

**Radiative magnification of neutrino mixings in split supersymmetry**

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Radiative corrections to neutrino mixings in seesaw models depend on the nature of new physics between the weak and the grand unified theory (GUT)-seesaw scales and can be taken into account using the renormalization group equations. This new physics effect becomes particularly important for models with quasidegenerate neutrino masses where small neutrino mixings at the seesaw scale can get magnified by radiative renormalization effects alone to match observations. This mechanism of radiative magnification which provides a simple understanding of why lepton mixings are so different from quark mixings was demonstrated by us for the standard supersymmetry (SUSY) scenario where the particle spectrum becomes supersymmetric above the weak scale. In this paper, we examine this phenomenon in split supersymmetry scenarios and find that the mechanism works also for this scenario provided the SUSY scale is at least 2–3 orders below the GUT-seesaw scale and one has larger values of  $\tan\beta$ .

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**I. INTRODUCTION**

An important question in particle physics is the nature of new physics beyond the astoundingly successful standard model (SM). One compelling scenario for this new physics is the weak-scale supersymmetry which provides an answer to a number of puzzles of the standard model such as a solution to the gauge hierarchy problem, the origin of electroweak symmetry breaking as well as providing a candidate for dark matter of the Universe. An added virtue is that it unifies the disparate strengths of the weak, electromagnetic, and strong forces at a scale of about  $10^{16}$  GeV, opening up another rich landscape of new physics around this scale. This high scale new physics can only manifest itself at low energies via large radiative correction effects which depend logarithmically on mass as well as through effects such as proton decay which are suppressed by this new high scale.

There is reason to believe that there might be a manifestation of this new physics effect in the domain of neutrinos. This arises from the consideration of seesaw mechanism [1] which provides a very natural way to understand the extreme smallness of neutrino masses and which requires the existence of massive right-handed neutrinos with at least one having mass close to the grand unified theory (GUT) scale. It is therefore suggestive that both the seesaw scale and the GUT scale are one and the same. This connection becomes particularly plausible in the context of

models based on the grand unifying groups such as  $SO(10)$  [2] or  $SU(2)_L \times SU(2)_R \times SU(4)_C$  [3] that predict the existence of the right-handed neutrino as part of the common fermion multiplet along with the fermions of the standard model.

In a recent paper, this concept of unification of GUT scale and seesaw scale was taken one step further by making the plausible hypothesis that quark and lepton mixings angles may be the same at the seesaw scale due to quark-lepton unification whereas the observed large mixings at the weak scale are a consequence of radiative corrections. Using the formulae for renormalization group evolution for neutrino mass [4,5], it was shown that this possibility can be realized if the neutrino masses are quasidegenerate and have the same  $CP$  [6,7]. Models based on  $SU(2)_L \times SU(2)_R \times SU(4)_C$  group were presented where both the quasidegeneracy as well as mixing unification at the seesaw scale arose from the so-called type II seesaw formula [8]. Detailed renormalization group evolution of mixing angles then revealed that one indeed gets the desired bilarge mixing pattern at low energies which are in agreement with the solar and the atmospheric neutrino data. The third angle also undergoes radiative magnification, but remains small and well within the CHOOZ-Palo-Verde limit [9] due to the smallness of the corresponding initial value which is the quark mixing,  $\sin\theta_{13}^0 = 0.0025$ .

Furthermore, our hypothesis has the interesting prediction that the common value for the quasidegenerate neutrino mass is larger than 0.2 eV, which can be tested in the next round of the neutrinoless double beta decay experiments [10] and is in the range claimed in Ref. [11]. It also overlaps with the range accessible to the KATRIN experi-

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ment [12] and consistent with the bound obtained from the Wilkinson microwave background probe (WMAP) [13]. Threshold corrections at the low-energy supersymmetry (SUSY) scale have been found to improve agreement [14] with the most recent data including KamLAND and SNO [15] for a particular choice of the SUSY particle spectrum.

As mentioned, two important ingredients that allow the radiative magnification to work are low-energy supersymmetry and large values of  $\tan\beta$  which contribute large logarithmic factors for radiative magnification via amplified values of  $\tau$ -Yukawa coupling.

Recently, a new unification scenario has been advocated [16] which is based on the observation that one can maintain gauge-coupling unification and neutralino dark matter without necessarily “buying” the entire machinery of weak-scale supersymmetry but rather keeping only a subset of SUSY particles i.e. the gauginos and the Higgsino at the weak scale and pushing the rest of the superpartners to high intermediate scales ( $M_S$ ). This has been called split supersymmetry. This approach discards the naturalness requirement for the Higgs mass that motivated the weak-scale supersymmetry—thus one has to fine-tune the Higgs mass to every loop order. One also has no simple mechanism to understand the electroweak symmetry breaking. Despite these disadvantages, one might consider this as an interesting minimal extension of the standard model that preserves gauge-coupling unification and provides a neutralino dark matter and discuss how neutrinos fit into it.

Clearly, there is no inherent obstacle to implementing the seesaw mechanism and quark-lepton unification in these models. The only thing one has to keep in mind is that since the seesaw physics and quark-lepton unification generally tend to add new contributions to the gauge-coupling evolution, if we want to retain the simple unification of three standard model gauge couplings, the seesaw scale must be at or above the GUT scale. In this paper, we work within a framework of this type and see if we can understand the large neutrino mixings via the radiative magnification mechanism.

Below the high SUSY scale ( $M_S$ ) in split supersymmetry, apart from the presence of gauginos and Higgsinos, the effective theory is governed by nonSUSY standard model with one light Higgs doublet. As the two important ingredients in the radiative magnification scenario i.e. the long intervals of running in the presence of weak-scale SUSY and the enhancement factor due to  $\tan\beta$  are missing in the split supersymmetry below  $M_S$ , a natural apprehension emerges about the validity of this mechanism [6,7] for large neutrino mixings. Working within a model with quark-lepton unification based on the group  $SU(2)_L \times SU(2)_R \times SU(4)_C$  and degenerate masses for neutrinos obtained via the type II seesaw mechanism [8,17] and the usual split SUSY spectrum, we show that this model does indeed lead to bilarge mixing pattern at low energies. The

$SU(2)_L \times SU(2)_R \times SU(4)_C$  symmetry guarantees mixing unification between the quark and the lepton sectors at the seesaw scale (i.e.  $\theta_{ij}^q(M_R) = \theta_{ij}^\ell(M_R)$ ).

The new point which is different from the work of Ref. [7] is that running of neutrino masses is now different. Furthermore, in order to maintain gauge-coupling unification, we must have the seesaw scale equal to the GUT scale, as already noted. We carry out this running effect and show that despite the high split SUSY scale, the radiative magnification mechanism operates successfully to give the bilarge mixings at the weak scale provided that, in addition to the quasidegenerate Majorana neutrino masses with the same  $CP$ , we have the scalar SUSY partners with masses starting at least 2–3 orders below the seesaw scale. Whereas with low-energy SUSY, the radiative magnification occurs at  $\mu \leq 1$  TeV, in split supersymmetry it occurs at the high value of the SUSY scale which may be anywhere between  $10^5$  GeV– $10^{15}$  GeV. Below this scale, the renormalization group equation (RGE) running does not affect the neutrino mixings significantly. What is interesting is that radiative magnification occurs for the scalar SUSY partner masses close to even the scale  $10^{15}$  GeV so that a major part of the running is nonsupersymmetric. Including small threshold effects needed to fit the details like the solar mass-squared difference, the extrapolated low energy values of  $\Delta m_{32}^2$ ,  $\Delta m_{21}^2$ , and the mixing angles are in excellent agreement with the current solar, atmospheric, and the reactor neutrino data.

This paper is organized as follows: in Sec. II we discuss the relevant RGEs in the context of split supersymmetry. In Sec. III, we present allowed perturbative region of  $\tan\beta$ , radiative magnification of mixing angles at high SUSY scales, and low-energy extrapolation of masses and mixings. In Sec. IV, we discuss threshold effects and their estimations. Discussion of the results and conclusions are presented in Sec. V.

## II. RENORMALIZATION GROUP EQUATIONS IN SPLIT SUPERSYMMETRY

In this section while discussing renormalization group equations for neutrino masses and mixings we point out some special features of split supersymmetry that contribute to radiative magnification of the mixings at high SUSY scales. Since we will follow the bottom-up approach to determine initial conditions at the seesaw scales for RG evolutions of neutrino parameters in the top-down approach, we use the standard RGEs for the SUSY, split supersymmetry, and nonSUSY cases as applicable in appropriate domains [16,18,19]. In the context of split supersymmetry where the scale ( $M_S$ ) of the scalar superpartner masses could be much larger than the weak scale [16], the seesaw scale ( $M_R$ ) is clearly always larger than the SUSY scale ( $M_S$ ) i.e.  $M_R \gg M_S$ . We assume the neutrino mass eigenstates to be quasidegenerate and possess the same  $CP$ . We also ignore all  $CP$  violating phases in the mixing

matrix, and adopt the mass ordering to be of type  $m_3 \gtrsim m_2 \gtrsim m_1$ . Parametrizing the  $3 \times 3$  mixing matrix as

$$U = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23} & -c_{12}s_{23} - c_{23}s_{13}s_{12} & c_{13}c_{23} \end{bmatrix}, \quad (1)$$

where  $c_{ij} = \cos\theta_{ij}$  and  $s_{ij} = \sin\theta_{ij}$  ( $i, j = 1, 2, 3$ ), the RGEs for the mass eigenvalues and mixing angles can be written as [4,5]

$$\frac{dm_i}{dt} = -2F_\tau m_i U_{\tau i}^2 - m_i F_u = b_i^{(m)} m_i, \quad (i = 1, 2, 3), \quad (2)$$

$$\frac{ds_{23}}{dt} = -F_\tau c_{23}^2 (-s_{12} U_{\tau 1} D_{31} + c_{12} U_{\tau 2} D_{32}), \quad (3)$$

$$\frac{ds_{13}}{dt} = -F_\tau c_{23} c_{13}^2 (c_{12} U_{\tau 1} D_{31} + s_{12} U_{\tau 2} D_{32}), \quad (4)$$

$$\begin{aligned} \frac{ds_{12}}{dt} = & -F_\tau c_{12} (c_{23} s_{13} s_{12} U_{\tau 1} D_{31} - c_{23} s_{13} c_{12} U_{\tau 2} D_{32} \\ & + U_{\tau 1} U_{\tau 2} D_{21}). \end{aligned} \quad (5)$$

Here  $D_{ij} = (m_i + m_j)/(m_i - m_j)$  and, for the minimal supersymmetric standard model (MSSM) with  $\mu \geq M_S$ ,

$$\begin{aligned} F_\tau &= -h_\tau^2 / (16\pi^2 \cos^2 \beta), \\ F_u &= \left( \frac{1}{16\pi^2} \right) \left( \frac{6}{5} g_1^2 + 6g_2^2 - 6 \frac{h_t^2}{\sin^2 \beta} \right), \end{aligned} \quad (6)$$

but, for  $\mu \leq M_S$ ,

$$\begin{aligned} F_\tau &= 3h_\tau^2 / (32\pi^2), \\ F_u &= (3g_2^2 - 2\lambda - 6h_t^2 - 2T) / (16\pi^2) \end{aligned} \quad (7)$$

with the definitions of the couplings at  $M_S$ ,

$$\begin{aligned} g_u(M_S) &= g_2(M_S) \sin\beta, \\ g_d(M_S) &= g_2(M_S) \cos\beta, \\ \tilde{g}_u(M_S) &= (3/5)^{1/2} g_1(M_S) \sin\beta, \\ \tilde{g}_d(M_S) &= (3/5)^{1/2} g_1(M_S) \cos\beta. \end{aligned} \quad (8)$$

The additional term present in Eq. (7) which is specific to the split SUSY below  $M_S$  and the Higgs quartic coupling are

$$\begin{aligned} T &= \frac{3}{2} (g_u^2 + g_d^2) + \frac{1}{2} (\tilde{g}_u^2 + \tilde{g}_d^2), \\ \lambda(M_S) &= \frac{[g_2^2(M_S) + (3/5)g_1^2(M_S)]}{4} \cos^2 2\beta. \end{aligned} \quad (9)$$

The basic mechanism responsible for radiative magnification of mixings of quasidegenerate neutrinos with same  $CP$  in MSSM which has been pointed out earlier [6,7] is also applicable with split SUSY but with high value of  $M_S$ .

Since  $m_i$  and  $m_j$  are scale dependent, the initial difference existing between them at  $\mu = M_R$  is narrowed down during the course of RG evolution as we approach the SUSY scale. This causes  $D_{ij} \rightarrow$  large, and hence large magnification to the mixing angle due to radiative effects through Eqs. (3)–(5). Another major contribution to radiative magnification is due to amplified negative value of  $F_\tau$  in Eqs. (3)–(5) in the presence of SUSY by a factor  $\approx \tan^2 \beta$  in the large  $\tan \beta$  region.

An approximate estimation of values of  $\tan \beta$  needed for radiative magnification in split SUSY is possible by noting that the magnification has been realized in MSSM with weak-scale SUSY for  $\tan \beta \approx 50 - 55$  and in that case the major factor controlling magnification is due to amplification of  $\tau$ -Yukawa coupling [6,7]. Expecting similar amplification in split SUSY gives the approximate relation

$$(h_\tau^0 \tan \beta^0)^2 \log(M_R^0/M_S^0) = (h_\tau \tan \beta)^2 \log(M_R/M_S), \quad (10)$$

where quantities with zero superscript (without superscript) refer to weak-scale (split) supersymmetry. Noting that the ratio of the two  $\tau$ -Yukawa couplings in Eq. (10) is  $\approx O(1)$ , and using  $\tan \beta^0 = 50 - 55$  [6,7], and  $M_R^0 = M_R = M_U = 2 \times 10^{16}$  GeV, the above relation gives the approximate requirements for split supersymmetry case as  $\tan \beta = 75, 110$  for  $M_S = 10^9$  GeV, and  $10^{13}$  GeV, respectively. In fact, it has been noted that split SUSY permits larger values of  $\tan \beta$  than the MSSM with weak-scale SUSY [20].

It would be shown in Sec. III that the allowed perturbative upper limit of  $\tan \beta$  increases with increase of the SUSY scale. Thus even though the supersymmetric part of the running of neutrino parameters is much less than in the weak-scale supersymmetry case, larger values of  $\tan \beta$  increases the value of  $F_\tau$  and this in turn leads to the desired enhancement of the radiative magnification effect on the mixing angles.

In addition to the above, here we point out that there is a specific mechanism which operates in the allowed parameter space of split SUSY that causes mass eigenvalues to approach one another faster and thus, drives radiative magnification even if the SUSY scale is many orders larger than the weak scale. Noting that the  $\beta$ -function coefficient for the evolution of mass eigenvalue  $m_i(\mu)$  in Eq. (2) is  $b_i^{(m)} = -2F_\tau U_{\tau i}^2 - F_u$ , near the seesaw scale  $U_{\tau i}^2 \sim 0$ , for  $i = 1, 2$  because of high-scale mixing unification constraint, but  $U_{\tau 3}^2 \sim 1$ . Therefore, the evolution of the third mass eigenvalue would be different from the first two. Further, since supersymmetric  $F_\tau$  is negative for  $\mu > M_S$  and, depending upon the allowed values of the gauge and the Yukawa coupling constants contributing to  $F_u$  in Eq. (6) above the high SUSY scale, the  $\beta$ -function coefficient may be positive or negative thus resulting in the decreasing or increasing behavior of the mass eigenvalue below the seesaw scale down to  $M_S$ . In particular with split

supersymmetry  $F_u$  may be positive due to dominance of gauge couplings over the top-Yukawa coupling resulting in negative values of  $b_{1,2}^{(m)}$  for  $\mu > M_S$ . But since  $F_\tau$  is negative, the positive value of the first term,  $-2F_\tau U_{\tau i}^2$ , may partly or largely cancel with the second term leading to a large or small positive value of  $b_3^{(m)}$ . As a result, the first two mass eigenvalues may be expected to increase but the third may decrease, rapidly or slowly, from their input values at  $M_R$ . This increasing behavior of some mass eigenvalues ( $m_1, m_2$ ) in the region  $M_S < \mu < M_R$  would be in sharp contrast to that in MSSM with weak-scale SUSY where the allowed high-scale values of the gauge, the Yukawa couplings, and the mixing unification constraint have been found to make all the three  $\beta$ -function coefficients positive resulting in the decreasing behavior of the three mass eigenvalues below the seesaw scale although with different rates [7]. Thus, with split SUSY the allowed values of the parameter space may cause three mass eigenvalues to approach sufficiently closer to one another even after much shorter interval of running. This may occur at high values of  $M_S$  which may even be only 2–3 orders smaller than  $M_R$ . Then the functions  $D_{ij} \rightarrow$  large near  $M_S$  causing magnification of the mixing angles.

On the other hand for  $\mu < M_S$ , the  $\tau$ -Yukawa coupling has negligible contribution and the  $\beta$ -function coefficients are nearly the same for all the three eigenvalues and approximately equal to  $-2F_u$ . From Eq. (7) it is clear that in the presence of nonSUSY SM,  $F_u$  is negative below  $M_S$  resulting in positive beta-function coefficients for all mass eigenvalues causing them to decrease approximately in the same manner down to  $M_Z$ . It is easy to check, using boundary conditions given in Eqs. (8) and (9), that  $F_u$  is negative at the starting point  $M_S$  of the nonSUSY theory, with  $16\pi^2 F_u(M_S) = -\frac{3}{5}g_1^2(M_S) - 2\lambda(M_S) - 6h_t^2(M_S)$ . Although this feature is also common to MSSM with the weak-scale SUSY, in split SUSY the running interval for nonsupersymmetric theory is much larger.

Our numerical solutions to masses and mixing angles reported in Sec. III with split SUSY are found to corroborate these properties of the RG evolution.

### III. RADIATIVE MAGNIFICATION AT HIGH SUSY SCALES

In this section we discuss the realization of radiative magnification of neutrino mixing angles at high SUSY scales and low-energy extrapolations of mass eigenvalues and mixing angles for comparison with the available experimental data.

#### A. Perturbative limit on $\tan\beta$

The fact that grand unification with split supersymmetry allows larger values of  $\tan\beta$  than those with weak-scale SUSY has been also noted in Ref. [20]. Since the value of  $\tan\beta$  is an important ingredient, at first we estimate the

maximum allowed values of  $\tan\beta$  for every value of  $M_S = 10^3$ - $10^{15}$  GeV. For this purpose we use the standard Particle Data Group (PDG) values of masses and couplings [21] at  $\mu = M_Z$  and follow the bottom-up approach scanning the values of the three gauge couplings and the third generation Yukawa couplings while varying input values of  $\tan\beta$  over a wide range at all values of  $\mu$  up to the GUT scale using the RGEs in the appropriate regions [16,18,19]. The lowest allowed value of  $\tan\beta$  is determined when the top-quark Yukawa coupling reaches the perturbative limit ( $h_t = 3.54$ ). Similarly the upper limit is determined by noting that the  $\tau$ -lepton Yukawa coupling attains its perturbative limit at the GUT scale. These are shown in Fig. 1. It is clear that the upper limit on  $\tan\beta$  is larger compared to the weak-scale SUSY model and this limit also increases quite significantly with increasing value of  $M_S$  in split supersymmetry. For example with  $M_S = 10^7$  GeV,  $10^9$  GeV,  $10^{13}$  GeV, and  $10^{15}$  GeV, the upper limits are found to be  $\tan\beta = 85, 100, 132,$  and  $160,$  respectively. Through this bottom-up approach we also noted the values of different coupling constants and Cabibbo-Kobayashi-Maskawa (CKM) mixings at different values of the seesaw scales starting from their low-energy values. Some of the coupling constants at the seesaw scales are presented in Table I where the factors like  $\sin\beta$  or  $\cos\beta$  are included in the values of SUSY Yukawa couplings. With large values of  $\tan\beta$  allowed near the upper limit, the Higgs mass prediction in split SUSY tends to be independent of this parameter and the two Yukawa couplings,  $g_d$  and  $\tilde{g}_d$ , have negligible effects below  $M_S$ .

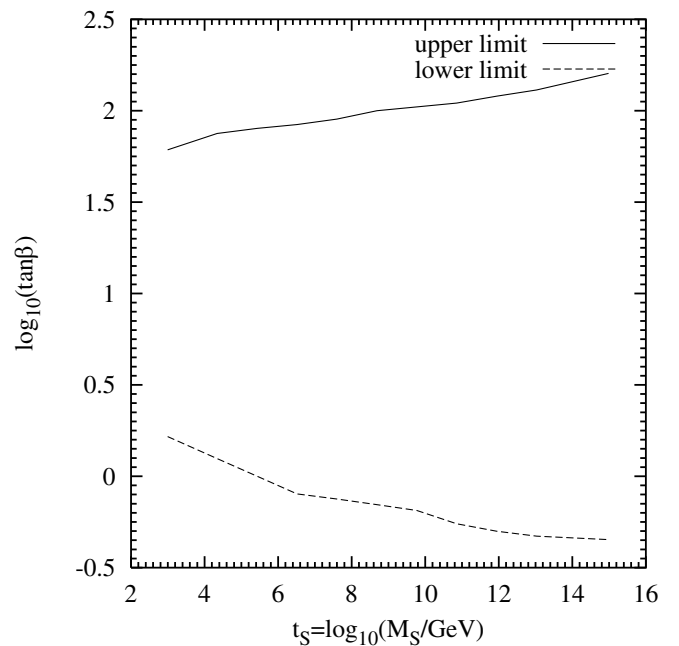


FIG. 1. Perturbative upper and lower limits on the allowed values of  $\tan\beta$  as a function of SUSY scale  $M_S$ , taken in unit of GeV, in split supersymmetry.

TABLE I. Radiative magnification to bilarge neutrino mixings for input values of  $m_i^0$  ( $i = 1, 2, 3$ ),  $\sin\theta_{23}^0 = 0.038$ ,  $\sin\theta_{13}^0 = 0.0025$ , and  $\sin\theta_{12}^0 = 0.22$  at the high scale  $M_R$ . The initial values of SUSY Yukawa couplings at seesaw scales include factors  $\sin\beta$  or  $\cos\beta$  as applicable.

$M_R$ (GeV)	$2 \times 10^{16}$	$2 \times 10^{16}$	$2 \times 10^{16}$	$2 \times 10^{18}$
$M_S$ (GeV)	$10^{13}$	$10^{13}$	$10^9$	$10^{15}$
$\tan\beta$	90	130	90	140
$g_1^0$	0.6206	0.6200	0.6518	0.6540
$g_2^0$	0.6203	0.6198	0.6522	0.6125
$g_3^0$	0.6262	0.6260	0.6565	0.5935
$h_t^0$	0.3943	0.4035	0.4450	0.3625
$h_b^0$	0.5325	0.9627	0.8052	0.9399
$h_\tau^0$	1.0181	2.2676	1.7592	2.5684
$m_1^0$ (eV)	0.4483	0.2965	0.2267	0.3648
$m_2^0$ (eV)	0.4500	0.30	0.2300	0.3700
$m_3^0$ (eV)	0.4911	0.3926	0.3188	0.5060
$m_1$ (eV)	0.2934	0.1938	0.1766	0.2301
$m_2$ (eV)	0.2937	0.1944	0.1773	0.2310
$m_3$ (eV)	0.2956	0.1983	0.1816	0.2364
$(\Delta m_{21}^2)_{RG}$ (eV <sup>2</sup> )	$1.693 \times 10^{-4}$	$2.28 \times 10^{-4}$	$2.31 \times 10^{-4}$	$3.96 \times 10^{-4}$
$(\Delta m_{32}^2)_{RG}$ (eV <sup>2</sup> )	$1.25 \times 10^{-3}$	$1.53 \times 10^{-3}$	$1.55 \times 10^{-3}$	$2.53 \times 10^{-3}$
$M_{\tilde{r}}/M_S$	1.1	0.92	0.50	1.03
$(\Delta m_{21}^2)_{th}$ (eV <sup>2</sup> )	$-0.84 \times 10^{-4}$	$-1.48 \times 10^{-4}$	$-1.51 \times 10^{-4}$	$-3.16 \times 10^{-4}$
$(\Delta m_{32}^2)_{th}$ (eV <sup>2</sup> )	$-0.61 \times 10^{-4}$	$-0.11 \times 10^{-3}$	$-0.10 \times 10^{-3}$	$-0.19 \times 10^{-3}$
$(\Delta m^2)_{sol}$ (eV <sup>2</sup> )	$8.5 \times 10^{-5}$	$8.0 \times 10^{-5}$	$8.0 \times 10^{-5}$	$8.0 \times 10^{-5}$
$(\Delta m^2)_{atm}$ (eV <sup>2</sup> )	$1.19 \times 10^{-3}$	$1.42 \times 10^{-3}$	$1.45 \times 10^{-3}$	$2.34 \times 10^{-3}$
$\sin\theta_{12}$	0.545	0.5474	0.549	0.526
$\sin\theta_{23}$	0.701	0.703	0.707	0.707
$\sin\theta_{13}$	0.101	0.101	0.103	0.104

## B. Radiative Magnification of Mixing Angles

The values of gauge and Yukawa couplings, the CKM mixing angles obtained at the seesaw scale from the bottom-up approach and fine-tuned values of light neutrino masses ( $m_i^0$ ,  $i = 1, 2, 3$ ) are used as inputs to obtain solutions from the neutrino RGEs in the top-down approach. These input parameters are given in Table I. The unknown parameters  $m_i^0$  are determined in such a way that the magnified values of mixing angles are at first obtained at  $M_S$  along with the mass eigenvalues. These are further extrapolated down to  $\mu = M_Z$  through the corresponding RGEs to give closest agreement with the experimental data:

$$\begin{aligned}
 \Delta m_{21}^2 &= (5 - 8) \times 10^{-5} \text{ eV}^2, \\
 \Delta m_{32}^2 &= (1.2 - 3) \times 10^{-3} \text{ eV}^2, \\
 \sin\theta_{23} &= 0.67 - 0.707, \\
 \sin\theta_{12} &= 0.5 - 0.6, \\
 \sin\theta_{13} &\leq 0.16.
 \end{aligned}
 \tag{11}$$

As noted in Sec. II the signs and values of the three  $\beta$ -functions for the mass eigenvalues at the respective seesaw scales are easily checked from the initial values of couplings given in Table I using Eq. (2). For example for the case with  $M_R = 2 \times 10^{16}$  GeV,  $M_S = 10^9$  GeV,

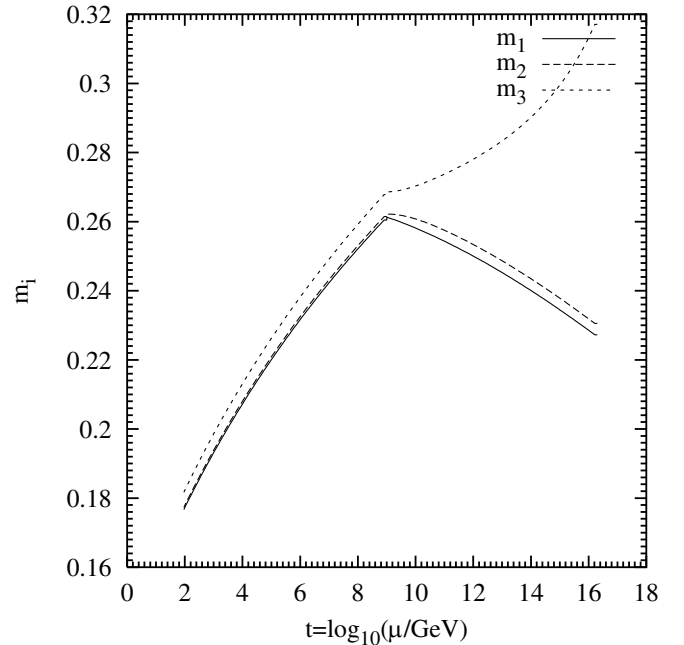


FIG. 2. Renormalization group evolution of light Majorana neutrino mass eigenvalues showing both the increasing and decreasing behaviors between SUSY scale  $M_S = 10^9$  GeV and the GUT-seesaw scale  $M_R = 2 \times 10^{16}$  GeV in split supersymmetry where  $t = \log_{10}(\mu)$  and  $\mu$  is in unit of GeV. The input values are given in the fourth column of Table I.

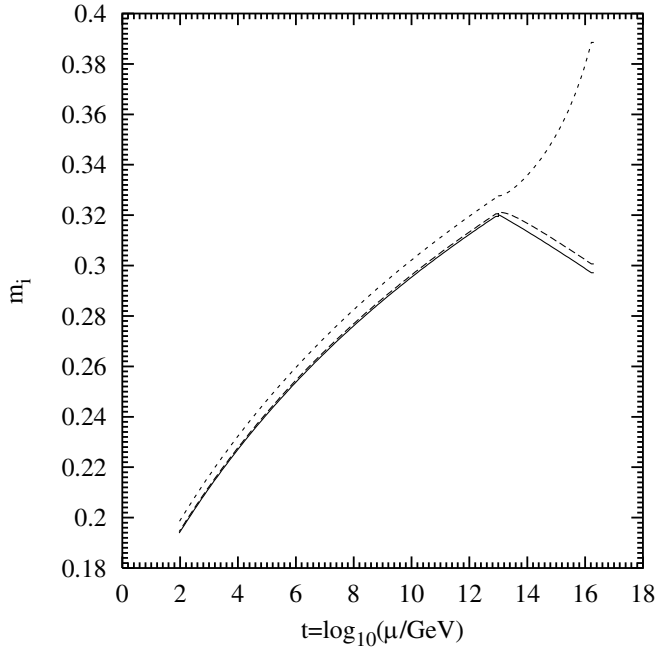


FIG. 3. Same as Fig. 2 but with  $M_R = 2 \times 10^{16}$  GeV, SUSY scale  $M_S = 10^{13}$  GeV, and  $\tan\beta = 130$ . The input values and low-energy extrapolations are given in the third column of Table I.

$\tan\beta = 90$ , the values of coupling constants given in Table I yield the  $\beta$ -function coefficients at  $M_R$  to be  $b_3^{(m)} = 0.0016$ ,  $b_1^{(m)} = b_2^{(m)} = -0.0115$  which, as per the predictions of Sec. II, are responsible for the slow decrease of  $m_3$  and rather faster increase of  $m_1$  and  $m_2$  below the seesaw scale. These features are clearly displayed in Figs. 2 and 3. Similarly for  $M_R = 2 \times 10^{16}$  GeV,  $M_S = 10^{13}$  GeV,  $\tan\beta = 90$ , these coefficients are  $b_3^{(m)} \approx 0.0015$ ,  $b_1^{(m)} = b_2^{(m)} = -0.01165$  at the GUT-seesaw scale. It is quite interesting to note that the mutual differences are narrowed down by the increase of the first and the second mass eigenvalues and simultaneous decrease of the third eigenvalue leading to large  $D_{ij}$  and the magnification at  $M_S$ . This may be contrasted with the low-energy SUSY case where all the three masses showed decreasing behavior although with different rates [7]. This increasing behavior of neutrino masses  $m_{1,2}$  in contrast to the decreasing behavior in low-energy SUSY is due to different relative values of the gauge and Yukawa couplings at higher scales in the large  $\tan\beta$ -region in split SUSY some of which are given in Table I. Below  $M_S$  the decrease is almost uniform due to approximate equality of the corresponding  $\beta$ -function coefficients of the three mass eigenvalues in this region as discussed in Sec. II. The solutions shown in the Fig. 4 and Fig. 5 clearly exhibit radiative magnification to bilarge mixings at the high values of  $M_S = 10^9$  GeV, and  $M_S = 10^{13}$  GeV corresponding to the evolutions of mass eigenvalues as shown in Fig. 2 and Fig. 3, respectively.

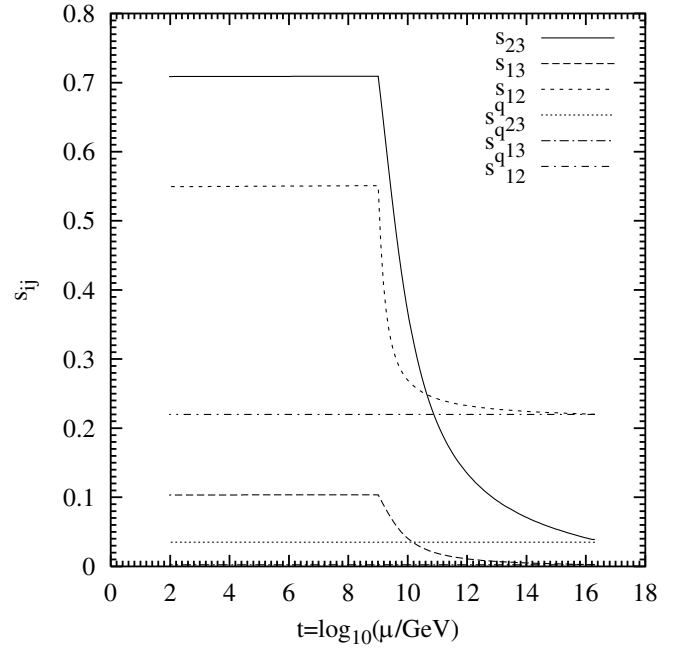


FIG. 4. Evolution of small quarklike mixings at the GUT-seesaw scale,  $M_R = 2 \times 10^{16}$  GeV to bilarge neutrino mixings at the SUSY scale  $M_S = 10^9$  GeV, and extrapolation to low energies for the input and output mass eigenvalues and mixing angles given in the fourth column of Table I. The solid, long-dashed, and short-dashed lines represent the sines of neutrino mixing angles  $\sin\theta_{23}$ ,  $\sin\theta_{13}$ , and  $\sin\theta_{12}$ , respectively, as defined in the text. Almost horizontal lines originating from the seesaw scale represent the sines of the corresponding CKM mixing angles in the quark sector. The mass scale  $\mu$  is in GeV.

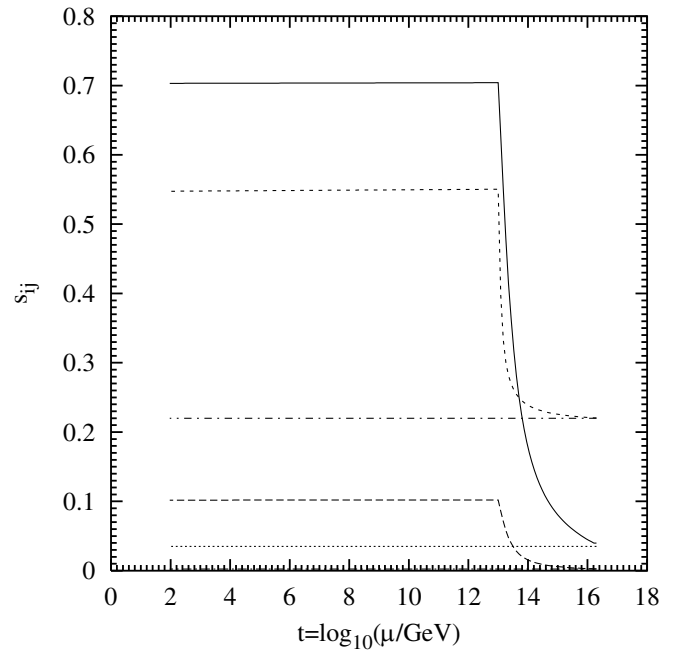


FIG. 5. Same as Fig. 4 but with  $M_R = 2 \times 10^{16}$  GeV,  $M_S = 10^{13}$  GeV,  $\tan\beta = 130$  and with input and output masses and mixing angles given in the third column of Table I.

Choosing the seesaw scale to be the reduced Planck scale  $M_R = M_{\text{Pl}} = 1.2 \times 10^{18}$  GeV we find that the magnification of mixings takes place even at such high SUSY scale as  $M_S = 10^{15}$  GeV. We have also checked that even if we start with the quasidegeneracy assumption and the mixing unification hypothesis at the Planck scale [21] with  $M_R = 1.2 \times 10^{19}$  GeV, then also the radiative magnification to bilarge mixings is possible at such high scale as  $M_S = 10^{15} - 10^{16}$  GeV in the split supersymmetry scenario.

It is clear that due to larger allowed values of  $\tan\beta$  and also due to opposite signs of  $\beta$ -function coefficients of mass eigenvalues at high scales, the radiative magnification of neutrino mixings is possible in split SUSY at high SUSY scales even if the SUSY scale is only 2–3 orders smaller than the seesaw scale. We find that although enhancement due to RG evolution occurs in the  $\nu_e$ - $\nu_\tau$  sector also, the predicted low-energy value remains at  $\sin\theta_{13} = 0.08 - 0.1$  which is well within the CHOOZ-Palo Verde bound [9] and can be tested in the planned  $\theta_{13}$  search experiments.

#### IV. THRESHOLD EFFECTS

In the previous section we considered the mass-squared differences obtained at lower energies purely by RG evolution and these, as shown in Table I, are in agreement with gross features of the experimental data on atmospheric neutrinos but falls somewhat on the higher side of solar neutrino data. Since threshold effects have been shown to make significant contribution on quasidegenerate neutrinos [14,22,23] in this section we estimate them to supplement our RG solutions of Sec. III. In particular we note that the loop factor at the high-SUSY-scale threshold ( $M_S$ ), where the heavy superpartners are located, assumes a very simple form. The threshold loop factor for neutrino mass at the electroweak scale is similar to the SM [23].

In contrast to the weak-scale SUSY where the superpartners contribute predominantly near  $M_Z$  [22,23], in split SUSY the dominant threshold corrections to neutrino masses occur near high SUSY-scale threshold  $\mu_0 \sim M_S$ . Since the mixing angles in the radiative magnification scenario are very nearly the same at both the thresholds, it is quite convenient to use the approximation for the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix  $U_{\alpha i}(M_S) \sim U_{\alpha i}(M_Z)$  and evaluate them in the limit  $\theta_{13} \rightarrow 0$  and  $\theta_{23} \rightarrow \pi/4$  which are very closely consistent with our solutions [14].

Using the loop factors  $T_{ij}$  in the mass basis with  $(i, j = 1, 2, 3)$  the threshold effects on mass eigenvalues at the threshold  $\mu_0$  are expressed as [22,23],

$$m_{ij}(\mu_0) = m_i(\mu_0)\delta_{ij} + m_i(\mu_0)T_{ij}(\mu_0) + m_j(\mu_0)T_{ji}(\mu_0),$$

$$(\mu_0 = M_Z, M_S). \quad (12)$$

Transforming the loop factors in terms of those in the flavor basis,  $T_{\alpha\beta}(\alpha, \beta = e, \mu, \tau)$ , it is straightforward to

evaluate the effects in the limiting case and for quasidegenerate neutrinos with slightly different values of the common mass  $m(\mu_0)$  at  $\mu_0 = M_S, M_Z$  leading to the formula [14]

$$(\Delta m_{21}^2)_{th}(\mu_0) = 4m^2(\mu_0) \cos 2\theta_{12} [-T_e(\mu_0) + (T_\mu(\mu_0) + T_\tau(\mu_0))/2], \quad (13)$$

$$(\Delta m_{32}^2)_{th}(\mu_0) = 4m^2(\mu_0) \sin^2 \theta_{12} [-T_e(\mu_0) + (T_\mu(\mu_0) + T_\tau(\mu_0))/2]. \quad (14)$$

When corrections at both the thresholds are included along with the contribution due to RG evolution, the analytical formula for the mass-squared differences is expressed as,

$$(\Delta m_{ij}^2)(M_Z) = (\Delta m_{ij}^2)_{\text{RG}} + (\Delta m_{ij}^2)_{\text{th}}(M_Z) + (\Delta m_{ij}^2)_{\text{th}}(M_S), \quad (i > j = 1, 2, 3), \quad (15)$$

where the first term in the right-hand side of Eq. (15) has been already estimated from RG evolution and the second and the third terms are estimated through Eqs. (13) and (14). As the RG evolution effects estimated in Sec. III have already given approximately the correct values of  $\Delta m_{32}^2$  for atmospheric neutrino data and  $\Delta m_{21}^2$  only  $4\sigma$ - $5\sigma$  larger than the KamLAND and SNO data, small and simpler threshold effects might be sufficient to account for these deviations. In split supersymmetry such contributions can easily arise from the high-scale SUSY-threshold effects at  $\mu_0 = M_S$ .

Since  $M_S \gg M_Z$  in split supersymmetry, left-handed charged sleptons and sneutrinos have almost identical masses. Denoting the masses of the left-handed sleptons by  $M_{\tilde{\nu}_\alpha}$  and the right-handed charged sleptons mass by  $M_{\tilde{\nu}_\alpha^c}$  we obtain a simple formula for the loop factors at the high SUSY scale ( $\mu_0 = M_S$ ),

$$16\pi^2 T_\alpha(M_S) = (3/8)(g_1^2(M_S)/5 + g_2^2(M_S))(\ln(M_{\tilde{\nu}_\alpha}^2/M_S^2) - 1/2) + \delta_{\alpha\tau}(1/4)h_\tau^2(M_S)(1 + \tan^2\beta) \times (\ln(M_{\tilde{\nu}_\alpha^c}^2/M_S^2) - 1/2). \quad (16)$$

Ignoring the low-energy threshold effect at  $M_Z$  we find that with  $M_{\tilde{\nu}_\alpha}$  or  $M_{\tilde{\nu}_\alpha^c}$  few times lighter than  $M_S$  gives the desired threshold corrections with negative sign. For example, using a simple and plausible choice of approximate degeneracy in the scalar superpartner spectrum around  $M_S$  with  $M_{\tilde{\nu}_\alpha} = M_{\tilde{\mu}_L} = M_{\tilde{\nu}_L} = M_{\tilde{\nu}_R} = M_{\tilde{\mu}_R}$ , we have  $[-T_e(M_S) + (T_\mu(M_S) + T_\tau(M_S))/2] \simeq (h_\tau^2(M_S)\tan^2\beta) \times [\ln(M_{\tilde{\nu}_R}^2/M_S^2) - 1/2]/(128\pi^2)$ . Then with  $M_{\tilde{\nu}_R}/M_S = 0.5 - 1.1$ , numerical values of threshold corrections are obtained as shown in Table I. Such threshold contributions, when added to the RG effects, bring the theoretical predictions in concordance with all the available neutrino data including those from KamLAND and SNO.

## V. DISCUSSION AND CONCLUSION

In this section we discuss the implications of our results briefly and state our conclusions. Our solutions require quasidegenerate neutrino mass eigenvalues in the range  $0.15 \text{ eV} < m_i(M_Z) < 0.5 \text{ eV}$  leading to the effective mass in neutrinoless double beta decay given by  $| < M_{ee} > | = |\sum_i m_i U_{ei}^2| = 0.15 - 0.5 \text{ eV}$ . This is accessible to all the experiments being planned to search for the neutrinoless double beta decay. Furthermore, this range of neutrino masses also overlaps with the range accessible to the KATRIN [12] Tritium beta decay experiment. The prediction  $U_{e3} \equiv \sin\theta_{13} = 0.08 - 0.10$  is also accessible to several planned long-baseline neutrino experiments [24] as well as the planned reactor experiments [25]. As discussed in Ref. [7] the range of eigenvalues of neutrino masses is consistent with WMAP observations and also with the combined analysis of WMAP + 2dF GRS data [13].

In summary, we had shown previously that in the MSSM with weak-scale SUSY [7], the hypothesis of quark-lepton mixing unification and quasidegenerate neutrino spectrum at the seesaw scale successfully explains the observed

mixing pattern for neutrinos i.e. two large mixings needed for  $\nu_e - \nu_\mu$  and  $\nu_\mu - \nu_\tau$  and small mixing for  $U_{e3}$  at low energies. In this paper we have extended this discussion to the case of split supersymmetry where we find that, despite the absence of low-energy SUSY and the corresponding absence of RG running with an amplified value of  $\tau$ -Yukawa coupling over a considerable mass interval ( $M_Z$ - $M_S$ ), radiative magnification of neutrino mixings does occur. These deficits seem to be adequately compensated by the larger values of  $\tan\beta$  allowed in this case and the positive and negative values of  $\beta$ -function coefficients for the running of mass eigenvalues at high scales. The key tests of the model still remain the common mass of neutrinos above 0.15 eV and  $U_{e3}$  between 0.08–0.1, as in [7].

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