Theorem on Vanishing Multiloop Radiative Corrections to $\sin^2\theta_W$ in Grand Unified Theories at High Mass Scales

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In contrast to the conventional belief that the gauge-boson-propagator renormalizations to every m-loop order contribute to $\sin^2\theta_W$ at all scales (μ) below the grand unification mass (M_U), for the first time we prove that all higher-order multiloop ($m \geq 2$) corrections are absent for $\mu = M_I - M_U$ in a class of grand unified theories where the Pati-Salam intermediate gauge symmetry breaks at the highest intermediate scale (M_I). This theorem holds also with supersymmetry and in string-inspired models.

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Measurements of the $W^{\pm}$ and Z boson masses at high-energy accelerators [1] combined with the radiative corrections [2] of the standard model, $G_{st} = SU(3)_C \times SU(2)_L \times U(1)_Y$, have resulted in the determination of the electroweak mixing angle with profound precision. The standard model being incomplete on various counts, several grand unified theories (GUTs) have been proposed to unify strong, weak, and electromagnetic interactions, and, in addition, the gravitational interaction through the introduction of higher-dimensional unification of the Kaluza-Klein-type, the $N=1$ supergravity, or superstrings. Very attractive predictions of GUTs including the unification mass (M_U) consistent with the experimental lower limit on the proton lifetime, $\sin^2\theta_W$, intermediate scales, fermion masses at low energies, strong and weak CP violations, and inflationary big-bang cosmology are possible in models with one or more intermediate symmetries. Since the final step of spontaneous symmetry breaking in every GUT has to proceed through G_{st}, the current theoretical and experimental determinations of $\sin^2\theta_W$ near the M_W scale are meaningful if the GUT predictions are consistent with such accuracies. The GUT predictions are obtained using the standard procedure [3,4] of solving the renormalization-group equations (RGEs) including the two-loop [5] and threshold contributions [4]. It is well known that the threshold and loop contributions are very much dependent upon the particle content (gauge bosons, Higgs scalars, fermions, and their superpartners in supersymmetric GUTs) of the theory. Since the heavy particle masses are not known precisely, the GUT predictions, including $\sin^2\theta_W$, are likely to be uncertain [6] and such uncertainties would persist until a deeper theory is available on these masses. Even in the standard model a similar uncertainty, although much smaller in magnitude, exists because of want of precise knowledge on the top quark and the Higgs-scalar masses.

In search of models with more precise predictions, recently we have proved that a major source of uncertainty in the $\sin^2\theta_W$ prediction due to the GUT-threshold effects is absent in the following class of models [7]:

$$G \xrightarrow{M_U} G_{224P} \xrightarrow{M_I} \cdots \rightarrow G_{st} \xrightarrow{M_W} G_{13}\,, \qquad (1)$$

where $G=$SO(10), SO(12), SO(14), SO(16), SO(18), SU(16), E_6, etc., $G_{224P} = SU(2)_L \times SU(2)_R \times SU(4)_C \times P$ (P=parity, the left-right discrete symmetry) [8], $G_{13} = U(1)_{em} \times SU(3)_C$, M_I is the highest intermediate scale, and ellipses denote other possible intermediate symmetries.

The purpose of the present Letter is to establish a theorem that shows for the first time that the presence of parity and the quark-lepton unification symmetries in G_{224P} triggers a natural cancellation mechanism leading to the absence of all higher-order multiloop radiative corrections to $\sin^2\theta_W$ arising out of gauge-boson-propagator renormalizations for $M_I < \mu < M_U$. This is a completely new development, counter to the conventional belief that contributions to $\sin^2\theta_W$ exist to all higher orders for all mass scales below M_U.

Theorem.— In all GUTs where the "custodial" Pati-Salam gauge symmetry G_{224P} breaks at the highest intermediate scale M_I, the gauge boson renormalization to every m-loop ($m \geq 2$) order has a vanishing contribution to $\sin^2\theta_W$ for $\mu = M_I$ to M_U.

Proof.— Let $g_i(\mu)$ be the gauge coupling of the subgroup G_i and $\alpha_i(\mu) = g_i^2(\mu)/4\pi$. The RGEs are

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M)} + \frac{a_i}{2\pi}\ln\frac{M}{\mu} + K_i(\mu,M)\,, \qquad (2)$$

with $i = Y, 2L, 3C$ for G_{st}. Here a_i is a coefficient in one-loop β function and K_i includes all higher-order m-loop contributions due to the gauge-boson-propagator renormalization,

$$K_i(\mu,M) = \sum_{m>2} K_i^{(m)}(\mu,M)\,.$$

For example,

$$K_i^{(2)}(\mu,M) = \sum_j B_{ij}\int_\mu^M \alpha_j(\mu')\frac{d\mu'}{\mu'}\,,$$
$$K_i^{(3)}(\mu,M) = \sum_{j,k} C_{ijk}\int_\mu^M \alpha_j(\mu')\alpha_k(\mu')\frac{d\mu'}{\mu'}\,, \qquad (3)$$

etc., where the constants B_{ij} and C_{ijk} depend upon the particle content of the theory. In a manner analogous to (2), the coupling constants for G_{224P} satisfy RGEs with the corresponding coefficient (multiloop term) denoted as $a_i'(K_i')$ ($i=2L,2R,4C$). The superheavy particle effects near $\mu=M_U$ on $a_i(\mu)$ are well known to have the form [4]

$$\frac{1}{\alpha_i(\mu)}=\frac{1}{\alpha_G}-\frac{\lambda_i(\mu)}{12\pi},\quad i=2L,2R,4C\,, \tag{4}$$

where α_G corresponds to the GUT coupling. At first we prove the theorem for the model (1) with single intermediate symmetry G_{224P}. Denoting $\lambda_i^U=\lambda_i(M_U)$, $K_i^U=K_i(M_I,M_U)$ ($i=2L,2R,4C$), and $K_i^I=K_i(M_W,M_I)$ ($i=Y,2L,3C$), we obtain, using (2) and (3) and the standard procedure,

$$\ln\frac{M_U}{M_I}=\frac{1}{a_{2L}'+a_{2R}'-2a_{4C}'}\left[2\pi(\alpha^{-1}-\tfrac{8}{3}\alpha_s^{-1})-(\tfrac{5}{3}a_Y+a_{2L}-\tfrac{8}{3}a_{3C})\ln\frac{M_I}{M_W}\right]+\Delta_{1U}+\Delta_{2U}+\Delta_{\lambda U}\,, \tag{5}$$

$$\sin^2\theta_W=\frac{1}{a_{2L}'+a_{2R}'-2a_{4C}'}\Bigg\{a_{2L}'-a_{4C}'+\frac{\alpha}{\alpha_s}(a_{2R}'+\tfrac{2}{3}a_{4C}'-\tfrac{5}{3}a_{2L}')$$
$$+\frac{3\alpha}{16\pi}[(a_{2R}'+\tfrac{2}{3}a_{4C}'-\tfrac{5}{3}a_{2L}')(\tfrac{5}{3}a_Y+a_{2L}-\tfrac{8}{3}a_{3C})$$
$$-\tfrac{5}{3}(a_{2L}'+a_{2R}'-2a_{4C}')(a_Y-a_{2L})]\ln\frac{M_I}{M_W}\Bigg\}+\Delta_{1S}+\Delta_{2S}+\Delta_{\lambda S}\,, \tag{6}$$

where the GUT-threshold contributions are

$$\Delta_{\lambda U}=\frac{1}{6}\frac{\lambda_{2L}^U+\lambda_{2R}^U-2\lambda_{4C}^U}{a_{2L}'+a_{2R}'-2a_{4C}'}\,,$$

$$\Delta_{\lambda S}=\frac{\alpha}{96\pi}[3(a_{2R}'+\tfrac{2}{3}a_{4C}'-\tfrac{5}{3}a_{2L}')(2\lambda_{4C}^U-\lambda_{2L}^U-\lambda_{2R}^U)+(a_{2L}'+a_{2R}'-2a_{4C}')(2\lambda_{4C}^U+3\lambda_{2R}^U-5\lambda_{2L}^U)]/(a_{2L}'+a_{2R}'-2a_{4C}')\,. \tag{7}$$

All the higher-loop contributions are contained in Δ_{1U} and Δ_{1S} for the mass range $\mu=M_W-M_I$, and in Δ_{2U} and Δ_{2S} for the range $\mu=M_I-M_U$,

$$\Delta_{1U}=\frac{2\pi}{a_{2L}'+a_{2R}'-2a_{4C}'}(\tfrac{8}{3}K_{3C}^I-K_{2L}^I-\tfrac{5}{3}K_Y^I)\,, \tag{8a}$$

$$\Delta_{2U}=\frac{2\pi}{a_{2L}'+a_{2R}'-2a_{4C}'}(2K_{4C}^U-K_{2L}^U-K_{2R}^U)\,, \tag{8b}$$

$$\Delta_{1S}=\frac{3\alpha}{8(a_{2L}'+a_{2R}'-2a_{4C}')}[(a_{2R}'+\tfrac{2}{3}a_{4C}'-\tfrac{5}{3}a_{2L}')(\tfrac{5}{3}K_Y^I+K_{2L}^I-\tfrac{8}{3}K_{3C}^I)-\tfrac{5}{3}(a_{2L}'+a_{2R}'-2a_{4C}')(K_Y^I-K_{2L}^I)]\,, \tag{9a}$$

$$\Delta_{2S}=\frac{3\alpha}{8(a_{2L}'+a_{2R}'-2a_{4C}')}[(a_{2R}'+\tfrac{2}{3}a_{4C}'-\tfrac{5}{3}a_{2L}')(K_{2L}^U+K_{2R}^U-2K_{4C}^U)-(a_{2L}'+a_{2R}'-2a_{4C}')(K_{2R}^U+\tfrac{2}{3}K_{4C}^U-\tfrac{5}{3}K_{2L}^U)]\,. \tag{9b}$$

The present theorem is addressed to the contribution Δ_{2S}. The presence of left-right discrete symmetry in G_{224P} for $\mu\geq M_I$ demands

$$a_{2L}'=a_{2R}'\,, \tag{10a}$$

$$K_{2L}^U=K_{2R}^U\,, \tag{10b}$$

$$\lambda_{2L}^U=\lambda_{2R}^U\,. \tag{10c}$$

Using (10a) and (10c) in (7) leads to

$$\Delta_{\lambda U}=\frac{\lambda_{2L}^U-\lambda_{4C}^U}{6(a_{2L}'-a_{4C}')}\,,\quad \Delta_{\lambda S}=0\,, \tag{11}$$

establishing the theorem on vanishing GUT-threshold contribution to $\sin^2\theta_W$ as carried out in Ref. [7]. But using (10a) and (10b) in (9b) leads to

$$\Delta_{2S}=0\,, \tag{12}$$

which proves this new theorem on the absence of multiloop contributions to $\sin^2\theta_W$ for $\mu\gtrsim M_I$. With more intermediate symmetries at lower scales, the expressions $\Delta_{\lambda U}$, $\Delta_{\lambda S}$, Δ_{2U}, and Δ_{2S} remain unchanged and the conditions in (10) lead to $\Delta_{2S}=0$ and $\Delta_{\lambda S}=0$.

The useful lesson learned from this theorem and Ref. [7] is that the presence of the left-right discrete symmetry (= parity) and quark-lepton unification in G_{224P} triggers natural cancellation mechanisms, unique to this gauge group, leading to elimination of uncertainties in $\sin^2\theta_W$ predictions due to two sources: GUT-threshold and multiloop radiative contributions for $\mu\gtrsim M_I$. Also we have checked that no other intermediate symmetry replacing G_{224P} achieves these objectives and hence the name custodial. We emphasize precise predictions of $\sin^2\theta_W$ by such models with few intermediate symmetries. To our knowledge this is the first discovery of the absence of such multiloop radiative corrections to $\sin^2\theta_W$ in a gauge theory. In simplifying predictions in such GUTs, the theorem guarantees accuracy of one-loop calculations for $\sin^2\theta_W$ for $\mu > M_I$ as in the original Georgi-Quinn-Weinberg [3] approach, although the multiloop effects contribute for $\mu < M_I$. In deriving the theorem, no specific particle content has been used, and, as such, it holds also in supersymmetric GUTs, or those models based upon $N=1$ supergravity, superstrings, or higher-dimensional unification. We hope this theorem will serve as an important guideline for future model building through GUTs. Some numerical estimations in SO(10) including two-loop contributions in the appropriate mass ranges suggest that the multiloop corrections to $\sin^2\theta_W$ are absent in the simplest case of single G_{224P} intermediate symmetry for $\mu = 10^{13}$–10^{15} GeV. In the case suggested to be experimentally interesting in Ref. [9],

$$\mathrm{SO}(10) \xrightarrow{\mathbf{54}} G_{224P} \xrightarrow{\mathbf{210}} \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_C$$
$$\xrightarrow{\mathbf{210}} \mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_{B-L} \times \mathrm{SU}(3)_C \xrightarrow{\mathbf{126}} G_{\mathrm{st}},$$

where **54**, **210**, and **126** are the Higgs representations, the multiloop contributions are absent for $\mu = 10^{12}$–10^{16} GeV.

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