Class. Quantum Grav. 10 (1993) 2307–2315. Printed in the UK

Effective (1 + 1)-dimensional cosmological model and fundamental constants

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Received 4 August 1992, in final form 27 April 1993

Abstract. An anisotropic cosmological model is investigated by solving Einstein's field equations in (1 + 3)-dimensional spacetime with topology $R^1 \otimes R^1 \otimes T^2$ (R^1 is the real line and T^2 is a two-dimensional torus). Through the process of Kaluze–Klein-type compactification on T^2 and one-loop quantum corrections to a scalar field, an effective action for (1 + 1)-dimensional gravity (with time-dependent gravitational constant G_{eff} as well as a cosmological constant, Λ_{eff}) are obtained.

PACS number: 9880

1. Introduction

Up until the past few years, the existence of gravity in (1 + 1)-dimensional spacetime has not been accepted due to the triviality of general relativity in these dimensions. Although Einstein's equations do not have any dynamical content in two-dimensional spacetime, many authors have shown very interesting features of (1 + 1)-dimensional gravity during the last few years [1–3]. Furthermore, two-dimensional gravity has important relationships with conformal field theory [4], the Liouville model [1], random lattice models [5] and non-linear sigma models [6–9]. Moreover, many interesting implications have also been noted for these theories [10–12].

Here, a different approach is adopted to get two-dimensional gravity via the usual method of Kaluza–Klein theory employing the idea of spontaneous compactification [13, 14]. Recently, this method was used by McGuigan [15] to get two-dimensional gravity from (1 + 3)-dimensional theory having spatial topology $S^1 \otimes T^2$, where S^1 is a circle and T^2 is the two-dimensional torus. In the present paper, an anisotropic (1 + 3)-dimensional cosmological model is derived. This model has spatial topology $R^1 \otimes T^2$. The isometry group $[U(1)]^2$ acts transitively on the compact manifold T^2. The number of Killing vectors corresponding to $[U(1)]^2$ is 2 as $U(1)$ has only one generator. The maximum number of Killing vectors in a 2-dimensional space is 3. However, as T^2 is isomorphic to $[U(1)]^2$, it can admit only two Killing vectors. Hence T^2 is homogeneous but not maximally symmetric. Kaluza–Klein-type compactification is done on T^2. As a result, (1 + 1)-dimensional gravity with a time-dependent background is obtained through one-loop quantum corrections to scalar fields. On adding the contributions of these quantum corrections to two-dimensional gravity to the dimensionally reduced two-dimensional gravity, one obtains an effective action for the

0264-9381/93/112307+09$07.50

same. Subsequently, a time-dependent effective gravitational constant and cosmological constant are obtained.

The focus of this paper is first getting a (1 + 3)-dimensional anisotropic cosmological model by solving Einstein's field equations exactly and second deriving a two-dimensional effective action for gravity. The paper is organized as follows. Section 2 contains a sketch of the cosmological model and an exact solution of the Einstein field equations for a background spacetime with topology $R^1 \otimes R^1 \otimes T^2$. Section 3 deals with the dimensional reduction of gravity and scalar fields. One-loop quantum corrections to a dimensionally reduced scalar field are computed in section 4. In section 5, the time-dependence of the gravitational constant as well as cosmological constants are discussed.

Natural units $\hbar = c = 1$ are used throughout the paper, where $\hbar$ and c have their usual meaning. A dot over a variable denotes the derivative with respect to a dimensionless parameter $\tau = t/t_{\mathrm{P}}$, where t is the cosmic limit and t_{P} is the Planck time.

2. (1 + 3)-dimensional anisotropic cosmological model

The cosmological model having topology as $R^1 \times R^1 \times T^2$ has the line-element

$$ds^2 = dt^2 - a^2(\tau)\, dx^2 - b^2(\tau)(\rho_1^2\, d\theta_1^2 + \rho_2^2\, d\theta_2^2) \tag{2.1}$$

where $0 \leq \theta_1, \theta_2 \leq 2\pi$, and ρ_1 and ρ_2 are physical radii of circles whose product is T^2; $a(\tau)$ and $b(\tau)$ are scale factors.

The energy–momentum tensor for the anisotropic fluid can be written as

$$T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu - (\delta p + \delta' p')g_{\mu\nu} \tag{2.2}$$

where $\mu, \nu = 0, 1, 2, 3$, ε is the energy density, p is the pressure on one-dimensional space R^1, p' is the pressure on T^2 and $\delta' = 1 - \delta$ with

$$\delta = \begin{cases} 1 & \text{for} \quad \mu, \nu = 0, 1 \\ 0 & \text{for} \quad \mu, \nu = 2, 3. \end{cases}$$

Thus

$$T^0_0 = \varepsilon \qquad T^1_1 = -p \qquad T^2_2 = T^3_3 = -p'. \tag{2.3}$$

In the background geometry given by line element (2.1), Einstein's field equations are

$$\frac{1}{2} G^0_0 = 2\frac{\dot a}{a}\frac{\dot b}{b} + \left(\frac{\dot b}{b}\right)^2 = -4\pi G t_{\mathrm{P}}^2 \varepsilon \tag{2.4a}$$

$$R^1_1 = \frac{d}{dc}\left(\frac{\dot a}{a}\right) + \frac{\dot a}{a}\left(\frac{\dot a}{a} + 2\frac{\dot b}{b}\right) = -4\pi G t_{\mathrm{P}}^2 (p + \varepsilon - 2p') \tag{2.4b}$$

$$R^2_2 = R^3_3 = \frac{d}{d\tau}\left(\frac{\dot b}{b}\right) + \frac{\dot b}{b}\left(\frac{\dot a}{a} + 2\frac{\dot b}{b}\right) = -4\pi G t_{\mathrm{P}}^2 (\varepsilon - p) \tag{2.4c}$$

where G is the four-dimensional Newtonian gravitational constant and G^μ_ν is the Einstein tensor. Moreover, conservation of the energy momentum tensor $T^\mu_{\nu;\mu} = 0$

implies that

$$\dot{\varepsilon} = \varepsilon\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right) + p\frac{\dot{a}}{a} + 2p\frac{\dot{b}}{b} = 0 \tag{2.5}$$

with the definition of $T_{\mu\nu}$ given by equation (2.2).

It is assumed that the physical quantities ε, p and p' obey the equations

$$\varepsilon = \frac{(1 + f^2\tau^2)(1 + 2f^2\tau^2)}{(6f^4\tau^4 + 11f^2\tau^2 + 5)}\, p = \tilde{p} \tag{2.6a}$$

$$\varepsilon = p'. \tag{2.6b}$$

$\tilde{p}$ is the pressure on one-dimensional space R^1 which was obtained absorbing

$$\frac{(1 + f^2\tau^2)(1 + 2f^2\tau^2)}{6f^4\tau^4 + 11f^2\tau^2 + 5} \qquad \text{in } p.$$

For the purpose of compactification to the effective two-dimensional gravity, a suitable form of the scale factor $b(\tau)$ is required. Firstly, it should be a decreasing function of time and secondly it should be free from the 'crack of doom' singularity (the reason for these conditions will be made clear in section 5). Keeping these requirements in mind, one can take an ansatz for $b(\tau)$ as

$$b^2(\tau) = f^2 + \bar{a}^2 \tag{2.7}$$

where f is a non-zero real constant.

Putting $b(\tau)$ given above by equation (2.7) in equations (2.4b) and (2.4c), adding these two equations and using conditions (2.6) one obtains

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\dot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 = 0 \tag{2.8}$$

which yields as a first integral

$$\dot{a} = c \text{ (constant)}. \tag{2.9}$$

Equation (2.9) is again integrated to

$$a = c\tau \tag{2.10}$$

with the condition $a(\tau = 0) = 0$. The integration constant c can be absorbed in a through rescaling $a \to ca$. Thus, one gets

$$a = \tau. \tag{2.11}$$

As a result, from equation (2.7)

$$b^2 = f^2 + \tau^{-2} \tag{2.12}$$

which is free from a 'crack of doom' singularity because

$$\lim_{\tau \to \infty} b = f.$$

Using equations (2.6), (2.11) and (2.12) in (2.5), one gets, on integrating,

$$\varepsilon = \varepsilon_0 \frac{(1 + 2f^2\tau^2)}{\tau^2(1 + f^2\tau^2)^2} \tag{2.13}$$

where ε_0 is an arbitrary integration constant. Now inserting $a(\tau)$ and $b(\tau)$ from equations (2.11) and (2.12) and ε from (2.13), one finds that the constraint (2.4a) is satisfied for all time if ε_0 is chosen as

$$\varepsilon_0 = \frac{1}{4\pi G t_P^2}. \tag{2.14}$$

G, being the Newtonian gravitational constant, is equal to M_P^{-2} in natural units (M_P is the Planck mass which is also equal to t_P^{-1} in natural units). Hence $\varepsilon_0 = M_P^4/4\pi$. Now one gets from equation (2.13)

$$\varepsilon = \frac{M_P^4(1 + 2f^2\tau^2)}{4\pi\tau^2(1 + f^2\tau^2)^2} \tag{2.15}$$

Thus, it is found that equations (2.11) and (2.12) provide the exact solutions of Einstein's field equations (2.4).

3. Dimensional reduction

3.1. Gravity

The four-dimensional action for gravity is given as

$$S_g^{(4)} = -\frac{1}{16}\pi G \int d^4x \sqrt{-g^{(4)}}\, R^{(4)} \tag{3.1}$$

where G is the gravitational constant, $g^{(4)}$ is the determinant of $g_{\mu\nu}$ and $R^{(4)}$ is the Ricci scalar with respect to $g_{\mu\nu}$ for the four-dimensional theory.

For the sake of convenience, the metric tensor given by the line element (2.1) is written as

$$g_{\mu\nu} = \begin{pmatrix} g_{ij} & 0 \\ 0 & -b^2(\tau) g_{mn} \end{pmatrix} \tag{3.2}$$

where $g_{ij} = \mathrm{diag}(1, -a^2)$ and g_{mn} ($m, n = 2, 3$) is the metric tensor on T^2. $g_{\mu\nu}$ may be conformally transformed to $\tilde{g}_{\mu\nu}$ as

$$g_{\mu\nu} = b^2(\tau)\tilde{g}_{\mu\nu} = b^2(\tau)\begin{pmatrix} \tilde{g}_{ij} & 0 \\ 0 & -g_{mn} \end{pmatrix} \tag{3.3}$$

where $\tilde{g}_{ij} = \mathrm{diag}(b^{-2}, -a^2 b^{-2})$. So, on ignoring terms of total divergence and integrating over θ_1 and θ_2, one gets

$$S_{\mathrm{ind}}^{(2)(m)} + S_g^{(2)} = -\frac{1}{16\pi\bar{G}^{(2)}} \int d^2x \sqrt{-\tilde{g}^{(2)}}\, b^4 \left[\tilde{R}^{(2)} - \frac{20}{t_P^2}\left(\frac{\dot{b}}{b}\right)^2\right] \tag{3.4}$$

where

$$\bar{G}^{(2)} = \frac{G}{4\pi^2 \rho_1 \rho_2}$$

($\bar{G}^{(2)}$ is the two-dimensional gravitational constant and is a dimensionless number) and $\tilde{g}^{(2)}$ is the determinant of $\tilde{g}_{ij}$.

To undo the earlier conformal transformation on $\tilde{g}_{ij}$, another conformal transformation may be employed:

$$\tilde{g}_{ij} = \bar{b}^2(\tau) g_{ij}. \tag{3.5}$$

Under (3.5), the action (3.4) is rewritten as

$$S_{\text{ind}}^{(2)(m)} + S_g^{(2)} = -\frac{1}{16\pi\bar{G}^{(2)}} \int \mathrm{d}^2x\, ab^2 \left[R^{(2)} - \frac{2}{t_p^2} \left(\frac{\dot{b}}{b}\right)^2 \right] \tag{3.6}$$

where

$$S_{\text{ind}}^{(2)(m)} = \frac{1}{2} \int \mathrm{d}^2x\, a \frac{\dot{b}^2}{8\pi\bar{G}^{(2)} t_p^2}$$

which is a contribution to matter fields, induced due to the compactification of toroidal component of space.

3.2. *Scalar fields*

In the background geometry, the existence of some scalar fields ϕ with bare mass m_0 is assumed. The action for ϕ is given as

$$S_0^{(4)} = \frac{1}{2} \int \mathrm{d}^2x\, \mathrm{d}\theta_1\, \mathrm{d}\theta_2\, p_1 p_2 ab^2 [g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi - (\xi R^{(4)} + m_0^2) \phi^* \phi] \tag{3.7}$$

where ξ is non-minimal coupling constant, which is equal to $\frac{1}{6}$ in four-dimensional space for conformal scalar fields (for which $m_0 = 0$). $R^{(4)}$ in equation (3.7) is given as

$$R^{(4)} = R^{(2)} + 2t_{\mathrm{P}}^{-2} \left\{ \frac{\mathrm{d}}{\mathrm{d}\tau} \left(\frac{\dot{b}}{b}\right) + \frac{\dot{b}}{b} \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right) \right\}. \tag{3.8}$$

On the spacetime, with topology $R^1 \otimes R^1 \otimes T^2$, ϕ can be decomposed as

$$\phi = [(2\pi)^2 \rho_1 \rho_2 b^2]^{-1/2} \sum_{n_1, n_2 = -\infty}^{\infty} \phi_{n_1 n_2}(t, x) \exp\left[\mathrm{i} \sum_{j=1}^{2} \frac{(n_j + d)}{p_j} y_j \right] \tag{3.9}$$

where $\alpha = 0(\frac{1}{2})$ for untwisted (twisted) fields [16]. The reason for the untwisted (twisted) fields in the non-simply connected nature of internal manifold T^2 which is the product of two circles. Untwisted fields are periodic in y, whereas twisted fields are anti-periodic in y. Here $y_1 = \rho_1 \theta_1$ and $y_2 = \rho_2 \theta_2$. Later on, only untwisted fields will be considered.

Now, substituting the decomposed form of ϕ given by equation (3.9) in the four-dimensional action (3.7) and integrating over T^2, one gets the two-dimensional action for the scalar field as

$$S_0^{(2)} = -\frac{1}{2} \sum_{n_1,n_2=-\infty}^{\infty} \int d^2x\, a\phi^*_{n_1n_2}[\Box^{(2)} + \xi R^{(2)} + m^2_{n_1n_2}]\phi_{n_1n_2} \tag{3.10}$$

where

$$m^2_{n_1n_2} = \tilde{m}_0^2 + \left(\frac{2\pi}{b}\right)^2 \sum_{n_1,n_2=-\infty}^{\infty} \left(\frac{n_1^2}{p_1^2} + \frac{n_2^2}{p_2^2}\right) \tag{3.11a}$$

with

$$\tilde{m}_0^2 = m_0^2 + 2\xi t_P^{-2}\left\{\frac{d}{d\tau}\left(\frac{\dot{b}}{b}\right) + \frac{\dot{b}}{b}\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right)\right\}. \tag{3.11b}$$

4. One-loop quantum corrections to $\phi_{n_1n_2}$

For one-loop quantum corrections to the field $\phi_{n_1n_2}$ the operator regularization method [17] (which is, in some sense, an extension of zeta-function regularization) is employed. This method is a convenient tool for regularization in curved spaces as it leads to finite results without renormalization.

$\phi_{n_1n_2}$ is a dimensionally reduced two-dimensional scalar field; hence the significant contribution of one-loop corrections can be found only up to second order in the adiabatic limit. Now, the effective action is

$$\Gamma = S_0^{(2)} + \sum_{n_1,n_2=-\infty}^{\infty} \frac{d}{ds}\left[\left(\frac{\mu^2}{m^2_{n_1n_2}}\right)^s \int d^2x\, a\left\{\frac{m^2_{n_1n_2}}{(s-1)} + \left(\frac{1}{6} - \xi\right)R^{(2)}\right\}\right]\Bigg|_{s=0}. \tag{4.1}$$

To evaluate the summation, the generalized ζ_D-function is defined as [18]

$$\zeta_D(p) = \sum_{n_1,n_2\ldots n_D=-\infty}^{\infty} \left(\frac{n_1^2}{p_1^2} + \frac{n_2^2}{p_2^2} + \cdots + \frac{n_D^2}{p_D^2}\right)^{-p}. \tag{4.2}$$

The properties of the ζ_D-function lead to the result

$$\zeta_D(-r) = 0 \tag{4.3}$$

for any even or odd integer r. This result can be easily checked for $D = 1, 2$ and can be proved for arbitrary D by induction. Now, using equations (4.2) and (4.3)

$$\begin{aligned}\sum_{n_1,n_2=-\infty}^{\infty} (m^2_{n_1n_2})^{1-s} &= \sum_{n_1,n_2=-\infty}^{\infty} (\tilde{m}_0^2)^{1-s} \\ &\quad + \sum_{r=1}^{1-s} \frac{(1-s)!}{r!(1-s-r)!}\left(\frac{2\pi}{b}\right)^{2r} \zeta_2(-r)(\tilde{m}_0^2)^{1-s-r} \\ &= (\tilde{m}_0^2)^{1-s}.\end{aligned} \tag{4.4a}$$

Also, similarly

$$\sum_{n_1,n_2=-\infty}^{\infty} (m^2_{n_1n_2})^{-s} = (\tilde{m}_0^2)^{-s} \tag{4.4b}$$

as

$$\sum_{n_1,n_2=-\infty}^{\infty} = [2\zeta(0)]^2 = 1.$$

Actually $\zeta(0)$ (Riemann-zeta function for zero argument) is divergent; however by employing the method of analytic continuation a finite value of $\zeta(0)$ (which is equal to $-\frac{1}{2}$) can be obtained.

Thus, using equations (4.4) in equation (4.1) one obtains

$$\Gamma = S_0^{(2)} + \frac{d}{ds}\left[\left(\frac{\mu^2}{\tilde{m}_0^2}\right)^s \int d^2x\, a\left\{\frac{\tilde{m}_0^2}{(s-1)} + \left(\frac{1}{6} - \xi\right)R^{(2)}\right\}\right]\Bigg|_{s=0}$$

$$= S_0^{(2)} + \int d^2x\, a\left[\left(\frac{1}{6} - \xi\right)\ln\frac{\mu^2}{\tilde{m}_0^2}R^{(2)} - \tilde{m}_0^2 - \tilde{m}_0^2 \ln\frac{\mu^2}{\tilde{m}_0^2}\right]. \tag{4.5}$$

Thus, one finds from equation (4.5) the one-loop correction to $\phi_{n_1n_2}$ contributes to gravity yielding induced terms given as [19]

$$\frac{1}{16\pi G_{\text{ind}}^{(2)}} = \left(\frac{1}{6} - \xi\right)\ln\frac{\mu^2}{\tilde{m}_0^2} \tag{4.6a}$$

and

$$\frac{\Lambda_{ind}}{8\pi G_{\text{ind}}^{(2)}} = \tilde{m}_0^2 + \tilde{m}_0^2 \ln\frac{\mu^2}{\tilde{m}_0^2}. \tag{4.6b}$$

5. Fundamental constants

Adding to the contribution of one-loop quantum corrections to gravity from the effective action (4.5), an effective action for gravity is obtained as

$$S_{(g)\text{eff}}^{(2)} = -\int d^2x\, a\left[\left\{\frac{b^2}{16\pi\bar{G}^{(2)}} + \left(\xi - \frac{1}{6}\right)\ln\frac{\mu^2}{\tilde{m}_0^2}\right\}R^{(2)} + \left\{\tilde{m}_0^2 + \tilde{m}_0^2 \ln\frac{\mu^2}{\tilde{m}_0^2}\right\}\right]. \tag{5.1}$$

So, from action (5.1) one obtains [20]

$$\frac{1}{16\pi G_{\text{eff}}^{(2)}} = \frac{b^2}{16\pi\bar{G}^{(2)}} + \left(\xi - \frac{1}{6}\right)\ln\frac{\mu^2}{\tilde{m}_0^2} \tag{5.2a}$$

where $G_{\text{eff}}^{(2)}$ is the effective two-dimensional gravitational constant which is time-dependent. The effective time-dependent cosmological constant is given as

$$\frac{\Lambda_{\text{eff}}}{16\pi G_{\text{eff}}^{(2)}} = \frac{1}{2}\left\{\tilde{m}_0^2 + \tilde{m}_0^2 \ln\frac{\mu^2}{\tilde{m}_0^2}\right\}. \tag{5.2b}$$

Using equation (2.13) for $b(\tau)$ in equation (5.2a), one gets

$$\frac{1}{16\pi G_{\text{eff}}^{(2)}} = \frac{f^2}{16\pi\bar{G}^{(2)}} + \left(\xi - \frac{1}{6}\right)\ln\frac{\mu^2}{\tilde{m}_0^2} \tag{5.2c}$$

where,

$$\tilde{m}_0^2 = m_0^2 + \frac{4\xi t_P^{-2}}{\tau^2(1+f^2\tau^2)}. \tag{5.2d}$$

So, at late times, one finds that

$$\frac{1}{16\pi G_{\rm eff}^{(2)}} = \frac{f^2}{16\pi \bar{G}^{(2)}} + \left(\xi - \frac{1}{6}\right) \ln \frac{\mu^2}{m_0^2}.$$

Now if $f = 0$, $G_{\rm eff}^{(2)} \to \infty$ in the case $\mu \to m_0^2$. To avoid this problem, f is assumed non-zero. It is better to determine the actual value of f; this can be done by going back to the four-dimensional gravitational constant on multiplying (5.2a) or (5.2c) by $(4\pi^2\rho_1\rho_2)^{-1}$ according to equation (3.4). Thus one obtains

$$\frac{1}{16\pi G_{\rm eff}} = \frac{b^2}{16\pi G} + \frac{(\xi - \frac{1}{6})}{4\pi^2 p_1 p_2} \ln \frac{\mu^2}{\tilde{m}_0^2}. \tag{5.3}$$

At late times, one gets from (5.3a)

$$\frac{1}{16\pi} G_{\rm eff} = \frac{f^2}{16\pi} G \tag{5.4}$$

on taking $\mu^2 = m_0^2$. At late times G_{eff} is supposed to be equal to the Newtonian gravitational constant G. So, according to equation (5.4)

$$f^2 = 1. \tag{5.5}$$

If ϕ is conformal i.e. $\xi = \frac{1}{6}$, from equations (5.2c), (5.2d) and (5.5)

$$G_{\rm eff}^{(2)} = \frac{\tau^2 \bar{G}^{(2)}}{1+\tau^2} \tag{5.6}$$

and

$$\Lambda_{\rm eff} = \frac{8\pi\bar{G}^{(2)}}{(1+\tau^2)^3}\left[m_0^2\tau^2(1+\tau^2)^2 + \frac{2}{3}M_P^2 + \left\{m_0^2\tau^2(1+\tau^2)^2 + \frac{2}{3}M_P^2\right\} \times \ln \frac{3\mu^2\tau^2(1+\tau^2)^2}{2M_P^2 + 3m_0^2\tau^2(1+\tau^2)^2}\right] \tag{5.7}$$

showing that $\lim_{\tau\to\infty} \Lambda_{\rm eff} = 0$, in case $\mu^2 = m_0^2$.

If ϕ is a non-conformal field, $\xi \neq \frac{1}{6}$, and from equations (5.2)

$$\Lambda_{\rm eff} = \frac{\left[m_0^2 + \dfrac{4\xi M_P^2}{\tau^2(1+\tau^2)}\right] \ln e\,\mu^2 \left\{m_0^2 + \dfrac{4\xi M_P^2}{\tau^2(1+\tau^2)}\right\}^{-1}}{2\left[\dfrac{1+\tau^{-2}}{16\pi\bar{G}^{(2)}} + \left(\xi - \dfrac{1}{6}\right) \ln \mu^2 \left\{m_0^2 + \dfrac{4\xi M_P^2}{\tau^2(1+\tau^2)}\right\}^{-1}\right]}$$

showing that

$$\lim_{\tau\to\infty} \Lambda_{eff} = \frac{8\pi\bar{G}^{(2)} m_0^2}{1 + 16\pi\bar{G}^{(2)}(\xi - \frac{1}{6})} = \frac{2M_P^{-2} m_0^2}{\pi\rho_1\rho_2 + 4M_P^{-2}(\xi - \frac{1}{6})}$$

in the case $\mu^2 = m_0^2$. Thus, it is found that Λ_{eff} asymptotically approaches extremely small values at late times (in the case $\mu^2 = m_0^2$) when $\xi \neq \frac{1}{6}$.

Thus, here (1 + 1)-dimensional gravity is obtained through Kaluza–Klein type compactification.

Acknowledgments

The author thanks the Inter University Centre for Astronomy and Astrophysics, Pune for hospitality where this work was started.

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