

A CROSSOVER SCALING STUDY OF THE
QUASI - TWO - DIMENSIONAL ISING MODELS

ABSTRACT

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I certify that the thesis entitled " A Crossover Scaling Study of the Quasi-two-dimensional Ising Models ", submitted by Miss Wendy Loslet Basaiawmoit for the Degree of Doctor of Philosophy of the North-Eastern Hill University, Shillong embodies the record of original investigation carried out by her under my supervision. She has been duly registered and the thesis presented is worthy of being considered for the Award of the Ph.D. Degree. This work has not been submitted for any Degree of any other University.

Date : 10th January, 1983.

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Signature of the Supervisor

Place : Shillong.

Abstract

Quasi-two-dimensional models have been of recent experimental and theoretical interest (1,2). These are really three-dimensional, but the interactions in the off-plane directions are much weaker than the intraplanar bonds. The crucial parameter determining the critical behaviour of such a system is g , the ratio of the two types of interactions. For relatively large values of g , say $\gtrsim 10^{-2}$, the system shows a three-dimensional behaviour. In this case, standard Padé approximant methods can be used to describe the system for the whole temperature range. On the other hand, if g is smaller, the system shows the true three-dimensional behaviour only on approaching the critical temperature very closely. If one moves away from the critical temperature, one sees a crossover to the two-dimensional behaviour for a range of temperatures. This can be described nicely in terms of the crossover scaling theory of Riedel and Wegner (3) as extended by Pfeuty, Fisher and Jasnow (4).

The aim of this dissertation was *two-fold*. *Firstly*, it was proposed to check the predictions of the crossover scaling theory in detail using the high-temperature series expansions. The second aim was to construct accurate approximants for thermodynamic quantities valid within the critical region. It was decided to restrict this study to the ordering susceptibility because, being the most singular thermodynamic quantity, it is highly suitable for studying the critical behaviour. Also, the information gotten from this about critical temperatures etc. can be used gainfully for weakly divergent quantities like specific heat etc.. The lattice models studied are the s.q. to s.c. and the s.q. to f.c.c. Ising models.

The plan of this dissertation is as follows. In Chap-

ter 1, a brief history of the Ising model, of the scaling and universality and also of crossover phenomena are given. An introduction to the present model is included in this Chapter. The scaling theory and its predictions and also a summary of the previous works on the present model is described in Chapter 2. Chapter 3 contains a summary of the extrapolation methods and also our study of the isotropic behaviour of the model. We have used the standard Padé and graphical ratio methods for extrapolation from high-temperature series expansions. By using the exact values of the exponents and $T_c(0)$, we obtain the first five universal amplitudes. This gives us a low- x expansion of the scaling function $X(x)$. In Chapter 4, we obtain the anisotropic behaviour of our model. We first obtain the range in which the crossover scaling theory applies. Then we use Fisher's method (4) and other standard methods to obtain $T_c(g)$ and $A(g)$ for $g \neq 0$ for a range of values of g . These help us to obtain the values of the universal parameters x , X . Construction of the isotropic and anisotropic scaling functions using two-point Padé approximants is carried out in Chapter 5. Also included in this Chapter is the calculation of the effective exponent of the susceptibility. We present detailed graphs of this exponent versus temperature in the *critical* region for various values of anisotropy for both the lattices. Chapter 6, which is the concluding Chapter contains remarks about other lattices and comparison with other results.

The following results emerge from this study.

1. The detailed universal behaviour of the model near $g = 0$ is described by the extended scaling theory (4) quite accurately. We verify this by calculating the first five universal amplitude ratios.

2. The predictions of the theory about the double-power scaling laws, scale-factor universality etc. are verified and the universal parameters are calculated within an accuracy of 5%.

3. The universal scaling function for the dimensional cross-over for the susceptibility is obtained for all temperatures in the critical region. The graphs of the effective exponent are presented for a range of values of anisotropy in this regime. (5)

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