. Bartel et al, Phys. Lett. **99**B, 281 (1981).

51. P.G. Rentens et al, Phys Lett. **152**B, 404, (1985).

52. D. Bogert et al, Phys. Rev. Lett. **55**, 1969 (1985).

53. F. Bergsma, Phys. Rev. Lett. **22**, 2401 (1985).

54. L.A. Ahrens et al, Phys. Rev. Lett. **54**, 18 (1985).

55. K. Abe, et al, Phys. Rev. Lett. **56**, 1107 (1986).

56. J.V. Allaby et al, Phys. Lett B, **177**, 446 (1986).

57. A. Abramowicz et al, Phys. Rev. Lett. **57**, 298 (1986).

58. A. Abramowicz et al, Z. physics, **C28**, 51 (1985).

59. G.T. Jones, et al, Phys. Lett B**178**, 329 (1986).

60. T. Kafka et al, Phys. Rev. Lett **48**, 910 (1982).

61. L.A. Ahrens et al, Phys. Rev. D35, 785 (1987).

62. F. Bergsma et al, Phys. Lett. 147B 481, (1984).

63. U. Amaldi et al., A comprehensive analysis of data pertaining to the weak neutral current and the intermediate vector boson masses, UPR-0331T. Also, G. Costa et al, Neutral currents within and beyond the standard model, CERN - TH 4675/87, MAD/TH/87-07 LBL 23271.

64. M. Fritshi et al., Phys. Lett. 173B, 485 (1986).

65. G. Senjanovic, Nucl. Phys. **B153**, 334 (1979).

66. G.C. Branco and G. Senjanovic, Phys. Rev. **D18**, 1621 (1978).

67. E. Witten, Phys. Lett. B91, 81 (1980).

68. R.N. Mahapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

69. M. Gell-Mann, P. Ramond and R. Slansky in Supergravity, ed. by Van Nieuwenhuizen and Freedman (North Holland, Amsterdam, 1979).

70. R.N. Mahapatra and G. Senjanovic, Phys. Rev. D23, 165 (1981).

71. A. Halprin, P. Minokowski, H. Primakoff and J.P. Rosen, Phys. Rev **D13**, 2567 (1976).

72. W.C. Haxton and G.J. Stephenson Jr., Prog. in part. and Nucl. Phys. **12**, 409 (1984).

73. K. Grotz and H.V. Klapdro, Phys. Lett **153B**, 1 (1985).

74. P. Vogel and P. Fisher, Phys. Rev C, **32**, 1362 (1985).

75. M.K. Parida and C.C. Hazra, Phys. Lett. **121B**, 355 (1983).

76. T.G. Rizzo and G. Senjanovic, Phys. Rev. D24, 704 (1981).

77. Talk at Europhysics study Conf. on Electroweak interaction at HEP (1983).

78. Particle Data gr., Rev. Mod. Phys. **56**, S1 (1984).

79. K.E. Bergkvist, Nucl. Phys. **B39**, 317 (1972).

80. P.R. Burchat et al, Phys. Rev. Lett **54**, 2489 (1985).

81. S. Boris et al., Phys. Rev. Lett. **58**, 2019 (1987). V.A. Lyubimov, Proc. XXIIth Int. Conf. on H.E.P. ed by Meyer and Wieczorek, Vol II, 108 (1984).

82. S.E. Csorna et al, Phys. Rev. **D35**, 2747 (1987).

83. S. Abachi et al, Phys. Rev. **D35**, 2888 (1987).

84. R. Cowsik and J. Mc. Clelland, Phys. Rev. Lett. **29**, 669 (1979).

85. J.N. Bahcall and S.L. Glashow, Nature, **326**, 426 (1987).

86. K.V.L. Sharma, Resonant Conversion of neutrino and the solar neutrino puzzle, Proc. of the HEP. Symp. (1986) ed. by M.K. Pal and G. Bhattacharjee.

87. H. Kawakami et al, Phys. Lett. **B187**, 198 (1987).

88. J.F. Wilkerson et al, Phys. Rev. Lett **58**, 2023 (1987).

89. M. Roos, Neutrino Mass .... from SN 1987A Bursts. Session P15. also J.A. Cowan and W.W. Repko Subm. to H.E.P. 87.

90. ARGUS collaborations presented by H. Kolanoski, Univ. of Dortmund, in Proc. of Intern. Europhysics Conf. on H.E.P. Vol. 1, p.577 (1987), ed. by O. Botner, Uppsala Univ.

91. F. del Agula and L. Ibanez. Nucl. Phys. **B177**, 60 (1981); R.N. Mahapatra and G. Senjanovic, Phys. Rev. **D27**, 1601 (1983).

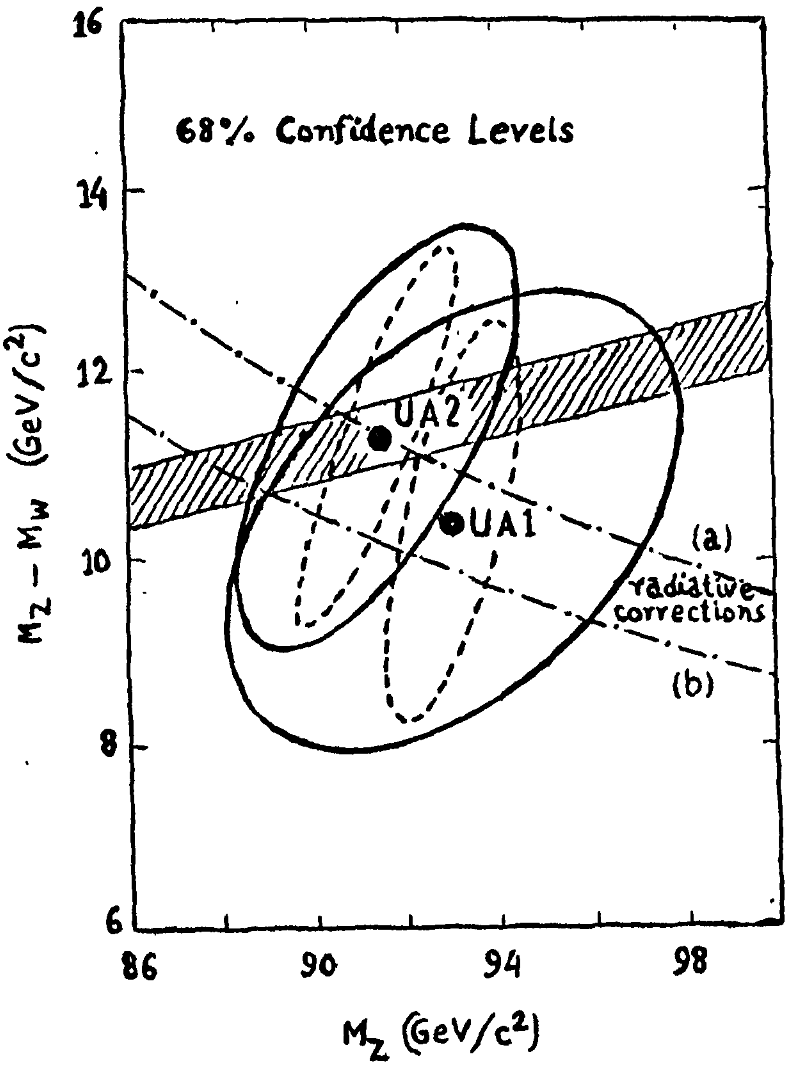


Fig. 3.1 Confidence contours (68% CL) in the (mz-mw, mz) plane, taking into account the statistical erros only (dashed curve) and with statistical and systematic erros combined in quadrature (solid curve). The dashed region is allowed by the average of recent low-energy measurement. Curve (a) is the SM prediction for $\rho=1$ with known radiative corrections, and curve (b) is the expectation without radiative corrections.

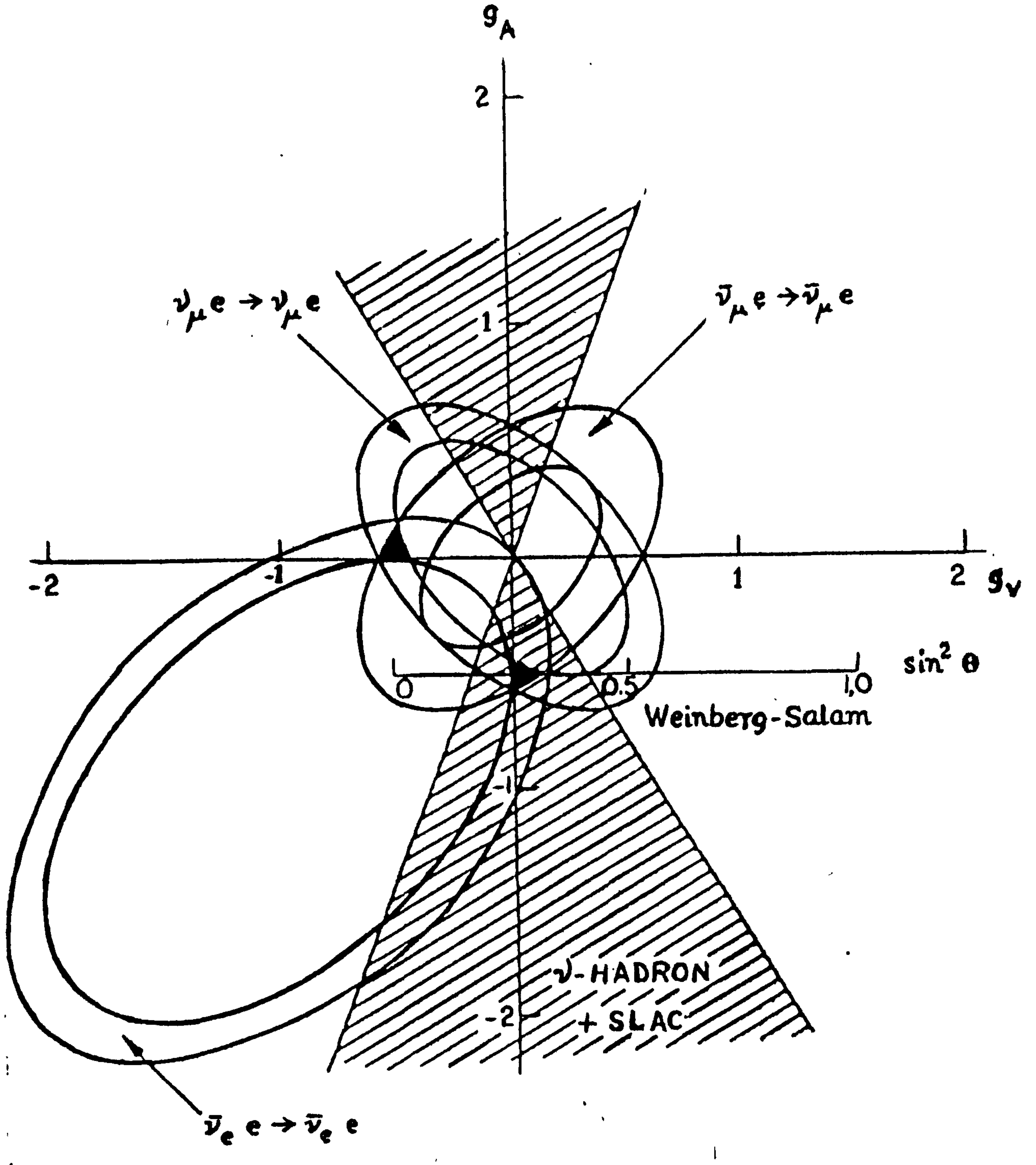


Fig. 3.3 Determination of the neutrino-electron parameters g_V and g_A. The shaded region indicates the constraints imposed by factorization.

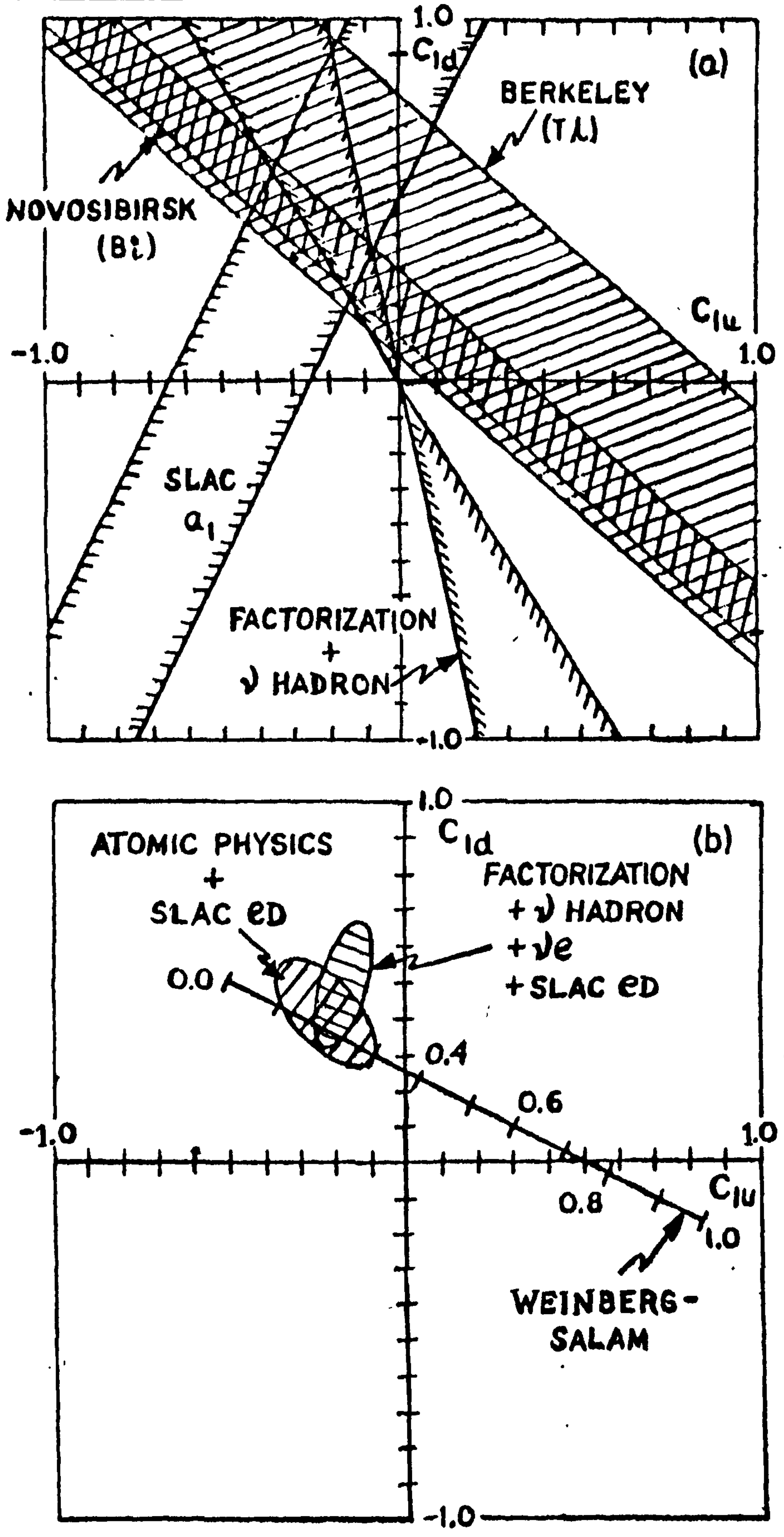


Fig. 3.4 Region of C_{1u} and C_{1d} allowed
(a)(i) the SLAc eD experiment
(ii) Novosibirsk & Berkeley atomic physics experiments
(iii) ν-hadron scattering, (b) allowed regions for a simultaneous fit t
(i) the eD and atomic physics results, or (ii) the ν-hadron, νe
and eD results. (assuming factorization)

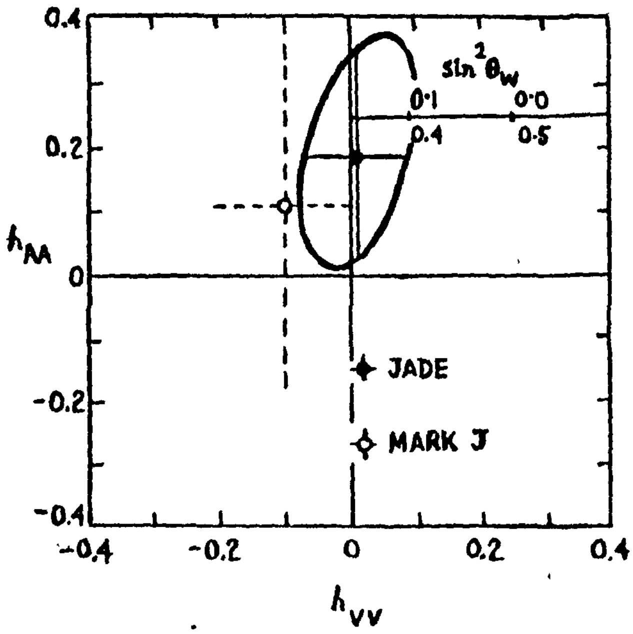


Fig. 3.5 Determination of the $(\bar{e}e)$ $(\bar{\mu}\mu)$ parameters, h_{VV} and h_{AA}

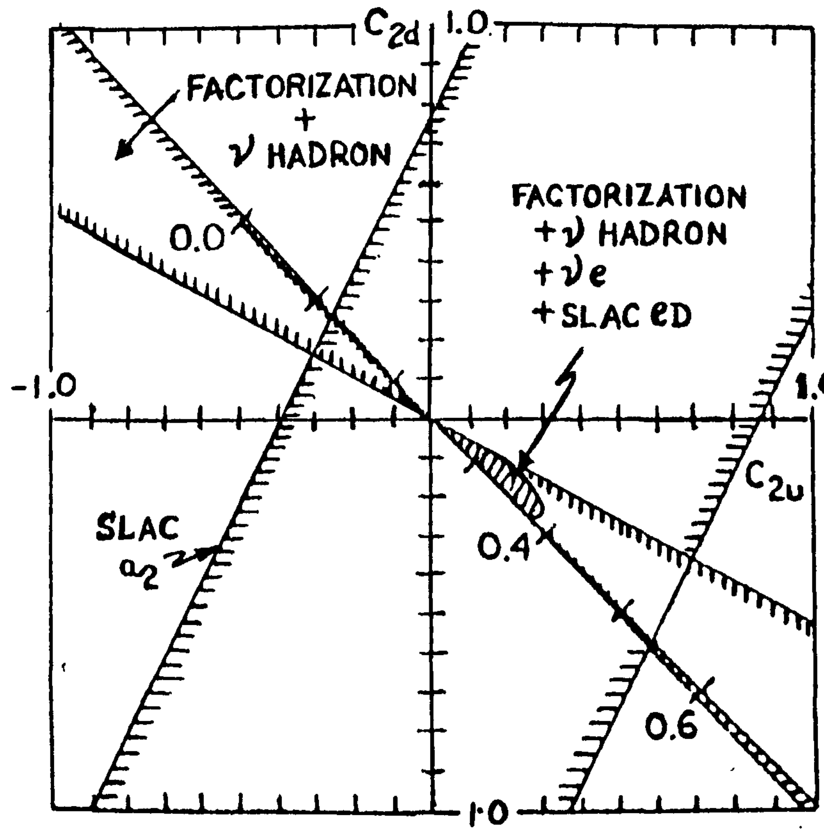


Fig.3.6 Region of C_{2u} and C_{2d} allowed

(i) the SLAC eD experiment

(ii) ν-Hadron scattering

(iii) a simultaneous fit to the ν-hadron, νe and SLAC eD results (assuming factorization)

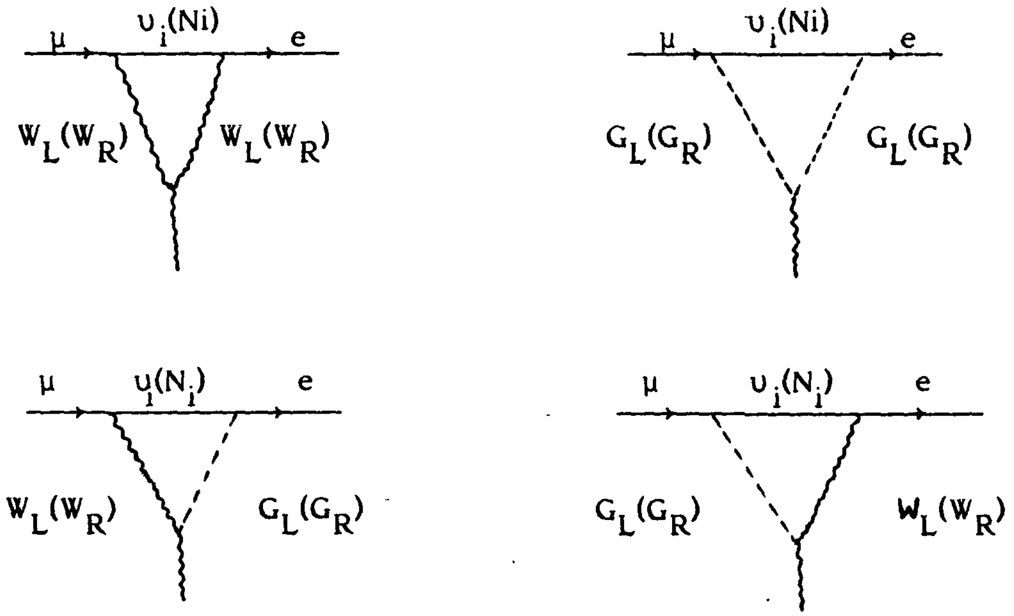


Fig.4.2 The leading diagram for a lepton-flavour changing process $\mu \to e\,\gamma$. Again the process goes through the exchange of υ_i and W_L or N_i and W_R. In addition, due to the GIM mechanism, the Goldstone boson exchange (denoted by G_L and G_R) are comparable in strength to gauge boson mediated amplitude. The physical Higgs particle exchanges are ignored by assuming $m_H >> m_W$.

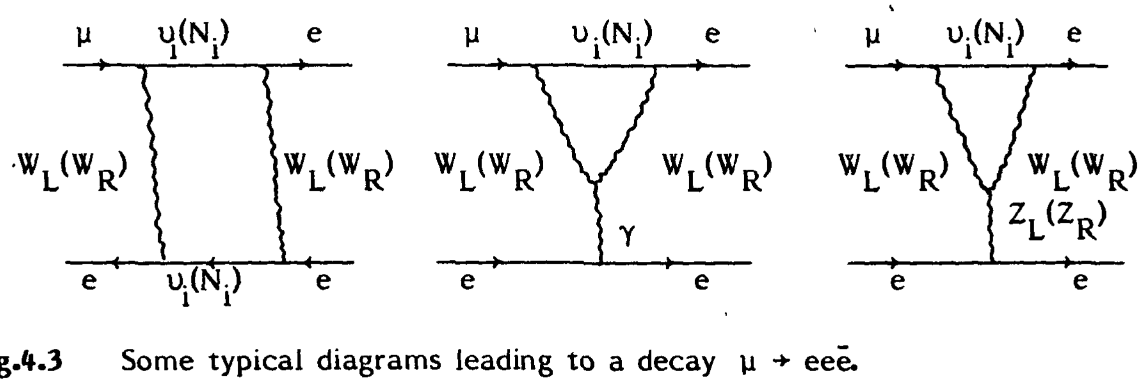


Fig.4.3 Some typical diagrams leading to a decay $\mu \to e e \bar{e}$.

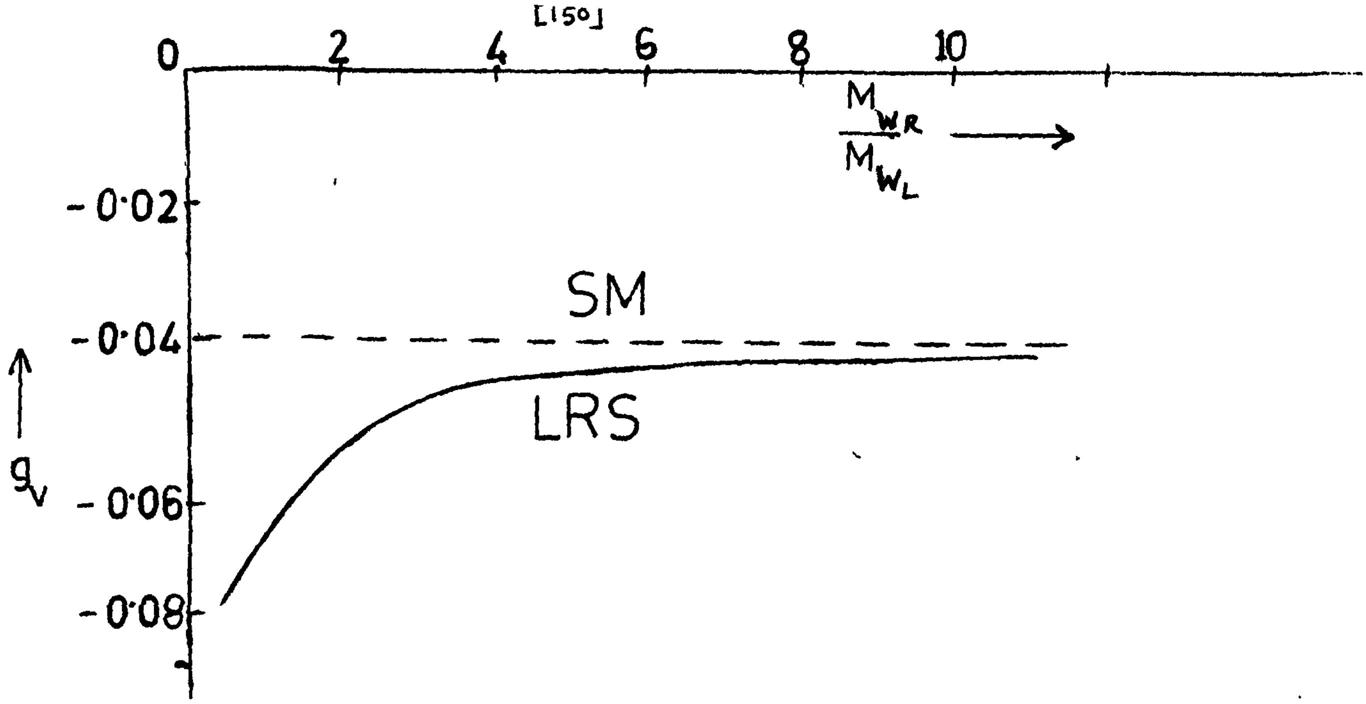


Fig.6.1 Comparison of left-right symmetric (LRS) model prediction on g_V (solid line) with the standard model (SM) prediction (dashed line)

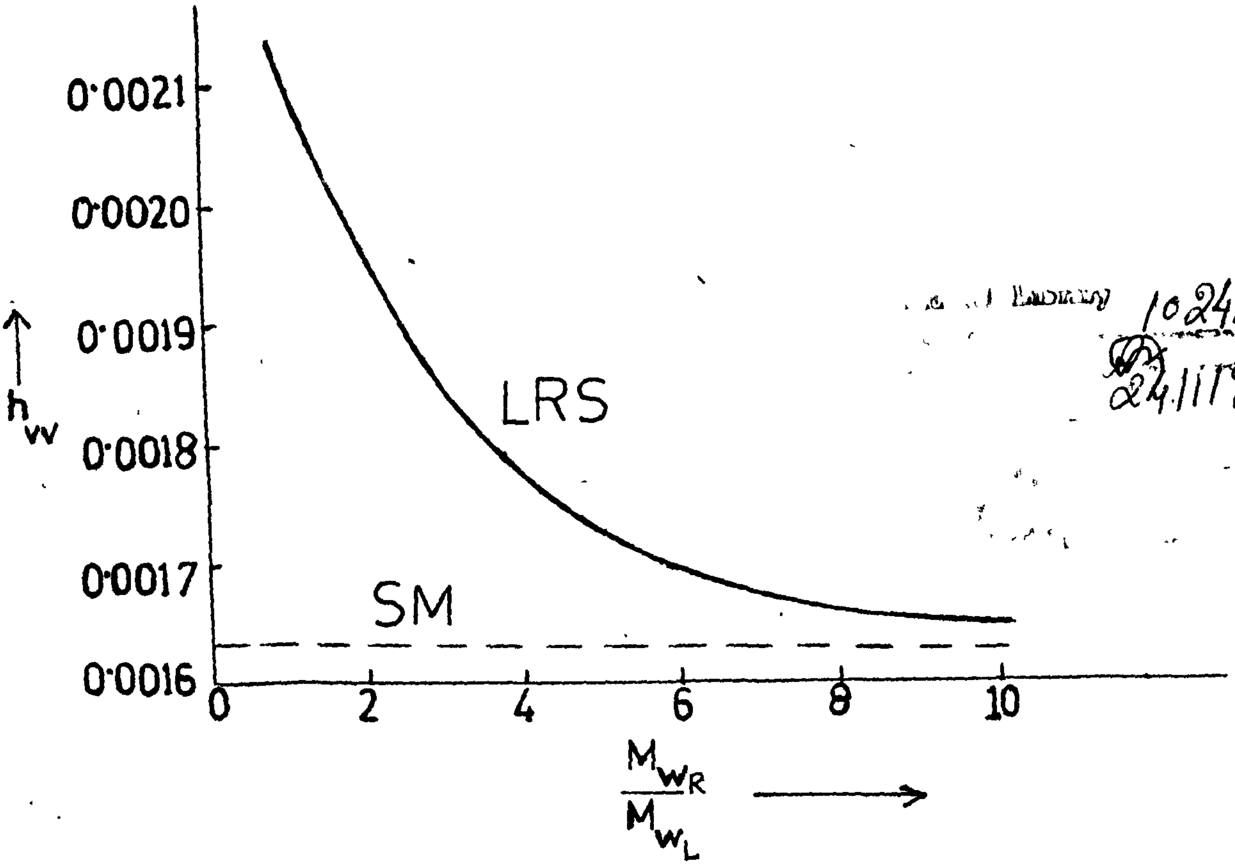


Fig. 6.2 Comparison of LRS model prediction on h_{VV} (solid line) with the standard-model (SM) prediction (dashed line).