

## Nonstatistical Structures in $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}$ Reaction (\*).

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**Summary.** — The data on the  $\theta_{\text{lab}} = 7^\circ$  excitation functions of  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}$  reaction between  $E_{\text{c.m.}} = 10.15$  and  $15.88$  MeV for 20 states of  $^{22}\text{Na}$  up to an excitation energy of  $5.800$  MeV have been subjected to a statistical analysis consisting of the calculations of the distributions of fluctuating cross-sections, energy-dependent correlation function, deviation function, summed excitation function, and the coherence widths. The number of effective channels was calculated exactly. The analysis shows that, in addition to the statistical fluctuations, there are nonstatistical structures at  $10.5$ ,  $11.62$ ,  $13.02$ ,  $13.3$ ,  $13.6$  and  $14.86$  MeV.

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### 1. – Introduction.

The scattering and reactions involving combinations of non- $\alpha$ -conjugate even-even nuclei such as  $^{14}\text{C}+^{14}\text{C}$  [1-4], and other even-even nuclei like  $^{14}\text{C}+^{12}\text{C}$  [4,5] and  $^{14}\text{C}+^{16}\text{O}$  [6,7] exhibit pronounced resonant and other nonstatistical effects. In addition, the collisions involving combinations of other non- $\alpha$ -conjugate (at least one partner) nuclei like  $^{12}\text{C}+^9\text{Be}$  [8, 9],  $^{12}\text{C}+^{11}\text{B}$  [10],  $^{12}\text{C}+^{13}\text{C}$  [11, 12] and  $^{12}\text{C}+^{15}\text{N}$  [13-15] lead to scattering and/or reactions that also show resonant and nonstatistical structures. A statistical model analysis of  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}$  reaction data by Cordell *et al.* [16] gave an average experimental coherence width of  $163$  keV in  $^{26}\text{Al}$  that was in very good agreement with the systematics of  $\Gamma$  vs. mass number  $A$  [17]. In a subsequent analysis Dennis and Thornton [18] noted that there were pronounced nonstatistical effects in the energy regions

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(\*) To speed up publication, the author of this paper has agreed to not receive the proofs for correction.

around  $E_{c.m.} = (10.5 \pm 0.1) \text{ MeV}$ , and 13.3 and 13.7 MeV. Two other energy regions around  $E_{c.m.} = 11.6$  and 12.9 MeV displayed anomalies by exhibiting a number of correlated minima [18] with probabilities of occurrence of about  $10^{-4}$ . Even at around  $E_{c.m.} = 15.2 \text{ MeV}$  there appears a correlated minimum with probability of occurrence less than 0.01. The presence of these correlated minima perhaps pointed to the nonstatistical nature of this reaction in the energy region of interest. Some statistical tests [19] gave evidence of nonstatistical structures at  $E_{c.m.} = 10.6, 13.3$  and 13.7 MeV. In view of the prevailing uncertain situation about the nature of this reaction we carried out a statistical analysis of the experimental data on  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}$  reaction consisting of 20 excitation functions (up to an excitation energy of 5.800 MeV in  $^{22}\text{Na}$ ) following the approach of Ericson [20] and Brink and Stephen [21]. The analysis consisted of the calculations of distributions of the fluctuating cross-sections, energy-dependent correlation function, deviation function, summed excitation function and the coherence widths.

## 2. – Analysis.

2.1. *Data reduction.* – The data, that consisted of 20 excitation functions measured at  $\theta_{\text{lab}} = 7^\circ$  between  $E_{c.m.} = 10.15$  and 15.88 MeV in steps of 92.3 keV (c.m.) for  $^{22}\text{Na}$  states up to an excitation energy of 5.800 MeV, are from Cordell *et al.* [16]. The experimental energy resolution was  $\sim 75 \text{ keV}$  and uncertainty in the absolute cross-sections was up to about 20%. Before subjecting the data to a statistical analysis, the energy-dependent gross structure should be removed from the excitation functions. This is usually done by following the approach, first suggested by Pappalardo [22], of dividing the experimental cross-sections,  $d\sigma(E)$ , by the running average  $\langle d\sigma(E) \rangle$  taken over a suitable energy interval  $\Delta E$ . The usual criterion of choosing  $\Delta E$  is  $\Gamma_{\text{fine}} \ll \Delta E \ll \Gamma_{\text{gross}}$ , where  $\Gamma_{\text{fine}}$  and  $\Gamma_{\text{gross}}$  indicate the fine and gross structure widths (observed in the excitation functions) respectively [23]. However, if the values of  $R(0)$ , the normalised variance of the trend reduced excitation function ( $R(0) = \langle x^2 \rangle / \langle x \rangle^2 - 1$ ;  $x = d\sigma(E) / \langle d\sigma(E) \rangle$ ), is plotted as a function of the averaging interval  $\Delta E$ , we should obtain a plateau that begins when  $\Delta E$  very well exceeds the coherence width [24]. The appropriate value of  $\Delta E$  can therefore be taken from such a plot. The variation of  $R(0)$  with  $\Delta E$  for two typical excitation functions is shown in fig. 1. It can be noted that for  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}(0.000, 3^+)$  excitation function the plateau starts at 1 MeV and for  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}(0.583, 1^+)$  it starts at 1.65 MeV. Similar behaviour was observed for most other excitation functions. We, therefore, decided to choose an averaging interval of  $\Delta E = 1.68 \text{ MeV(c.m.)}$  for carrying out the data reduction, and it was this trend reduced data that was subjected to the analysis. The percentage deviations of the reduced data,  $x \equiv d\sigma(E) / \langle d\sigma(E) \rangle$  from unity were then calculated for obtaining the distribution

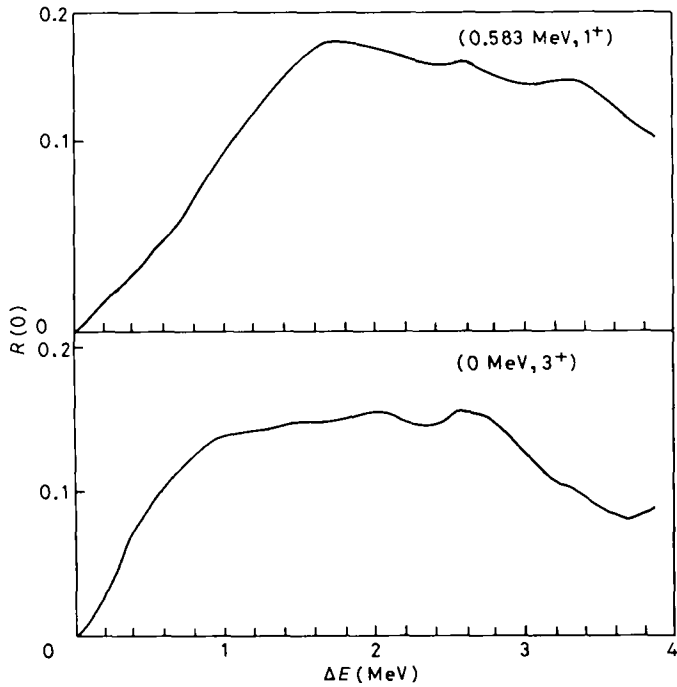


Fig. 1. – The variation of the normalized variance,  $R(0)$ , with the averaging interval,  $\Delta E$ , for the indicated excitation functions of  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}$  reaction.

of experimental cross-sections (to be compared with the corresponding theoretical distributions) as discussed in the following subsection.

2.2. *Number of effective channels and the distribution of cross-sections.* – The number of effective channels  $N_{\text{eff}}$ , the quantity that determines the statistically independent cross-sections which contribute to the total cross-section, is crucial for calculating the theoretical distributions of the cross-sections. The approximate expression that is often used to estimate the maximum value of  $N_{\text{eff}}$ ,  $N_{\text{max}}$

$$(1) \quad N_{\text{max}} = \begin{cases} g/2 & \text{(for even } g\text{),} \\ (g+1)/2 & \text{(for odd } g\text{),} \end{cases}$$

with

$$g = (2i+1)(2I+1)(2i'+1)(2I'+1),$$

where  $i$  and  $I$  are the spins of the projectile and target and  $i'$  and  $I'$  are the spins of the final fragments involved in the reaction [20], frequently breaks down for the reactions involving heavy ions [25]. This number should be calculated exactly by using the appropriate detailed expression [20]. We calculated the number of

effective channels by using the code STAT2[26] as described by Dayras *et al.*[25]. The required (for these calculations) optical-model transmission coefficients were calculated by the code HOP2[27] and the necessary parameters were taken from the literature[28]. The n, p, d,  $^3\text{He}$  and  $^4\text{He}$  exit channels were included in these calculations and the required level density parameters were taken from ref.[28]. In table I we give some typical values of the number of effective channels obtained from these calculations.

TABLE I. - Some typical values of the exactly calculated  $N_{\text{eff}}$  at  $E_{\text{c.m.}} = 14.00$  MeV and  $\theta_{\text{c.m.}} = 10^\circ$  for the indicated states of  $^{22}\text{Na}$  for  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}$  reaction.

$^{22}\text{Na}$ state	$N_{\text{eff}}$
(0.000 MeV, $3^+$ )	1.2
(0.583 MeV, $1^+$ )	1.3
(1.582 MeV, $5^+$ )	1.1
(1.937 MeV, $1^+$ ; 1.983 MeV, $3^+$ )	2.1
(2.571 MeV, $2^-$ )	1.5
(2.969 MeV, $3^+$ )	1.2
(3.944 MeV, $1^+$ )	1.3
(5.317 MeV, $1^+$ )	1.3
(5.404 MeV, $2^-$ )	1.4
(5.604 MeV, $1^+$ )	1.3

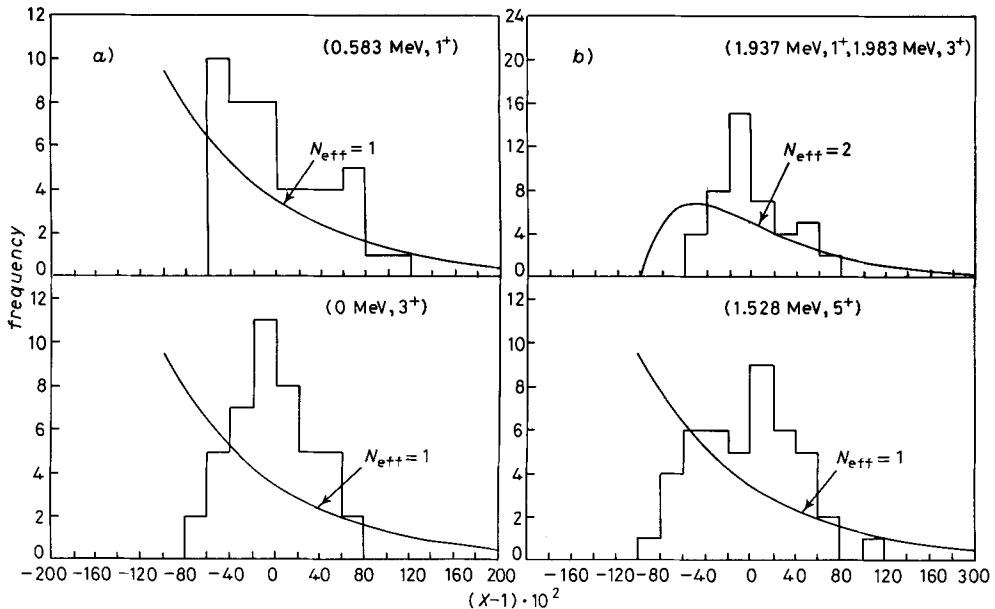


Fig. 2. - a) Experimental (histogram) and theoretical (continuous curve) distributions of cross-sections for the indicated excitation functions of  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}$  reaction  $y_d = 0$ ; b) the same as in fig. 2a) but for the indicated excitation functions of  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}$  reaction  $y_d = 0$ .

The distribution of the fluctuating cross-section in the absence of direct reaction contributions ( $Y_d \equiv$  ratio of direct to total cross-sections = 0; see fig. 2a) and b)) is given by [29]

$$(2) \quad P(x) = N_{\text{eff}} (N_{\text{eff}} x)^{N_{\text{eff}}-1} \exp[-N_{\text{eff}} x] / (N_{\text{eff}} - 1)!$$

where  $x \equiv d\sigma(E) / \langle d\sigma(E) \rangle$  and  $N_{\text{eff}} \equiv$  number of effective channels as mentioned earlier. Some typical comparisons of experimental and theoretical distributions of cross-sections are given in fig. 2a) and b). The disagreement between the experimental and theoretical distributions shows the nonstatistical nature of the observed structures in the  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}$  excitation functions.

2.3. *Deviation function, cross-correlation function, and the summed excitation function.* – The deviation function and energy-dependent cross-correlation function are very useful to locate nonstatistical structures. We calculated these functions by using the following formulae [19]:

$$(3) \quad D(E) = \frac{1}{N} \sum_{i=1}^N (x_i(E) - 1),$$

$$(4) \quad C(E) = \frac{2}{N(N-1)} \sum_{i>j=1}^N (x_i(E) - 1)(x_j(E) - 1) [R_i(0)R_j(0)]^{-1/2},$$

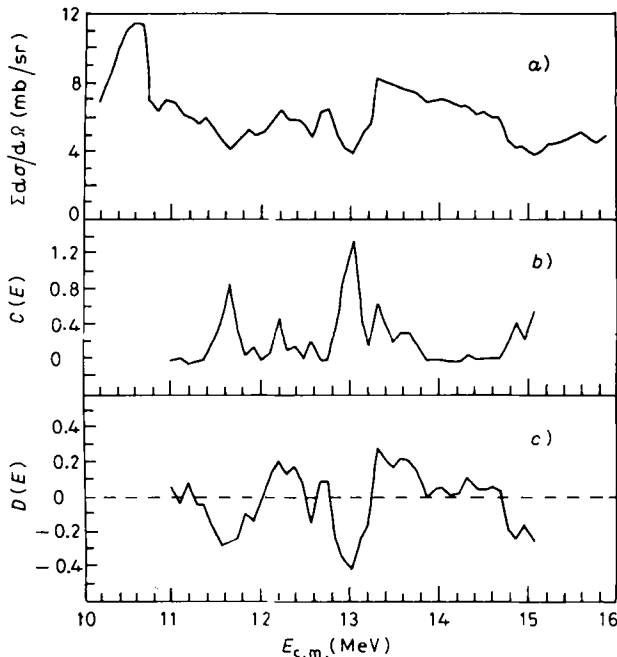


Fig. 3. – The summed excitation function,  $\sum_{i=1}^N d\sigma_i(E)/d\Omega$ , the energy-dependent correlation function,  $C(E)$ , and the deviation function,  $D(E)$ , for the  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}$  reaction.

where  $N$  is the total number of excitation functions and subscripts  $i$  and  $j$  denote  $i$ -th and  $j$ -th excitation functions. The other symbols have their usual meanings as indicated earlier. The deviation function,  $D(E)$ , and the correlation function  $C(E)$  are given in fig. 3, in which we have plotted the summed excitation function  $\sum_{i=1}^{20} (d\sigma_i/d\Omega)(E)$  also. It may be mentioned that due to the procedure followed for the data reduction, the  $C(E)$  and  $D(E)$  cannot be accounted for properly below 11 MeV and above 15 MeV. We, therefore, cannot see the strongest structure around 10.5 MeV in  $C(E)$  and  $D(E)$  that manifests itself so prominently in the summed excitation function. The maxima at  $E_{c.m.} = 11.62, 12.2, 13.02, 13.3, 13.6$  and 14.86 MeV in  $C(E)$  lie very well outside the statistical limit of 3 standard deviations (standard deviation  $= [2/N(N-1)(n-1)]^{1/2} = 0.0176$ ; here  $n \equiv$  number of experimental points in the averaging interval, see ref. [29]). Corresponding to the maxima at 11.62 and 13.02 MeV (in  $C(E)$ ), the summed excitation function and the deviation function exhibit correlated minima. The maxima at  $E_{c.m.} = 12.2$  and 13.3 MeV are very well correlated in all the three functions. This correlated nature indicates the nonstatistical origin of these structures.

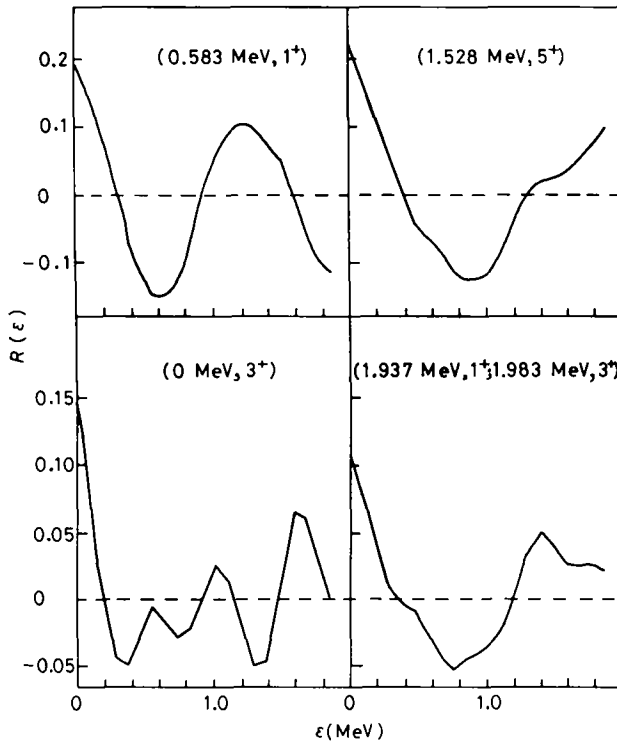


Fig. 4. - The autocorrelation functions,  $R(\epsilon)$ , for the indicated excitation functions of  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}$  reaction.

2'4. *Coherence widths.* – The coherence widths in  $^{26}\text{Al}$  were obtained by the autocorrelation analysis, empirical estimates, and by counting the number of maxima in the excitation functions. The autocorrelation function is given by [30]

$$(5) \quad R(\varepsilon) = \frac{\langle x(E) \cdot x(E + \varepsilon) \rangle}{\langle x(E) \rangle \cdot \langle x(E + \varepsilon) \rangle} - 1 = \frac{R(0)}{1 + \varepsilon^2/\Gamma^2},$$

where  $\varepsilon$  is a variable energy interval, and  $\Gamma$  and  $\langle \rangle$  denote the coherence width and the energy average, respectively. The autocorrelation function for some typical excitation functions are shown in fig. 4. The average value of  $\Gamma$  obtained this way was  $\Gamma = (135 \pm 60)$  keV, after applying correlations due to finite range of data and finite energy resolution [31]. The empirical estimates [17] in the energy range of interest ranged between 120 and 190 keV. The method of counting the number of maxima in the excitation functions [32], after correcting for the target thickness and finite spacing of experimental points [33], gave a value of  $\Gamma = (370 \pm 80)$  keV. We regarded  $i$ -th point as a maximum if  $d\sigma_{i\pm 2}(E) < d\sigma_{i\pm 1}(E) < d\sigma_i(E)$ . Thus the  $\Gamma$ -value obtained by autocorrelation analysis agrees very well with the empirical estimates. The large difference in the  $\Gamma$ -values obtained by autocorrelation analysis and by the method of counting the maxima indicates that the nonstatistical mechanism contributes significantly to this reaction in addition to the usual statistical fluctuation phenomenon.

### 3. – Conclusion.

Disagreement between the experimental and theoretical distributions of cross-sections and the difference in the  $\Gamma$ -values obtained from autocorrelation analysis and by the method of counting the maxima indicate the presence of nonstatistical structures. The structures at  $E_{\text{c.m.}} = 11.62, 12.2, 13.02, 13.3, 13.6$  and  $14.86$  MeV, that stand out rather clearly beyond statistical limits of the cross-correlation function, are of nonstatistical origin. The most prominent structure at  $E_{\text{c.m.}} = 10.5$  MeV that is seen in the summed excitation function is clearly of nonstatistical origin. The present analysis, therefore, shows that the above-mentioned structures observed in the  $^{12}\text{C}(^{14}\text{N}, ^4\text{He})^{22}\text{Na}$  excitation functions arise from some nonstatistical mechanism.

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