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Statistical Analysis of the Elastic and Inelastic Scattering of $^{12}C + ^{28}Si$ and $^{16}O + ^{28}Si$

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A statistical model analysis has been performed on the $\theta_{c.m.} = 180°$ elastic and inelastic scattering excitation functions in the energy range $27.8 \leqq E_{c.m.} \leqq 31.5$ MeV for the $^{12}C + ^{28}Si$ system and in the energy range $30.0 \leqq E_{c.m.} \leqq 32.7$ MeV for the $^{16}O + ^{28}Si$ system. The exact calculation of the number of effective channels for inelastic excitation gives a value of ~ 1.5 corresponding to a change of 5° in the angle at which the cross sections are measured. The Hauser-Feshbach cross-sections, when compared to the experimental data, indicate very large ($\geqq 90\%$) direct reaction contributions to the observed cross sections. Good agreement between the theoretical and experimental distributions of the fluctuating cross sections together with the insignificant values of the cross correlation coefficients indicate that the fluctuating component of the experimental cross sections is consistent with the statistical model predictions.

1. Introduction

The results of the first measurement of elastic and inelastic cross sections of $^{16}O + ^{28}Si$ at most backward angles exhibited strong rise towards 180° which was taken to be an indication of the existence of an orbiting resonance [1]. In order to have more information on the scattering behaviour of such heavy-ion systems Barrette et al. [2] carried out measurements at $\theta_{c.m.} = 180° \pm 5°$ for investigating the energy dependence of elastic and inelastic scattering of $^{12}C + ^{28}Si$ and $^{16}O + ^{28}Si$ in the energy range $17 \leqq E_{c.m.} \leqq 37$ MeV. In this energy range the elastic and inelastic scattering data were found to exhibit gross structure resonances of widths between 1 and 2 MeV, which were fragmented into finer structure of about 250 keV(c.m.) or less for $^{12}C + ^{28}Si$. This was explained in terms of shape resonances with different principal quantum numbers in the ion-ion potential [2].

Recently the same group of authors [3] has reported the results of a more detailed measurement of the fine structure in the elastic and inelastic scattering excitation functions at $\theta_{c.m.} = 180°$ for $^{12}C + ^{28}Si$ and $^{16}O + ^{28}Si$ systems. Based on an statistical analysis of this data in terms of the level width and the fluctuating part of the average cross section it has been pointed out that within the "standard" statistical model it is impossible to reproduce the observed level width and compound nucleus cross sections simultaneously [3]. This was taken to be an indication of the possibility that the entrance channel did not couple directly to the compound nucleus and, consequently, at least partly, the fine structure resulted because of coupling of the entrance channel configurations to various doorway states. However, at the same time it was mentioned [3] that it was not easy to decide whether the above mentioned disagreement was due to deficiencies in the statistical model and/or due to the parameters used in the investigation, or whether it reflected the presence of a nonstatistical mechanism contributing to the observed fine structure.

In order to have a more definite idea of the fine structure in the elastic and inelastic scattering excitation functions of $^{12}C + ^{28}Si$ and $^{16}O + ^{28}Si$ we have performed a statistical analysis following Ericson [4] and Brink and Stephen [5] type of approach. Such an analysis can provide a quantitative estimate of the likelihood that the structures in the data arise from formation of strongly overlapping compound states in ^{40}Ca and ^{44}Ti or whether some non-statistical mechanism is responsible for their presence.

The analysis consists of the calculations of the percentage deviations of the reduced data, the Hauser-Feshbach cross sections, the number of effective channels, the distribution of the fluctuating cross sections and their comparison with the experimental distributions, 1% and 10% probability limits, experimental and empirical coherence widths, and the cross correlation coefficients.

2. Analysis

2.1. Data Reduction

The data subjected to the analysis are from [3]. They were taken by using self-supporting C and Al_2O_3 targets (^{28}Si beam was used and ^{12}C and ^{16}O ions were detected at $\theta_{lab}=0°$ by a *QDDD* spectrometer. This angle of observation corresponds to $\theta_{c.m.}=180°$) with thicknesses of 10 and 20 µg/cm², respectively. This corresponds to an energy averaging of about 30 keV(c.m.) for $^{12}C+{}^{28}Si$ and of about 80 keV(c.m.) for $^{16}O+{}^{28}Si$. Elastic and inelastic scattering excitation functions for $^{12}C+{}^{28}Si$, in the energy range $27.8 \leq E_{c.m.} \leq 31.5$ MeV, were measured in steps of 45 keV(c.m.) and for $^{16}O+{}^{28}Si$, in the energy range $30.0 \leq E_{c.m.} \leq 32.7$ MeV, were measured in steps of 75 keV(c.m.).

For comparing the behaviour of the experimental cross sections with the predictions of the statistical model, we have removed the energy dependent gross structure from the excitation functions. This was done by dividing the individual data points by the running average of the cross-sections $\langle d\sigma(E)\rangle$ taken over an energy interval of $\Delta E=0.630$ MeV(c.m.) for $^{12}C+{}^{28}Si$ and over an energy interval of $\Delta E=0.873$ MeV(c.m.) for $^{16}O+{}^{28}Si$. The percentage deviations of the reduced data, $d\sigma(E)/\langle d\sigma(E)\rangle$, from unity for $^{12}C+{}^{28}Si$ are shown in Fig. 1. It may be seen that no gross structure remains in the data. A similar figure (not shown here) is obtained for $^{16}O+{}^{28}Si$ system also.

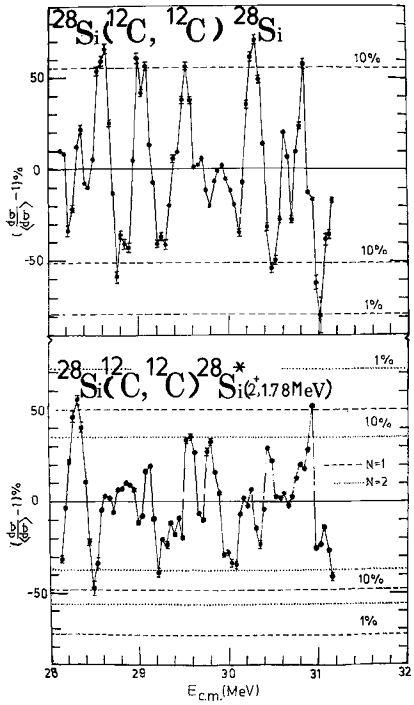

Fig. 1. The percentage deviations from unity of the quantity $d\sigma(E)/\langle d\sigma(E)\rangle$ for $^{28}Si(^{12}C, ^{12}C)^{28}Si$ and $^{28}Si(^{12}C, ^{12}C)^{28}Si^{*}(2^{+}, 1.78$ MeV) at $\theta_{c.m.}=180°\pm5°$. The curves marked 1% and 10% denote deviations from the average for which the probability of finding a larger deviation is 1% and 10% respectively (see Sect. 2.3). For $^{28}Si(^{12}C, ^{12}C)^{28}Si$ the 1% curve corresponding to positive deviation lies at 118% (not shown in the figure) and the same for $^{28}Si(^{12}C, ^{12}C)^{28}Si^{*}(2^{+}, 1.78$ MeV) corresponding to $N=1$ lies at 104% (not shown in the figure)

2.2. Hauser-Feshbach Cross Sections and the Number of Effective Channels

The cross sections for compound elastic and inelastic scattering were calculated by the statistical model code STATIS [6] which employed the Hauser-Feshbach expression [7] for evaluating energy averaged differential cross sections for population of specific final states. For the $^{12}C+{}^{28}Si$ system, $n+{}^{39}Ca$, $p+{}^{39}K$, $^{4}He+{}^{36}Ar$ and $^{8}Be+{}^{32}S$, and for the $^{16}O+{}^{28}Si$ system, $n+{}^{43}Ti$, $p+{}^{43}Sc$, $^{4}He+{}^{40}Ca$, $^{8}Be+{}^{36}Ar$ and $^{12}C+{}^{32}S$ exit channels were included in the calculations. For calculating the transmission coefficients, the optical model parameters for $^{16}O+{}^{28}Si$, $^{12}C+{}^{28}Si$ and $^{12}C+{}^{32}S$ (same set used for all three systems) were taken from [8], for $n+{}^{39}Ca$, $p+{}^{39}K$, $n+{}^{43}Ti$, $p+{}^{43}Sc$ from [9], for $^{4}He+{}^{36}Ar$, $^{4}He+{}^{40}Ca$ from [10] and for $^{8}Be+{}^{32}S$ and $^{8}Be+{}^{36}Ar$ (like $^{7}Li+{}^{28}Si$ and $^{7}Li+{}^{40}Ca$) from [11]. The level density parameters were obtained from the empirical formula $a=(2.40+0.067\,A)\,\mathrm{MeV}^{-1}$ as given by Dilg et al. [12] and the pairing energies were taken from [13]. All these parameters used in the calculations are summarised in Tables 1 and 2. The spin cutoff factor was evaluated using a rigid body moment of inertia $\mathscr{J}\sim\frac{2}{5}mR^2$ where $R=roA^{1/3}$ and $ro=1.5$ fm. It may be mentioned that in addition to their dependence on the transmission coefficients the Hauser-Feshbach cross sections are very sensitive to the level density parameters which are really not precisely known at high excitation energies involved

Table 1. Optical model parameters used for calculating the transmission coefficients (which go as input to the statistical model code STATIS)

Channel	V_{Re}	V_{Im} (Vol) (MeV)	V_{Im} (surf) (MeV)	$r_{0_{Re}}$ (fm)	a_{Re} (fm)	$r_{0_{Im}}$ (fm)	a_{Im} (fm)
$^{12}C+^{28}Si$	10.0	23.4		1.35	0.618	1.23	0.552
$^{8}Be+^{32}S$	42.2		9.86	1.02	0.78	1.07	0.64
$^{4}He+^{36}Ar$	183.7	26.6		1.40	0.564	1.40	0.564
$p+^{39}K$	$54.0-0.32E_{c.m.}+24.0\frac{N-Z}{A}+\frac{0.4Z}{A^{1/3}}$	$0.22E_{c.m.}-2.7$	$11.8-0.25E_{c.m.}+12\frac{N-Z}{A}$	1.17	0.75	1.32	0.53
$n+^{39}Ca$	$56.3-0.32E_{c.m.}-24.0\frac{N-Z}{A}$	$0.22E_{c.m.}-1.56$	$13.0-0.25E_{c.m.}-12\frac{N-Z}{A}$	1.17	0.75	1.26	0.58
$^{16}O+^{28}Si$	10.0	23.4		1.35	0.618	1.23	0.552
$^{12}C+^{32}S$	10.0	23.4		1.35	0.618	1.23	0.552
$^{8}Be+^{36}Ar$	173.0	20.7		1.0	0.62	0.69	0.97
$^{4}He+^{40}Ca$	183.7	26.6		1.40	0.564	1.40	0.564
$p+^{43}Sc$	$54.0-0.32E_{c.m.}+24.0\frac{N-Z}{A}+\frac{0.4Z}{A^{1/3}}$	$0.22E_{c.m}-2.7$	$11.8-0.25E_{c.m.}+12\frac{N-Z}{A}$	1.17	0.75	1.32	0.53
$n+^{43}Ti$	$56.3-0.32E_{c.m.}-24.0\frac{N-Z}{A}$	$0.22E_{c.m.}-1.56$	$13.0-0.25E_{c.m.}-12\frac{N-Z}{A}$	1.17	0.75	1.26	0.58

Table 2. The level density parameters (a/A) and the pairing energies (Δp) used for Hauser-Feshbach calculations

	^{28}Si	^{39}Ca	^{39}K	^{36}Ar	^{32}S	^{40}Ca	
a/A (MeV^{-1})	0.116	0.129	0.129	0.134	0.142	0.127	
Δp (MeV)	3.89	3.66	4.08	3.48	3.29	3.90	
E_{cut} (MeV)	15.224	6.200	7.739	10.85	12.050		
	^{28}Si	^{43}Ti	^{43}Sc	^{40}Ca	^{32}S	^{36}Ar	^{44}Ti
a/A (MeV^{-1})	0.116	0.123	0.123	0.127	0.142	0.134	0.122
Δp (MeV)	3.89	3.46	3.28	3.90	3.29	3.48	3.40
E_{cut} (MeV)	15.224	3.22	6.710	10.129	12.050	10.850	

Note: Here E_{cut} = energy upto which explicit levels were used in the calculation of the denominator

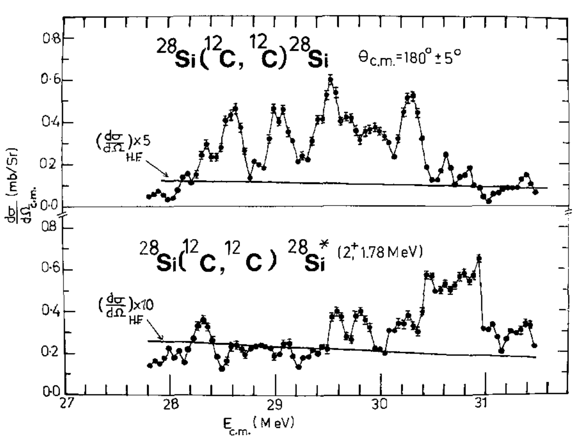

Fig. 2. The experimental and the corresponding Hauser-Feshbach cross sections for $^{28}Si(^{12}C,^{12}C)^{28}Si$ and $^{28}Si(^{12}C,^{12}C)^{28}Si^{*}(2^+, 1.78\,MeV)$. The Hauser-Feshbach cross sections (shown by continuous lines) have been multiplied by the indicated number for the purpose of plotting

in the present type of situations. The influence of these parameters on the cross sections and widths has been examined for some systems by Braun-Munzinger and Barrette [14] (which appeared after the present analysis was over). Contrary to their earlier investigation [3] Braun-Munzinger and Barrette [14] could get good agreement between the experimental values of widths and cross-sections with the corresponding values obtained from the statistical model predictions based on standard Hauser-Feshbach theory for $^{12}C+^{28}Si$ and $^{16}O+^{28}Si$ both. This was obtained by using different level density formula and parameters. We, however, did not make any attempt to this effect.

The experimental values of the cross sections for elastic and inelastic scattering of $^{12}C+^{28}Si$ along with the Hauser-Feshbach cross sections are given in Fig. 2. It can be seen that the theoretical cross sections are upto more than an order of magnitude lower than the experimental ones. The situation is similar for $^{16}O+^{28}Si$ system, as can be seen in Fig. 3. It is, therefore, clear that the measured cross sections have very large direct reaction contributions for both the systems.

Noting that the detecting system had, rather large solid angle ($\theta_{c.m.}=180\pm5°$) in these measurements and the involved angular momenta are also large and, therefore, the $m \neq 0$ magnetic sub-states can contribute appreciably to the inelastic scattering excitation functions, we used the code STATIS to calculate the number of effective channels N. This quantity determines the statistically independent cross sections which contribute to the measured cross section. The details of the evaluation of N are given by Dayras et al. [15]. The variation of N with

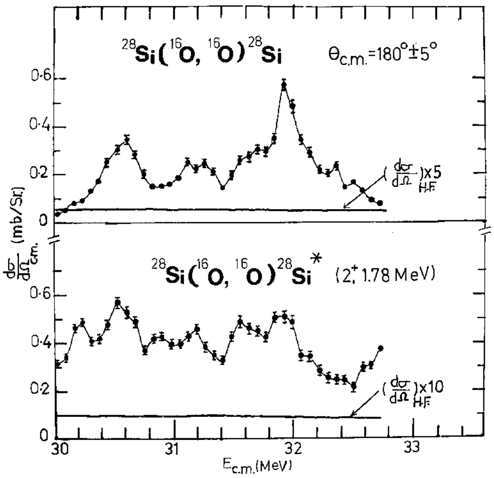

Fig. 3. The same as in Fig. 2 but for $^{28}Si(^{16}O, ^{16}O)^{28}Si$ and $^{28}Si(^{16}O, ^{16}O)^{28}Si^{*}(2^+, 1.78$ MeV)

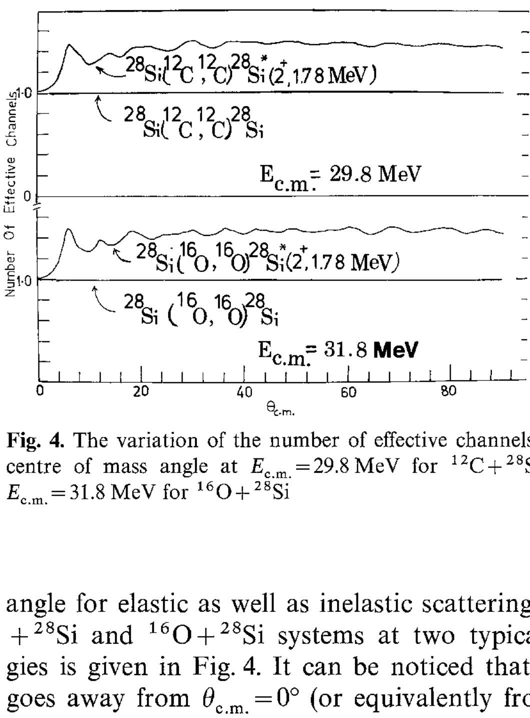

Fig. 4. The variation of the number of effective channels, N, with centre of mass angle at $E_{c.m.} = 29.8$ MeV for $^{12}C + ^{28}Si$ and at $E_{c.m.} = 31.8$ MeV for $^{16}O + ^{28}Si$

angle for elastic as well as inelastic scattering of $^{12}C + ^{28}Si$ and $^{16}O + ^{28}Si$ systems at two typical energies is given in Fig. 4. It can be noticed that as one goes away from $\theta_{c.m.} = 0°$ (or equivalently from $\theta_{c.m.} = 180°$) the value of N, for inelastic excitation, tends to become larger than unity. At $\theta_{c.m.} = 5°$ it becomes 1.47 for $^{12}C + ^{28}Si$ at $E_{c.m.} = 29.8$ MeV and 1.48 for $^{16}O + ^{28}Si$ at $E_{c.m.} = 31.8$ MeV. Thus, as mentioned in [3], the average fluctuating cross section in the inelastic channel obtained by Barrette et al. [3] by using $N = 1$ (N will actually be somewhat larger) is subjected to a large uncertainty. It must be mentioned that in calculating N it is assumed that the reactions proceed purely via compound nuclear process. In using N in this analysis a uniform m-substate distribution is assumed for the direct reaction contributions.

2.3. Distribution of Cross Sections

In presence of the direct reaction contributions, the distribution of the fluctuating cross sections is given by [5, 16]

$$P(x) = \left(\frac{N}{1 - Y_d}\right)^N x^{N-1} \exp\left(-N \frac{x + Y_d}{1 - Y_d}\right) + \frac{I_{N-1}[2N\sqrt{x Y_d}/(1 - Y_d)]}{[N\sqrt{x Y_d}/(1 - Y_d)]^{N-1}} \tag{1}$$

where $x = d\sigma(E)/\langle d\sigma(E)\rangle$, N is the number of effective channels, Yd is the ratio of direct to total cross section, and I_{N-1} is the modified Bessel function of order $N-1$. The average direct reaction contribution Yd was determined as

$$\text{Yd} = \left\langle \frac{\langle d\sigma(E)\rangle - d\sigma_{\text{H.F.}}}{\langle d\sigma(E)\rangle} \right\rangle \tag{2}$$

where $d\sigma_{\text{H.F.}}$ is the calculated Hauser-Feshbach cross section, and $\langle\ \rangle$ denotes the average over the entire energy range. For $^{12}C + ^{28}Si$ this procedure gave Yd $= 0.90$ (for elastic scattering) and Yd $= 0.92$ (for inelastic scattering) and for $^{16}O + ^{28}Si$, Yd $= 0.95$ (for elastic scattering it was slightly less and for inelastic scattering slightly more but we took it to be 0.95 for both). These values of Yd agreed very well with the ones obtained by using the normalised variances of the data and calculated values of the number of effective channels as described by Singh et al. [17]. The theoretical distributions (calculated for $N = 1$ and $N = 2$ both in case of inelastic scattering since N is some what larger than 1) of cross sections are compared with the corresponding experimental ones in Figs. 5 and 6 for $^{12}C + ^{28}Si$ and $^{16}O + ^{28}Si$ respectively. It can be seen that the agreement between the theoretical and experimental distributions is quite good.

The probability of observing a cross section fluctuation which is greater than certain value x' is given by [15]

$$Q(x') = \int_{x'}^{\infty} P(x)\, dx. \tag{3}$$

The probability of observing a fluctuation which is less than some value x' is obviously $1 - Q(x')$. The curves marked as 10% and 1% in Fig. 1 for different values of N correspond to the deviations from the average value of cross sections for which probability of finding a larger deviation is 10% and 1% respectively.

It turns out that in case of elastic scattering of $^{12}C + ^{28}Si$ a few structures just cross the 10% probabili-

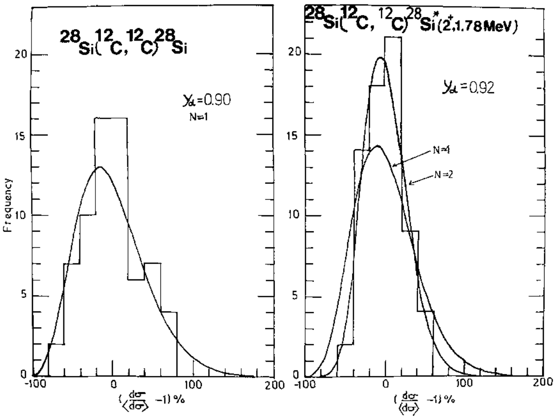

Fig. 5. The distributions of the experimental cross sections (shown in Fig. 1) about the average value. The curves show the theoretical distributions for the indicated values of Yd and N for the $^{12}C+^{28}Si$ system

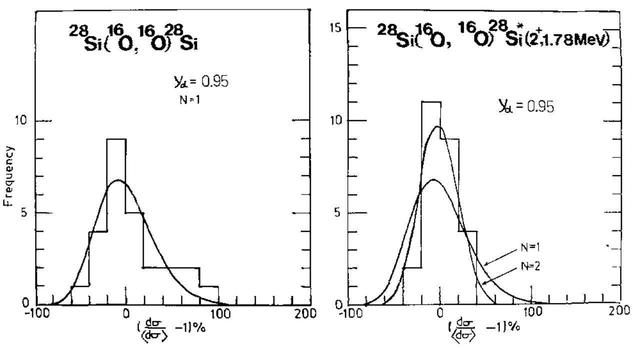

Fig. 6. The same as in Fig. 5 but for the cross sections for $^{16}O+^{28}Si$ system

ty limit and only one touches the 1% probability limit. Thus all (except one with rather large error bar) structures occur with much larger than 1% probability. In case of inelastic scattering for this system only few touch the 10% probability limit and all occur with probabilities much larger than 1% and are, therefore, more likely to be the Ericson type of fluctuations. This type of figure for $^{16}O+^{28}Si$ system exhibits the situation that is even more favourable to the fluctuation phenomenon.

2.4. *Coherence Widths and Correlation Coefficients*

The coherence widths of fluctuations were determined by the usual method of counting the maxima in the excitation functions as described in [17]. The values thus obtained were $\Gamma=(127\pm9)$ keV and $\Gamma=(119\pm8)$ keV for elastic and inelastic scattering of $^{12}C+^{28}Si$ respectively. For $^{16}O+^{28}Si$ the respective values were found to be $\Gamma=(187\pm27)$ keV and $\Gamma=(162\pm27)$ keV. These values are in good agreement with the ones obtained by Barrette et al. [3]. The coherence widths were also estimated using the empirical formula [18, 19]

$$\Gamma=14\exp(-4.69\sqrt{A/E_x})\,\mathrm{MeV}. \quad (4)$$

Where A is the mass number and E_x is the excitation energy of the compound nucleus in MeV. The empirical widths thus obtained turned out to be in the same range of values as the experimental ones for both the systems.
The cross correlation coefficient calculated from the reduced data for elastic and inelastic channels of $^{12}C+^{28}Si$ was 0.05 ± 0.20 and those of $^{16}O+^{28}Si$ was 0.29 ± 0.30. This indicates that there are hardly any correlations between the structures observed in the elastic and inelastic excitation functions in case of $^{12}C+^{28}Si$ and some but insignificant (because of large error resulting from small range of data - for both the systems) in case of $^{16}O+^{28}Si$.

3. Conclusion

The compound elastic and inelastic scattering cross sections predicted by Hauser-Feshbach theory turn out to be more than an order of magnitude smaller than the respective experimental cross sections for both the systems, $^{12}C+^{28}Si$ and $^{16}O+^{28}Si$. This implies that there are very large direct reaction contributions to the cross sections. This is in accordance with the values of $Y_d(\geqq 0.90)$, the direct to total cross section ratios, obtained from the normalised variances of the reduced data, the exactly calculated number of effective channels, and the parameter involving the sample size [17]. The theoretical and experimental probability distributions in Figs. 5 and 6 agree very well with each other. All the deviations occur with much more than 1% probability and majority of them with more than 10% probability in case of both the systems. The experimental coherence widths are in the same range of values as obtained from the empirical estimates. The values of the cross correlation coefficients do not really indicate any significant correlations between the two channels for both the systems.
According to the present analysis the fluctuating features of the experimental cross sections are consistent with statistical model expectations. It is possible that Barrette et al. [3] could not explain the observed width and average fluctuating cross section quantitatively by using the statistical model because of inadequate parameters, specially the ones used to estimate the fusion cross sections.

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