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Scalar and Spinor Fields in the Very Early Universe

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Here it is shown how the vacuum energy may dominate the energy density of the very early universe even when the Higg's field in the Coleman–Weinberg potential is confined near the origin at extremely high temperature and the inflationary scenario may start. Also it is shown that supersymmetry breaking may be responsible for this phenomenon. Thus it provides another support for the hypothesis of primordial inflation proposed by Ellis et al. [4].

1. INTRODUCTION

In the past few years, the chaotic state of the early universe has attracted the attention of many physicists. In 1981, Guth [1] proposed an inflationary model of the universe to resolve three cosmological problems of the standard scenario: (i) the horizon problem, (ii) the flatness problem, and (iii) the monopole problem. But Guth's model suffered from the problem of the "graceful exit" from the de Sitter phase. In 1982, Linde [2] and Albrecht and Steinhardt [3] proposed a modified scenario known as "new inflationary universe." In this model, the problem of the original inflationary universe is evaded by using the Coleman–Weinberg potential for the Higg's field in place of the generic Higg's potential. The key feature of the "new inflationary model" is the realization that, under spcial circumstances, it is possible for the observed universe to have evolved from a single bubble or fluctuation region. This kind of evaluation is possible for Coleman–Weinberg potential but it is not possible for a generic Higg's potential. The universe expands exponentially as the Higg's field Φ begins to oscillate about its minimum [1].

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In these models it is considered that the vacuum energy $V(0)$ (which introduces the cosmological constant) dominates other types of energy. Now one can ask: "How could the vacuum energy reach such a dominating stage after the big-bang?" To answer this question, Guth [1] suggested that the temperature falls rapidly due to the expansion of the universe, so that other types of energy may be neglected in comparison to the vacuum energy. In this paper we examine this question in a quite different manner.

It is well-known that due to its extremely high temperature the very early universe contains matter in the ionized state. So it is reasonable to consider the contribution of fermions also, along with scalar fields, to the energy density of the early universe. We consider only spin-$\frac{1}{2}$ fermions since most of the fermions found in nature are spin-$\frac{1}{2}$ particles. Motivated by this idea, an action integral for a spin-$\frac{1}{2}$ field ψ and a scalar field ϕ, is written as

$$J=\frac{1}{2}\int(-g)^{1/2}[\bar{\psi}(i\gamma^{\mu}\psi_{;\mu}-m\psi)+g^{\mu\nu}\partial_{\mu}\phi^{*}\partial_{\nu}\phi-f\bar{\psi}\psi(\phi^{*}\phi)-V(\phi)]\,d^{4}x \quad (1)$$

where f is the coupling constant, $f\bar{\psi}\psi(\phi^*\phi)$ is an interaction term between ψ and ϕ, γ^{μ} are dirac matrices, $\bar{\psi}$ is the conjugate of ψ, m is the mass of spinor field ψ, $g^{\mu\nu}$ is the metric tensor, and g is the determinant of $g_{\mu\nu}$. Here $V(\phi)$ is the finite temperature Coleman–Weinberg potential in $SU(5)$ gauge symmetry given as [4]

$$V(\phi,T)=A\phi^{4}\left(\log\frac{\phi^{2}}{v^{2}}-\frac{1}{2}\right)+\frac{1}{2}CT^{2}\phi^{2} \quad (2)$$

with

$$A=\frac{1}{64}\pi^{2}v^{4}\left[\sum_{\beta}g_{\beta}m_{\beta}^{4}-\sum_{F}g_{F}m_{F}^{4}\right] \quad (3)$$

where $g_{\mathrm{B(F)}}$ is the number of boson (fermion) helicity states of mass $m_{\mathrm{B(F)}}$, $c=(75/8)\,g^{2}$ [$g^{2}=0.3$ in minimal $SU(5)$], v stands for the vacuum expectation value of an adjoint multiplet of ϕ and is equal to 1.2×10^{15} Gev, approximately.

The aim of this paper is to introduce a cosmological model of the very early universe taking the above action as a source of energy density. The measurement of time (t) is from the Planck epoch, because due to our ignorance of quantum gravity it is difficult to investigate before Planck time. Here it is also assumed that, in the beginning, the model obeys supersymmetry (SUSY), but some time after the Planck epoch SUSY breaks down. It is interesting to note that as a result of SUSY breaking, ϕ decreases expontially and $V(\phi)\to V(o)=\frac{1}{2}Av^{4}$. Due to the high temperature, the expansion of the model also is very fast and the fermion energy density decreases rapidly with the expansion of the universe. Thus it is seen that even prior to the GUT phase transition, the breaking of SUSY yields an inflationary scenario as the energy density is dominated by $V(o)$. This result supports the idea of supersymmetric inflation or primordial inflation proposed by Ellis, Nanopoulous, Olive, and Tamvakis [4].

The paper is organized as follows. In Section 2 we solve the Einstein's equation using the energy density derived from the action (1) and taking a supersymmetric $V(\phi)$. Section 3 contains investigations on the time development of ϕ in the case when SUSY breaks down. In Section 4 we discuss our results briefly.

Here we set $\hbar = c = 1$, a semicolon (;) denotes covariant differentiation. A dot (.) over the variable denotes differentiation with respect to t. $M_p = 1/G^{1/2}$ (G is the Newtonian gravitation constant) $= 1.22 \times 10^{19}$ GeV.

2. CLASSICAL SOLUTION

The model of the universe is described by a Robertson–Walker (RW) line element

$$ds^2 = dt^2 - \left[a^2(t) \Big/ \left(1 + \frac{\varepsilon r^2}{4} \right)^2 \right] [dx^2 + dy^2 + dz^2] \tag{4}$$

where the cosmic time t is measured from the Planck epoch $t = 10^{-45}$ s, $r^2 = x^2 + y^2 + z^2$, ε is the scalar curvature having possible values $+1$, -1, and 0 for closed, open, and flat models, respectively.

The background model given by equation (4) is homogeneous and isotropic, hence the spinor field ψ and the scalar field ϕ in the action integral (1) will depend on t only. Hence, on applying the action principle, the action integral (1) yields

$$\left[\gamma^o \partial_o + \frac{3}{2} \gamma^o \frac{\dot{a}}{a} + i(m + f\phi^*\phi) \right] \psi = 0 \tag{5}$$

for the spinor field ψ and

$$\ddot{\phi} + (3\dot{a}/a)\dot{\phi} + f\bar{\psi}\psi\phi = -(\partial v/\partial \phi) \tag{6}$$

for the scalar field ϕ.

The equation (5) is integrated to yield

$$\psi a^{3/2} = \psi_o a_o^{3/2} \exp - i\gamma^0 \int (m + f\phi^*\phi)\, dt \tag{7}$$

and $\bar{\psi}$ its hermitian conjugate is

$$\bar{\psi}a^{3/2} = \bar{\psi}_o a_o^{3/2} \exp i\gamma^0 \int (m + f\phi^*\phi)\, dt \tag{8}$$

where $\psi_o(\bar{\psi}_o)$ is the spinor $\psi(\bar{\psi})$ at $t=0$ and $a_o = a(o)$.

From equations (7) and (8) we have

$$\bar{\psi}\psi a^3 = \bar{\psi}_o \psi_o a_o^3 \tag{9}$$

If the model obeys SUSY, fermions and bosons will be alike. It implies that $g_B = g_F$ and $m_B = m_F$. So from equation (3) we have $A = 0$ in SUSY [4]. Now we have

$$V(\phi) = \tfrac{1}{2} CT^2 \phi^2 \tag{10}$$

Connecting equations (6) and (10) we get

$$\ddot{\phi} + (3\dot{a}/a)\dot{\phi} + (fa^{-3}\bar{\psi}_o\psi_o a_o^3 + CT^2)\phi = 0 \tag{11}$$

Under the transformation

$$\tau = \int^t \frac{dt'}{a^3(t')} \tag{12}$$

Equation (11) reduces to

$$(d^2\phi/d\tau^2) + (fa^3\bar{\psi}_o\psi_o a_o^3 + CT^2 a^6)\phi = 0 \tag{13}$$

The WKB solution of equation (13) is like

$$\phi = \frac{\alpha_\pm}{(fa^3\bar{\psi}_o\psi_o a_o^3 + CT^2 a^6)^{1/4}} \exp\left[\pm i \int d\tau (fa^3\bar{\psi}_o\psi_o a_o^3 + CT^2 a^6)^{1/2} \right] \tag{14}$$

Temperature is supposed to be extremely high, even greater than 10^{17} GeV; hence, we can approximate equation (14) as

$$\phi \simeq \frac{\alpha_\pm}{c^{1/4} a^{3/2} T^{1/2}} \exp\left(\pm i \int c^{1/2} T\, dt \right) \tag{15}$$

The total energy density for ϕ is

$$\begin{aligned} \rho_\phi &= \frac{1}{2}\dot{\phi}^2 + v(\phi) \\ &= \left(\alpha^2 + \frac{1}{2}\right)\frac{c^{1/2}T}{a^3} + \frac{9\alpha^2}{4c^{1/2}Ta^3}\left(\frac{\dot{a}}{a}\right)^2 \end{aligned} \tag{16}$$

and energy density for ψ is

$$\rho_\psi = \frac{\bar{\psi}_o \psi_o a_o^3}{2a^3}\left(m + \frac{f\alpha^2}{c^{1/2}a^3 T}\right) \tag{17}$$

In the early universe we can safely assume $\varepsilon = 0$; hence, Einstein's equation, governing the dynamics during the era, is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_p^2}(\rho_\phi + \rho_\psi) \tag{18}$$

Connecting equations (16)–(18) and neglecting $f\alpha^2/c^{1/2}a^3T$ in equation (17), due to the extremely high value of temperature T in the denominator, we get

$$\left(a^3 - \frac{6\pi\alpha^2}{c^{1/2}M_p^2 T}\right)\left(\frac{\dot{a}}{a}\right)^2 = \frac{4\pi}{3M_p^2}(m\bar{\psi}_o\psi_o a_o^3 + \alpha^2 c^{1/2} T) \tag{19}$$

Here also we find that $[a^3 - (6\pi\alpha^2/c^{1/2}M_p^2 T)]$ is effectively equal to a^3 due to the presence of M_p^2 and T in the denominator. So, we get the differential equation (19) as

$$a\dot{a}^2 = \beta^2 \tag{20}$$

where

$$\beta^2 = \frac{8\bar{n}}{3M_p^2}(m\bar{\psi}_o\psi_o a_o^3 + \alpha^2 c^{1/2} T) \tag{21}$$

Equation (20) yields the solution

$$a = a_o(1 + \beta t)^{2/3} \tag{22}$$

3. SUSY BREAKING AND SCALAR FIELD

As SUSY is not a natural symmetry, it is assumed that it breaks down at a certain epoch after the Planck time. So in this case [4]

$$m_{\mathrm{B}}^2 - m_{\mathrm{F}}^2 = m_{\mathrm{s}}^2$$

where m_{s} is the effective supersymmetric mass splitting quantity. It implies that

$$A \neq 0$$

Hence

$$V(\phi) = A\phi^4[\log(\phi^2/v^2) - \tfrac{1}{2}] + \tfrac{1}{2}CT^2\phi^2 \tag{23}$$

At high temperature, ϕ is confined near the origin; hence, one can approximate $V(\phi)$ as

$$V(\phi) \simeq V(0) + \tfrac{1}{2}CT^2\phi^2 - \tfrac{1}{2}A\phi^4 \tag{24}$$

Connecting equation (6) with equations (22) and (24), we get the differential equation for ϕ, when SUSY breaks, as

$$(1+\beta t)^2\ddot{\phi} + 2\beta(1+\beta t)\dot{\phi} + f\bar{\psi}_o\psi_o\phi = 2A(1+\beta t)^2\phi^3 - CT^2\phi \tag{25}$$

taking the approximation $CT^2(1+\beta t)^2 \simeq CT^2$ because from equation (21) β is $O(T^{1/2}/M_p) \ll 1$ and t is also small. The differential equation (25) is integrated to

$$\phi^2 = c_1(1+\beta t)^{-1+\beta^{-1}(2c)^{-1/2}T} + c_2(1+\beta t)^{-1-\beta^{-1}(2c)^{1/2}T} - \frac{4A(1+\beta t)^2}{8\beta^2 - 2f\bar{\psi}_o\psi_o - 2CT^2} \tag{26}$$

Due to the very high temperature and small t, the solution (26) may be approximated as

$$\phi^{-2} \simeq c_1 \exp\{(2c)^{1/2}Tt\} + (2A/CT^2)\exp(2\beta t) \tag{27}$$

Equation (27) yields that effectively

$$\phi^2 \simeq \phi_0^2 \exp[-(2c)^{1/2}Tt] \tag{28}$$

because $C_1CT^2 \exp[(2C)^{1/2}Tt]$ will dominate the term $2A\exp(2\beta t)$. ϕ_0 is ϕ at $t=0$.

4. DISCUSSION

When SUSY breaks, the total energy density is given by

$$\rho = \frac{\bar{\psi}_o\psi_o}{2(1+\beta t)^2}\left[m + \frac{f\alpha^2}{c^{1/2}a_o^3(1+\beta t)^2T}\right] + \frac{\phi_0^2}{2T}\exp[-(2c)^{1/2}Tt] + V(o) - \frac{1}{2}A\phi^4\exp[-(2c)^{1/2}Tt] + \frac{1}{2}CT^2\phi_0^2\exp[-(2c)^{1/2}Tt] \tag{29}$$

Due to high temperature, terms containing exponential function in equation (29) will damp rapidly. The second term within the square bracket also will loose its effect after some time for the same reason. So, we find that after some time the energy density

$$\rho \simeq \frac{m\bar{\psi}_o \psi_o}{2(1+\beta t)^2} + V(o) \tag{30}$$

As time increases, the first term on the right-hand side of equation (30) also becomes less effective in comparison to $V(o)$. Thus $\rho \to V(o)$. Substituting this value of ρ in Einstein's equation we find that the universe expands as the de Sitter model like

$$a(t) \sim \exp\left(\frac{8\pi V(o)}{3M_p^2}\, t\right)$$

Thus one can see that the inflationary situation may start earlier than the GUT phase transition provided that SUSY breaks.

This inference is another support for the hypothesis of primordial inflation proposed by Ellis et al. [4].

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