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Resonant structures in $^{24}Mg + ^{28}Si$ elastic and inelastic scattering

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The data on the excitation functions of $^{24}Mg + ^{28}Si$ elastic and inelastic ($2^+ - 0^+$, $2^+ - 2^+$, $4^+ - 0^+$ and $4^+ - 2^+$) scattering from $E_{c.m.} = 48.97$ to 57.21 MeV have been subjected to a statistical analysis consisting of the calculations of deviation function, cross-correlation function, summed excitation function, cross-channel correlation coefficients, coherence widths, and the distribution of cross sections. Based on the outcome of the analysis resonant structures at $E_{c.m.} = 49.23$, 50.02, 50.51, 52.10, 52.53, 53.27 and 54.14 MeV have been confirmed and three new structures of the same nature have been identified at $E_{c.m.} = 51.42$, 54.88 and 55.60 MeV.

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1. Introduction

Among the heavy systems experimentally investigated so far [1–7] only $^{28}Si + ^{28}Si$ [1–3] and $^{24}Mg + ^{24}Mg$ [4, 8] exhibit a number of very narrow and rather prominent resonances in their elastic and inelastic scattering excitation functions in the energy region of about twice the Coulomb barrier energy. Remarkably the inelastic channels are very strongly populated, particularly in $^{24}Mg + ^{24}Mg$ system [4]. The noteworthy characteristic features of these resonances are: narrow widths ~ 150–250 keV (c.m.), very high excitation energies ~ 60–80 MeV, high spins ~ 34–40 $\hbar$ in the vicinity of the grazing angular momenta in the entrance channel, and strong correlations in energy in the elastic and several inelastic channels. In spite of the availability of additional experimental information of different nature (spin alignment) [9–10], the mechanism of occurrence of these resonances lacks clear understanding although the spin alignment data provide some indications of the existence of dinuclear molecule type of configurations and also tend to support the concept of superdeformed intermediate complex. Novertheless the observation of these resonances at excitation energies where the level densities are in the range of $\sim 5 \times 10^3$ to 10^5/MeV points to a qualitatively new aspect of nuclear structure [11] that is yet to be understood.

A recent measurement of elastic and inelastic scattering excitation functions of $^{24}Mg + ^{28}Si$ reveals that this system does show resonance behaviour [12] similar to what has been observed for $^{24}Mg + ^{24}Mg$ and $^{28}Si + ^{28}Si$ systems. The characteristic features of the observed resonances are also very similar to the ones mentioned above. On the basis of a statistical fluctuation analysis, consisting of the summed deviation function, cross-correlation function and summed excitation function (-7 MeV $\leqq Q \leqq 0$), Wuosmaa et al. [12] located resonat structures at $E_{c.m.} = 49.23$, 50.02, 50.51, 52.20, 52.53, 53.17 and 54.06 MeV and estimated their parameters from a resonance analysis assuming them to be single isolated resonances. From the measurement of elastic angular distributions at (two strong resonances) $E_{c.m.} = 50.51$ and 52.53 MeV the respective dominance of $l = 34\ \hbar$ and 37 $\hbar$ was established.

In view of this rather restricted analysis of the data we have carried out a detailed statistical analysis of the $^{24}Mg + ^{28}Si$ elastic and inelastic excitation functions following the approach of Ericson [13], and Brink and Stephen [14] in order to have quantitative estimate of the likelihood that the observed structures are statistical in nature or they originate from some nonstatistical mechanism that prevails when ^{24}Mg and ^{28}Si collide and interact in the given energy region. The analysis consists of the calculations of the deviation and energy dependent cross-channel correlation functions, summed excitation function, cross-channel correlation coefficients, coherence widths, and the distribution of fluctuating cross sections.

2. Analysis

2.1. Data reduction

The data on the excitation functions of $^{24}Mg + ^{28}Si$ consisted of elastic scattering ($0^+ - 0^+$) and inelastic scatter-

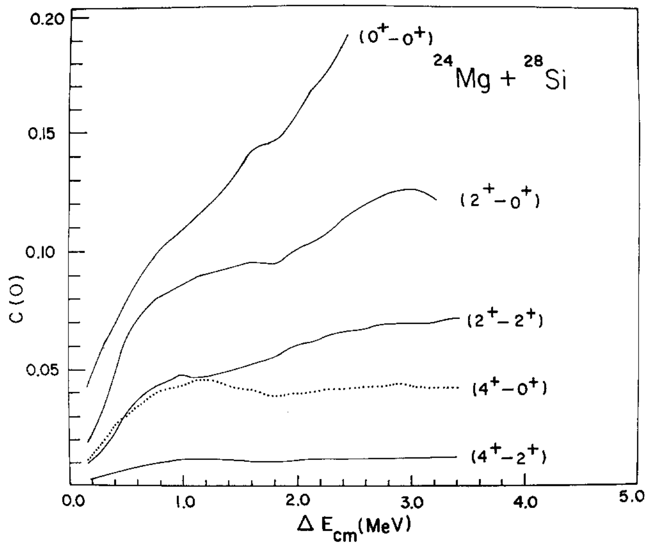

Fig. 1. Variation of variance, $C(0)$, with the variation of averaging interval, $\Delta E_{\text{c.m.}}$ (MeV) for the indicated excitation functions of ^{24}Mg + ^{28}Si scattering

ing (2^+-0^+, 2^+-2^+, 4^+-0^+ and 4^+-2^+) processes such that 2^+-0^+ excitation function has contributions from both 2^+ states (1.37 MeV of ^{24}Mg as well as 1.78 MeV of ^{28}Si), 4^+-0^+ one has contributions from both 4^+ states (4.12 MeV of ^{24}Mg as well as 4.62 MeV of ^{28}Si) and the 4^+-2^+ excitation function from both 2^+ and 4^+ states of the two nuclei [12]. Since we want to compare the behaviour of the data with the statistical model predictions the energy dependent gross structure should be removed from the excitation functions before subjecting them to the analysis. For doing this we followed the approach of Pappalardo [15] according to which the experimental cross sections, $\mathrm{d}\sigma(E)$, are divided by the running average, $\langle \mathrm{d}\sigma(E)\rangle$, taken over a suitable energy interval $\Delta E_{\text{c.m.}}$. The resulting data, $x(E) = \mathrm{d}\sigma(E)/\langle \mathrm{d}\sigma(E)\rangle$ are subjected to the analysis.

The usual criterion for choosing $\Delta E_{\text{c.m.}}$ is that $\Gamma_{\text{fine}} \ll \Delta E_{\text{c.m.}} \ll \Gamma_{\text{gross}}$ where Γ_{fine} & Γ_{gross} are the fine and gross structure widths observed in the excitation functions [16]. However, if $C(0)$, the normalised variance of the reduced excitation function ($C(0) = \langle x^2\rangle/\langle x\rangle^2 - 1$), is plotted as a function of the averaging interval $\Delta E_{\text{c.m.}}$, a plateau should be seen when $\Delta E_{\text{c.m.}}$ very well exceeds the coherence width [17]. The suitable value of $\Delta E_{\text{c.m.}}$ can then be taken from such plots. The $\Delta E_{\text{c.m.}}$ versus $C(0)$ plots for all the five excitation functions are given in Fig. 1. Except for the 0^+-0^+ excitation function all other data exhibit plateau type of behaviour. Noting this behaviour, and also the widths, Γ_{fine} and Γ_{gross} in the excitation functions we decided to take $\Delta E_{\text{c.m.}} = 1.131$ MeV for data reduction. It might be mentioned that the corresponding value in [12] was taken as 1.050 MeV.

2.2. *Deviation and cross-correlation functions*

In order to locate resonant/nonstatistical structures in the excitation functions the deviation function, $D(E)$, and energy dependent cross-channel correlation function, $C(E)$, are usefully exploited. These functions are respectively defined as [18].

$$D(E) = \frac{1}{M}\sum_{i=1}^{M}\left(\frac{\mathrm{d}\sigma_i(E)}{\langle \mathrm{d}\sigma_i(E)\rangle} - 1\right) \tag{1}$$

$$C(E) = \frac{2}{M(M-1)}\sum_{i>j=1}^{M}\left(\frac{\mathrm{d}\sigma_i(E)}{\langle \mathrm{d}\sigma_i(E)\rangle} - 1\right) \cdot \left(\frac{\mathrm{d}\sigma_j(E)}{\langle \mathrm{d}\sigma_j(E)\rangle} - 1\right)[C_i(0)\,C_j(0)]^{-1/2} \tag{2}$$

where $\mathrm{d}\sigma_i(E)$ is the differential cross section for the i^{th} excitation function at bombarding energy E and $\langle\ \rangle$ denotes the corresponding running average taken over the energy interval mentioned in Sect. 2.1. The $C_i(0)$ and $C_j(0)$ are the variances of the i^{th} and j^{th} excitation functions and M the total number of exciation functions. Both these functions are shown in Fig. 2, where the summed excitation function, $\sum_{i=1}^{5} \mathrm{d}\sigma_i(E)/\mathrm{d}\Omega$, is also plotted. It can be noted from this figure that the correlated structures at $E_{\text{c.m.}} = 50.02$, 50.51, 51.42, 52.10, 52.53, 53.27, 54.14, 54.88 and 55.60 stand out clearly in all the three functions and can certainly be called to be orginating from some nonstatistical mechanism. It may be remarked that the structure at $E_{\text{c.m.}} = 49.23$ can not be seen in the present deviation as well as cross-correlation functions because of our averaging procedure. But this structure is seen with extraordinary prominence in the summed excitation function (see Fig. 2). It might be mentioned that slight differences in the locations of resonances at $E_{\text{c.m.}} = 52.10$, 53.27 and 54.14 MeV here and in [12] are within the experimental energy uncertainty.

The standard deviation, σ_c, of $C(E)$ due to finite energy range of the excitation functions is given by [19]

$$\sigma_c = \left[\frac{2}{M(M-1)(n-1)}\right]^{1/2}, \tag{3}$$

where M is the number of excitation functions involved in the calculation of $C(E)$ and n the number of data points in the averaging energy interval. For the present data $\sigma_c = 0.0845$. For a statistical ensemble with no correlations the values of $C(E)$ are expected to be within $3\sigma_c = 0.253$. The above mentioned correlated maxima are very well outside (5.92 to 78.11 times the σ_c) the statistical limits and are, therefore, resonant structures. The structures at $E_{\text{c.m.}} = 51.42$, 54.88 and 55.60 have been identified in the present analysis and are being reported for the first time.

2.3. *Cross-channel correlation coefficients*

On the basis of the standard statistical model [13, 20] no cross-channel correlations are expected and hence large values of the cross-channel correlation coefficients should be taken as an evidence for the observed structures being resonances. We calculated the cross-correla-

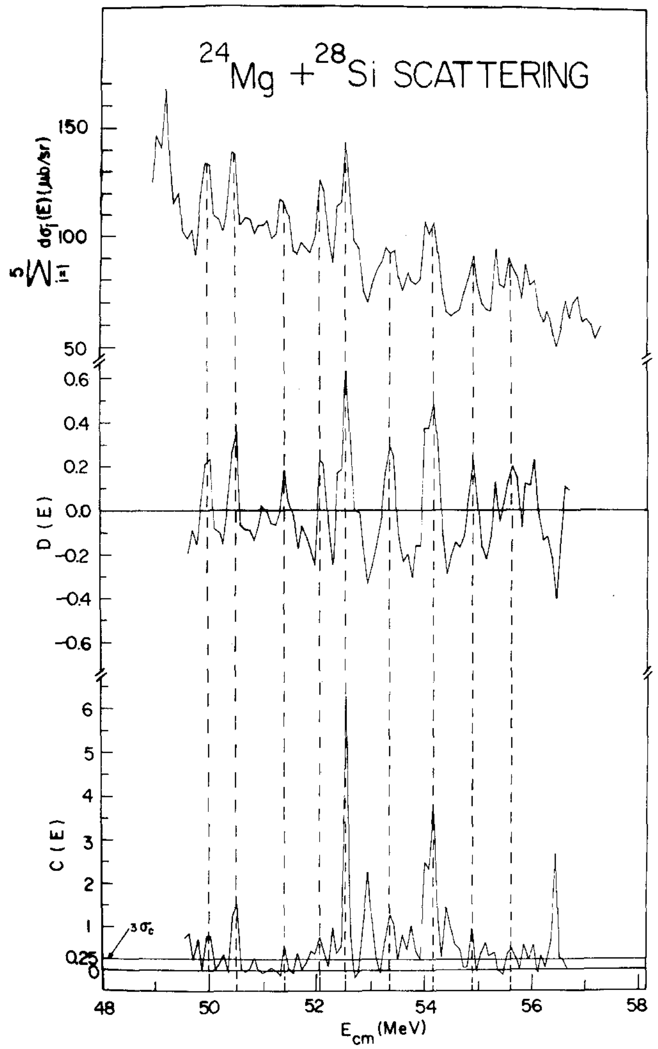

Fig. 2. Summed excitation function $\left(\sum_{i=1}^{5} \frac{\mathrm{d}\sigma_i(E)}{\mathrm{d}\Omega}\right)$, deviation function ($D(E)$), and the energy dependent cross-correlation function ($C(E)$) for the ^{24}Mg$+^{28}$Si scattering data. The vertical dashed lines indicate the location of the resonant structures. The structure at $E_{\text{c.m.}} = 49.23$ MeV (most prominent peak in the summed excitation function) cannot be seen in $D(E)$ and $C(E)$ due to our energy averaging procedure (see text)

tion coefficients between different (elastic and inelastic) channels by using the expression [18].

$$C_{ij} = \left\langle \left(\frac{\mathrm{d}\sigma_i(E)}{\langle \mathrm{d}\sigma_i(E) \rangle} - 1 \right) \left(\frac{\mathrm{d}\sigma_j(E)}{\langle \mathrm{d}\sigma_j(E) \rangle} - 1 \right) \right\rangle \cdot [C_i(0)\, C_j(0)]^{-1/2} \tag{4}$$

Here the subscripts i & j refer to particular scattering channels and the other symbols have already been explained earlier. The cross-channel correlation coefficients so obtained have been listed in Table 1. The indicated errors include a single maximum error in the cross sections in addition to the ones due to the finite range of data [20]. From this table it can be noted that there are large positive correlations between different channels, specially the 0^+-0^+ and 2^+-0^+, 0^+-0^+ and 2^+-2^+, 2^+-0^+ and 2^+-2^+, 2^+-0^+ and 4^+-0^+, 2^+-2^+ and 4^+-0^+, and 4^+-0^+ and 4^+-2^+ combinations. These large values of the cross-channel correlation coefficients obviously favour the observed structures to be the resonant structures.

2.4. Coherence widths

The coherence widths (Γ) in ^{52}Fe were obtained by autocorrelation analysis, empirical estimates and by counting the maxima in the excitation functions. The autocorrelation function is given by [21].

$$C(\varepsilon) = \frac{\langle x(E) \cdot x(E+\varepsilon) \rangle}{\langle x(E) \rangle \cdot \langle x(E+\varepsilon) \rangle} - 1 = \frac{C(0)}{(1+\varepsilon^2/\Gamma^2)}, \tag{5}$$

where ε is a variable energy interval and $\langle\ \rangle$ denotes the energy average. The auto-correlation functions (obtained by using (5)) for all the five excitation functions are presented in Fig. 3. The values of Γ deduced from these functions, after appropriately correcting for the finite range of data effects [22] are also given in Table 1.

The empirical estimates of the coherence widths were made by using the relation [23]

$$\Gamma = 14 \exp(-4.69 \sqrt{A/E_x})\ \text{MeV}, \tag{6}$$

where A denotes the mass and E_x the excitation energy of the compound nucleus ^{52}Fe. The values of Γ thus obtained ranged between 190 & 247 keV. The coherence widths were also determined by counting the maxima in the excitation functions [14]. These values of Γ, appropriately corrected for the finite target thickness and finite spacing of experimental points [24], are also listed in Table 1. The empirical estimates of the Γ-values are larger than the Γ-values obtained from the autocorrelation analysis. The peak counting method gives even larger values. Since for a purely statistical ensemble of the data the two methods should give identical results (within errors, see [25]), the difference in the values of Γ obtained by the autocorrelation method and the peak counting method indicates the presence of resonant structures in addition to the statistical fluctuations.

2.5. Distribution of cross sections

When direct reactions also contribute the distribution of fluctuating cross sections is given by [14, 26]

$$P(x) = \left(\frac{N}{1-Y_d}\right)^N x^{N-1} \exp\left(-N \frac{x+Y_d}{1-Y_d}\right) \cdot \frac{I_{N-1}[2N\sqrt{x Y_d}/(1-Y_d]}{[N\sqrt{x Y_d}/(1-Y_d)]^{N-1}}, \tag{7}$$

where, as mentioned earlier, $x = \mathrm{d}\sigma(E)/\langle \mathrm{d}\sigma(E) \rangle$, N = the number of effective channels, Y_d = the ratio of the average direct (noncompound) to total cross section and I_{N-1} is the modified Bessel function of order $N-1$. The quan-

Table 1. Cross-channel correlation coefficients for $^{24}Mg + ^{28}Si$ elastic and inelastic channels (upper half) and the coherence widths in ^{52}Fe obtained from the autocorrelation analysis and peak counting method (lower half)

Pair of channels	Correlation coefficients	Pair of channels	Correlation coefficients
(0^+-0^+)-(2^+-0^+)	0.62 ± 0.18	(2^+-0^+)-(4^+-0^+)	0.70 ± 0.10
(0^+-0^+)-(2^+-2^+)	0.61 ± 0.12	(2^+-0^+)-(4^+-2^+)	0.55 ± 0.08
(0^+-0^+)-(4^+-0^+)	0.58 ± 0.15	(2^+-2^+)-(4^+-0^+)	0.71 ± 0.10
(0^+-0^+)-(4^+-2^+)	0.50 ± 0.10	(2^+-2^+)-(4^+-2^+)	0.54 ± 0.07
(2^+-0^+)-(2^+-2^+)	0.77 ± 0.11	(4^+-0^+)-(4^+-2^+)	0.66 ± 0.09

Coherence widths

Excitation function	$\Gamma_{\text{(autocorrelation function)}}$ (keV)	$\Gamma_{\text{(counting the maxima)}}$ (keV)
0^+-0^+	61 ± 14	292 ± 31
2^+-0^+	72 ± 18	347 ± 42
2^+-2^+	82 ± 22	246 ± 24
4^+-0^+	82 ± 22	246 ± 24
4^+-2^+	72 ± 18	316 ± 36

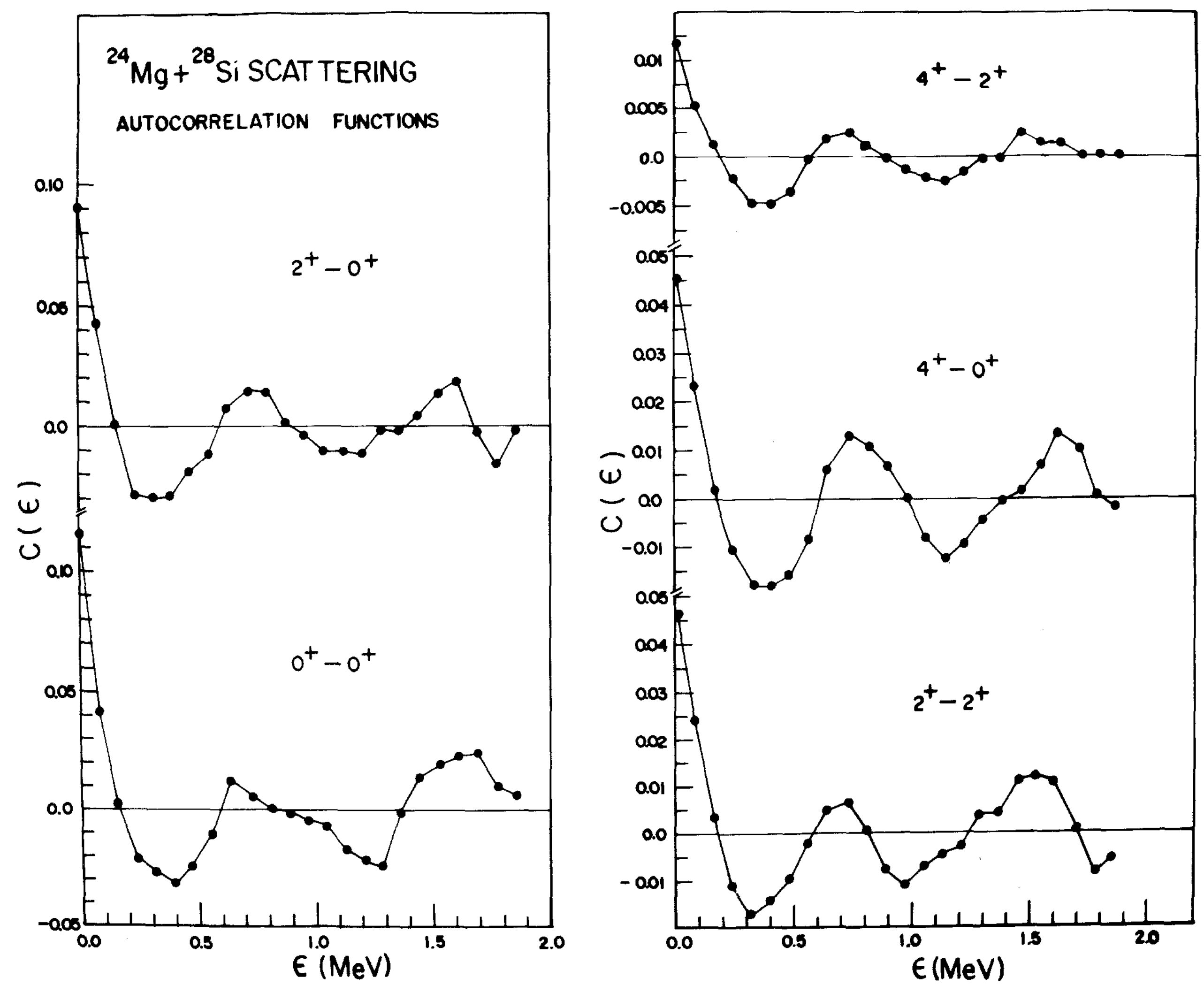

Fig. 3. Autocorrelation functions for the indicated excitation functions of $^{24}Mg + ^{28}Si$ scattering

tities N and Y_d are interrelated through the relation [17]

$$C(0)=\frac{1-Y_d^2}{N}, \tag{8}$$

where $C(0)$ is the variance of the data, as stated earlier. Thus knowing $C(0)$ and N, Y_d can be deduced from this relation. Noting that $N=1$ for elastic scattering excitation function a value of $Y_d=0.94$ was obtained. Such a value indicates that as much as 94% of the cross section is of nonstatistical origin. The theoretical and experimental distributions of cross sections for elastic excitation function are compared in Fig. 4. Good agreement between the two distributions clearly points to a large nonstatistical component in the cross sections [27]. For the excitation functions involving inelastic processes the maximum value of N, N_{max}, can be estimated by using the approximate expression

$$N_{max} = g/2 \quad \text{(for even } g\text{)}$$
$$= (g+1)/2 \quad \text{(for odd } g\text{)} \tag{9}$$

with $g=(2i+1)(2I+1)(2i'+1)(2I'+1)$.

Here i and I are the spins of the projectile and target and i' and I' are the spins of the final fragments [13]. Using these values of N_{max} the corresponding estimates of Y_d obtained from (8) also indicated large nonstatistical components in the cross section but since the approximation (9) frequently breaks down for the reaction processes involving heavy ions [26], such estimates of Y_d will certainly be ambiguous. Therefore, the distributions for the inelastic excitation functions are not given here.

3. Conclusion

The correlated structures at $E_{c.m.}=49.23$ (only seen in the summed excitation function due to the averaging procedure), 50.02, 50.51, 51.42, 52.10, 52.53, 53.27, 54.14, 54.88 and 55.60 stand out rather clearly in the cross-channel correlation function, deviation and summed excitation functions with $C(E)$ values much beyond the statistical limits. Large values of the cross-channel correlation coefficients between various pairs of exit channels support resonant nature of these correlated structures. A comparison of the experimental and theoretical distributions of cross sections for elastic channel ($N=1$) clearly indicates rather large nonstatistical component in the cross sections. Rather large differences in the coherence widths obtained from auto-correlation analysis and peak counting method indicates the presence of resonant structures. This way the present analysis confirms the existence of resonant structures at $E_{c.m.}=49.23$, 50.02, 50.51, 52.10, 52.53, 53.27 and 54.14 MeV (reported in [12]) and suggests three new structures of the similar nature at $E_{c.m.}=51.42$, 54.88 and 55.60 MeV. Like $^{28}Si+{}^{28}Si$ and $^{24}Mg+{}^{24}Mg$ it may be possible to attempt to understand the behaviour of $^{24}Mg+{}^{28}Si$ system also in terms of high-spin fissioning shape isomers which are theoretically expected to occur in the Cr-Ni region [5]. Of course, in case of $^{24}Mg+{}^{24}Mg$ system the spin alignment data in the region of strong resonances indicate the presence of a large molecular component in the wave function of 45.70 MeV resonance [9], and also support the concept of superdeformed intermediate complex [10]. The situation may as well be similar in case of $^{24}Mg+{}^{28}Si$ system.

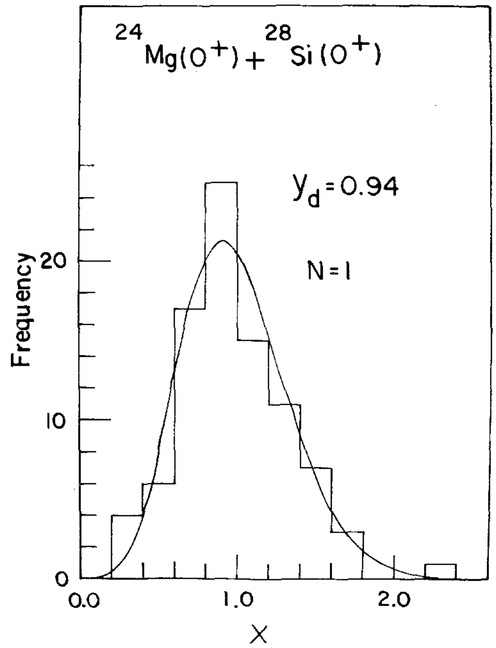

Fig. 4. Comparison of experimental and theoretical distributions of cross sections for $^{24}Mg+{}^{28}Si$ elastic scattering excitation function (see text)

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