PHYSICAL REVIEW D VOLUME 39, NUMBER 7 1 APRIL 1989

Spontaneous compactification effects, low-energy signature of quark-lepton unification, and small neutrino masses in SO(10)

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(Received 15 August 1988)

Signatures of quark-lepton unification can be experimentally verified by rare-kaon-decay processes provided the $SU(4)_C$-breaking scale $M_C \sim 10^5-10^6$ GeV. With the single intermediate symmetry, $SU(2)_L \times U(1)_R \times SU(4)_C$ in SO(10), we find, for the first time, that such a scale and small Majorana neutrino masses are allowed when gravity-induced corrections due to a five-dimensional operator, arising out of spontaneous compactification of extra dimensions, are included. For such lower values of M_C, the predicted proton lifetime is large depending upon the value of $\sin^2\theta_W$ in the range 0.22–0.24. For still larger values of M_C, neutrino masses and proton lifetime decrease, and the latter saturates the Irvine-Michigan-Brookhaven limit when $M_C \sim 10^{11}-10^{12}$ GeV.

I. INTRODUCTION

Two of the most important theoretical developments in particle physics have been the Kaluza-Klein-type unification with gravity[1,2] and grand unified theories[3–5] (GUT's) of strong, weak, and electromagnetic interactions. Originally proposed to unify gravitation with electromagnetism, the Kaluza-Klein method[1] has been applied with the standard, or grand unifying symmetries,[2] in higher dimensions with a view to unifying all basic forces of nature. In such cases, effective gauge theories in four-dimensional space-time emerge as a result of compatification of extra dimensions. In theories employing spontaneous compactification, nonrenormalizable higher-dimensional operators involving gauge and Higgs fields are always generated and the coefficients of these operators are scaled by the powers of the compactification scale (M_G) (Refs. 2 and 6). Even without invoking the idea of dimensional reduction, such operators scaled by powers of the Planck mass ($M_{\rm Pl}=10^{19}$ GeV) can also be present as the gravity-induced corrections to the GUT Lagrangian.

Compared to many other GUT's, SO(10) has many attractive features.[5] It is the minimal left-right-symmetric extension of SU(5), and contains all known fermions (plus the right-handed neutrino) of one generation in a single spinorial representation. It can attribute the origins of parity (P) and CP violations[7] as arising out of spontaneous symmetry breaking. It can explain neutrino masses over a wide range of values. With the decoupling of parity (P) and $SU(2)_R$-breaking scales, the new SO(10) model[8] provides a natural solution to the domain-wall problem.[9] With one or more intermediate symmetries, besides explaining the observed proton stability, SO(10) promises experimental verification of interesting theoretical ideas such as the quark-lepton unification based upon $SU(4)_C$, and left-right symmetry.[10] To date, possible low-energy signatures of $SU(4)_C$ breaking in SO(10), SU(16), and $SU(8)_L \times SU(8)_R$ GUT's have been predicted,[8,10,11] within the context of decoupling P and $SU(2)_R$ breakings, in the presence of the gauge group $SU(2)_L \times SU(2)_R \times SU(4)_C (\equiv G_{224})$ with $g_L \neq g_R$, as one of the intermediate symmetries, at a scale $M_C \sim 10^5-10^6$ GeV, such that both free neutron oscillations[12] and rare-kaon decays are expected to be experimentally observable. Since the first class of experiments are very difficult, because of nonavailability of free neutron sources, it might be useful to search for a grand unified theory where the consequences of $SU(4)_C$ breaking can be testified by a relatively easier class of experiments, such as the rare-kaon decays only. Even with two intermediate symmetries in the SO(10) GUT (Ref. 13),

$$\begin{aligned} SO(10) &\xrightarrow{M_U} SU(2)_L \times U(1)_R \times SU(4)_C (\equiv G_{214}) \\ &\xrightarrow{M_C} SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C \\ &\xrightarrow{M_R^0} SU(2)_L \times U(1)_Y \times SU(3)_C (\equiv G_{213}) \,, \end{aligned} \tag{1}$$

it has not been possible to obtain $M_C \simeq 10^5-10^6$ GeV, for the presently accepted values[14] of $\sin^2\theta_W = 0.23 \pm 0.005$. Two or more intermediate symmetries populating the grand desert provide possibilities of richer physical structure; but predictions with a single intermediate symmetry are very appealing because of the minimal nature of the underlying GUT scenario.

In this paper we note that, with the single intermediate symmetry $SU(2)_L \times U(1)_R \times SU(4)_C (\equiv G_{214})$, the SO(10) GUT is ruled out as it predicts a proton lifetime lower than the Irvine-Michigan-Brookhaven (IMB) limit[15] for the $p \rightarrow e^+\pi^0$ mode. But, when the twin ideas underlying grand unification and Kaluza-Klein theory are combined together, gravity-induced corrections by a five-dimensional operator[6,16] allow the chain

$$SO(10) \underset{M_U}{\overset{54+45}{\longrightarrow}} G_{214} \underset{M_C}{\overset{126}{\longrightarrow}} G_{213} \tag{2}$$

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with $M_C \sim 10^5$–10^{11} GeV, $M_U \sim 10^{15}$–10^{17} GeV, and $\sin^2\theta_W = 0.22$–0.24. For $M_C \sim 10^5$–10^6 GeV, corresponding to observable rare-kaon decays, ν_e mass is negligible; but ν_μ and ν_τ masses could be measured in the laboratory. The proton lifetime is significantly larger than the IMB limit depending upon the values of $\sin^2\theta_W$ and M_C. For still larger values of $M_C \sim 10^7$–10^{11} GeV, corresponding to undetectable rare-kaon decays, ν masses decrease further and the proton lifetime also decreases, saturating the IMB limit for $M_C \sim 10^{11}$ GeV.

This paper is organized in the following manner. In Sec. II we summarize earlier contributions in SO(10) where gravity-induced corrections have been included and discuss their implications in the context of cosmological domain-wall problem and neutrino masses. In Sec. III our new results are reported. The paper is summarized in Sec. IV.

II. MODIFICATIONS IN SO(10) WITH $SU(2)_L \times SU(2)_R \times SU(4)_C$ AND $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ INTERMEDIATE SYMMETRIES

In this section we summarize earlier contributions in SO(10) grand unified theory including gravity-induced corrections and discuss their significance in the context of domain walls in the early Universe and neutrino masses. Hill[16] and Shafi and Wetterich[6] (SW) introduced five-dimensional operators, involving gauge and Higgs fields, and scaled by the Planck mass ($M_{\rm Pl} = 10^{19}$ GeV) (Ref. 16) or the compactification scale ($M_G < M_{\rm Pl}$) (Ref. 6), with a view to obtain modified predictions on τ_p and $\sin^2\theta_W$. Detailed analysis in the minimal GUT has been made by them and others.[17] Besides SU(5), SW investigated[6] possible changes in SO(10) predictions in the presence of Pati-Salam intermediate symmetry:

$$\mathrm{SO}(10) \underset{M_U}{\overset{54}{\longrightarrow}} \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_C \underset{M_C}{\overset{126}{\longrightarrow}} G_{213} \ . \qquad (3)$$

In the absence of a $d=5$ operator, purely renormalizable interactions permit $M_C \simeq 10^{13}$ GeV, $M_U \simeq 10^{15}$ GeV for $\sin^2\theta_W \simeq 0.23$. When a nonrenormalizable term

$$\mathcal{L}_{\rm NR} = -\frac{\eta}{2M_G} \mathrm{Tr}(F_{\mu\nu}\Phi_{54}F^{\mu\nu}) \qquad (4)$$

is added, the following changes were noted:

$$\sin^2\theta_W \simeq 0.22, \quad \tau_p(p \to e^+\pi^0) \gtrsim (10\text{–}100)\tau_p(\mathrm{IMB}) \ , \qquad (5)$$

where τ_p(IMB) is the IMB limit.[15] In Eq. (4),

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \ , \\ (A_\mu)^a_b &= A^i_\mu(\lambda_i)^a_b \ , \\ \mathrm{Tr}(\lambda_i\lambda_j) &= \tfrac{1}{2}\delta_{ij} \ , \end{aligned} \qquad (6)$$

and A_μ's are the gauge field matrices, λ_i's are the SO(10) generators, η is an unknown parameter, M_G is the compactification scale, and Φ_{54} is the scalar field $\mathbf{54} \subset \mathrm{SO}(10)$. As in SU(5), SW noted that their modifications are consistent with $M_G \simeq 10^{-2} M_{\rm Pl}$, compatible with the parameter $\epsilon \simeq 0.01$–0.02 where

$$\epsilon = \frac{1}{\sqrt{30}} \frac{\eta\Phi_0}{M_G} \qquad (7)$$

and Φ_0 is related to the vacuum expectation value

$$\langle \Phi_{54} \rangle = \frac{\Phi_0}{\sqrt{30}} \mathrm{diag}(1,1,1,1,1,1,-\tfrac{3}{2},-\tfrac{3}{2},-\tfrac{3}{2},-\tfrac{3}{2}) \ . \qquad (8)$$

Without introducing the idea of spontaneous compactification, as in Hill's approach,[16] Rizzo[18] investigated the possibility of low-mass purity restoration in SO(10),

$$\mathrm{SO}(10) \underset{M_U}{\overset{210}{\longrightarrow}} \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \times \mathrm{SU}(3)_C \underset{M_R}{\overset{126}{\longrightarrow}} G_{213} \qquad (9)$$

using a $d=5$ operator scaled by the Planck mass:

$$\mathcal{L}_{\rm NR} = \frac{C}{2M_{\rm Pl}} \mathrm{Tr}(F_{\mu\nu}\Phi_{210}F^{\mu\nu}) \ . \qquad (10)$$

With a purely renormalizable Lagrangian, $M_R \sim M_W$ in the chain requires[19] $\sin^2\theta_W \simeq 0.27$–$0.28$, which are much larger than the accepted world average. Addition of (10) has been found to allow $M_R \sim M_W$ with $C \simeq -0.5$ and $\sin^2\theta_W$ within acceptable limits, for different combinations of Higgs triplets and doublets.

In the symmetry-breaking chain (3), the parity-violating scale $M_P = M_C \simeq 10^{13}$ GeV, and in (9) $M_P = M_R \sim M_W$. Several years before, it was noted by Kibble, Lazarides, and Shafi,[9] that in a model such as (3), where parity breaks at a scale lower than M_U, domain walls, bounded by strings, are created in the early Universe. Such domain walls contribute to the mass density of the Universe, much larger than observed values, unless $M_P = M_C \gtrsim 10^{13}$ GeV. With gravity-induced corrections,[6] or otherwise,[9] $M_P = M_C \simeq 10^{13}$ GeV, such that problematic domain walls are likely to be absent in chain (3). But in (9), since $M_P = M_R \sim M_W \ll 10^{13}$ GeV, the domain walls created are supposed to be extremely problematic.

Since the scalar representation $\mathbf{126} \subset \mathrm{SO}(10)$ is used in the chains (3) and (9), to break the intermediate gauge symmetry spontaneously to the standard group, Majorana neutrino masses are generated by a seesaw mechanism satisfying the formula[20]

$$m_{\nu_i} \simeq \frac{m_i^2}{M_{W_R}}, \quad i = e, \mu, \tau \ , \qquad (11)$$

where m_i is the charged-lepton mass of ith generation and M_{W_R} is the $W_R^\pm$ gauge-boson mass. In case (3), investigated by SW (Ref. 6) $M_{W_R} = M_C \simeq 10^{13}$ GeV, such that

$$m_{\nu_e} \sim 10^{-11}\ \mathrm{eV}, \quad m_{\nu_\mu} \sim 10^{-6}\ \mathrm{eV}, \quad m_{\nu_\tau} \sim 10^{-4}\ \mathrm{eV} \ . \qquad (12)$$

Such neutrino masses are too small to be detected by laboratory experiments, although they might be compatible with the solution to the solar-neutrino puzzle. In model (9), observable low-mass parity-restoration requires $M_{W_R} \sim M_{W_L}$, leading to rather larger values of Majorana neutrino masses:

$$m_{\nu_e} \sim 1 \text{ eV}, \quad m_{\nu_\mu} \sim 100 \text{ keV}, \quad m_{\nu_\tau} \sim 10 \text{ MeV} . \tag{13}$$

In this case, although the masses could be measured in the laboratory, ν_μ and ν_τ masses might be too large.

In the next section we study the possibility of SO(10) grand unification with single G_{214} intermediate symmetry. Since $SU(2)_R$ breaks at the GUT scale, there is no domain-wall problem in this model. When the effects of the $d=5$ operator are included, the allowed solutions for M_C are such that we obtain neutrino masses larger than (12) but smaller than (13) by 3–9 orders of magnitude.

III. GRAVITY-INDUCED CORRECTIONS WITH $SU(2)_L \times U(1)_R \times SU(4)_C$ INTERMEDIATE SYMMETRY

In this section we study the modifications caused by the $d=5$ operator in the predictions of the symmetry-breaking pattern (2), with G_{214} intermediate symmetry. It is usually stated that the vacuum expectation value of the Higgs field $\chi(1,0,1) \subset \mathbf{45} \subset SO(10)$, where the transformation properties of χ are under G_{214}, might achieve the spontaneous symmetry breaking at the first stage of the chain (2). But, according to observations made by Yasue[21] several years ago, both **54** and **45** are needed to break $SO(10) \to G_{214}$. As **45** is antisymmetric, it does not contribute to the gravity-induced corrections through the $d=5$ operator disussed in Sec. II; but the necessary presence of **54** is sufficient to induce significant modifications to the GUT predictions through the $d=5$ operator of Eq. (4). Following the techniques of Refs. 6 and 16, and using Eqs. (4)–(8), the Lagrangian $\mathcal{L}=\mathcal{L}_R+\mathcal{L}_{NR}$, with $\mathcal{L}_R=-\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$, is at first decomposed into kinetic energies of the $SU(4)_C$, $SU(2)_L$, and $U(1)_R$ gauge fields:

$$\mathcal{L}=-\tfrac{1}{2}(1+\epsilon)\mathrm{Tr}(F^{(C)}_{\mu\nu}F^{(C)\mu\nu})-\tfrac{1}{2}(1-\tfrac{3}{2}\epsilon)\mathrm{Tr}(F^{(L)}_{\mu\nu}F^{(L)\mu\nu}) -\tfrac{1}{4}(1-\tfrac{3}{2}\epsilon)F^{(R)}_{\mu\nu}F^{(R)\mu\nu} , \tag{14}$$

where the superscripts (C), (L), and (R) stand for the $SU(4)_C$, $SU(2)_L$, and $U(1)_R$, respectively. Now rescaling of the gauge fields changes their coupling constants as

$$g_C^2(M_U)\to g_C^2(M_U)(1+\epsilon), \quad g_L^2(M_U)\to g_L^2(M_U)(1-\tfrac{3}{2}\epsilon) ,$$
$$g_R^2(M_U)\to g_R^2(M_U)(1-\tfrac{3}{2}\epsilon) ,$$

where $g_C(M_U)$, $g_L(M_U)$, and $g_R(M_U)$ denote the coupling constants of $SU(4)_C$, $SU(2)_L$, and $U(1)_R$, respectively, without gravity-induced corrections. In order to achieve unification of strong, weak, and electromagnetic interactions for $\mu \geq M_U$, the GUT condition is imposed through the equations

$$g_C^2(M_U)(1+\epsilon)=g_L^2(M_U)(1-\tfrac{3}{2}\epsilon) =g_R^2(M_U)(1-\tfrac{3}{2}\epsilon)=g_0^2 , \tag{15}$$

where g_0 is the bare-GUT coupling constant. With the boundary conditions modified as in (15), we solve one-loop renormalization group equations for the chain (2).

$M_W \leq \mu \leq M_C$:

$$\frac{1}{\alpha_i(M_W)}=\frac{1}{\alpha_i(\mu)}+\frac{a_i}{2\pi}\ln\frac{\mu}{M_W}, \quad i=Y,L,3C \tag{16}$$

with

$$\alpha_i(\mu)=g_i^2(\mu)/4\pi, \quad a_Y=\tfrac{41}{10}, \quad a_L=-\tfrac{19}{6}, \quad a_{3C}=-7 .$$

$M_C \leq \mu \leq M_U$:

$$\frac{1}{\alpha_i(M_C)}=\frac{1}{\alpha_i(\mu)}+\frac{a_i}{2\pi}\ln\frac{\mu}{M_C}, \quad i=R,L,4C , \tag{17}$$

with $a_R=\frac{15}{2}$, $a_L=-\frac{19}{6}$ and $a_{4C}=-\frac{29}{3}$.

Note that we have been confined to the minimal fine-tuning condition and used three fermion generations, and the minimal number of Higgs scalars, needed for spontaneous symmetry breaking. The Higgs scalars used in the two different mass ranges are $M_W \leq \mu \leq M_C$, $\Phi(1,2,1)$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$, $M_C \leq \mu \leq M_U$, $\Phi(2,\frac{1}{2},1)$ and $\Delta_R(1,1,10)$ under $SU(2)_L \times U(1)_R \times SU(4)_C$. These are present in **10** and **126** of SO(10). Using Eqs. (15)–(17) and the combinations, $e^{-2}(M_W)-\frac{8}{3}g_3^{-2}(M_W)$, $e^{-2}(M_W)-\frac{8}{3}g_L^{-2}(M_W)$, $e^{-2}(M_W)=\frac{5}{3}g_Y^{-2}(M_W)+g_L^{-2}(M_W)$, yields the following constraints on the unification mass, $\sin^2\theta_W$, and the GUT coupling constant ($\alpha_G=g_0^2/4\pi$):

$$\ln\frac{M_U}{M_W}=\frac{6\pi}{71-74\epsilon}\left[\frac{1}{\alpha}-\frac{8}{3\alpha_s}+\left[\frac{7}{3\alpha_s}+\frac{1}{\alpha}\right]\epsilon\right] +\left[\frac{4-36\epsilon}{71-74\epsilon}\right]\ln\frac{M_C}{M_W} , \tag{18}$$

$$\sin^2\theta_W=\frac{1}{71-74\epsilon}\left[\left[\tfrac{39}{2}+(19-\tfrac{38}{3}\epsilon)\frac{\alpha}{\alpha_s}-53\epsilon\right] -\frac{\alpha}{\pi}(\tfrac{245}{3}-170\epsilon)\ln\frac{M_C}{M_W}\right] , \tag{19}$$

$$\frac{1}{\alpha_G}=\frac{1}{71-74\epsilon}\left[\frac{29}{\alpha}-\frac{19}{3\alpha_s}-\frac{226}{3\pi}\ln\frac{M_C}{M_W}\right] . \tag{20}$$

In Eqs. (18)–(20), $\alpha_s=g_3^2(M_W)/4\pi$ and $\alpha=e^2(M_W)/4\pi$.

For the chain (3), or (9), where formula (11) is applicable, $SU(2)_R$ breaks along with $U(1)_R$ at the same scale such that $M_{W_R}=M_{Z_R}$. But, in the present case, $SU(2)_R$ breaks at $M_{W_R}=M_U$, keeping $U(1)_R \times SU(4)_C$ unbroken down to $\mu=M_C$. It is at the second stage of the spontaneous symmetry breaking that the Majorana neutrino mass is generated when $\Delta_R(1,1,\overline{10})$ under G_{214} acquires vacuum expectation value. In this case $M_{Z_R}=M_C \ll M_{W_R}=M_U$, and the corresponding seesaw mechanism, worked out by Parida and Hazra,[22] yields a different formula for the Majorana neutrino mass:

$$m_{\nu_i}\simeq\frac{m_i^2}{M_C}, \quad i=e,\mu,\tau . \tag{21}$$

Using Eqs. (18)–(20), $\alpha_s=0.1088$ ($\Lambda_{\overline{\rm MS}}=160$ MeV, where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme), $\alpha^{-1}=127.54$, we compute numerically allowed regions for $M_C(\equiv 10^{n_c})$ and ϵ within the available experimental constraints (Ref. 14) on M_U and $\sin^2\theta_W$. Note that $\epsilon=0$ corresponds to the absence of gravity-induced effects and such solutions are presented in Table I. It is clear that, with a purely renormalizable Lagrangian, chain (2) is ruled out as it yields a maximum $M_U=3\times10^{14}$ GeV and the corresponding proton lifetime $\tau_p\simeq10^{29\pm2}$ yr which is significantly less than the IMB limit.[15]

Interesting solutions are obtained when $\epsilon>0$ and are presented in Figs. 1–3, and Tables II and III. At first, Fig. 1 is plotted using Eq. (18), and Fig. 2 using Eq. (19). In Fig. 1 the horizontal lines are the IMB and the Planck limits on the unification mass. The projection of the line PQ onto Fig. 2 has been denoted as the IMB limit in the latter. The horizontal lines in Fig. 2 represent the 2σ limits of the world average,[14] $\sin^2\theta_W=0.230\pm0.005$. The projection of the Planck limit from Fig. 1 onto Fig. 2 does not provide any useful boundary for the allowed region. But, a much better limit exists[23] from the experimentally observed bounds on the rare-kaon decay mode, $K_L\rightarrow\bar{\mu}e$, corresponding to $M_C\gtrsim3\times10^5$ GeV. Specifying the four sides of the quadrilateral in Fig. 2 in this fashion, the allowed solutions are shown by the shaded area.

The numerical values of M_C, ϵ, M_U, $\sin^2\theta_W$, and α_G^{-1} are shown in Table II for $M_C=10^5-10^6$ GeV and, in Table III for $M_C=10^7$–10^{11} GeV.

We find, ignoring the uncertainty in $\Lambda_{\overline{\rm MS}}$ and proton decay matrix elements,[24] that the modifications caused by the $d=5$ operator permit $10^5\lesssim M_C\lesssim10^{11}$ GeV. For every M_C, the parameter ϵ and the unification mass M_U are allowed over a wider range depending upon the 2σ or 1σ limit of $\sin^2\theta_W$. The solutions with smaller (larger) values of $\sin^2\theta_W$ are associated with larger (smaller) values of M_U and τ_p. Our solutions include the values of $M_C\sim10^5$–10^6 GeV, which predict rare-kaon decays to be observable for any value of $\sin^2\theta_W$ in the range of 0.22–0.24. The highest value of $M_U\simeq3\times10^{17}$ GeV is possible for $M_C=10^5$ GeV and $\sin^2\theta_W=0.22$. This has been shown by the point R in Fig. 1 which has been obtained by the projection of the corresponding point in Fig. 2.

As M_C increases, the unification mass, for a fixed value of $\sin^2\theta_W$, and the proton lifetime for the $p\rightarrow e^+\pi^0$ mode

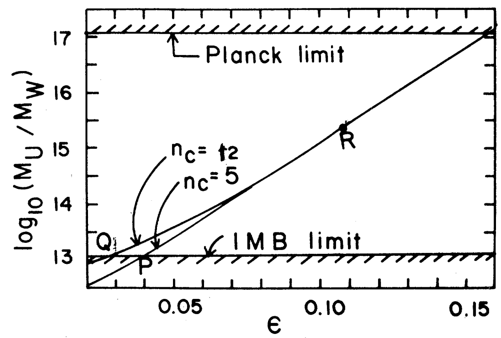

FIG. 1. Solutions of one-loop renormalization-group equations for M_U as a function of ϵ, and for $M_C=10^{n_c}$, $n_c=5$–12. The horizontal lines are the IMB (lower) and the Planck (upper) limits. The allowed upper limit, for $n_c=5$ shown as point R is obtained as the projection of the corresponding point in Fig. 2.

decreases. This has been shown in Fig. 3 for the 1σ and 2σ boundaries, and the central value of $\sin^2\theta_W=0.230$. For $M_C>10^8$ GeV, the allowed range of τ_p also decreases being restricted by the IMB limit form below. The IMB limit is found to be saturated nearly at $M_C\sim10^{10}$ (10^{11}) GeV if the value of $\sin^2\theta_W$ is allowed to be 0.225 (0.220). Including uncertainty in τ_p by a factor $10^{\pm2}$, arising out of uncertainties in the proton decay matrix element and the QCD parameter,[24] we find that the maximum allowed value of M_C can be increased by 1 order, (i.e., 10^{11}–10^{12} GeV) unless $\sin^2\theta_W$ is allowed to be significantly lower than 0.220.

Using Eq. (21) and the allowed range $M_C\sim10^5$–10^{12} GeV, we now calculate neutrino masses as shown in Fig. 4. They are found to vary over a wider range:

$$
\begin{aligned}
m_{\nu_e} &\sim (2\times10^{-10}\text{–}2\times10^{-3})\ \text{eV}\ ,\\
m_{\nu_\mu} &\sim (10^{-5}\text{–}100)\ \text{eV}\ ,\\
m_{\nu_\tau} &\sim (3\times10^{-2}\ \text{eV–}30\ \text{keV})\ ,
\end{aligned}
\qquad (22)
$$

where the lower (upper) limit corresponds to $M_C=10^{12}$ (10^5) GeV. The observable signatures of $SU(4)_C$ breaking by rare-kaon decay processes predict

$$
\begin{aligned}
m_{\nu_e} &\sim (2\times10^{-4}\text{–}2\times10^{-3})\ \text{eV}\ ,\\
m_{\nu_\mu} &\sim (11\text{–}110)\ \text{eV},\quad m_{\nu_\tau}\sim 3\text{–}30\ \text{keV}\ .
\end{aligned}
\qquad (23)
$$

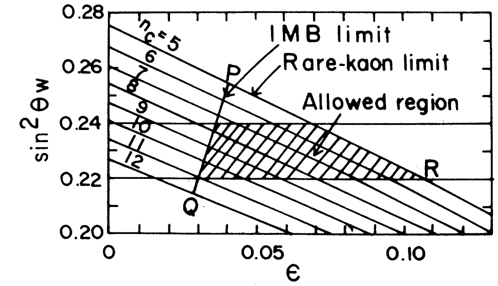

FIG. 2. Solutions of one-loop renormalization-group equations for $\sin^2\theta_W$ as a function of M_C and ϵ.

TABLE I. One-loop solutions for SO(10) with single intermediate symmetry, $SU(2)_L\times U(1)_R\times SU(4)_C$, in the absence of gravity-induced corrections.

M_C (GeV)	M_U (GeV)	$\sin^2\theta_W$
10^5	9.2×10^{13}	0.273
10^7	1.2×10^{14}	0.260
10^9	1.5×10^{14}	0.247
10^{11}	2.0×10^{14}	0.233
10^{13}	2.57×10^{14}	0.220
10^{14}	2.95×10^{14}	0.214

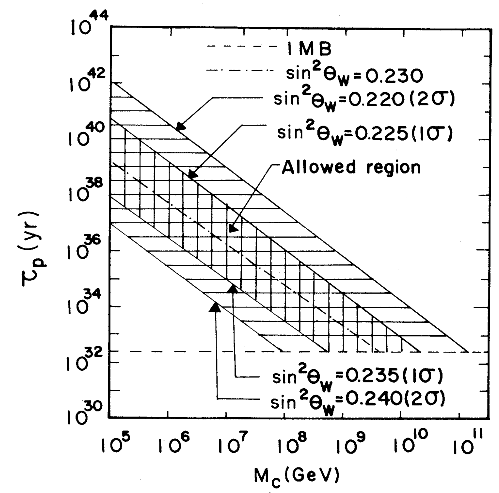

FIG. 3. Predictions on proton lifetime for the $p \to e^+\pi^0$ mode with $\Lambda_{\overline{MS}}=160$ MeV, as a function of M_C and $\sin^2\theta_W$. The solid lines are for values of $\sin^2\theta_W$ corresponding to 1σ and 2σ limits. The dot-dashed line is for $\sin^2\theta_W=0.230$. The IMB limit is shown by the dashed line.

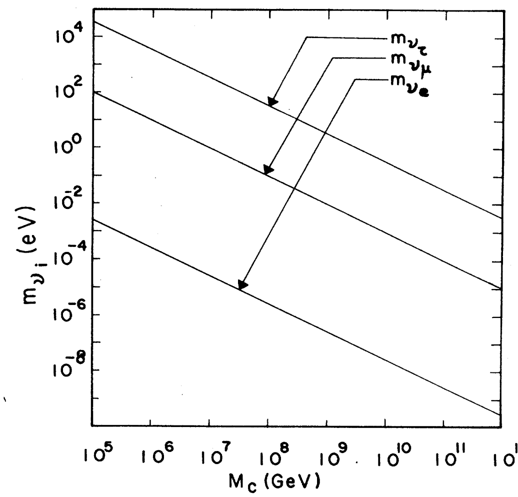

FIG. 4. Predictions on neutrino masses as a function of M_C.

Out of these, ν_μ and ν_τ masses are measurable by laboratory experiments. The masses are about 6–7 orders of magnitude larger than those obtained with single Pati-Salam intermediate symmetry,[6] but they are 3–4 orders of magnitude smaller than the models having low-mass $W_R^\pm$ and Z_R gauge bosons.[18,19,22]

To estimate, approximately, the order of magnitude of M_G that makes these gravity-induced corrections important, we use $\eta=(25\pi\alpha_G/2)^{1/2}\epsilon M_G/M_U$. Our estimation depends, crucially, on the assumption that $|\eta|\simeq 1$ as in the SW case.[6] Solutions having $\epsilon\simeq 0.03$–0.05 are found to be associated with lower values of the unification mass, $M_U\sim 10^{15}$ GeV. They require compactification scale nearly 2 orders smaller than $M_{\rm Pl}$. These solutions belong to the same class as noted[6] by SW in the context of SU(5) and SO(10) models. The other class of solutions found in this model are associated with $\epsilon\simeq 0.07$–0.10, and $M_U\sim 10^{16}$–10^{17} GeV. They require compactification scale $M_G\sim 10^{17}$–10^{18} GeV. In particular, the observable predictions for rare-kaon decay corresponding to $M_C\sim 10^5$–10^6 GeV are found to be also possible with $\sin^2\theta_W\simeq 0.22$–$0.23$, $\epsilon\simeq 0.1$, and $M_U\sim 10^{17}$ GeV, requiring $M_G\sim 10^{18}$ GeV. This scale is generally expected from the Kaluza-Klein-type compactification, where $M_G=M_{\rm Pl}/2\pi\simeq 1.6\times 10^{18}$ GeV. If, on the other hand, η is allowed to be $|\eta|\simeq 0.1$ (10), our estimation would require M_G 1 order less (more) for every value of ϵ. For example, with $M_C\sim 10^5$ GeV, and $M_U\sim 10^{17}$ GeV, consistency of the solutions with $\epsilon\simeq 0.1$ requires $M_G\sim 10^{17}$ (10^{19}) GeV, if $|\eta|\simeq 0.1$ (10), instead of $|\eta|\simeq 1$.

IV. SUMMARY AND CONCLUSION

In theories exploiting Kaluza-Klein-type unification with gravity, nonrenormalizable terms involving higher-dimensional ($d>4$) operators, scaled by suitable powers of the compactification scale, are usually present. We have investigated the modifications caused by a $d=5$ operator on the SO(10) GUT with single G_{214} intermediate symmetry. In the absence of gravity-induced corrections, such a model is ruled out as it predicts τ_p significantly below the IMB limit. Including gravity-induced corrections, the $SU(4)_C$-breaking scale is found to be permitted over a wide range, $M_C\sim 10^5-10^{12}$ GeV, leading to predictions on neutrino masses:

$$m_{\nu_e}\sim(10^{-10}\text{–}10^{-3})\ \text{eV},\quad m_{\nu_\mu}\sim(10^{-5}\text{–}100)\ \text{eV},$$
$$m_{\nu_\tau}\sim 10^{-2}\ \text{eV}-30\ \text{keV}.$$

TABLE II. Predictions for M_U, $\sin^2\theta_W$, and values of M_C, corresponding to observable rare-kaon decay, as a function of ϵ, in the presence of gravity-induced corrections, for the same chain as Table I. The proton lifetime is for the $p\to e^+\pi^0$ mode excluding uncertainties.

M_C (GeV)	ϵ	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_p (yr)
10^5	0.10	1.3×10^{17}	0.225	54.56	4.8×10^{40}
	0.09	5.9×10^{16}	0.230	53.93	2.0×10^{39}
	0.08	2.7×10^{16}	0.235	53.32	8.6×10^{37}
10^6	0.09	6.0×10^{16}	0.224	53.08	2.1×10^{39}
	0.08	2.8×10^{16}	0.229	52.47	9.6×10^{37}
	0.07	1.3×10^{16}	0.234	51.88	4.4×10^{36}
	0.06	6.3×10^{15}	0.239	51.30	2.4×10^{35}

TABLE III. Same as Table II but for larger values of M_C.

M_C (GeV)	ϵ	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_p (yr)
10^7	0.07	1.3×10^{16}	0.228	51.04	4.2×10^{36}
	0.06	6.7×10^{15}	0.233	50.48	2.9×10^{35}
	0.05	3.3×10^{15}	0.238	49.92	1.7×10^{34}
10^8	0.06	7.2×10^{15}	0.227	49.65	3.8×10^{35}
	0.05	3.6×10^{15}	0.232	49.10	2.3×10^{34}
	0.04	1.8×10^{15}	0.236	48.57	1.4×10^{33}
10^9	0.05	3.8×10^{15}	0.225	48.28	2.8×10^{34}
	0.04	1.9×10^{15}	0.230	47.75	1.7×10^{33}
10^{10}	0.04	2.1×10^{15}	0.223	46.94	2.4×10^{33}
10^{11}	0.03	1.2×10^{15}	0.221	45.63	2.0×10^{32}

Although m_{ν_e} is too small to be detected, m_{ν_μ} and m_{ν_τ} could be measured by laboratory experiments depending upon M_C.

For the first time, we have obtained interesting SO(10) predictions, with single intermediate symmetry, for the observable $SU(4)_C$ breaking by rare-kaon decay modes at low energies with $M_C \sim 10^5$–10^6 GeV, and any value of $\sin^2\theta_W$ in the range 0.22–0.24. For such lower values of M_C, the predicted ν masses are 6–7 orders of magnitude larger than the SO(10) prediction with Pati-Salam intermediate symmetry,[6] but 3–4 orders smaller than models with low-mass right-handed gauge bosons.[18,19,22] The proton lifetime is large depending upon the value of $\sin^2\theta_W$. For larger values of $M_C > 10^8$ GeV, the allowed range of τ_p decreases with increasing M_C. For a fixed $\sin^2\theta_W$, τ_p decreases with M_C and the IMB limit is saturated when $M_C \sim 10^{11}$–10^{12} GeV. The order of magnitude of the compactification scale, estimated in this model, is found to be in the range 10^{17}–10^{18} GeV, unless the parameter in the nonrenormalizable term has the value $|\eta| \simeq 10$, or larger.

With the $SU(2)_L \times U(1)_R \times SU(4)_C$ gauge symmetry, existing at a scale $\mu \geq M_C = M_{Z_R} \geq 10^5$ GeV, and $M_{W_R} = M_U \sim 10^{15}$–$10^{17}$ GeV, there is negligible contribution to the $V+A$ structure of charged and neutral currents, in this model. Similarly, the K_L-K_S mass difference and other *CP*-violating parameters have, essentially, the same prediction as the standard model. At low energies, this model does not seem to predict any other detectable signatures, except rare-kaon decays and neutrino masses.

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