# IMPLICATION
# AND
# ENTAILMENT

BY

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# CERTIFICATE

Certified that the subject matter of this dissertation is the record of work done by Ajit Kumar Basak, that the contents of this dissertation did not form a basis of the award of any previous degree to him, or, to the best of my knowledge to anybody else, and the dissertation had not been submitted by him for research degree in any other University.

In habit and character, Ajit Kumar Basak is a fit and proper person for the degree of MASTER IN PHILOSOPHY ( in Philosophy ).

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# CHAPTER-I

## INTRODUCTION

Chiefly logic is characterised as the study of the methods and principles used in distinguishing between correct and incorrect reasonings or arguments. An argument is defined as any group of propositions, in which one is claimed to follow from the others. Every argument has a structure, in the analysis of which the terms 'premise' and 'conclusion' are usually employed. The conclusion of an argument is that proposition which is claimed to follow from the other propositions and the propositions from which it is claimed to follow are called premises. Both the premises and the conclusion are thus propositions; and for the purpose of logic, a proposition is defined as anything which can be said to be either true or false.

Propositions are divided into two classes; simple and compound. A simple proposition is defined as one which does not contain any other proposition as a part of it. For example, 'Ram is a student of Philosophy' is a simple proposition, because it does not contain any other proposition as a part of it. While 'It is not the case that Ram is a student of Philosophy' is a compound proposition, because it does contain another proposition as a part of it, that is, 'Ram is a student of Philosophy'.

Compound propositions are classified in propositional logic in various ways acccording to their logical connectivews or constants. When the logical connetives operate upon the simple propositions, they generate or form compound propositions. This is the reason why logical connectives are called proposition forming operators. Logical connectives are of two types; monadic and diadic. The logical connectives '∼', 'L', 'M', are monadic and '&', '∨', '⊃', '≡', '⥽' are diadic. The symbols '∼', 'L', 'M' are interpreted as 'It is not the case that', 'It is necessary that', 'It is possible that' respectively and the symbols '&', 'v', '⊃', '≡', '⥽' are interpreted as 'and', 'or', 'if-then', 'if and only if', 'entails' respectively. Compound propositions formed by the use of the connective '⊃ are called material conditional propositions. A conditional proposition is formed by the operation of the 'If-then' phrase. For example, 'If the barometer is falling, then there will be a storm' is a conditional proposition. The meanings of logical connectives are given by stating precisely the conditions under which sentences containing them will be true or false.

In the truth-functional language, propositions are characterised as true or false. From this point of view, they differ from questions, commands and exclamations. Grammarians classifiy the linguistic formulations of propositions, questions, commands, exclamations as declarative, interrogative, imperative and exclamatory sentences respectively. These are familiar notions. All propositions are expressed through declarative sentences.

But declarative sentences and propositions are different. A declarative sentence is part of a language, the language in which it is enunciated, while propositions are not part of any particular language though they are always expressed through language. Declarative sentences are linguistic entities. Propositions are not linguistic entities. Different sentences may express the same peoposition, and different propositions may be expressed by the same sentence. But the logicians mainly deal with the propositions, not the sentences in which they are expressed and formulated. Every proposition has a truth-value./ In case of truth function, truth-value is determined solely by the truth-values of its component propositions. Function is a corelative notion. The numerical value of a simple mathematical function is determined by the variable occuring in the function. An expression is said to be a function of a given variable or variables, if the value of the expression is uniquely determined when the variable or variables take a determinate value.) For example;

$$y = 2x + 2$$

In the above equation, y is a function of x, because its value is determined by the value of variable x. Thus if x takes the value 0, the value if y is 2; if x takes the value 4, the value of y is 10; if x takes the value -4, then the value of y is -6, and so on. In logic we extend this notion of function by taking either of the two values, tha is, 'true' or 'false'. (The truth-value of the compound proposition is uniquely detemined by the truth-values of its component propositions, just as the numerical value or a simple mathematicalfunction is determined by the values taken by the variables occuring in the function. Consider the following examples:

(i) Either the crisis of Janata Dal will be discontinued or the National Front Government will become powerless.

(ii) If V.P.Singh fails to show his majority in Lok Sabha, then Chandrashekhar will become Prime Minister.

In the above examples, the first proposition is a disjunctive proposition and it is true when one of its component propositions is true, while the second propositions is a conditional proposition. It is false when its antecedent is true and the consequent is false, otherwise, true. Truth and falsity are the properties of propositions, not of arguments. Validity and invalidity of an argument and truth and falsity of its premises and conclusion are closely connected. Some valid arguments contain only true propositions. For example:

All men are mortal - True

Mandela is a man - True

∴ Mandela is mortal - True

But an argument may contain only false propositions and yet be valid. For example:

All mathematicians are scientists - False

All philosophers are mathematicians - False

∴ All philosophers are scientists - False

In the above argument, all the premises and the conclusion are false, yet the argument is valid because it is not possible for its premises to be true without conclusion being true.

The relation that holds between the premises and the conclusion of a valid deductive inference is generally called implication. It is an objective relation and as such does not depend upon the knowledge of any individual thinker.In the relation of implication neither the antecedent nor the consequent is asserted to be true. '$\supset$ ' does not assert that p is true or q is true. It only assert that if p is true then q cannot be false. When we assert that 'If it rains, the road will be muddy', we do not assert either that 'it will rain' or that 'the road will be muddy'. On the contrary, in an inference we assert both the premise and conclusion. In order to convert implication into an inference we have to assert three things, namely, implication, implicans and implicate.

In asserting 'p therefore q', we assert that:

(i) p implies q;

(ii) p is true, and therefore

(iii) q is true.

'If-then' phrase expresses a relation of implication and 'since........therefore', expresses the form of inference.

Thus in an inference implication is dissolved and the implicate is separated from its mere hypothetical form and is asserted to be true.) That is why Russell states that inference is the dissolution of implication[1]. According to Johnson, what in the relation of implication is put forward merely hypothetically, is in the inference asserted categorically[2].

Let us consider the following examples:

(a) All men are mortal. Gandhi is a man. Therefore Gandhi is mortal.

(b) If all men are mortal and Gandhi is a man; then Gandhi is mortal.

The statement of (a) and (b) are quite different. The statement of (b) is a statement of logic to the effect that if certain conditions are fulfilled, certain consequence results. It says nothing as to whether or not the conditions referred to in the "It-then" clause are, in fact, fulfilled. But in the statements of (a), that is, in the premises, certain assertions are made and as a logical consequence of these assertions a further assertion is made, namely, the fact stated in the conclusion. To mark this important distinction, the argument of (a) is called as inference and statement of (b) is called as implication.

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1. Russell,B.: Principia Mathematica, Vol.I, 1910, p.9.

2. Johnson,W.E.: Logic, Part II, Dover publications Inc., New York, 1964, p.8.

When we make an inference, we assume the truth of the premises in asserting them; and as a consequence of the truth of the premises and of the logical validity of the argument, we assert the truth of conclusion. But when we commit ourselves to an implication, we do not thereby commit ourselves to the truth of the premises. In inferring we do not only pass from the assertion of the premise to the assertion of the conclusion, but also implicitly assert that the assertion of the premise does justify the assertion of the conclusion.

Whether an inference is deductive or inductive, depends upon the relation holding between the premises and the conclusion. All arguments involve the claim that something follows from something else, but the notion of 'follows from' does not carry the same meaning in all. A valid deductive argument involves the claim that its conclusion follows from the premises conclusively. The truth of the premises provide absolute guarantee for the truth of its conclusion. If the premises of a valid deductive argument are true, its conclusion cannot be false. But in inductive argument the position is different. A valid inductive argumen does not involve the claim that its conclusion follows from the premises conclusively. In it there always remains some gap between the premises and the conclusion. As a result of which, the premises do not provide absolute guarantee for the truth of its conclusion. In both the forms of argument, thus, the notion of 'follows from' carries different meanings.

There is a common misconception about the distinction between deductive and inductive inferences. A deductive argument is defined as one in which we always proceed from the general to the particular or from more general to the less general while in an inductive argument, it is said , we always proceed from the particular to the general or from the less general to the more general.

Take for example, the following arguments:

(1)

All men are mortal

I am a man

∴ I am mortal.

(2)

Ram is mortal

Shyam is mortal

I am mortal

∴ All men are mortal.

The first argument is said to be a deductive argument because it proceeds from the general to the particular. The second argument is said to be inductive argument because it proceeds from the particular to the general.

But both these views of deductive and inductive inference are erroneous. Because deductive arguments do not always proceed from the general to the particular. Take, for example, the following arguments:

(i)

All men are mortal

∴ All non-mortals are non-men.

(ii)

This pen is red

This pen is a ball pen

∴ This pen is red ball pen.

(iii)

Ram is an Indian

∴ Given anything, if that thing is human being, then either that thing is mortal or else Ram is an Indian.

All these examples are the examples of valid deductive arguments. But in case of (i), argument proceeds from the general to the general. In the case of (ii), argument proceeds from the particular to the particular. In the case of (iii), argument proceeds from particular to the general. Likewise, inductive arguments do not always proceed from the particular to the general. Sometimes they proceed from the particular to the particular. For example:

The class that Dr. Ghosh taught yesterday was interesting.

∴ The class that Dr. Ghosh will teach today will be interesting.

Sometimes they proceed from the geneal to the particular. For example:

All classes that Dr. Ghosh taught in the past were interesting.

∴ The class that Dr. Ghosh will teach today will be interesting.

So there is no sherd of truth to the view that a valid deductive reasoning always proceeds from the general to the particular, and ~~the~~ a valid inductive reasoning from the particular to the general. Deductive and inductive arguments differ from one another only in regard to the relationship of their premises and conclusion, not in the way as they depart from one proposition (or set of propositions) to another proposition. Premises and conclusion of a valid deductive argument always stand in the relationship of implication or entailment. It is because of this reason thier premises with the negation of conclusion always imply contradiction, and provides absolute guarantee, if true, for the truth of its conclusion. While in the case of inductive argument, position is different. Premises and conclusion of inductive argument do not stand in the relationship of implication or entailment. Their relationship always remains contingent. As a result, neither their premises with the negation of conclution imply contradiction nor logical oddity of any sort. Their premises do not provide absolute guarantee, even if true, for the truth of its conclusion. This is the distinctive mark of all inductive arguments opposed to deductive ones. This is the reason why the notion of 'validity' is used in both the cases in different senses.

No doubt, inferring is a kind of activity but this activity is different from other sorts of activities like proving, arguing, solving etc.. Proving is different from arguing. Because, someone can argue without proving even in support of invalid argument and may convince others that the premises of a valied argument are false. A man may prove something without arguing, withuot seeking to convince. For example, in proving mathematical theorem in an examination, the examinee does not argue or try to convince the examiner of its truth. He rather exhibits his mathematical knowledge by writing down a set of statements. Inferring is different from both proving and arguing. In inferring, one proposition is always drown from another proposition (or set of propositions), no matter whether it is done deductively or inductively without arguing or proving. Inferring, proving and arguing no doubt are different activities and have different purposes, but in all these activities truths are connected with truths.

The validity of the steps of an argument, in general, are prized for the sake of the truth of the conclusions to which they lead. But the common purpose of arguing, proving and inferring is not of a logical concern. The logical question of the validity of the steps, is one that can be raised and answered independently of the question of whether these purposes are achieved. The steps are valid when the conclusion follows from the premises. And the conclusion follows from the premises only when it is impossible to accept the premises and deny the conclusion, that is, the truth of the premises is inconsistent with the falsity of the conclution. The validity of the steps does not alone guarantee the truth of the conclusion. We often signalize a claim to be making a valid step in resoning by the use of certain expressions to link one statement or set of statements with another. These are words and phrases like 'so', 'consequently', 'therefore', 'for', 'it follows that' etc.. Strawson has analysed these notions. in his analysis he has mentioned that " where the steps are numerous and intricate, we unhesitatingly apply such words as 'inference' or 'argument'; where something that has been said is simply repeated, in whole or in part, we unhesitatingly withhold these words"[3]. The logicians take into account only the relationship that holds between the statements, irrespective of whether or not the transition from one statement to another so related to it is a transition which we should signify by the name 'step in reasoning'; irrespective even of whether it is something we should acknowledge as a transition. From this point of view, we can say that the definition of logic as .the study of the principles of valid deductive reasoning is too narrow.

The relation that holds between the premises and the conclusion of a valid deductive inference is generally characterised as implication. But when we analyze the notion of 'implication' or 'follows from', we find that it

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3. Strawson, P.F.: Introduction to Logical Theory, Mathuen and Co.Ltd., London, 1952, p.12-13.

generates a lot of questions. For example, What is the nature of implication ? Is it formal or material ? Is it related to the structure of the propositions or to their meanings and truth-values ? Is meaning of proposition relevant for it ? These are some of the important questions which need enquiry from the logical point of view. In this proposed dissertation we have addressed ourselves to these questions. So from this point of view, the objective of our dissertation is limited. We have analysed the notions of 'implication' and 'entailment', and other related concepts within the framework of deductive system.

# CHAPTER-II

## IMPLICATION

Consider the proposition'If the train is late then we will miss our connection'. In this compound proposition there are two atomic propositions, namely,'The train is late' and 'We shall miss our connection' joined together by the sentential connective "If-then". Any compound proposition which results by the operation of "If-then" phrase is called a conditional or implicative proposition. The component which comes between the words 'If' and 'then' is called the antecedent or implicans, and the component which follows the word 'then' is called the consequent or implicate. Thus the general form of a conditional or implicative statement can be written as,

"If the antecedent, then the consequent"

A conditional statement does not assert that either its antecedent is true or its consequent is true. It only asserts that if the antecedent is true then the consequent is also true, that is, its antecedent implies its consequent.

Implication is, usually, paraphrased as follows :

(a) "If..................then......................."

(b) "....................not................without"

(c) "Either..............not......................."

(d) "....................if........................"

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(e) "....................only if .................."

(f) "....................provided that............."

(g) "...........is a sufficient condition for......"

(h) "...........is a necessary condition for......."

*Implication* as logical operator is symbolized in the following way by the different logicians :

(i) 'p ⊃ q' - by Russell.

(ii) 'p → q' - by Hilbert, Bennett etc.

(iii) 'pIq' - by T.J.Smiley.

(iv) 'p q' - by C.I.Lewis..

(v) 'Cpq' - by Polish Logicians.

(vi) 'p * q' - by G.E.Moore..

Russell characterises the relation that holds between the premises and the conclusion of a valid deductive inference as implication. In his book, 'Principles of Mathematics',he employs the notion of 'material implication'; but in 'Principia Mathematica', he refers the same notion by calling it simply 'implication'[1]. In order to infer one propositon from another, it is necessary that the two propositions should have that relation which makes the one as the consequence of the other. When a proposition q is a consequence of a proposition p, we say that 'p implies q', or proposition q folows from p.

Russell defines the notion of 'material implication' in terms of negation '-' and disjunction 'v'. For him, 'p materially implies q' means 'either p is false or q is

1. *Russell,B.:* Principia Mathematica, *Vol.I, p.90.*

true'. For example, 'It is raining' materially implies 'the road is wet', which is equivalent to say, either 'It is raining' is false or 'the road is wet' is true.

$$(p \supset q) =df. \; - p \vee q$$

'Material implication' is also defined in terms of negation, '-' and conjunction, '&'. To say 'p implies q' is to say 'it is not the case that p is true and q is false'.Both the definitions of material implication are logically equivalent.

$$(p \supset q) =df. \; - (p \;\&\; q)$$
$$=df. \; - p \vee q$$

In the above expressions the term "df." is shorthand for "is defined equivalent of".

The relation in virtue of which it is possible for us to infer validly, Russell says, is a relation of material implication[2]. By 'formal implication' Russell means general material implication. We get a formal implication on his view when we assert that for every value of x, 'x is a man' materially implies 'x is mortal'. This can be expressed in the logical schema of '$(x)(\phi x \supset \psi x)$'. This schema is the schema of formal implication. For him, formal implication is the class of material implication. That is to say, it is the general material implication. The view which interprets 'implication' in terms of 'the meanings of propositions' is called intensional view - and this view is different from

2. Russell,B.: Introduction to Mathematical Philosophy, 1918 p. 163.

the extensional view of Russell. According to him, implication holds between the truth-values of the propositions. It is relevant here to elaborate both the views.

According to the intensional view, implication holds between the meanings of concepts or propositions. Take, for example, the proposition 'All human are mortal'. In this proposition 'To be human being' implies 'to be mortal'. When we consider this proposition, we consider it, on this acount, in a contemplative way without taking note of their exemplifications in actual cases. We take the proposition intensionally, as asserting a connection of meaning. When we assert it, we assert that there is a meaning connection between the two concepts, that is, 'human being' and 'mortality' can be apprehended without examining vast collections of human beings and finding out in each case that this, that, and the other human beings are mortal. It is because of this reason the word 'implies' in the statement'To be human being implies to be mortal' cannot be interpreted as 'materially implies'. Take another example.The proposition 'If Mandela is a man then India is in Europe', is a false proposition according to the truth-functional analysis, because the first component proposition of this conditional is true and the secind is false. As a result, the first part of the proposition cannot be said to be materially implying the second part of the proposition. The facts being what they are we discover that

'Mandela is a man' does not materially imply 'India is in Europe'. If Asia and Europe unite together and make a continent as Europe then the first proposition would materially imply the other. Thus, it is what is actually the case that determines whether a material implication holds. Another way of saying this is that whether a proposition is materially true or false depends upon what the facts are. When we consider a proposition merely from the point of view of whether it is true or false without taking into account the meaning, we consider it in extensional rather than intensional sense.In an intensional consideration of a proposition only the meaning of proposition is taken into account, not its truth-values.Material implication holds in the following cases:

(i) p is true and q is true.

(ii) p is false and q is true.

(iii) p is false and q is false.

Material implication does not hold in a case where p is true and q is false. The horseshoe ' ⊃ ' symbol is a truth-functional connective and its exact meaning is indicated by the truth-table in the following way :

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| p | q | -p | -q | p&-q | -(p&-q) | -pvq | p ⊃ q |
| T | T | F | F | F | T | T | T |
| T | F | F | T | T | F | F | F |
| F | T | T | F | F | T | T | T |
| F | F | T | T | F | T | T | T |

In the above truth-table, the first two columns represent all the possible combinations of truth-values for the component statements p and q. The columns 4,5,6 and 2,3,7 represent the successive stages in determining the truth-values of the compound statements, -(p&-q) and (-pvq) respectively. The truth-values of column number 8 are identical with the truth-values of 6 and 7, since their formulas are defined to express the same propositions and they take the same truth-values of the components, p and q. We can exemplify this true conditional by the use of different statements.

Conditional statements are of different types. Different types of conditional statements state different types of implication. But not all conditional statements are truth-functional in character. Implication may be either a logical or definitional or causal or subjunctive or factual or material. Consider, for example, the following propositions :

(i) If all cats like milk and Furi is a cat, then Furi likes milk.

(ii) If the figure is triangle then it has three sides.

(iii) If you take Novalgin, your toothache will disappear.

(iv) If Ram has done his job properly, he would not have been fired.

(v) If Ram goes to the movie then I will go to the movie.

(vi) If 2+2=4 then Geetanjali was written by R.N.Tagore.

The proposition (i) expresses a logical implication between the antecedent and the consequent, because the consequent follows from the antecedent by the rule of logic. The proposition (ii) expresses a definitional implicative relation between the antecedent and the consequent, because the consequent follows from the antecedent by the rule of definition. The proposition (iii) expresses a causal relation betwen the antecedent and the consequent, because the consequent follows from the antecedent by the law of causality. The proposition (iv) expresses a counter-factual causal relation between the antecedent and the consequent, because the consequent follows from the antecedent by the rule of counter-factual conditional. The proposition (v) expresses a factual implicative relation between the antecedent and the consequent. The consequent does not follow from the antecedent by the rule of logic or definition or causality. The proposition (vi) expresses a material implication which is different from other types of implication. But since all conditional statements assert that if the antecedent is true then the consequent is true, they could be expressed in the form of material implication. The proposition 'If 2+2=4, then Geetanjali was written by Tagore' is materially true because its antecedent and consequent both are true. Although both the components of proposition are not themselves connected but the proposition gives the impression that the consequent 'Tagore's writing' is related to the mathematical truth '2+2=4'. The truth of the mathematical statement has nothing to do with the historical facts. This means that a conditional statement can be materially true even if its antecedent and consequent are not themselves connected. The use of the phrase 'if-then' in the propositions of (i), (ii), (iii), (iv) and (v) carries more meaning than does ' $\supset$ '. The symbol ' $\supset$ ' expresses only material implication which is different from other sorts of implication, namely, logical, definitional, causal and counter-factual. The symbol ' $\supset$ ' does not capture their whole meanings. It only captures a part of their meanings. A material conditional statement, therefore, should not be confused with a logical, definitional, causal or counter-factual conditional.

A material conditional statement is true in all cases where its consequent is true, irrespective of the truth-value of its antecedent. Take, for example, the following propositions :

(1) If 2+2=5 then Geetanjali was written by R.N.Tagore.

(2) If 2+2=4 then Geetanjali was written by R.N.Tagore.

Here both the propositions are materially true, because their consequents are true. In both the propositions, the antecedent materially implies the consequent. Material implication also holds in cases where the antecedent is false, irrespective of the truth-value of the consequent. Take, for example, the following propositions :

(1) If the tigers are reptiles then Geetanjali was written by R.N.Tagore.

(2) If the tigers are reptiles then Geetanjali was not written by R.N.Tagore.

In both the propositions, the antecedent implies the consequenct. They are true by the definition of material implication. A material implication is always true if its corresponding disjunctive proposition is true; and its corresponding disjunctive proposition becomes true when one of its disjunct is true. And this condition is satisfied in both the cases of material implication cited above, because their corresponding disjunctive propositions contain one true statement in them, that is, the antecedent. But in the whole analysis of material implication one should not forget that when it is define in terms of the truth-values of propositions, propositions are always taken as atomic whole. In other words, truth and falsity of the propositions are taken in complete, not in partial sense. In this respect, truth-functional logic is different from monistic logic. No matter, whether a proposition is true or false, its ~~the~~ truth-value is always determinable and justifiable only in reference to its possible world of discourse.

But when we fail to distinguish material implication from other sorts of implications and think that it expresses the whole meaning of all conditional statements, it leads to paradoxes. A pardox is the lexical sense of the word, is a statement that goes against generally accepted opinion. In logic, this word is given a more precise meaning. A logical paradox consists in contradictoy propositions which seems to be apparently sound because when they are used in other contexts they do not seem to create any difficulty. It is only in the particular combination in which the paradox occurs, the argument leads to a puzzle. In this more extreme form, a paradox consists in the apparent equivalence of two propositions, one of which is the negation of the other[3].

It could be expressed in the following symbolic form :

(i) $A \supset - A$

(ii) $(A \supset - A) \supset - A$

The paradoxes of material implication are expressed in the forms of '$p \supset (q \supset p)$' and '$-p \supset (p \supset q)$'.Both the forms are forms of tautology. But when they are expressed in natural language as 'a true proposition is implied by any proposition whatever' and as 'A false proposition implies any proposition whatever', they seem strange though in fact they are not. Because when it is kept in mind that the horseshoe '$\supset$' is a truth-functional symbol which stands for material implication rather than implication in general, these propositions do not seem strange. The paradoxical position arises only when we take material implication in the sense of general implication. But once we realise that material implication is a relation which takes the values from the truth-values of its component propositions, no matter

3. Edwards,P.: Encyclopedia of Philosophy, McMillan Pub. Co. Inc. New York Vol.5, 1967, p. 15.

whether its component propositions themselves are connected or not, the paradoxes cease to exist. Take, for example, the following propositions :

(a) 'The sun is cold' implies '2+2=4'

(b) 'The sun is cold' implies '2+2=5'

(c) '2+2=5' implies 'Mother Teresa is a social worker'.

(d) '2+2=4' implies 'Mother Teresa is a social worker'.

In the cases of (a) and (b), a false proposition, that is, 'Thw sun is cold' implies true and false propositions, that is, '2+2=4' and '2+2=5' respectively, and yet material implication holds (a) and (b) both the propositions are true because a material conditional propsition is true when its antecedent is false which is true of (a) and (b), as a matter of fact.Likewise, in the case of (c) and (d), a true proposition, that is , 'Mother Teresa is a social worker' is implied by false as well as true propositions, that is, '2+2=5' and '2+2=4' respectively and material implication holds. (c) and (d) both the propositions are true because a material conditional proposition is true when its consequent is true, which is true of (c) and (d), as a matter of fact, although their components themselves are not connected. The truth of consequent cannot be said to be the result of the truth of antecedent. Formal logicians, like Langer, argue that the paradoxical cases of material implication do not minimize its importance for inference. In paradoxical cases like the cases of (a) and (b), where the antecedent is known to be false, its intensional implication is as useless as its material implication. In case, like (c) and (d), where the consequent is known to be true, the inference is unnecessary. The only case where inference is in question, is the case when a true proposition implies another proposition and in this case material implication guarantees that this other proposition will be true. Whenever, 'real' implication exists, Langer argues, we have 'p implies q' and whenever 'p ⊃ q' holds and real implication does not, inference is irrelevant any way[4]. This argument of Langer does not convincingly establish the point that material implication is a necessary basis for inference. Because in his argument, there is an explicit admission that material implication is wider than real implication. It gives rise to inference only when it coincides with real implication and as such cannot be taken as a sure basis for valid inference. Further, whether inference is relevant or not can be decided only with reference to the meaning or content of propositions. Again, even if we know that a proposition p is true, we cannot assert that it implies another, q, unless we know also that q is true.

The relation of material implication cannot be said to be the one that holds between the premises and the conclusion of a vcalid inference. Moore has rightly pointed out that one proposition may materially imply other, but the

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4. Langer, S.K.: An Introduction to Symbolic Logic, Dover Pub. Inc., New York, 1967, p.278.

second may not be deducible from the first[5]. The proposition 'Mandela is a man' materially implies the proposition 'Cheralite is a metal'. But the second proposition, that is, 'Cheralite is a metal' is not deducible from the first proposition, that is, 'Mandela is a man'. The first proposition 'Mandela is a man' is not related to the second proposition 'Cheralite is a metal' in the way "Kumar is a bachelor" is related to "Kumar is an unmarried male". But it could be said to be an instance of a valid formal implication, because it is materially true statement. Both the antecedent and the consequent are true.'Mandela is a man' materially implies '2+2=4'. But to say this is not to say that the latter is deducible from the first. Neither material implication nor formal implication, as conceived by Russell, could be said to be the logical ground for an inference to be valid. Moore rightly characterises the relaion that holds between the premises and the conclusion of a valid argument is a relation of entailment.

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5. Moore,G.E.: Philosophical Studies, Routledge and Kegan paul, London, 1922, p.291.

# CHAPTER-III

## ENTAILMENT

In the preceeding chapter we discussed the notion of implication. And in that connection we had seen Russell's notion of material and formal implication does not involve, strictly speaking, the notion of necessity which is the logical requirement of any argument to be valid. Therfore, neither material nor formal implication would be useful as the basis of inference. To capture the notion of necessity many logicians introduced the notion of strict implication and entailment.Let us examine their views.

Russell employs the term 'entailment' in his book, "Principia Mathematica", in the sense of material implication[1]. According to him, to say that 'p entails q' is to say that 'It is not the case that p is true and q is false' or 'p is false or q is true'. But Russell's definition of entailment cannot be accepted because when we say that 'p entails q' we not only say that 'It is not the case that p is true and q is false' but also 'It is impossible that p is true and q is false'. Russell commited a mistake by identifying the notion of 'entailment' with the notion of 'material implication'. Russell was trying to develop extensional logic, because he wanted to eliminate the notion

1.Russell and Whitehead :Principia Mathmatica, Vol.I,1910, p.94.

of 'necessity' which is quite compatible with his empiricism. To reduce entailment to material implication is to eliminate 'necessity'.

C.I.Lewis defines the notion of 'entailment' in terms of 'strict implication' in order to remove the paradoxes of material implication ; and the notion of strict implication defines in terms of negation, possibility, necessity and product :

$$p \;\text{-3}\; q = \text{df. } L(p \supset q)$$ [2]

$$= \text{df. } -M(p.-q)$$

For him[3], to say that 'p strictly implies q' is to say that "It is necessary that 'p materially implies q'". In other words, 'p strictly implies q' means 'It is impossible that p is true and q is false', or "The statement 'p is true and q is false' is not self-evident". Lewis recognizes the element of necessity involve in the notion of entailment which Russell sought to eliminate in his theory. But Lewis' definition too cannot be said to be satisfactory, because it contains in it the notion of a contradiction of the form (p.-p). To assert that "It is impossible that 'p is true and q is false'" is to assert implicitly a contradiction, because the notion of 'impossible' involves in it the notion

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2. Lewis does not use the symbol, L and M for necessity and possibility. He uses the symbols □ and <> for necessity and possibility. But for the sake of convenience we are using L and M for □ and <> respectively.

3. Lewis and Langford: Symbolic Logic, Dover Pub.Inc.,New York, 1956, p.123.

of contradiction. How it involves the notion of contradiction, we can see this in the following ways:
The negation of material implication is equivalent to the conjunction of antecedent and the negation of consequent.

$$-(p \supset q) = df. - ( - p \vee q )$$
$$= df. - - p . - q$$
$$= df. \; p . - q$$

Since - (p ⊃ q) is logically equivalent to (p . - q), to say that 'p entails q'is to say that according to Lewis' definition, (p . - q) is impossible. If p is impossible, it contains in it a conradiction according to the Lewis' idea of impossibility. And if p contains in it a contradiction, then whatever q might be, 'p . - q' will contain a contradiction. But if q is necessary, - q is would be impossible. And if - q is impossible, then whatever p might be, 'p . - q' will contain in it a contradiction. So, no matter whether p is impossible or q is necessary the conjunction 'p . -q' always contains in it a contradiction. And if (p . - q) contains in it a contradiction, Lewis' definition of 'entailment' in terms of 'strict implication' cannot be said to be satisfactory. Besides, Lewis' definition of strict implication also leads to the paradoxical conclusion, as Russell's definition does, which can be stated in the following symbolic forms :

(i) $(p.-p) \prec q$

(ii) $q \prec (p \vee -p)$

These formulas are unplausible as principles of entailment of deducibility[4]. When ' $\prec$ ' is interpreted as 'entails', the formula (i) means that 'an impossible proposition (a proposition that contains a contradiction) entails any proposition whatsoever'; and the formula (ii) means that 'a necessary proposition is entailed by any proposition whatsoever'.

Lewis introduced the notion of strict implication to remove the paradoxes of material implication and capture the notion of necessity. But he failed on this account. If we take the truth of strict implication, we find that it leads to the paradoxical conclusions. Because, according to the definition of Lewis, both the statement forms of (i) and (ii) are necessary truths of logic. Lewis was aware of this paradoxical consequences of the identification of strict implication with entailment. But he did not consider them to be paradoxical at all except in the sense of being strange and unfamiliar. He constructed proofs for the paradoxical consequences by deducing 'q' from 'p.-p' and 'qv-q' from 'p' by valid principles of definitive inference[5]. The following principles Lewis took intuitively to be valid principles of definitive inference :

(a) Any conjunction entails each of its conjuncts. (Simplificaton)

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4. See Hughes and Cresswell: Introduction to Modal Logic, Barnes and Noble Inc, London, 1968, p.335.

5. Ibid, op.cit.p.337.

(b) Any proposition, p entails pvq, no matter whatever q may be. (Addition)

(c) The principles pvq and -p together entails the conclusion q. (The principle of the disjunctive syllogism)

(d) Whenever p entails q, and q entails r; p entails r. (The principle of the transitivity of entailment)

(e) The proposition p is logically equivalent with the form '(p.q) v (p.-q).

(f) Any proposition of the form (p.q) entails (q.p).

Lewis shows that by using these principles we can validly derive any arbitrary proposition[6]. He derives by these principles the following paradoxical consequences :

(i) An impossible proposition (that is, a proposition and its negation) entails any proposition.

| | |
|---|---|
| | (1) p.-p |
| (1), entails | (2) p |
| (1), entails | (3) -p.p |
| (3), entails | (4) -p |
| (2), entails | (5) pvq |
| (5),(4), entails, | (6) q |

Thus, (p.-p) entails q.

(ii) A necessary proposition is entailed by any proposition whatsoever.

| | |
|---|---|
| | (1) p |
| (1), entails | (2) (p.q) v (p.-q) |

6. Lewis and Langford: Symbolic Logic, Dover Pub.Inc., New York, 1956, pp. 250-251.

| | |
|---|---|
| (2), entails | (3) p.(qv-q) |
| (3), entails | (4) (qv-q).p |
| (4), entails | (5) qv-q |

Thus, (qv-q) is entailed by p.

Deducibility is a relation that holds between the conclusion and the premises of a valid deductive inference. And in a valid deductive inference, what we require is the logical guarantee that we shall not have that premises true and the conclusion false. From this point of view, paradoxes could be said to be sound principles of deducibility. So, it is not their absence from a system which would tell against its claim to be a correct logic of entailment. Take the case of (i). To say that '(p.-p) entails q', on this account, is to say that it is logically impossible for (p.-p) to be true and q false. This is equivalent to say that (p.-p.-q) is logically impossible. (p.-p.-q) is logically impossible because (p.-p) itself is logically impossible. And to say this , is not to say that (p.-p.-q) is impossible because of the conjunction of -q. In any valid deductive reasoning, the logical guarantee that we require is not the guarantee of the former kind but of the latter kind,that is, premises with the negation of conclusion must not be true. The same kind of comment applies to the other paradoxes also. Moreover, this account will guarantee that ' ⥽ ' can be interpreted as 'entails'; for in all the standard systems ' ⥽ ' is defined as $-M\ (\alpha.-\beta)$, or $L(\alpha \supset \beta)$, where 'M' is interpreted as "It is logically possible that" and 'L' is interpreted as "It is necessary that".[7]

Let us consider the inconsistency involved in Lewis' proof of the argument for the validity of the paradoxical statements. In case of (i), the final step (6) of the deduction, q, is deduced from the two premises, that is, step (5) and (4). Why did Lewis do it ? Obvioulsly, because -p is contradictory of p. Since -p is contradictory of p; if -p is true, p is false. And if p is false, the other component of the disjunction (pvq), namely, q must be true or else the argument cannot be valid. It is this conjunction which Lewis took as the primary premise of the deduction. In an inference which is confessedly per impossible, we can start with an impossible proposition as a premise but we cannot contrast this premise, either explicitly or implicitly, within the the context of the same inference. In every inference there is a demand for consistency which Lewis' proof fails to satisfy. Not only this, to accept a contradictory proposition is to suspend temporarily the use of the law of contradiction. And when it is done, no inference can be drawn at all. In the proof of (i) argument, since we start with a contradictory premises and proceed to deduce consequence from its proof, it cannot be said to be a genuine proof. P.K. Sen rightly pointed out that the reductio absurdum method is useful only for equivalent

7. Hughes and Cresswell: Introduction to Modal Logic, p.336.

transformatiom[8]. In an argument the steps which consist equivalent transformation, we merely pass from one (linguistic) expression to another. But in all other steps we pass from one proposition to another with the help of certain valid principles of inference. It is only a step of latter kind that could be said to be the genuine step of inference, not the former kind.

G.E.Moore[9] defines the notion of entailment in terms of deducibility. For him, 'p entails q', when and only when, 'q follows from p' or 'q is deducible from p' in the sense in which the conclusion of syllogism in BARBARA follows from the two premises, taken as one conjunctive proposition; or in which the proposition "This is coloured" follows from "This is red". "p entails q" will be related to "q follows from p", in the same way in which 'a is greater than b' is related to 'b is less than a'. When two propsosition p and q are so related that 'it is not the case that p is true and q is false' the relation between them is that of entailing.... Moore's notion of entailment does not involve in it subjective element. He makes the distinction between entailment and self-evident relation. Fir him, entailment is purely objective and logical relation which holds between the concepts and propositions. Self-evident notion involves

8. Sen,P.K.: 'The Problem of Entailment' Ed. by Daya Krishna in the book Modern Logic, Delhi,p.38.

9. Moore,G.E.: Philosophical Studies Routledge and Kagen paul Ltd., London, 1960, p.291.

in it subjective or psychological elements; and hence cannot define the notion of entailment. Moore's notion of entailment involves in it the notion of necessity which is neither captured by material nor by formal implication of Russell.

To avoid the problem arising out of Lewis' notion of entailment, P.F. Strawson[10] makes some amendments in Lewis' notion of entailment. According to him, entailment relation holds between contingent propositions. It does not hold between a contradictory and a contingent or a contingent and a tautologous propositions. For him, P entails Q, if and only if,

(1) P $\rightarrow$ Q

(2) P is contingent

(3) Q is contingent.

Strawsonian account of entailment is different from Lewis' account. According to Lewis, entailment relation holds between any two sorts of propositions whatever. It holds between a contradictory and a contingent or a contingent and a tautologous propositions. For him, a contradictory proposition entails any proposition and a necessary proposition is entailed by any proposition whatsoever. But according to Strawson, an impossible proposition does not entail any proposition nor a necessary proposition is entailed by a contingent proposition. In other words, on

10. Strawson,P.F.: Introduction to Logical Theory, Methuen and Co. Ltd, London, 1952. p.25.

Strawsin's view, no necessary or contradictory proposition entails or is entailed by any other proposition whatsoever. Strawson's theory of entailment, no doubt, removes the paradoxes of entailment which arise from Lewis' definition of entailment. Because, on his account, entailment relation always holds only between contingent propositions. And if entailment relation always holds only between contingent propositions, the question of a contradictory proposition's entailing any other proposition whatsoever and a necessary proposition's being entailed by any proposition whatsoever, does not arise at all like in Lewis' definition.

T.J. Smiley's[11] account of entailment is similar to Strawsonian account. Like Strwson he also maintains that entailment relation neither holds between tautologous propositions nor a contingent and a tautologous propositions. For him, P entails q, if and only if, $P \supset Q$ is a substitution instance of $P' \supset Q'$ such that, (1) $P'$   $Q'$ is a tautology, (2) Q is a non-tautologous and -P is non-tautologous. To say that entailment relation always holds between non-tautologous propositions is not equivalent to say that it always holds between contingent propositions. Because, propositions could be non-tautologous without their being contingent, for example, contradictory propositions. So from this point of view it could be said very well that Strwsonian account of entailment is different from Smiley's

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11. Smiley,T.J.: "Entailment and Deducibility" in Proc. of Aristotelian Society, 1958-1959.

account. Because, on Smiley's account, entailment relations holds between a contradictory and a contingent propositions which Strwsonian account denies. Smiley's definition merely rules out the entailment relation between two tautologous propositions or a tautologous and another sorts of propositions. But it does not rule out entailment relation between a contradictory and a contingent propositions which Strawson's definition of entailment rules out. This shows that on Smiley's account, 'an impossible proposition implies any proposition' is not against the notion of entailing which according to Strawson is against the notion of entailing. Smiley's account does not remove completely the paradixical consequences of Lewis's account. In this respect Strawsonian account is different from Smiley's account.

Von Wright[12] offers the following definition of entailment: P entails Q, if and only if, by means of logic, it is possible to come to know the truth of P  Q without coming to know the falsehood of P or the truth of Q. Though Von Wright and P.T. Geach[13] express the notion of entailment in different language, what they say is the same. Both the ogicians hold the view that P entails Q, if and only if, a material conditionals statement of P and Q is a tautology. Tautologous statements are of such kind that their truth-values can be known and determined on apriori grounds by the rules of logic alone. In fact Von Wright's and

12. Von Wright,G.H.:Logical Studies, London,1957.

13 Geach,P.T.:"Entailment"in Arist. Soc.Suppl.,Vol.32,1958.

Geach's definition of entailment does not differ from Lewis' definition. Because, Lewis too holds the view that P entails Q, if and only if, their material conditional is a tautology. Von Wright's and Geach's account of entailment suffer from the same kind of difficulty from which Lewis' account suffers.

E.J. Nelson[14] defines 'entailment' in the sense of 'follows from'. For him, 'P entails Q' means there is inner connection between propositions, it relates. And the relevancy of the one to other cannot hold merely on account of same property (falsehood or truth or impossibility or ncessity) that one of the propositions may posses on its own. Nelson suggested that in defining 'P entails Q' as "P is inconsistent with the denial of Q", where inconsistency again means a relation involving both propositions and not merely the impossibility of their joint truth, the paradoxical conclusions cease to exist.

Belnap's and Anderson's[15] definition of entailment satisfies Nelson's demand for 'relevance'. Anderson and Belnap define the notion of 'entailment' in terms of 'deducibility and relevance'. For them, a statement 'P' logically entails another statement 'Q', if and only if, 'Q' can be deduced from 'P' in a non-trivial way. The restriction which Anderson and Belnap impose on deductive

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14. Nelson,E.J.: "Intentional Relations", Mind, Vol.39,1930.

15. Anderson and Belnap: "The Pure Calculus of Entailment" J. of Symbolic Logic, Vol.27, 1962.

proof constitute a guarantee of relevance between the entailing and entailed statements. Thus on their account, P entails Q, if and only if, P is relevant to Q and Q is deducible from P. P is said to be relevant to Q when P and Q have common content. The definition yields no laws of entailment except ones in which the entailing and the entailed forms have atleast one variable in common.For example, it is law that for any P and Q, (P & Q) entails P (here P is common to antecedent and consequent), but not that for any P and Q, (P & -P) entails Q (in which the consequent-variable Q does not appear in the antecedent). The entailment relation on their account, demands that the entailing statement must be relevant to the entailed statement. Gary Iseminger[16] criticises Anderson and Belnap by saying that relevance is not necessary for an inference to be valid and to define the notion of entailment which is not correct. Because, in the absence of relevancy, proofs becomes meaningless.

From the foregoing discussions, it is quite clear that the notion of 'entailment' is used by different logicians in different senses. Russell used this term in the sense of material implication. Lewis used 'strict implication' to define the notion of entailment. When 'entailment' is define interms of 'material implication', as Russell did it, the notion of entailment does not involve in it the notion of necessity. Because, the

16. Iseminger,G.: "Is Relevance Necessary for Validity?" in Mind, Vol.IXXXIX, N.354, April, 1980, pp.196,213.

notion of material implication does not involve in it the notion of necessity. And this view, to say that 'p entails q' is to say "It is not the case that p is true and q is false". Russell's view of entailment could be called as an extensional view of entailment devoid of necessity element. Lewis defines 'entailment' in terms of 'strict implication'. According to him, to say that 'p strictly implies q' is to say "It is necessary that 'p materially implies q'". When the notion of entailment is defined in terms of strict implication, surely it does contain in it the notion of necessity but it is not free from truth-value element. Because 'strict implication' is defined in terms of 'necessity' and 'material implication'. When the notion of 'entailment' is defined in terms of 'L(p ⊃ q)'; it only asserts that material conditional statement is necessary when it is tautology. But modal necessity is different from tautological necessity. Tautological necessity is truth-value based necessity. It is a meaning based necessity. The antecedent and the consequent are used as atomic whole. They are related by ' ⊃ ' truth-functional connective and ' ⊃ ' connective does not involve in it the notion of necessity. But when 'p ⊃q' is tautologous, it does not involve in it the notion of necessity. Modal notion of necessity is not a truth-functional nor is it an extensional notion of necessity.

Moore defines the notion of entailment in terms of deducibility. And his notion of deducibility involves the notion of necessity. Strawson and Smiley also used the term entailment in terms of deducibility, but they impose some restrictions on entailing and entailed propositions. According to Strawson, entailment relation always holds between contingent propositions. And for Smiley, entailment relation holds between non-tautologous propositions. Von Wright and P.T. Geach also hold the similar view. But Anderson, Belnap and Nelson add the notion of relevance to the notion of entailment apart from the notion of deducibility. They take both the features, that is, deducibility and relevance to define the notion of entailment. Material, formal and strict implication are implications of truth-values and not of meanings. They are extensional and intensional. But the notion of entailment is not merely a relation of truth-values but also the relation of meaning. Meaning element cannot be divorced from the relation of entailing. When it is divorced and only truth-value elements are taken into consideration, it leads to paradoxical conclusions; as we have seen in Lewis' interpretation of strict implication. The difficulty arises when attempts are made to formalise the meaning element of entailment relation.

# CHAPTER-IV

## A CRITICAL EXAMINATION

In the preceeding chapters, I discussed the notions of implication and entailment. And while discussing these notions we had seen that they involve difficulties, some of which have already been pointed out. In this chapter we will discuss some the other logical difficulties involve in these notions.

According to the Lewis' definition of entailment, the expression, 'p → q' q is equivalent to 'L(p⊃q). The expression 'L(p⊃q)' is interpreted as "it is necessary that 'p materially implies q'" or 'p materially implies q' is necessary. To say that 'p materially implies q' is necessary is to say 'p⊃q' is a tautology, which means that whenever p is true, q is also true This amounts to saying that 'p is true and q is false' is imposssible according to the definition of Lewis. But to say that 'p⊃q' is necessary in tautological sense is not to say that p and q themselves are necessarily connected Nor does it mean that q is deducible from p. It only means that 'p⊃q' is true in all its extensional interpretations And to say this is to say that whenever p is true, q is also true or it is not the case that p is true and q is false. This means that the true truth-value of p and q are coextensive. And to say this is not to say that q is deducible from p, nor does it mean that q is necessarily connected. The horseshoe symbol '⊃' does not involve in it the notions of connectedness, necessity and deducibility when it is used in truth-functional sense. The expression 'L(p⊃q)' involves in it the notion of tautological necessity. But the modal operator 'L', operating on 'p⊃q' is not a truth-functional operator. Therefore, the notion of modal necessity expressed by the symbol 'L' cannot be said to be identical with the extensional notion of necessity. If 'p⊃q' is a thesis, that is, 'p⊃q' is a tautology, then 'L(p⊃q)' is also a thesis according to the rule of Necessitation. It also holds vice versa according to the axiom of Necessity. But if 'p⊃q' is not a thesis, it does not imply 'L(p⊃q)' according to the rule of Necessitation. A contingent proposition does not imply a necessary proposition. '(p⊃q)⊃L(p⊃q)' is not a valid thesis when 'p⊃q' is not a tautology. But when Lewis interprets 'p ⥽ q' [illegible] in terms of 'L(p⊃q)', he fails to maintain the distinction between the tautological and modal necessities, because he interprets the expression 'L(p⊃q)' in tautological sense, that is, 'p materially implies q' is a tautology. And in doing so he transforms modal necessity into tautological necessity. This we can also see from another point of view. According to Lewis, if 'p⊃q' is a thesis, 'L(p⊃q)' is also a thesis, that is, ⊢ (p⊃q) → ⊢ L(p⊃q). (In this expression, 'thesis' is abbreviated by the symbol '⊢ ' and to express the derivability of one thesis to another by the symbol '→'). It holds vice versa also, that is, if 'L(p⊃q)' is a thesis, then 'p⊃q' is also a thesis.

$\vdash$ L(p⊃q) → $\vdash$(p⊃q), which means propositional and modal theses not only imply each other but also are logially equivalent. But to accept this is to rule out the distinction between modal necessity and tautological neccessity which is not correct. Tautological necessity holds between truth-values of the propositions. It holds good only in a particular system of thought, a system of thought which presupposes that a proposition must be either true or false but not both. Any system of logical thought which does not presuppose that true and false truth-values are mutually exclusive and collectively exhaustive and admits to a third possibility, the truth-functional tautological notion of necessity does not hold good in that system. Modal necessity, on Lewis' account, operates on truth-functional necessity but is not identical with it, although while interpreting he does not maintain this distinction clearly. Lewis wanted to remove the paradoxes of material implication by introducing the notion of strict implication. But he could not succeed. He derives a necessary proposition 'qv-q' from a contingent proposition which goes against the rule of Necessitation. The rule of Necessitation does not permit to deduce a necessary formula from a contingent one.

Lewis interpretes the notion of entailment not only in the sense of tautological necessity but also in the sense of logical deducibility. He defines the notion of entailment in terms of 'deducibility'. For him to say 'p entails q' is to say 'q is logically deducible from p' or 'q logically follows from p'. The notion of 'deducibility' or 'logically follows from' involves in it the notion of logical necessity (or tautological necessity). Inferential modal notion of necessity is not only different from the tautological notion of necessity but also from the necessity of the 'L' operator. Inferential modal notion of necessity (that is, entailment) is a binary operator, while the operator of modal necessity 'L' is a monadic operator. Both the operators involve in them the notion of necessity but the entailment notion of necessity consists in between the premises and the conclusion of a valid deductive inference, whereas the necessity of 'L' operator does not. Lewis interprets the notion of entailment in terms of tautological necessity and deducibility but does not maintain their distinction clearly in his logical system. He transforms them interchangably which he should have not done.

For any inference to be valid, premises and conclusion must be necessarily connected, no matter whether it is expressed by the notion of implication or entailment. This is the logical requirement of validity. In fact, the validity of an argument is defined and judged in terms of this relationship. But from Lewis' proof of 'p/∴ qv-q', it follows that an argument can be valid without premises and conclusion being necessarily connected. Premise 'p' is quite irrelevant to the conclusion 'qv-q', and yet the conclusion is claimed to follow from the premise according to the rule of inference. If the premise is irrelevant to the conclusion, the premise and the conclusion cannot be said to be necessarily connected. If the premise and the conclusion are not necessarily connected, it makes no sense to say that the conclusion logically follows from the premises. Premises of a valid deductive argument cannot be said to provide absolute guarantee for the truth of conclusion. In fact, when there is no connection between the premises and the conclusion, the question of deducing conclusion from premises does not simply arise at all. We cannot say that premises entail or imply the conclusion when there is no connection between them. But since Lewis has done it, his proof cannot be said to be a legitimate one.

In a valid deductive argument the conclusion does not contain anything which is not present in the premises implicitly or explicitly. The relation that holds between the premises and the conclusion of a valid deductive argument is characterised as analytic. Since 'validity' is defined in terms of 'analyticity', it is claim that nothing can appear in the conclusion of a valid deductive argument which is not present in the premises. But when we examine Lewis' both the proof, 'p/∴ (qv-q)' and '(p.-p)/∴ q',we find that the conclusion of a valid deductive argument contains what is not present in the premises, which shows that the premises and the conclusion of a valid deductive argument do not stand in the relationship of analyticity. If the relation between the premises and the conclusion of a valid deductive argument is analytic, then the conclusion must contain what is present in the premises and must not contain what is not present in the premises. The premise cannot be irrelevant to its conclusion. This follows from the notion of analyticity itself. But in Lewis' argument we find that this conditions are not satisfied. An argument is held to be valid even when there is no analytic relation between the premises and the conclusion. Even when premises are irrelevant to conclusion, Lewis still maintain the thesis that the premises entail the conclusion and that proofs are valid. And this procedure of Lewis is quite meaningless.

For the sake of argument, if we admit that 'an impossible proposition entails any proposition whatsoever', is a thesis then a system which contains it as a thesis could never be consistent and complete. That system would lack the chracteristics of consistency and completeness which are the requirements of the logistic system. A consistent and complete system does not contain in it both a formula and its negation as theses. But Lewis' system of thought contains both a formula and its negation as theses and that we can see in the following way. Consider Lewis' proof of paradoxical consequences :

(i) $(p.-p) \to q$

(ii) $(p.-p) \to -q$

Proof of (i):

| | |
|---|---|
| | (1) p.-p |
| (1), entails, | (2) p |
| (1), entails, | (3) -p.p |
| (3), entails, | (4) -p |
| (2), entails, | (5) pvq |
| (5),(4), entails, | (6) q |

Thus, (p.-p) entails q.

Proof of (ii):

| | |
|---|---|
| | (1) p.-p |
| (1), entails, | (2) p |
| (1), entails, | (3) -p.p |
| (3), entails, | (4) -p |
| (2), entails, | (5) pv-q |
| (5),(4), entails, | (6) -q |

Thus, (p.-p) entails -q.

From the above given proofs it is quite clear that (p.-p) —3 q and (p.-p) —3 -q both are valid theses on Lewis' account. But both the theses are not mutually consistent since they contain a contradictory formula and no consistent and complete system contains any contradictory formula as its theses. Therefore, Lewis' system cannot be said to be a consistent and complete one. A system which allows to deduce any proposition from a contradictory proposition fails to satisfy the logical requirement of the system. No doubt, both the argument forms, '(p.-p)/∴ q' and '(p.-p)/∴ -q' on

Lewis' account are valid. But since their premises can never be true, it would be absurd for anyone to propound an argument of this form.

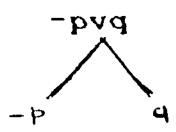
The notion of entailment in Lewis' formal system is defined in terms of deducibility and the notion of deducibility is defined in terms of tautological necessity. A tautological necessity is a priori which can be determined and known independently of experience. Take, for example, the form of 'pv-p'. This form is a form of tautology. It is true in all its interpretations. The expression 'pv-p' is equivalent to 'p⊃p'. And to say that 'p implies p' is to say 'p is identical with itself' or 'p repeats itself'. Here, the notion of implication or identity involves in the notion of necessity which is characterised as tautological necessity; though it is expressed by the symbol '⊃ ', which is a symbol of material implication. If this being so, and I think it is, the notion of analyticity is used in terms of repeatability. 'pv-p' and 'p⊃q'expressions are analytic or tautologous in this sense of the term and their truth-values hold good in all their interpretations. This kind of analyticity is called syntactical analyticity. This is the reason why 'validity' is defined in terms of 'deducibility'. Validity is characterised as a purely formal feature of argument which can be known and determined by the rules of logic alone without knowing the contents of the propositions. A proposition is said to be analytic or tautologous, if it is not contraictory. This means that ‘non-contradictoriness’ is a sign of analyticity. And this theory of analyticity rules out the logical possibility of any proposition being both analytic and synthetic. But the notion of necessity (except tautological necessity) must not be identified with the notion of analyticity. The notion of analyticity no doubt involves in it the notion of necessity, but the notion of necessity does not involve in it the notion of analyticity. Because two propositions may have a necessary connection with each other without they being analytically connected. If this is possible, and I believe it is, all valid arguments whose premises necessarily imply conclusion cannot be characterised to be deductive in analytic sense of the term. An argument can be conclusively valid without its premises and conclusion being analytically connected. Take, for example, the following argument:

This is red

∴ This is not green.

This is a valid argument. Its conclusion is necessarily connected with its premise, but it is not connected in analytic sense of the term. Therefore, this argument cannot be said to be a deductive one despite its validity being conclusive.

According to the definition of material implication, if p is false than ‘$p \supset q$’is true, that is ‘$-p \supset (p \supset q)$’ is a tautology; and if q is true then ‘$p \supset q$’ is true, that is, ‘$q \supset (p \supset q)$’is a tautology. But in the both forms of tautologous statement, the relational symbol ‘⊃ ’ does not carry the same meaning. The first occurance of the notion, that is, between ‘-p’ and ‘(p⊃q)’ and between ‘q’ and ‘(p⊃q)’involves in it the notion of necessity. As a result of which, we can say that ‘p⊃q’ logically follows from ‘-p’ or ‘q’. In otherwords, ‘-p’ entails ‘p⊃q’ and ‘q’ entails ‘p⊃q’. While the second occurance of ‘⊃ ’ in both the forms, that is, between ‘p’ and ‘q’, does not involve in it the notion of necessity. We cannot say that p and q themselves are necessarily connected or q is entailed by p. p and q are materially connected and the notion of material implication does not involve in it the notion of necessity. But when they are expressed by the same symbol ‘⊃ ’, it leads to confusions and gives birth to many philosophical issues. One gets the impression from the use of the same symbol that, since in both the occurances the same symbol is used, it carries the same meaning, whereas which is not the case. In both the ocurances the same symbol does not .carry the same meaning. This also can be seen from another point of view. The expression ‘p⊃q’ is equivalent to ‘-pvq’ according to the definition of material implication. The relationship between the compound expression, ‘-pvq’, and its components, ‘-p’ and ‘q’, could be expressed in the following way:




The fork indicates that the compound expression is true if and only if at least one of the components displayed at the tips is true. Each component is such that its truth alone is sufficient, but not a necessary condition for the truth of the compound expression. This means that the relation between '-p' and '-pvq', 'q' and '-pvq' is a necessary relation which is expressed by the first occurance of '⊃ ' in the formulas of '-p⊃(p⊃q)' and 'q⊃(p⊃q)' respectively.To say that their relation is necessary is not to say that '-p' and 'q' are necessary conditions for 'p⊃q'. '-p' and 'q' are sufficient, but not necessary, conditions for the truth of the compound expression. Because no matter what the truth-value of q is, the first formula is tautologous when p is false, which means the truth-value of 'q' is not a necessary condition nor is it relevant for the truth of the compound. What is true at the first formula is also true of the second formula. If q is true, no matter what the value of p is, the formula is tautologous. But what holds good between the components and compound, that is, '-p' and 'p⊃q', 'q' and 'p⊃q' does not hold good between the components, that is, '-p' and 'q'.The relation between the components is not a necessary relation. It is a material conditional relation; and hence contingent. But the relation between the components and compound is a necessary relation; and hence non-contingent. But since in both the formulas the same symbol '⊃ 'is used,this distinction is not maintained. As a result, the expressions of formulas lead to confusions.

The same kind of difficulty also arises when the argument form is transformed in to a conditional statement form. Take, for example, the following argument form :

$$p \supset q$$

$$p \;/ \therefore\; q$$

This argument form is transformed into a conditional statement form by taking premises as antecedent and conclusion as consequent in the following way :

$$((p \supset q).p) \supset q$$

The argument form, $p \supset q, p / \therefore q$ , is valid because its corresponding conditional statement form $((p \supset q).p) \supset q$ is a tautology. But the use of the expression '$\supset$ ' in its both occurances in formula, $((p \supset q).p) \supset q$, is not the same. In first occurance the symbol '$\supset$ ' is used in material sense, while in second occurance it is used in inferential sense and both the senses are not the same. The first occurance of '$\supset$ ' does not involve in it the notion of necessity but the second occurance of '$\supset$ ' does involve in it the notion of necessity, Therefore, the second occurance of '$\supset$ 'must not be taken in material sense. Material implication, as it is defined by formal logicians, can not give rise to inference. Because, necessity which guarantees the validity of inference does not enter into the notion of material implication. The relation that holds between the premises and the conclusion of a valid deductive argument is an analytic relation and analytic realtion is a necessary relation opposed to contingent one. It is only because of this reason, in a valid deductive argument negation of the conclusion with the affirmation of the premises always leads to a contradiction. 'Ram is mortal' does not follow necessarily from 'The pen is green', although the latter could be said to materially imply the former. But the formal logicians, while transforming argument forms into conditional statement forms do not maintain this distinction. They use the term 'implication' ambiguously. They use the symbol horseshoe, '$\supset$ ', to characterise both the inferential and material implications, which is a mistake. Because, if inferential implication is taken in the sense of material implication, the validity of a deductive argument cannot be chracterised as analytic; nor its corresponding material conditional statement could be said to be tautologous one because material implication does not involve in it the notion of necessity, connectedness and deducibility. The distinction between inferential and material implications requires a different symbol. The same horseshoe symbol, '$\supset$ ', cannot be used to express both the implications, inferential and material. The symbol of therefore, '$\therefore$ ', cannot be used to express the inferential implication, because this symbol is used for both valid and invalid arguments and merely serves to set off the conclusion from the premises. The turnstile symbol '$\vdash$ 'could be used to indicate inferential implicatios in the sense of deducibiliy. To avoid confusion, the above formula should be expressed as follows :

$$((p \supset q).p) \vdash q$$

But unfortunately, in truth-functional analysis the distinction among the expressions of '⊃ ', '∴ ', '⊢ ' is not maintained. As a result of which, instead of solving the problem it generates the problem.

Russell makes a distinction between material and formal implications. 'Material implication' he defined in terms of 'negation' and 'disjunction'; and formal implication in terms of 'general material implication'. He expressed material implication in the logical schema of '$p \supset q$' and 'formal implication' in the schema of '$(x)(\emptyset x \supset \psi x)$'. On his account, to say that 'p materilly implies q' is to say, 'either p is false or q is true' or 'it is not the case that p is true and q is false'. While on the contrary, we get a formal implication, on his account, when we assert that every value of x, 'x is a human being' implies 'x is mortal'.

$$(x)(Hx \supset Mx)$$

But when we assert that for every value of x, 'x is a human being' implies 'x is mortal', we do not use the term 'implies' in truth -functional sense. The symbol '⊃ ' in the expression of '$(x)(Hx \supset Mx)$' is used in a formal, not in a material sense. And when it is used in a formal sense, it always holds between the propositional functions, not propositions according to Russell. Propositional functions do no involve in them truth-value notions, true or false. Propositional functions always involve in them individual variables, not individual constants; and individual variables are undetermined expressions. That is why propositional functions are characterised as neither true nor false. Since individual variable symbols are the symbols of classes and propositional functions involve in them individual variable symbols, they always assert about the classes, and classes are different from their elements. Universal quantification of a propositional function, is true if and only if all the substitution instances of propositional function are true. When a universal proposition is transformed into a conjunction of singular propositions, it is done by assuming the universe of discourse non-empty. It is from this point of view, Russell defines 'formal implication' in terms of 'a class of material implication', that is, a general material implication. But in doing so, Russell does not identify formal implication with material implication. He always maintains the distinction between them. But unfortunately, while expressing formal and material implications in symbolic form, he used the same symbol '⊃ ', which he should have not done. Because, '⊃ ' symbol is a truth-functional symbol. It operates only on those expressions which have truth-values, not which lack truth-values, that is, propositions. Propositional functions since they lack truth-values, '⊃ ' symbol cannot be inserted between them. But since Russell has done it, his use of '⊃ ' symbol cannot be said to be free from ambiguity. Russell though uses the same '⊃ ' symbol in both the expressions, 'p⊃q' and '(x)(Hx ⊃ Mx)', he does not use it in the same sense. He uses '⊃ ' symbol in two different senses. In the case of 'p⊃q' he uses '⊃ ' symbol as material implication. But in the case of '(x)(Hx ⊃ Mx)', he uses '⊃ ' symbol as formal implication. But when he defines 'formal implication' in terms of 'a general material implication',it creates problem. Because, his definitional statement could be interpreted in two different senses : formal and material. When it is interpreted in terms of 'quantifier' and 'propositional function', it expresses formal implication. But it is interpreted in terms of a conjunction of singular conditional propositions, it involves the notion of material implication. Both the senses of implication get mixed when the same symbol '⊃ ' is used to express them. From this point of view, we can say that Russell's definition of formal implication is not a neat and clean definition, because it is not free from ambiguity and confusion. He should have avoided ambiguity and confusion by using two different symbols for formal and material implications while expressing them which he did not do. Consider, for example, the symbolic expression of '(x)(Hx ⊃ Mx)'. This expression, on Russell's account, is an expression of a formal implication. It has infinite number of substitution instances. We get a conditional statement 'Ha ⊃ Ma' from '(x)(Hx ⊃ Mx)' by substituting individual constant 'a' for the individual variable 'x'. But when we get the expression 'Ha ⊃ Ma' from '(x)(Hx ⊃ Mx)' by the rule of Universal Instantiation, we do not in fact change the relation.

Relation always remains the same, whatever it is. In such a case, what really we do is that we only substitute individual constant ‘a’ for individual variable, ‘x’, and doing so, we do not change the relationship holding between the symbols. If this is so, and I believe it is, the use of ‘$\supset$ ’ in both the expressions, that is, ‘$(x)(Hx \supset Mx)$’ and ‘$(Ha \supset Ma)$’ cannot be said to be different. We cannot say that in the former case, ‘$\supset$ ’ symbol expresses formal implication and in latter case it expresses material implication, as Russell does it. If ‘$(x)(Hx \supset Mx)$’ is taken in the sense of a conjunction of material implication, the occurance of ‘$\supset$ ’ symbol in the expressions ‘$(x)(Hx \supset Mx)$’ cannot said to be formal.Because the conjunction of material conditional statements does not render material implication as formal. It always remains material conditional. But if the occurance of ‘$\supset$ ’ in the expression of ‘$(x)(Hx \supset Mx)$’ is taken in formal sense, then the occurance of ‘$\supset$ ’ in ‘$Ha\supset Ma$’ has to be taken only in formal sense. We cannot take it in one case in formal sense and in another case in material sense. So, Russell cannot define ‘formal implication’ in terms of ‘a conjunction of material implications’ and at the sametime maintain the distinction between them.

CHAPTER-V

# CONCLUSION

In all arguments, claim is made that one proposition follows from another proposition(s). But the notion of 'follows from' does not carry the same meaning in all. In a valid deductive argument claim is made that its premises provide absolute or conclusive guarantee for the truth of its conclusion. A valid deductive argument does not contain true premises and false conclusion But in inductive argument the position is different. In this kind of argumrnt no claim is made that its premises, if true, provide absolute guarantee for the truth of its conclusion. Nor is it said that the conclusion logically follows or is deducible from its premises. Inductive argument always involves some gap between the premises and the conclusion because of which its premises do not provide absolute guarantee for the truth of its conclusion. Both the deductive and inductive arguments, thus, the notion of 'follows from' does not carry the same meaning A valid deductive argument involves in it the notion of necessity which makes it impossible for the premises to be true and conclusion false. Inductive argument lacks this inferential characteristic Its premises and conclusion do not stand in the relation of necessity. Their relationship always remains contingent This is the distinctive mark of all inductive arguments opposed to deductive ones. Because of this reason the notion of validity in both the cases is defined in two different senses.

The validity of a deductive argument depends upon the relationship that holds between its premises and conclusion. If premises and conclusion are necessarily connected, the conclusion necessarily follows from the premise(s) and the argument is valid or else it is invalid. The relation of material implication, as conceived by Russell, does not capture this notion of inferential necessity. Because, Russell's notion of material implication does not involve in it any kind of necessity, connectedness and deducibility. When we say that 'p materially implies q', it does not mean that p and q themselves are connected or necessarily follows from p. It only means that if p is true, than q is true or it is not the case that p is true and q is false. And to say this is not to say that p and q themselves are connected; nor does it mean that q logically follows from p. Material implication holds between p and q even when there is no relation between p and q. Because of this reason its antecedent does not give the guarantee for the truth of its consequent which is guaranteed by the inferential notion of necessity.

Lewis tries to formalise inferential necessity by introducing the notion of strict implication, but he fails. Because his notion of strict implication does not involve in it inferential necessity, a necessity which is found between the premises and the conclusion of a valid deductive argument. He interprets the notion of strict implication in two different ways. According to his first interpretation, to say that ' p strictly implies q' is to say that 'p materially implies q' is necessary.

$$p \rightarrow q \equiv df.\ L(p \supset q)$$

To say that 'p materially implies q'is necessary is to say that '$p \supset q$' is a tautology which does not merely state that it is not the case that p is true and q is false. But to say that p materially implies q is necessary in tautological sense is not to say that p and q themselves are necessarily connected, nor does it mean that q is deducible from p. It only means that '$p \supset q$' is true in all its extensional interpretations. And to say this is to say wherever p is true, q is also true or true truth-value of p and q are coextensive.And to say this is not to say that q is deducible from p nor does it mean that q is necessarily connected with p. The notion of strict implication, no doubt,involves in it the notion of necessity in tautological sense but it does not involve in it the inferential notion of necessity. Tautological necessuty of '$p \supset q$' does not state that p and q themselves are necessarily connected. The modal necessity expressed by L-operator operating on '$p \supset q$' is not a truth-functional oprarator; hence cannot be identified with truth-functional notion of tautological necessity. '$L(p \supset q)$' is valid if and only if, '$p \supset q$' is a tautology or is necessarily true. If '$p \supset q$' is not tautologous, '$L(p \supset q)$' cannot be said to be a valid thesis according to the rule of Necessitation which Lewis holds. But when he defines 'p strictly implies q' in terms of 'L(p ⊃ q)' he fails to maintain the distinction between the tautological and modal necessities. He interprets the expression 'L(p ⊃ q)' in tautological sense, that is, 'p materially implies q' is a tautology . And in doing so he transforms modal necessity into tautological necessity which he should have not done.

According to his second interpretation, to say that 'p strictly implies q' is to say that 'q is deducible from p' or 'q logically follows from p'. But his notion of 'deducibility' or 'logically follows from' does not involve in it inferential notion of necessity. It involves in it tautological notion of necessity. For him, q is deducible from p or q logically follows from p only if 'p ⊃ q' is tautology. And this condition is satistied even if there is no connection between p and q. This is quite obvious from his proofs given for the paradoxes of strict implication.

Take, for example, Lewis' proof of paradoxical consequences :

(i) (p.-p) ⥽ q

| | | |
|---|---|---|
| Proof: | | (1) p.-p |
| | (1), entails, | (2) p |
| | (2), entails, | (3) pvq |
| | (1), entails, | (4) -p.p |
| | (4), entails, | (5) -p |
| | (3),(5), entails, | (6) q |

Therefore, (p.-p) entails q.

(ii) p $\rightarrow$ (qv-q)

| | | |
|---|---|---|
| Proof: | | (1) p |
| | (1), entails, | (2) (p.q) v (p.-q) |
| | (2), entails, | (3) p.(qv-q) |
| | (3), entails, | (4) (qv-q).p |
| | (4), entails, | (5) (qv-q) |

Therefore, p entails (qv-q).

From the above given proofs, it is quite clear that according to Lewis, q is deducible or logically follows from (p.-p) even when there is no connection between '(p.-p)' and 'q'. '(qv-q)' logically follows from 'p' even when there is no connection between 'p' and 'qv-q'. This shows that Lewis' notion of 'deducibility' or 'logically follows from' does not involve in it inferential necessity in terms of connectedness. 'q' and '(qv-q)' follow from '(p.-p)' and 'p' respectively even when '(p.-p)' and 'p' are irrelevant to 'q' and '(qv-q)' respectively. If 'q' and '(qv-q)' are not connected with '(p.-p)' and 'p' or the latter is irrelevant to the former, it makes no sense to say that 'q' and '(qv-q)' logically follow from '(p.-p)' and 'p' respectively, nor can it be said that '(p.-p)' and 'p' provide absolute guarantee for the truth of 'q' and '(qv-q)' respectively. In fact, where there is no connection between them the question of deducing or logically following from does not simply arise at all. The question of deducing or logically following from does arise only when there is a necessary connection between the entailing and entailed propositions or premise(s) and conclusion. The truth of (qv-q) does not need any guarantee from p and the truth of (p.-p) cannot give guarantee to q. Even if it is assumed that (p.-p) is true , it fails to give guarantee to q because of its unconnectedness. The formulas '(p.-p) ⥽ q' and 'p ⥽ (qv-q)' are tautologous not becquse there is necessary connection between the entailing and the entailed parts of the formulas or that the entailed parts logically follow from the entailing part but because '(p.-p)' in the former case is contradictory and '(qv-q)' in the latter case is tautology which can be seen in their PC-transforms in the following way :

The formula (i) (p.-p) ⥽ q

1. (p.-p) ⥽ q
2. L((p.-p) ⊃ q)
3. (p.-p) ⊃ q
4. - (p.-p) v q
5. (-pvp) v q

The formula (ii) p ⥽ (qv-q)

1. p ⥽ (qv-q)
2. L(p ⊃ (qv-q))
3. p ⊃ (qv-q)
4. -p v (qv-q)

The above given PC-transform of the formulas '(p.-p) ⥽ q' and 'p ⥽ (qv-q)' clearly shows that the formulas are not tautologous because of the relationship holding between the entailing and entailed parts but because ‘(p.-p)’ in the former case is contradictory and ‘(qv-q)’ in the latter case is tautology. This means that Lewis’ notion of validity does not involve in it the notion of necessity in inferential sense. It only involves the notion of necessity in tautologous sense which fails to capture the notions of relevancy and connectedness found between the premises and conclusion of a genuine valid deductive argument.

For the sake of argument, if we admit that ‘p entails (qv-q)’, then we have to admit that a contingent proposition entails a proposition of modal necessity. ‘p entails (qv-q)’ according to Lewis’ above given proof ‘(qv-q) entails L(qv-q)’ according to the rule of Necessitation. Therefore, ‘p entails L(qv-q)’ according to the rule of transitity of entailment. But this goes against the rule of Necessitation and the axiom of Necessity. If a proposition p is true, from this it does not follow that p is necessarily true. But if we accept p entails (qv-q), it does follow that if p is true, then p is necessarily true which is not correct. Therefore, Lewis’ proof given for ‘p ⥽ (qv-q)’ cannot be said to be a genuine proof.

A genuine deductive proof always involve in it the notion of connectedness and relevancy. The notion of validity involves in it the notion of connectedness which makes premise(s) relevant to its conclusion and the premise(s) provide the absolute guarantee for the conclusion (if true). In otherwords, it is the relation, holding between the premise(s) and the conclusion, which makes the premise(s) relevant to the conclusion and gives the guarantee that the conclusion necessarily follows from its premise(s), of a given argument. Lack of connectedness gives rise to the paradoxical conclusion. The truth-functional notion of tautological necessity fails to capture deductive notion of validity taken in terms of connectedness which I have called inferential necessity. The notion of entailment when it is taken in the sense of 'inner connection',it does not capture inferential notion of validity. The notion of deducibility and analyticity should be understood in this sense of the term.

## BIBLIOGRAPHY


1. Anderson & Belnap : " The Pure Calculus of Entailment", Journal of Symbolic Logic, Vol.27, 1962.

2. Barcan,R.C. & Marcus,R.B. :"Functional Calculus of First Order Based on Strict Implication" , in Journal of Symbolic Logic, Vol.II, 1946.

3. Basson & O'connor :Introduction to Symbolic Logic, University Tutorial Press, London, 1953.

4. Bennett,J.F. :"Entailment", in the Philosophical Review, Vol.78, 1969.

5. Bennett,J.F. :"Meaning and Implication", in Mind, Vol.63, 1954.

6. Blanshard,B. :The Nature of Thought, Vol.II, George Allen & ~~University~~ Unwin Press, London, 1964.

7. Boole,G. :Mathematical Analysis of Logic, Cambridge Univ.Press,1947.

8. Cohen & Nagel :An Introducion to Logic and Scientific Method, Allied Pub. Pvt.Ltd.,Calcutta,1972.

9. Copi,I.M. :Symbolic Logic,Macmillan co., New York,1965.

10. Downing,P. :"Entailment", Proceeding of Aristotelian Society,Vol.66, 1965-6.

11. Edwards, Paul :Encyclopedia of Philosophy,Mac millan Pub.Co.Inc.,New York, Vol.5,1967.

12. Emch,A.F. :"Implication and deducibility", Journal of Symbolic Logic, Vol.I,1936.

13. Geach,P.T. :"Entailment", Aristotelian Society Supplementary,Vol.XXXII,1958.

14. Halking, I. :"What is strict implication?", Journal of Symbolic Logic, Vol.28, 1963.

15. Hilbert &Ackerman:Principles of Mathmatical Logic, translated by Harmond,Leckie, and Steinhardt, New York, 1950.

16. Hintikka, K.J.J. :'Modality and Quantification', Theories,Vol.27,1961.

17. Hughes & Cresswell:Introduction to Modal Logic, Barnes and Noble Inc.,London, 1968.

18. Iseminger,Gary :"Is Relevance Necessary for Validity? in Mind,Vol.IXXXIX,No.354,1980.

19. Johnson,W.E. :Logic,Part I-II, Dover Pub.Inc., New York,1964.

20. Joseph,H.W.B. :Introduction to Logic, 2nd ed., Oxford, 1916.

21. Joseph,H.W.B. :Mind, Vol.XLII,1934.

22. Kahane,H. :Logic and Philosophy,Wadsworth Pub. Co.Inc., Belmont, California,1969.

23. Kant,I. :Critique of Pure Reason,Trans. by N.K.Smith,London,1933.

24. Klenk,V. :Understanding Symbolic Logic, Prentice-Hall Inc.,New Jersy,1983.

25.Langer,S.K. :An Introduction to Symbolic Logic, Dover Publications Inc.,New York,1967.

26. Lewis & Langford :Symbolic Logic, Dover Publications Inc., New York, 1959.

27. Lewy,C. :Meaning and Modality, Cambridge Univ.Press,Cambridge,1976.

28. Moore,G.E. :Philosophical Studies, Routledge & Kegan Paul Ltd.,London,1960.

29. Nelson,E.J. :'Intentional Relations', Mind, Vol.39,1930.

30. Prior,A.N. :Formal Logic,2nd ed.,Oxford Univ. Press, Oxford, 1962.

31. Quine,W.V. :Methods of Logic,New York,1950.

32. Russell,B. :Introduction to Mathematical Philosophy,London, 1918.

33. Russell & Whitehead: Principia Mathematica,Vol.I,1910.

34. Sen,P.K. :'The Problem of Entailment',ed.by Daya Krishna in his book Modern Logic-its relevance to phil., New Delhi,Impex India,1969.

35. Smiley,T.J. :"Entailment and deducibility", Proceedings of Aristotelian Soc.,Vol.59, 1959.

36. Snyder,D.Paul :Modal Logic and its applications, Van Nostrand Reinhold Co.,New York, 1971.

37. Stebbing,L.S. :A Modern Elementary Logic,Methuen & Co.Ltd.,London, 1952.

38. Strawson,P.F. :Introduction to Logical Theory, Mathuen and Co. Ltd.,London,1952.

39. Strawson,P.F. :Necessary Propositions and entailment statements, Mind, Vol.57,1948.

40. Strawson,P.F. :Philosophical Logic, Oxford Univ. Press, Oxford,1967.

41. Suppes,P. :Introduction to Logic,Van Nostrand Reinfold Co., New York,1957.

42. Wright,G.H.V. :The Concept of Entailment in Logical Studies, Routledge & Kegan Paul, London,1957.

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