SEARCH FOR MASS SCALES BEYOND THE STANDARD MODEL AND THEIR LOW-ENERGY SIGNATURES

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Dedicated to my father

Shri K.S. Dey

and

in the revered memory of my mother

Smt. Namita Dey

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This is to certify that the thesis entitled "Search for mass scales beyond the standard model and their low energy signatures" submitted by Miss Sudipta Dey for the fulfilment of the degree of Doctor of philosophy of the North-Eastern Hill University, Shillong embodies the record of an original investigation carried out by her under my supervision. She has been duly registered and the thesis submitted is worthy of being considered for the award of the Ph.D. degree.

This work has not been submitted for any other degree to any other institution.

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CONTENTS

List of papers produced under this thesis

1. `` Low mass right-handed gauge bosons, proton decay and other observable predictions in $SU(8)_L$ X $SU(8)_R$ and SU(16) " S.Dey and M.K.Parida, Phys. Rev.D 52, 518 (1995).

2. ``How to get $SU(2)_L$ X $SU(2)_R$ X $SU(4)_C$ breaking intermediate scale in SUSY SO(10)" M.K.Parida, S.Dey and B.Purkayastha, Proceedings of the XII DAE Symposium on High Energy Physics, 1,63 (1996)

3. `` Supersymmetric SO(10) with $SU(2)_L$ X $SU(2)_R$ X $SU(4)_C$ intermediate gauge symmetry" M.K.Parida, S.Dey and B.Purkayastha.ICTP Preprint IC/97/211; hep-ph / 9712424 (submitted to Phys. Rev. D)

CHAPTER 1

CHAPTER 1

INTRODUCTION

There has been a revolutionary change in the theoretical understanding of elementary particles and their interactions during the last two decades. Moreover, electrons, protons and neutrons have long been established as the building blocks of matter. Elementary particles participate in a virtually unlimited number of processes and their underlying interactions can be categorised into strong, electromagnetic, weak and gravitational interactions on the basis of their observed strength at low energies and varying scales. Fields are chosen to describe the fundamental particles in conformity with the symmetries of nature. Various interactions preserve different symmetries covering space – time symmetries of special or general relativity and the internal symmetries acting on the fields. Gell Mann and Neeman proposed a larger symmetry based on the group SU(3) wherein, symmetries grouped particles into larger multiplets of 8,10 or even more numbers with hadrons as composite particles. Since then, there has been a further simplification of this concept.

Symmetries have played a fundamental role in our understanding of particle physics. There are two kinds of distinct symmetries for physical systems:

global symmetries and local symmetries. The symmetry transformations in which the fields transform in the same way at every point in space and time are known as global symmetries,

$\phi_a(\vec{x}) \longrightarrow (e^{-i\beta.L})_{ab}\phi_b(x) = U(\beta)_{ab}\phi_b(x)$ where $U(\beta) = e^{-i\beta.L}$ is the same for all x. Here L constitutes the set of generators of transformation. If the symmetry is extended to allow independent transformations at different space time points, the symmetry is known as a local or gauge symmetry. Under a gauge transformation,

$\phi_a(x) \longrightarrow \left(e^{-i\beta(x).L}\right)_{ab}\phi_b(x) = U(\beta(x))_{ab}\phi_b(x)$ where$\beta(x)$ is now an arbitrary differentiable function of x.

A Lagrangian describing a physical system must be invariant under a set of symmetry transformations. The requirement that a Lagrangian be invariant under a local symmetry is more stringent than the requirement of global invariance and can only be met provided some new spin-1 fields A^a_μ, as dynamical variables are introduced into the theory. The fields A^a_μ are called gauge fields and its coupling to other fields in the Lagrangian is uniquely fixed by the requirement of gauge invariance. Thus, local symmetries dictate dynamics and therefore provide a more powerful theoretical tool for studies of particle interactions. Moreover in contrast to global symmetries, the current in the case of local symmetries participates in the interactions and is therefore a physical quantity which, in principle can be measured. The existence of a local symmetry implies the existence of massless gauge bosons, one for each generator of the local symmetry group. Theories in which field quanta may interact directly are called "non Abelian". The non Abelian case differs from the abelian in that the gauge fields themselves carry the charges associated with the generators of the group. This leads to the existence of off diagonal vertices in which a fermion or scalar field ϕ_a absorbs or emits a gauge boson and turns into a different field ϕ_b.

Gauge theories are very attractive in that the structure of the interactions is dictated by gauge invariance. Furthermore, they are believed to be the only field theories for vector mesons that are renormalisable, which means that all of the ultraviolet divergences in higher order diagrams can be removed from the theory by the redefinition of a finite number of masses and coupling constants. However, one cannot add vector meson mass terms to the Lagrangian because such terms would break the gauge invariance and lead to a nonrenormalisable theory. It therefore appears that the vector bosons must be massless and the forces which they mediate must be long ranged. This is, of course, desirable for quantum electrodynamics (QED), for which the gauge boson is the photon. The strong and weak interactions are not long ranged, however, and mainly they do not seem to fit into the gauge theory framework.

A way out of this difficulty was found through the mechanism of spontaneous symmetry breakdown. In this, the underlying Lagrangian is invariant under a symmetry group but the ground state of the system, the vaccum state is not invariant. It is possible for the symmetries for the equations of motion of a theory, to be broken by the stable solutions, which can pick out a specific direction in the symmetry space. This situation is known as spontaneous symmetry breaking. Spontaneous symmetry breaking occurs when the lowest energy state i.e. the vaccum state of the theory possesses a non-zero distribution of the charge associated with a symmetry generator. A gauge boson propagating through this vaccum state will constantly interact with this charge and develop an effective mass proportional to the vaccum expectation value of the charge. The associated force will be shielded becoming short ranged in much the same way the Coulomb force becomes short ranged in a Plasma due to shielding effects.

The Higgs mechanism is a simple explicit model for implementing spontaneous symmetry breaking. A set of spin-0 fields are introduced into the theory which transform in a nontrivial way under the gauge symmetry. If the vaccum expectation value of one of these fields is nonzero (this is essentially a Bose condensation), then all of the symmetry generators for which this field has a nonzero charge will be spontaneously broken and the associated gauge bosons will become massive. If the symmetry is global, then the spectrum of the theory contains a massless particle known as the Nambu-Goldstone boson. For a general group G of dimension N, which is spontaneously broken, leaving an unbroken subgroup H, of dimension M, there will be N-M massless Nambu-Goldstone bosons corresponding to the N-M broken generators of the coset space G/H. Spontaneous symmetry breaking has had its most significant application in the breakdown of a local not a global symmetry. In this case, the massless scalars do not appear, but supply the longitudinal components of massive vector gauge fields, This is known as Higgs mechanism. A massless vector field A_μ carries two degrees of freedom (transverse polarisation); when A_μ acquires mass, it picks up a third degree of freedom (longitudinal polarisation). This extra degree of freedom came from the Goldstone boson, which meanwhile disappeared from the theory. The gauge field "ate" the Goldstone boson, thereby acquiring both a mass and a third polarisation state.

It was shown by 't Hooft in 1971 that if gauge boson mass is generated by Higgs mechanism, then the theory is renormalisable. It is therefore very important in building gauge models that, the gauge symmetries that need to be broken (so that the corresponding gauge bosons pick up mass), must be broken by Higgs mechanism. Furthermore, to keep the theory renormalisable all terms in the Lagrangian must have dimensions less than or equal to four.

Another important criteria for renormalisability is the absence of triangle anamolies. In proving renormalisability of a theory, gauge invariance is a must. Gauge invariance means that all currents corresponding to gauge symmetries must be conserved to all orders in perturbation theory. It was pointed out by Adler, Bell and Jackiw that if a theory involves chiral interactions (i.e. interactions involving γ_5 currents) of fermions, triangular one-loop graph in general destroy the current conservation which was true at the tree level. This is called the axial anamoly. If such anamolies are not cancelled, then the theory loses its renormalisability. One must, therefore impose the constraint of anamoly cancellation on gauge theories. This imposes restrictions on the nature of fermion spectrum. In general, if in the space of all fermions, θ_i denotes the coupling matrix of fermions to the current J^i_μ, then *the condition for anamoly cancellation* $Tr(\theta_i[\theta_j,\theta_k])_L - Tr(\theta_i[\theta_j,\theta_k])_R = 0$

where the subscripts L and R denote left and right chirality states of fermions. This constraint plays an important role in understanding fermion spectrum, charge quantisation and helps reduce the arbitrariness of gauge theories. It is possible to avoid the anamoly by choosing a group G and a fermion representation so that anamoly is automatically rendered zero. Alternatively, it may happen that different representations would get cancelled. For a real representation, the anamoly vanishes because matrices are antisymmetric. Another obvious possibility, giving zero anamoly, is that there are equal couplings for left-handed and right-handed fermions (a vector like theory) then the contributions of left-handed and right-handed fermions to the anamoly cancel. But, the anamoly is independent of fermion mass.

All the above desirable features of a gauge theory, namely, renormalizability, anamoly cancellation with known fermions, massive gauge bosons mediating weak interactions, and electromagnetic gauge invariance after spontaneous symmetry breaking are achieved in the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ proposed by Glashow, Weinberg and Salam which breaks spontaneously to $U(1)_{em}$ by the Higgs mechanism,

$$SU(2)_L \times U(1)_Y \xrightarrow{\langle\phi\rangle_0 \simeq M_W} U(1)_{em}$$

where $< \phi >_0$ denotes the VEV of the neutral component in a Higgs-scalar doublet at a scale ($\mu \simeq M_W$) of the order of massive W-boson mass. The electroweak symmetry holds good for $\mu > M_W$.

It has been found that when proved at large momentum transfers, the strong interactions exhibit a property known as "asymptotic freedom". i.e. the strong interactions interact less strongly at high energies and start to exhibit the characteristic interactions of a gauge theory. The overwhelming weight of theoretical and experimental searches point to the conclusion that the strong interactions arise from the exchange of the eight gluons of an $SU(3)$ local gauge field theory, popularly known as QCD.

Including the strong interactions, the gauge group structure $SU(3)^C \times SU(2)_L \times U(1)_Y$ has come to be known as the "standard" model of the strong, weak and electromagnetic interactions which is good symmetry for $\mu \geq 100GeV$. When spontaneous symmetry breaking occurs the $SU(3)_C$ symmetry is left unbroken leading to $U(1)_{em} \times SU(3)_C$ as the low energy symmetry.

The limitations of the standard model suggest that the standard model is only a step towards a more fundamental theory and that, at best, it describes merely the low energy manifestations of an underlying unified theory. This led to the enunciation of Grand Unified Theories (GUT), in which the strong, weak and electromagnetic interactions are embedded in a large underlying gauge theory with a single gauge coupling constant g. But in order to unify the three interactions with gravity, grand unification is insufficient. Considerable attempts have been made to unify all basic interactions through superstring theories which manifest in SUSY GUTS or just the MSSM as effective low-energy theories. More recently the idea of underlying M-theory has been advanced.

The present thesis is organised in the following manner : Chapter -II and III are devoted to brief review of certain aspects of the non SUSY and SUSY standard models and grand unified theories respectively. In Chapter IV -V, the new research investigations carried out under this thesis are presented. The realisation of low-mass right-handed gauge bosons, accompanied by other observable predictions in $SU(8)_L \times SU(8)_R$ and $SU(16)$ GUTs are presented in detail in Chapter IV. In Chapter V we demonstrate how the intermediate gauge symmetry $SU(2)_L \times SU(2)_R \times SU(4)_C (g_{2L} \neq g_{2R})$ is achieved in SUSY SO(10) by two loop and threshold effects. A brief summary of the investigations is incorporated in Chapter VI alongwith conclusions.

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CHAPTER 2

CHAPTER II

THE STANDARD MODEL

II.1 INTRODUCTION

The development of the standard model of elementary particles and their interactions have been one of the greatest achievements of theoretical physics in recent years.In the standard model the $SU(3)_C$ colour gauge theory of the strong interactions is combined with the $SU(2)_L \times U(1)_Y$ leading to the SM gauge symmetry $SU(2)_L \times U(1)_Y \times SU(3)_C (\equiv G_{213})$ above the mass scale of 100 GeV.

Solving the weak gauge boson mass problem via the Higgs mechanism leads to the so called gauge hirerarchy problem - i.e. how to control the Higgs boson mass in the desired range of W and Z masses. This is because in the absence of a protecting symmetry the scalar masses have quadratic divergent quantum corrections making them infinitely heavy.By far the most promising solution to this problem is provided by supersymmetry (SUSY), which is a basic symmetry between bosons and fermions i.e. all fermions have bosonic superpartners and vice versa.It ensures cancellation between the divergent quantum corrections originating from the exchange of the standard particles and their superpartners.In the process however one predicts that a host of new particles,scalar partner of quarks and leptons as well as fermionic partners of gauge and Higgs boson should occur in the mass range of W and Z bosons. In this chapter we also discuss the supersymmetric extension of the Standard model which is called the minimal supersymmetric standard model (MSSM).

The particle representations in the non SUSY Standard model are briefly sketched in Section II.2 wherein the transformation properties under G_{213} have also been shown.In Section II.3 we discuss the MSSM.The limitations of the Standard model are discussed in Section II.4.

II.2 PARTICLE REPRESENTATION IN THE NON-SUSY STANDARD MODEL

All particles of the non susy standard model,except the neutral Higgs scalar have been experimentally detected.Even some of the weak gauge bosons and fermions were predicted in some theory or the other before the discovery of the Standard model.All particles of the SM follow the charge quantisation relation

$$Q = I_{3L} + \frac{Y}{2} \tag{2.1}$$

where Q, I_{3L}, Y denote the charge number,$SU(2)_L$ weak isospin and $U(1)_Y$

hypercharge respectively. All particles of SM fall into three different categories:(i)the spin 1 gauge or vector bosons

(ii) the spin 1/2 fermions and

(iii) the spin 0 Higgs scalar.

Gauge Bosons: The gauge bosons of $SU(2)_L, U(1)_Y$ and $SU(3)_C$ are $W_\mu^{1,2,3}, (3,0,1)$, $B_\mu(1,0,1)$ and $G^i_{\mu j}(1,0,8) i = 1,2,.....8$ respectively.

The kinetic energy term for $SU(2)_L \times U(1)_Y$ is given by,

$$\mathcal{L}^{Gauge} = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \tag{2.2}$$

where

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon_{abc} W^b_\mu W^c_\nu \tag{2.3}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.4}$$

g_2 is the $SU(2)$ gauge coupling.

Equation (2.3) includes the self coupling of the vector field W^a_μ, characteristic of a non-Abelian gauge theory.

Fermions: The charged weak currents couple the left-handed components of the charged leptons and the associated left-handed neutrino.All left-handed fermions transform as $SU(2)_L$ doublets while right-handed fermions as singlet.The μ and τ family repeat with exactly the same trans- formation properties under G_{213} as the electron family

First Generation

$$\begin{array}{llll}
\begin{pmatrix} u \\ d \end{pmatrix}_L & (2,1/3,3); & \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L & (2,-1,1) \\
u_R^{r,y,b} & (1,4/3,3); & e_R^- & (1,-2,1) \\
d_R^{r,y,b} & (1,-2/3,3) & &
\end{array}$$

Second Generation

$$\begin{array}{llll}
\begin{pmatrix} c \\ s \end{pmatrix}_L & (2,1/3,3); & \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L & (2,-1,1) \\
c_R^{r,y,b} & (1,4/3,3); & \mu_R^- & (1,-2,1) \\
s_R^{r,y,b} & (1,-2/3,3); & &
\end{array}$$

Third Generation

$$\begin{array}{llll}
\begin{pmatrix} t \\ b \end{pmatrix}_L & (2,1/3,3); & \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L & (2,-1,1) \\
t_R^{r,y,b} & (1,4/3,3); & \tau_R^- & (1,-2,1) \\
b_R^{r,y,b} & (1,-2/3,3); & &
\end{array} \tag{2.5}$$

The kinetic energy of the quarks and leptons and their interactions with $W^\pm$, Z and γ are given by,

$$\mathcal{L}^{fermions} = \bar{L}\gamma^\mu \left(i\partial_\mu - \frac{1}{2} g_2 \tau . W_\mu - g_1 \frac{Y}{2} B_\mu \right) L + \bar{R}\gamma^\mu \left(i\partial_\mu - g_1 \frac{Y}{2} B_\mu \right) R \tag{2.6}$$

where L denotes a left-handed fermion (lepton or quark) doublet and R denotes a right-handed fermion singlet.

Higgs scalar: The Higgs scalar (ϕ) in the SM is a complex doublet under $SU(2)_L$ having weak hypercharge $Y(\phi) = 1$. This scalar is needed to obtain massive gauge bosons through spontaneous symmetry breaking of $SU(2)_L \times U(1)_Y$ and Higgs mechanism.

$$\phi = \begin{pmatrix} \phi^{\dagger} \\ \phi^0 \end{pmatrix} \tag{2.7}$$

with

$$\phi^{\dagger} = \frac{(\phi_1 + i\phi_2)}{\sqrt{2}}$$

$$\phi^0 = \frac{(\phi_3 + i\phi_4)}{\sqrt{2}}$$

An $SU(2)_L \times U(1)_Y$ gauge invariant Lagrangian for scalar fields is given by,

$$\mathcal{L}^{scalar} = (\mathcal{D}^{\mu}\phi)^{\dagger}(\mathcal{D}_{\mu}\phi) - V(\phi^{\dagger}\phi) \tag{2.8}$$

where the covariant derivative is

$$\mathcal{D}^{\mu} = \partial_{\mu} + ig_1 B_{\mu}\frac{Y}{2} + ig_2\frac{\tau}{2}.W_{\mu} \tag{2.9}$$

and the scalar potential is

$$V(\phi^{\dagger}\phi) = \mu^2(\phi^{\dagger}\phi) + \lambda(\phi^{\dagger}\phi)^2 \tag{2.10}$$

To generate gauge boson masses through SSB, the parameters in (2.10) are chosen such that $\mu^2 < 0$ and $\lambda > 0$ with $\langle\phi_0\rangle = \frac{v}{\sqrt{2}} \neq 0$. The VEV is expressed as

$$\langle\phi\rangle = \phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad (2.11)$$

There are no charged-scalar fields in the standard model. The charged components of ϕ are absorbed by the massless $W^{\pm}$ gauge bosons as their longitudinal modes. Similarly the imaginary part of the neutral component of ϕ_0 provides the longitudinal mode to the neutral Z^0- boson.This generates the following tree-level masses for the vector bosons,

$$M_W = \frac{1}{2}g_2 v$$

$$M_Z = \frac{1}{2}(g_1^2 + g_2^2)^{1/2} v \qquad (2.12)$$

The mass eigenstates of the neutral Z-boson and the massless photon are,

$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$$

$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W \qquad (2.13)$$

The weak mixing angle,θ_W,is given by

$$\tan\theta_W = \frac{g_1}{g_2} \qquad (2.14)$$

and the electromagnetic coupling e is

$$e = \frac{g_1 g_2}{\sqrt{(g_1^2 + g_2^2)}} = g_2 \sin\theta_W \qquad (2.15)$$

After spontaneous symmetry breakdown, only one component of the Higgs field the physical neutral Higgs scalar mass is given by

$$M_H^2 = -2\mu^2 \qquad (2.16)$$

To generate fermion masses the Yukawa couplings in the Lagrangian are included

$$\mathcal{L}^{Yukawa} = -h[R(\phi^\dagger L) + (L\phi)R] + h.c. \qquad (2.17)$$

The complete Lagrangian of the standard model is given by,

$$\mathcal{L} = \mathcal{L}^{Gauge} + \mathcal{L}^{fermions} + \mathcal{L}^{scalar} + \mathcal{L}^{Yukawa} \qquad (2.18)$$

The $W^\pm$ and Z bosons have been discovered at the CERN pp collider experiment in 1983 and at CERN-LEP with masses,

$$M_{W^\pm} = 80.22GeV \pm 0.26GeV, M_{Z^0} = 91.187 \pm 0.007GeV \qquad (2.19)$$

The masses of quarks and the charged leptons are

m_u = 2 to 8 MeV; m_d =.5 to 15 MeV; m_e = 0.5109 MeV,

m_c = 1 to 1.6 GeV; m_s = 100 to 300 MeV; m_μ = 105.65 MeV,

$m_t = 176 \pm 24$ GeV; m_b = 4.1 to 4.5 GeV; m_τ = 1.777 GeV,

(2.20)

All other fermions are lighter than M_{Z^0}, but only the top quark appears to be heavier. A unique prediction of the standard electroweak theory is the neutral current interaction through Z exchange. The currently measured value of $\sin^2\theta_W$ is 0.23152 ± 0.00032.

II.3 SUSY VERSION OF THE STANDARD MODEL

Elementary particles consist of both fermions and bosons, and their ultimate unification would require them either to be composite of some basic set of fermions which can be unified within a Lie group framework or that there must exist a new symmetry that transforms bosons to fermions. This latter kind of symmetry, is known as supersymmetry, which relates particles of different spins. To see why symmetry between bosons and fermions may be of interest to the study of elementary particle physics, we point out that renormalisable quantum field theories with scalar particles (such as the Higgs sector of unified gauge theories) have a very disturbing feature in that the scalar masses have quadratic divergences associated with fermion masses which can be eliminated by taking advantage of chiral symmetries, there is no apparent symmetry that can control the divergences associated with scalar field masses. On the other hand, if we have a theory that couples fermions and bosons, the scalar masses have two sources of quadratic divergences, one from a scalar loop which comes with a positive sign and another from a fermion loop with a negative sign. It was then suggested that, if there was a symmetry that related the couplings and masses of fermions and bosons, all quadratic divergences from scalar field masses would be eliminated. The two sets of particles are distinguished by a multiplicative quantum number called R-parity, which is +1 for all standard particles and -1 for their superpartners. Conservation of R-parity implies that (i) the superpartners are produced in pairs and (ii) the lighest superparticle (LSP) resulting from their decay is stable. The LSP is also required to have no colour or electric charge. Consequently it is expected to interact weakly with matter and escape detection like the neutrino. This results in an apparent momentum imbalance (missing momentum), which serves as a powerful signature for superparticle production. In this section we briefly review some aspects of minimal supersymmetric extension of the Standard model. We call the supersymmetric partners of quarks and leptons as squark, slepton with a prefix s and denote them with a tilde over the symbol representing the corresponding particle. The particle spectrum along with their quantum numbers under G_{213} for one generation is given in Table II.1. All particles are chosen to be left-handed (or chiral) particles so that a right-handed field ($i.e. u_R$) will be denoted as a left-handed antiparticle field ($i.e. u_L^c$).

Table II.1

superfield	Component fields	$SU(2)_L \times U(1)_Y \times SU(3)_C$ quantum no.	Name
	Matter fields		
Q	$\begin{pmatrix} u_L \\ d_l \end{pmatrix} \equiv Q_L$	$(2, \frac{1}{3}, 3)$	Quark
	$\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix} \equiv \tilde{Q}_L$		Squark
U	u^c_L	$(1, \frac{-4}{3}, 3^*)$	Quark (denotes right handed up quark)
	$\tilde{u}^c_L$		
D	d^c_L	$(1, \frac{+2}{3}, 3^*)$	Squark (denotes right handed down quark)
	$\tilde{d}^c_L$		
L	$\begin{pmatrix} \nu_L \\ e^-_L \end{pmatrix}$	$(2, -1, 1))$	Lepton
	$\begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}^-_L \end{pmatrix}$		Slepton
E	e^c_L	$(1, +2, 1)$	Antilepton
	$\tilde{e}^c_L$		Antislepton
	Gauge fields		
V	$\begin{pmatrix} W^\pm \\ W^3 \end{pmatrix}$	$(3, 0, 1)$	Gauge boson
	$\begin{pmatrix} \tilde{W}^\pm \\ \tilde{W}^3 \end{pmatrix}$		Gaugino
B	B	$(1, 0, 1)$	Gauge Boson
	$\tilde{B}$		Gaugino
	Higgs field		
H_u	$\begin{pmatrix} \phi^\dagger_u \\ \phi^0_u \end{pmatrix}$	$(2, 1, 1))$	Higgs field
	$\begin{pmatrix} \tilde{\phi}^\dagger_u \\ \tilde{\phi}^0_u \end{pmatrix}$		Higgsino
H_d	$\begin{pmatrix} \phi^0_d \\ \phi^-_d \end{pmatrix}$	$(2, -1, 1))$	Higgs field
	$\begin{pmatrix} \tilde{\phi}^0_d \\ \tilde{\phi}^-_d \end{pmatrix}$		Higgsino

In order to give mass to fermions supersymmetry requires two Higgs doublets, H_u and H_d. The Lagrangian for MSSM is given by,

$$\mathcal{L} = \mathcal{L}^{Gauge} + \mathcal{L}^{matter} + \mathcal{L}^{soft} + \mathcal{L}^{Yukawa} - V \tag{2.21}$$

where

$$\mathcal{L}^{Gauge} = -\frac{1}{4}W_{\mu\nu}.W_{\mu\nu} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} - \tilde{W}\gamma.\Delta\tilde{W} - \tilde{B}\gamma.\partial\tilde{B} \tag{2.22}$$

$$\mathcal{L}^{matter} = \sum -\bar{\psi}\gamma_\mu \mathcal{D}_\mu \psi - \sum (\mathcal{D}_\mu \mathcal{A}_\psi)^\dagger (\mathcal{D}_\mu \mathcal{A}_\psi) + i\frac{g}{\sqrt{2}}\sum \bar{\psi}_L \tau.\tilde{W} A_\psi + h.c.$$

$$+i\frac{g'}{\sqrt{2}}\sum \bar{\psi}_L \tilde{B} Y A_\psi + h.c. \tag{2.23}$$

Summation goes over

$$\psi = Q, U, D, L, E, \tilde{H}_u, \tilde{H}_d, \psi_X$$

and

$$\mathcal{A}_\psi = \tilde{Q}, \tilde{U}, \tilde{D}, \tilde{L}, \tilde{E}, H_u, H_d, \tilde{\psi}_X$$

$$\mathcal{D}_\mu = \partial_\mu - ig'\frac{Y}{2}B_\mu - ig\frac{\tau}{2}.W_\mu \tag{2.24}$$

Y being the $U(1)_Y$ quantum number.

The term $\tau.W_\mu$ will be absent when a particle is $SU(2)_L$ singlet. ∂_μ denotes the covariant derivative for the appropriate gauge fields.

$$\mathcal{L}^{Yukawa} = h_u\left(Q_L^T C^{-1}\tau_2 H_u U_L^c + Q_L^T C^{-1}\tau_2 \tilde{H}_u + \tilde{U}^c \tilde{H}_c^T C^{-1}\tau_2 \tilde{Q} U_L^c\right) + u \rightarrow d$$

$$+h_e\left(L^T C^{-1}\tau_2 H_d e^c + L^T C^{-1}\tau_2 \tilde{H}_d \tilde{e}_c + \tilde{H}_d^T C^{-1}\tau_2 \tilde{L} e_L^c\right)$$

$$+\lambda\left(\tilde{H}_d^T C^{-1}\tau_2 H_d \psi_X + \tilde{H}_d^T C^{-1}\tau_2 H_u \psi_X + \tilde{H}_u C^{-1}\tau_2 \tilde{H}_d \tilde{\psi}_X\right) + h.c. \qquad (2.25)$$

Here C is the Dirac charge conjugation matrix and τ_2 is the second Pauli matrix.The first term in each of the first three bracketted expressions in the above equation is represented as the Yukawa couplings present in a two Higgs extension of the standard model. The remaining term involving fermions is the soft supersymmetry breaking Majorana mass term,i.e.

$$\mathcal{L}^{soft} = m'\tilde{W}^T C^{-1}\tilde{W} + m''\tilde{B}^T C^{-1}\tilde{B} + h.c. \qquad (2.26)$$

The potential V is given by,

$$V = |F|^2 + D^2 + V_{soft} \qquad (2.27)$$

$$|F|^2 = \left|h_u \tilde{Q}\tilde{U}^c + \lambda\tilde{\psi}_X H_d\right|^2 + \left|h_d \tilde{Q}\tilde{d}^c + h_e \tilde{L}\tilde{e}^c + \lambda\tilde{\psi}_X H_u\right|^2 + \left|h_u H_u \tilde{u}^c + h_d H_d \tilde{d}^c\right|^2$$

$$+h_u^2\left|\tilde{Q}^T\tau_2 H_u\right|^2 + h_d^2\left|\tilde{Q}^T\tau_2 H_d\right|^2$$

$$+h_e^2\left(H_d^\dagger H_d \tilde{e}^c e^c + \left|\tilde{L}^T\tau_2 H_d\right|^2\right) + \lambda^2\left(H_u^T\tau_2 H_d - \mu^2\right)^2 \qquad (2.28a)$$

$$V_{soft} = h_u\tilde{Q}^T\tau_2 H_u U^c + h_d\tilde{Q}^T\tau_2 H_d\tilde{d}^c + h_e\tilde{L}^T\tau_2 H_d\tilde{e}^c + \lambda\tilde{\psi}_X H_u^T\tau_2 H_d + h.c.$$

$$+m_Q^2\tilde{Q}^\dagger\tilde{Q} + m_L^2\tilde{L}^\dagger\tilde{L} + m_U^2\tilde{U}^{c*}\tilde{U}^c + m_D^2\tilde{d}^{c*}\tilde{d}^c + m_E^2\tilde{e}^{c*}\tilde{e}^c \qquad (2.28b)$$

$$\frac{1}{4}D^2 = \frac{1}{4}g^2 \sum_a \left| \sum_{A\psi} A_\psi^\dagger \tau_a A_\psi \right|^2 + \frac{1}{4}g'^2 \sum_a \left| \sum_{A\psi} A_\psi^\dagger Y A_\psi \right|^2 \qquad (2.28c)$$

Let us now study the spontaneous breaking of the gauge symmetry. For $m_Q^2, m_L^2, m_U^2, m_D^2, m_E^2$ positive, the minimum of V corresponds to

$$\langle \tilde{Q} \rangle = \langle \tilde{L} \rangle = \langle \tilde{e}^c \rangle = \langle \tilde{u}^c \rangle = \langle \tilde{d}^c \rangle = 0 \qquad (2.29)$$

$$\langle H_u^0 \rangle = \langle H_u^0 \rangle = \frac{v}{\sqrt{2}} = \mu \qquad (2.30)$$

The above equations break the $SU(2)_L \times U(1)_Y$ symmetry leaving the $SU(3)_C$ $U(1)_{em}$ symmetry intact This also gives nonzero masses to the quarks and leptons. As far as the Higgs sector is concerned (decomposing $H_u^0 = \mu + \sigma_a + i\chi_a$) the three Higgs-Kibble bosons are $\frac{1}{\sqrt{2}}(\chi_u - \chi_d), \frac{1}{\sqrt{2}}(H_u^\pm - H_d^\pm)$ which are absorbed to become longitudinial modes of the gauge bosons. The remaining physical Higgs fields $\frac{1}{\sqrt{2}}(\sigma_u + \sigma_d)$, $\frac{1}{\sqrt{2}}(\chi_u + \chi_d)$, $\frac{1}{\sqrt{2}}(H_u^\pm + H_d^\pm)$ all pick up mass λv. The mass of W and Z boson are given by,

$$m_W = \frac{1}{\sqrt{2}} g v \qquad (2.31)$$

$$m_Z = \frac{v}{\sqrt{2}} (g^2 + g'^2)^{1/2} \qquad (2.32)$$

The Higgs and the superparticles are the minimal set of missing pieces, required to complete the current picture of particle physics.Search of these particles is among the prime goal of the present and proposed high energy colliders.

II.4 LIMITATIONS OF THE STANDARD MODEL

Although the Standard Model has many impressive successes it falls short of a complete theory of the strong, weak and electromagnetic interactions for several reasons. Some of the reasons are discussed below.

(i) In the Standard model there are three different coupling constants associated with the gauge subgroup $SU(3)_C, SU(2)_L$ and $U(1)_Y$ respectively. Therefore the theory is not truly unified.

(ii) Standard model does not explain the origin of parity (P) and CP (C$\equiv$ charge conjugation) violations in weak interactions. Although the observed CP violation in weak interaction is parametrized in the Standard model framework through the Kobayashi-Maskawa (KM) approach, the model does not offer an origin of CP violations. Besides in the KM model one needs at least three fermion generations to get CP violations.

(iii) The Standard model contains a large number of parameters to explain physical phenomena including fermion masses. They are the three gauge coupling constants, six parameters for the six quarks plus three generalised Cabbibo angles, one CP violating phase, two parameters for the Higgs potential and either three or ten mass mixing and phase parameters for the leptons (corresponding to massless or massive neutrinos), for a total of 18 or 25 independent parameters. In addition, there can in principle be two CP violating vaccum angles, θ_{QCD} and θ_{QFD} associated with $SU(3)_C$ and $SU(2)_L$.

(iv) Electric charge is not quantised. The relation of quark to lepton charges is also not understood.

(v)The present experimental upper bound on the neutrino mass are $m_{\nu_e} \leq 18eV$,$m_{\nu_\mu} \leq 250KeV$ and $m_{\nu_\tau} \leq 35MeV$. There exists a limit of about 1eV on the Majorana mass of ν_e. Thus it is evident from the experimental bounds that if the neutrino has a mass, it is much less than the corresponding masses of the charged quarks and the leptons. On the other hand, in the standard model the neutrino is massless and the model prediction for the neutrino magnetic moment μ_{ν_l} is much smaller as might be needed for explaining fluxes from the sun and the supernova. The masslessness of the neutrino is due to the absence of the right-handed neutrino in the standard model. However, if a ν_R is added to the standard model the neutrino gets a large Dirac mass which is of the same order as the corresponding quark or charged lepton mass.Such a large mass is ruled out by available experimental limits and the big bang cosmology. Experimental measurements involving neutrinoless double beta decay, neutrino oscillations and the observation of neutrinos emitted from the sun and the 1987A supernova explosion are consistent with small neutrino masses.

(vi)Gravitational interaction is ignored in the standard model.

These limitations suggest that the standard model is only a step towards a more fundamental theory and that at best, it is an effective theory valid upto a mass scale, say M_X, at which the underlying theory will answer the above questions.

There are two main possibilities for this underlying theory if it exists. The first is that some or all of the fields of the standard model may be composite and there is some more fundamental level of structure. The second is that the fields of the standard model are themselves fundamental, but they are related by further symmetries, broken at the scale M_X. The latter approach leads to grand unified theories (GUTs) and to supersymmetric grand unified theories (SUSY-GUTs) which we discuss in the next chapter. In GUTs, the additional symmetries are gauge symmetries based on a larger Lie algebra than $SU(3)_C \times SU(2)_L \times U(1)_Y$, which may relate particles of the same spin. In the ideal GUT all the fundamental fields of a given spin will belong to a single irreducible representation of a gauge group G and hence their interactions will also be related by the (gauge) transformation of G. In SUSY GUTs the additional symmetry is based on a graded Lie algebra, which relates particles of different spin, and ideally may relate all particles and all interactions to the fundamental gauge bosons and gauge interactions.

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CHAPTER 3

CHAPTER III

GRAND UNIFIED THEORY

III.1.INTRODUCTION

Recent developments in particle physics have been dominated by theoretical and experimental investigations into the Standard model (SM) gauge theory for over two decades. Although, the idea of strong and electroweak unification based upon $SU(2)_L \times U(1)_Y \times SU(3)^C$ has been experimentally established, it is fraught with a number of difficulties most of which cannot be removed within the model itself. In the standard model, the strong weak and electromagnetic interactions are largely independent of each other as is illustrated by the fact that the gauge group $G_S = SU(3)^C X SU(2)_L X U(1)_Y$ is a direct product of three factors with different gauge coupling constants. The arbitrary and complicated pattern of fermion representation offers no fundamental explanation either for the repeatition of fermion families or violation of parity in the weak interactions least of all the strong interactions. Furthermore, the problem of quantization of electric charge remains unsolved both in this model as well as any unification group which has a $U(1)$ local symmetry. This means that there is no prior reason for the quark and lepton charges to be related by simple factors by 3. The standard model has many free parameters, notwithstanding the representations and electric charge assignments in the given group. This signifies that many observable quantities such as the fermion masses, mixing angles and CP violating phase are solely arbitrary. Finally, the standard model lacks the incorporation of gravity and an explanation for the empirical absence of a large cosmological term.

One way to constrain or determine some of the features that are arbitrary in the standard model (SM) is to consider the model with more or higher symmetry. The first attempt in this direction was made by Pati and Salam who unified the quarks and leptons within the group $SU(2)_L \times SU(2)_R \times SU(4)^C$ by extending the colour gauge group to include the leptons. They explained the quantization of electric charge although they had three coupling constants g_{2L}, g_{2R} and g_C, since there was no natural left-right symmetry in their model. This short coming was soon removed by making this theory a two coupling-constant partial unification theory. But more promising are grand unified theories in which the strong, weak and electromagnetic interactions are embedded in a larger underlying gauge theory with a single gauge coupling constant g. The idea is that the observed interactions of the standard model are merely the low energy manifestation of an underlying unified theory. This underlying theory possesses additional structure that appear arbitrary at the level of the Standard Model.

Grand unified theories typically have new symmetry generators which relate $SU(3)_C$ colour quantum numbers and $SU(2)_L \times U(1)_Y$ flavour quantum numbers. Quarks, antiquarks, leptons and antileptons are often placed together in irreducible representations of the underlying gauge group G and are related by the new symmetries. Associated with the new symmetry generators are vector bosons which carry both flavour and colour quantum numbers and can mediate baryon number violating processes such as proton or bound neutron decay. The very stringent experimental limits on the proton lifetime ($\tau_p > 10^{31}$ years) require that these new interactions must be extremely weak. Most grand unified theories explain the quantization of electric charge and many of the models predict $\sin^2\theta_w \simeq 0.23152$, which is reasonably close to the experimental value obtained from neutral curent experiments. Most models predict massive neutrinos, masses typically in the range $10^{-5} - 10^{+2}eV$. While ongoing experiments on solar and atmospheric neutrino oscillations and measurements on β-decay and neutrinoless double β-decay might confirm the existence of neutrino mass, massive neurinos provide a very interesting interpretation of the big bang cosmology and the dark matter of the universe. Finally some of the more complicated models predict flavour changing neutral currents (FCNC) at some level. One of the most exciting implications of grand unified theories is that they may explain the excess of matter over antimatter in the universe.

In a grand unified theory the standard model $G_S = SU(3)_C \times SU(2)_L \times U(1)_Y$ is embedded in a larger underlying group G, the so called grand unified group. One desirable constrain is to demand that the theory have one gauge coupling constant. In order to have only one gauge coupling constant the gauge group G must either be a simple group such as SU(N) or a direct product of identical simple groups such as $SU(N) \times SU(N)$. In addition to SU(N) there are four other classes of simple groups $SU(N+1), SO(2N), SO(2N+1)$ and $Sp(2N)$ where N is a positive integer, together with five exceptional groups G_2, F_4, E_6, E_7 and E_8. One important constrain is that the group should contain the Standard Model.Parity violation in the weak interaction requires either that the fermions be placed in a complex representation or that the number of fermions be doubled. So only complex representations are suitable for building a Grand Unified Theory with minimal particle content. These occur only for SU(N) with $N > 2$,E_6 and $SO(4N+2)$. Hence the desire for complex representations further limits the possible Grand Unified Groups.

The Standard Model gauge group $G_s = SU(3)^C \times SU(2)_L \times U(1)_Y$ has rank 4. Therefore any unified theory that contains G_S as a sub-group must have rank ≥ 4. Out of all the possible rank 4 groups that allow a single coupling constant the requirement for complex representations allow only $[SU(3)]^2$ and SU(5) as the possible groups for the Grand Unified theories. Among $[SU(3)]^2$and SU(5) only SU(5) has the complex representations necessary to accomodate the SU(3) (complex) triplet and SU(2) (complex) doublet fermion representations $SU(3)^C \times SU(2)_L \times U(1)_Y$ needed for the spectrum of the standard model. Thus SU(5) is the unique rank 4 candidate for a Grand Unified Theory. We shall discuss the structure of the model in Section 3.II.

In selecting an explicit grand unified gauge theory the leptons and quark appear to work together in families, with the first such family containing the familiar u_i and d_i quarks in three colours (i=R,Y,B) together with e and ν_e. The simplest way is to put the basic fermions only into the fundamental representation of some group since the obvious way to classify the ultimate building blocks of matter is in the fundamental of a group. Thus for the purpose of grand unification groups based on SU(8) are considered as they can classify the eight left-handed fermions in a common fundamental. However under a single SU(8) the seven right-handed fermions in the first family could only be classified as singlets, which is undesirable phenomenologically, since for instance the right-handed quarks would then have to be colour singlets. Thus, one further right-handed fermion, a right handed neutrino needs to be introduced to fill out another $\bar{8}$ and this leads to consider the local gauge group $SU(8)_L \times SU(8)_R$ based on families of 16 fermions. The model has anamolies, so heavy mirror families of fermions in which the L and R fields transform as (1,8) and (8,1) respectively, must be introduced to make the theory vectorlike. The grand unified model $SU(8)_L \times SU(8)_R$ include the Pati-Salam group $SU(2)_L \times SU(2)_R \times SU(4)^C$ as a subgroup and embody the twin features of quark-lepton unification. A brief review of the model is given in Section 3.III

As discussed above only complex representations are suitable for building a Grand unified theory with minimal particle content and this restricts us to the orthogonal groups SO(4N+2) with $N > 2$. The smallest such group with rank > 4 is SO(10), so the possibility of building a Grand unified theory based on SO(10) is considered. SO(10) is a group of rank 5 with the extra diagonal generator of SO(10) being B-L as in the left-right symmetric groups. The advantages of SO(10) as a grand unification group are:

1) only one 16-dimensional spinor representation of SO(10) has the right quantum number to accomodate all fermions (including the right-handed neutrino) of one generation.

2) the gauge interactions of SO(10) conserve parity thus making parity a part of a continuous symmetry: this has the advantage that it avoids the cosmological domain wall problem associated with parity symmetry breakdown.

3) it is the minimal left-right symmetric grand unified model that gauges the B-L symmetry and is the only other simple grand unification group that does not need mirror fermions. The model does not have any global symmetries. We describe the model in Section 3.III

III.2 SU(5) MODEL

SU(5) is defined by its adjoint representation which is the group of 5×5 complex unitary matrices with determinant 1. They are 25 independent real 5×5 matrices i.e.,50 independent complex matrices U. The unitary condition $U^{\dagger}U = 1$, and the unimodular condition, detU=1 give 25+1 constraints leaving the 24 independent matrices defining SU(5). U may be written in the form

$$U = \exp(-i \sum_{j=1}^{24} B^j L^j) \qquad (3.1)$$

where the 24 generators L^j are Hermitian (ensuring $UU^{\dagger} = 1$) and traceless (ensuring detU=1). Thus for generators normalised such that

$$Tr(L^a L^b) = 2\delta^{ab} \qquad (3.2)$$

In the matrix form we have,

$$L^a = \begin{pmatrix} & & & 0 & 0 \\ & \lambda^a & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix};$$

a=1,...,8 (generators of $SU(3)$)

(3.3)

λ^a are the usual Gell-Mann Zwig matrices acting on the colour indices. $V_\mu^{a=1,\ldots,8}$ are the gauge bosons of SU(5) which are to be identified with the gluons.

$$L^{9,10,11} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \multicolumn{2}{c}{\sigma_{1,2,3}} \\ 0 & 0 & 0 & & \end{pmatrix} \text{ generators of SU(2)}$$

$$L^{12} = \begin{pmatrix} 2/3 & 0 & 0 & 0 & 0 \\ 0 & 2/3 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \tag{3.4}$$

$\sigma_{1,2,3}$ are the Pauli spin matrices. Then $\frac{1}{\sqrt{2}}(V_\mu^9 \pm iV_\mu^{10})$ are the $W^\pm$ of the Standard Model. V_μ^{11} and V_μ^{12} are the weak gauge bosons W_μ^3 and B_μ respectively.

The remaining twelve additional Hermitian generators of SU(5) are

$$L^{13} = \begin{pmatrix} & & & 1 & 0 \\ & 0 & & 0 & 1 \\ & & & 0 & 0 \\ 1 & 0 & 0 & & \\ 0 & 0 & 0 & & 0 \end{pmatrix}, L^{14} = \begin{pmatrix} & & & -i & 0 \\ & 0 & & 0 & 0 \\ & & & 0 & 0 \\ i & 0 & 0 & & \\ 0 & 0 & 0 & & 0 \end{pmatrix}, L^{15} = \begin{pmatrix} & & & 0 & 1 \\ & 0 & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & & \\ 1 & 0 & 0 & & 0 \end{pmatrix}$$

$$L^{16} = \begin{pmatrix} & & & 0 & -i \\ & 0 & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & & \\ i & 0 & 0 & & 0 \end{pmatrix}, L^{17} = \begin{pmatrix} & & & 0 & 0 \\ & 0 & & 1 & 0 \\ & & & 0 & 0 \\ 0 & 1 & 0 & & \\ 0 & 0 & 0 & & 0 \end{pmatrix}, L^{18} = \begin{pmatrix} & & & 0 & 0 \\ & 0 & & -i & 0 \\ & & & 0 & 0 \\ 0 & i & 0 & & \\ 0 & 0 & 0 & & 0 \end{pmatrix}$$

$$L^{19} = \begin{pmatrix} & & & 0 & 0 \\ & 0 & & 0 & 1 \\ & & & 0 & 0 \\ 0 & 0 & 0 & & \\ 0 & 1 & 0 & & 0 \end{pmatrix}, L^{20} = \begin{pmatrix} & & & 0 & 0 \\ & 0 & & 0 & -i \\ & & & 0 & 0 \\ 0 & 0 & 0 & & \\ 0 & i & 0 & & 0 \end{pmatrix}, L^{21} = \begin{pmatrix} & & & 0 & 0 \\ & 0 & & 0 & 0 \\ & & & 1 & 0 \\ 0 & 0 & 1 & & \\ 0 & 0 & 0 & & 0 \end{pmatrix}$$

$$L^{22} = \begin{pmatrix} & & & 0 & 0 \\ & 0 & & 0 & 0 \\ & & & -i & 0 \\ 0 & 0 & i & & \\ 0 & 0 & 0 & & 0 \end{pmatrix}, L^{23} = \begin{pmatrix} & & & 0 & 0 \\ & 0 & & 0 & 0 \\ & & & 0 & 1 \\ 0 & 0 & 0 & & \\ 0 & 0 & 1 & & 0 \end{pmatrix}, L^{24} = \begin{pmatrix} & & & 0 & 0 \\ & 0 & & 0 & 0 \\ & & & 0 & -i \\ 0 & 0 & 0 & & \\ 0 & 0 & i & & 0 \end{pmatrix}$$

The associated vector bosons $V_\mu^{a=13,\ldots 18}$ and $V_\mu^{a=19,\ldots\ldots 24}$ are the twelve new gauge bosons of SU(5). They are called X and Y bosons.

In order to calculate the SU(5) gauge invariant kinetic energy term for the gauge bosons, the 5X5 matrix V_μ is defined as

$$\frac{1}{\sqrt{2}} V_\mu = \frac{1}{2} \sum_{a=1}^{n^2-1} V_\mu^a L^a \tag{3.6}$$

In terms of vector bosons introduced above V_μ is given by

$$V_\mu = \begin{pmatrix} G_1^1 - \frac{2B}{\sqrt{30}} & G_2^1 & G_3^1 & \bar{X}^1 & \bar{Y}^1 \\ G_1^2 & G_2^2 - \frac{2B}{\sqrt{30}} & G_3^2 & \bar{X}^2 & \bar{Y}^2 \\ G_1^3 & G_2^3 & G_3^3 - \frac{2B}{\sqrt{30}} & \bar{X}^3 & \bar{Y}^3 \\ X_1 & X_2 & X_3 & \frac{w^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} & W^+ \\ Y_1 & Y_2 & Y_3 & W^- & \frac{-w^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} \end{pmatrix} \tag{3.7}$$

The gauge invariant kinetic energy term is given by

$$\mathcal{L}_{kin} = \frac{-1}{4} F^a_{\mu\nu} F^{\mu\nu}_a = \frac{-1}{4} Tr(F_{\mu\nu} F^{\mu\nu} \qquad (3.8)$$

where

$$(F^{\mu\nu}_a)^i_j = \partial_\mu (V_\nu)^i_j - \frac{ig}{\sqrt{2}} (V_\mu)^j_k (V_\nu)^k_j + \partial_\nu (V_\mu)^i_j - \frac{ig}{\sqrt{2}} (V_\nu)^j_k (V_\mu)^k_j \qquad (3.9)$$

Each family of fermions fits neatly into the fifteen states available in the $\bar{5} + 10$ representations of SU(5). Also the anomalies in a $\bar{5}$ left-handed representation cancell the anomalies from a 10 left-handed representation. Thus $\bar{5} + 10$ is anomaly free.

$$\Psi_{\bar{5}} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ \nu_e \\ e^- \end{pmatrix}_L$$

$$\chi_{10} = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & E^+ & 0 \end{pmatrix} \qquad (3.10)$$

where the charge operator is identified as

$$Q = Diagonal(+1/3, +1/3, +1/3, 0, -1) = \frac{1}{2}\left(L^{11} + \sqrt{\frac{5}{3}} L^{12}\right) \qquad (3.11)$$

The coupling of the gauge bosons to the 5 of fermions is given by the invariant kinetic energy term

$$\mathcal{L}^5_{kin} = i\bar{\psi}_{5i}\gamma^\mu \left(D_\mu \psi_5\right)^i = i\bar{\Psi}_{5i}\gamma^\mu \left(\partial_\mu \delta^i_j - \frac{ig}{\sqrt{2}}\left(V_\mu\right)^i_j\right)\Psi^j_5 \tag{3.12}$$

The gauge invariant kinetic energy term for the 10 dimensional representation

$$\mathcal{L}^{10}_{kin} = \frac{i}{2}\left(\chi_{ac}\right)\left(\partial_\mu \delta^a_b - \frac{2ig}{\sqrt{2}}\left(V_\mu\right)^a_b\right)\gamma^\mu \chi^{bc} = \left(\chi_{10}\right)_{ac}\left(\partial_\mu \delta^a_b - \frac{2ig}{\sqrt{2}}\left(V_\mu\right)^a_b\right)\gamma^\mu \chi^{bc}_{10} \tag{3.13}$$

where for convenience a normalization factor of $\frac{1}{\sqrt{2}}$ is usually introduced $\left(\chi_{10} = \frac{1}{\sqrt{2}}\chi\right)$

Having assigned the fermions to multiplets of SU(5), the couplings of the gauge bosons are defined as in equations (3.12) and (3.13). The couplings of the neutral fields W^3_μ and B_μ, to matter are determined by the covariant derivative

$$D_\mu = \partial_\mu - \frac{ig}{2}\left(W^3_\mu L^{11} + B_\mu L^{12}\right) \tag{3.14}$$

where L^{11} and L^{12} are the diagonal generators given in (3.5)

Using the equations

$$A_\mu = B_\mu cos\theta_W + W_{\mu,3} sin\theta_W$$

$$Z_\mu = -B_\mu sin\theta_W + W_{\mu,3} cos\theta_W$$

in equation (3.14), D_μ in terms of A_μ and Z_μ is given by,

$$D_\mu = \partial_\mu - \frac{ig}{2}\left[(\sin\theta_W L^{11} + \cos\theta_W L^{12})A_\mu + (\cos\theta_W L^{11} - \sin\theta_W L^{12})Z_\mu\right] \tag{3.15}$$

$$= \partial_\mu - i\left[eQA_\mu + g_2 Q Z_\mu\right]$$

where the charge operator is defined by eqn.(3.15).Equation (3.15) gives,

$$\tan\theta_w = \sqrt{\frac{3}{5}}, \sin^2\theta_w = \frac{3}{8} = \frac{g_1^2}{g_1^2 + g_2^2}, \frac{g}{2} = \sqrt{\frac{2}{3}}e \tag{3.16}$$

The prediction for the $\sin^2\theta_W$ applies at the scale , M_U,at which SU(5) is a good symmetry and that it must be corrected while comparing with experimental measurements at low energies.
The interactions of the new "leptoquark" X and Y bosons which can couple leptons to quarks is given by,

$$\frac{1}{\sqrt{2}} g \bar{X}^i_\mu \left(\bar{d}_{iR}\gamma^\mu e^+_R + \epsilon_{ijk}\bar{u}^{Ck}_L\gamma^\mu u^j_L + \bar{d}_{iL}\gamma^\mu e^+_L\right) + h.c.$$

$+$

$$\frac{1}{\sqrt{2}} g Y^i_\mu \left(-\bar{d}_{iR}\gamma^\mu \nu^c_R + \epsilon_{ijk}\bar{u}^{Cj}_L\gamma^\mu d^k_L - \bar{u}_{iL}\gamma^\mu e^+_L\right) + h.c.$$

Each of these interactions changes both baryon and lepton number by -1 unit and changes the number of elementary fermions by -4 but conserves the quantum number B-L.

The couplings of equation (3.17) give the effective Lagrangian for baryon number violating four fermion interaction,

$$\frac{1}{4}\mathcal{L}_{\Delta B=1} = \frac{g^2}{8M_X^2}\left(\epsilon_{ijk}\bar{u}_L^{ck}\gamma^\mu u_L^j\right)\left(2\bar{e}_L^+\gamma^\mu d_R^i + \bar{e}_R^+\gamma^\mu d_R^i\right)$$

$$+\frac{g^2}{8M_Y^2}\left(\epsilon_{ijk}\bar{u}_L^{ck}\gamma^\mu d_L^j\right)\left(\bar{\nu}_{eR}^c\gamma^\mu d_R^i\right) + h.c. \tag{3.18}$$

leading proton decay of the type $p \to e^+\pi^0, n \to e^+\pi^-$ but not B-L violating processes such as $n \to e^-\pi^+$.

In order to achieve a phenomenologically acceptable model it is necessary to break SU(5) in two stages

$$SU(5) \xrightarrow{M_U} SU(3)^C \times SU(2)_L \times U(1)_Y$$

$$\xrightarrow{M_W} SU(3)^C \times U(1)_{em} \tag{3.19}$$

The first stage of symmetry breaking is achieved through the adjoint representation $\underline{24}$ of real scalar fields. At this stage the X and Y bosons receive a mass of order M_U leaving the other 12 gauge fields of $SU(3)^C X SU(2)_L X U(1)_Y$ massless. To accomplish the symmetry breaking the adjoint $\underline{24}$ representations of real scalar bosons acquire a vacuum expectation value

$$< \phi_{24} >_0 = v_{24} Diagonal(1,1,1,-3/2,-3/2) \tag{3.20}$$

where v_{24} is the parameter that will set the scale of masses of the X and Y bosons. The second stage of symmetry breaking is accomplished through a $\underline{5}$ dimensional representation of complex scalar fields. The complex $\underline{5}$ contains the complex $(1,2)_1$ representation used for spontaneous symmetry breaking in the standard model of weak and electromagnetic interactions. The vacuum expectation value is choosen as

$$< H >= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} v_5/\sqrt{2} \tag{3.21}$$

This gives mass of order M_W to the $W^{\pm}$ and Z bosons. The left handed fermions in $SU(5)$ transform as $\bar{5}+10$. As the fermion masses involve the product of two left handed fermion fields, so the representation content of these masses is obtained from the product $(\bar{5}+10)\times(\bar{5}+10)$. None of the product include a 24, so the adjoint ϕ does not couple to fermions. As a result the scale for fermion masses is $0(M_w)$, through their coupling to H and not M_U. The possible Yukawa coupling to H are

$$\mathcal{L}_{Yuk} = (\psi^{\dagger}_{Ri\alpha}) M^{D}_{ij} \chi^{\alpha\beta}_{Lj} H^{\dagger}_{\beta} - 1/4 \epsilon_{\alpha\beta\gamma\delta\epsilon} (\chi^{T})^{\alpha\beta}_{Li} \sigma^2 M^{U}_{ij} \chi^{\gamma\delta}_{Lj} H^{\epsilon} + hc \tag{3.22}$$

Here i,j are generation indices and $\alpha,\beta,\gamma,\delta,\epsilon$ are SU(5) indices. When H develops a vacuum expectation value, as in equation (3.21) it will generate down quark and charged lepton masses through the term proportional to M^D

and up quark masses through the term proportional to the M_U. M_D may be diagonalised by unitary rotations in the flavour space of the fermion fields.

$$\Psi_{Li} \longrightarrow [U_1]\,\psi_{Lj};\,\chi_{Li} \longrightarrow [U_2]_{ij}\,\chi_{Lj} \tag{3.23}$$

Then the masses obtained are $m_d = m_e = M_{11}^D v_5$

$m_s = m_\mu = M_{22} v_5$

$m_b = m_\tau = M_{33} v_5$

These predictations are corrected by radiative corrections. Up quark masses come from the term involving M^U, but because there are no further relations between quark and lepton masses. the unification mass M_U is obtained from the constraints that the three different gauge couplings g_{3c},g_{2L} and g_{1Y} at low energies unify to a single gauge coupling g_u above the unification scale
The SU(5) model predicts the grand unification masses $M_U \geq 10^{15} GeV$ which leads to $\tau_p \simeq 3 \times 10^{31}$ years.

III.3 $SU(8)_L \times SU(8)_R$ MODEL

In this model the basic fermions transform according to the $(8,1) + (1,8)$ representations of $SU(8)_L \times SU(8)_R$. Within each chiral sector the basis for each family are labelled as $(\nu_e, u_R, u_G, u_B, e, d_R, d_G, d_B)$. Each U(8) posseses eight diagonal λ matrices for which the basis are.

$$\lambda_A = \frac{1}{2}\begin{pmatrix} 1 &&&&&&& \\ & 1 &&&&&& \\ && 1 &&& 0 && \\ &&& 1 &&&& \\ &&&& 1 &&& \\ & 0 &&&& 1 && \\ &&&&&& 1 & \\ &&&&&&& 1 \end{pmatrix}, \lambda_B = \frac{1}{2}\begin{pmatrix} 1 &&&&&&& \\ & 1 &&&&&& \\ && 1 &&& 0 && \\ &&& 1 &&&& \\ &&&& -1 &&& \\ & 0 &&&& -1 && \\ &&&&&& -1 & \\ &&&&&&& -1 \end{pmatrix},$$

$$\lambda_C = \frac{1}{2}\begin{pmatrix} 3 &&&&&&& \\ & -1 &&&&&& \\ && -1 &&& 0 && \\ &&& -1 &&&& \\ &&&& 3 &&& \\ & 0 &&&& -1 && \\ &&&&&& -1 & \\ &&&&&&& -1 \end{pmatrix}, \lambda_D = \frac{1}{2}\begin{pmatrix} 3 &&&&&&& \\ & -1 &&&&&& \\ && -1 &&& 0 && \\ &&& -1 &&&& \\ &&&& -3 &&& \\ & 0 &&&& 1 && \\ &&&&&& 1 & \\ &&&&&&& 1 \end{pmatrix}$$

$$\lambda_E = \frac{1}{2}\begin{pmatrix} 0 &&&&&&& \\ & 1 &&&&&& \\ && -1 &&& 0 && \\ &&& 0 &&&& \\ &&&& 0 &&& \\ & 0 &&&& 0 && \\ &&&&&& 0 & \\ &&&&&&& 0 \end{pmatrix}, \lambda_F = \frac{1}{2}\begin{pmatrix} 0 &&&&&&& \\ & 1 &&&&&& \\ && 1 &&& 0 && \\ &&& -2 &&&& \\ &&&& 0 &&& \\ & 0 &&&& 0 && \\ &&&&&& 0 & \\ &&&&&&& 0 \end{pmatrix}$$

$$\lambda_G = \frac{1}{2}\begin{pmatrix} 0 &&&&&&& \\ & 0 &&&&&& \\ && 0 &&& 0 && \\ &&& 0 &&&& \\ &&&& 0 &&& \\ & 0 &&&& 1 && \\ &&&&&& -1 & \\ &&&&&&& 0 \end{pmatrix}, \lambda_H = \frac{1}{2}\begin{pmatrix} 0 &&&&&&& \\ & 0 &&&&&& \\ && 0 &&& 0 && \\ &&& 0 &&&& \\ &&&& 0 &&& \\ & 0 &&&& 1 && \\ &&&&&& 1 & \\ &&&&&&& -2 \end{pmatrix}$$

The local SU(8) chiral theory contains the generators $Y_L = 1/2(B_L - L_L)$, $Y_R = 1/2(B_R - L_R)$ where B_L, B_R, L_L and L_R are left and right handed baryon and lepton numbers respectively. The charge operator (which is contained in the theory since the sum of the electric charges within each fundamental is zero) can be expressed in terms of the above operators as

$$Q = T_L^3 + T_R^3 + Y_L + Y_R$$

where T_L^3 and T_R^3 are the third components of the weak isospins $SU(2)_L$ and $SU(2)_R$ contained within $SU(8)_L$ and $SU(8)_R$ respectively. In the Lagrangian of the $SU(8)_L \times SU(8)_R$ model the fermion, gauge boson sector of the theory actually possesses the larger global invariance $U(8)_L X U(8)_R$. With two additional conserved currents
$F_L = L_L + 3B_L, F_R = L_R + 3B_R$
when these operators act on the fundamental of each SU(8) we have the correspondence $T^3 \sim \lambda_B, Y \sim -\frac{1}{\sqrt{3}}\lambda_C, F \sim 2\lambda_A, Q \sim \lambda_B - \frac{1}{\sqrt{3}}\lambda_C$. While the diagonal $SU(3)_C$ generators transform as $\lambda_E + \lambda_G, \lambda_F + \lambda_H$

Within each SU(8) the gauge bosons can be represented as a rank two tensor

$$W_{ij} = \sum_{a=1}^{63} \lambda_{ij}^{a} W_{a}$$

In $SU(8)_L \times SU(8)_R$ there are triangle anamolies whose cancellation requires the introduction of a mirror family (Ψ_L^m, Ψ_R^m) corresponding to every standard family of quarks and leptons (Ψ_L, Ψ_r) and imposition of mirror symmetry $\Psi_L \longrightarrow \Psi_R^m, \Psi_R \longleftrightarrow \Psi_L^m$

In order to achieve the standard model phenomenology at low energies, when attempt is made to keep only three families light and the mirror masses near the grand unificaton mass M_U, the standard fermions also get bare Dirac masses of the order M_U. This is avoided by additional discrete symmetry $\Psi_L^m \rightarrow -\Psi_L^m, \Psi_R^m \rightarrow -\Psi_R^m$

$SU(8)_L \times SU(8)_R$ can descend to the low energy $SU(3)_c \times U(1)_{em}$ symmetry through various distinct chains. It also includes the Pati-Salam group $SU(4)_C \times SU(2)_L \times SU(2)_R$ as a subgroup and embody the twin features of quark lepton unification. The low energy $SU(3)_C X U(1)_{em}$ is obtained after the breaking of $SU(2)_L X SU(2)_R$ left-right symmetry. The parity violations in the weak interaction requires the right-handed weak bosons to be much heavier than the left-handed weak bosons. The model allows novel decay modes of the proton ($p \rightarrow e^- \pi^+ \pi^+$ etc) and depending on the chain of descent the different decay modes may co-exist and $n - \bar{n}$ oscillation may be allowed.

III.4 SO(10) MODEL

The SO_n groups are convenient for grand unification because they admit complex representations for n=2m (m odd) and are anomaly free ($n \neq 6$). The smallest such group with rank $\geq$ 4is SO(10). SO(10) is a group of rank 5 with each family of left-handed fermions assigned to a 16 dimensional complex spinor χ^+ An interesting aspect of SO(10) is that it contains the left-right symmetric gauge group and therefore it contains the right-handed neutrino. Since it has quarks and leptons in the same irreducible representation, the mechanism responsible for generating quark lepton masses automatically makes the neutrino massive. Thus, unlike the SU(5) model, in SO(10) model massive neutrinos arise naturally. To begin with, the two maximal subgroups of SO(10) are :

$G_{224} = SU(2)_L \times SU(2)_R \times SU(4)_C$

and $G_5 = SU(5) \times U(1)$

The 16-dimensional spinor represenation of SO(10) decomposes under G_{224} and G_5 as follows:

$G_{224} : \{16\} \supset \{2,1,4\} + \{1,2,\bar{4}\}$

$G_5 : \{16\} \supset \{10\}_1 + \{\bar{5}\}_{-3} + \{1\}_5$

From the SU(5) content of the spinor representation, it is clear that all the known fermions of a single generation can be fitted into a spinor. The SU(5) singlet piece of the spinor is identified with the right-handed neutrino. In the SO(10) model there 45 gauge bosons which transform as the adjoint representation. Under SU(5) the adjoint representation transforms as

$45 = 24 + 1 + 10 + \bar{10}$ Of the 45 generators of $SO(10)$, 24 are those of the $SU(5)$ subgroup. The other 21 generators connect or distinguish between the $\bar{5}, 10$ and 1. Under $SU(2)_L \times SU(2)_R \times SU(4)_C$ the adjoint representation 45 transform as

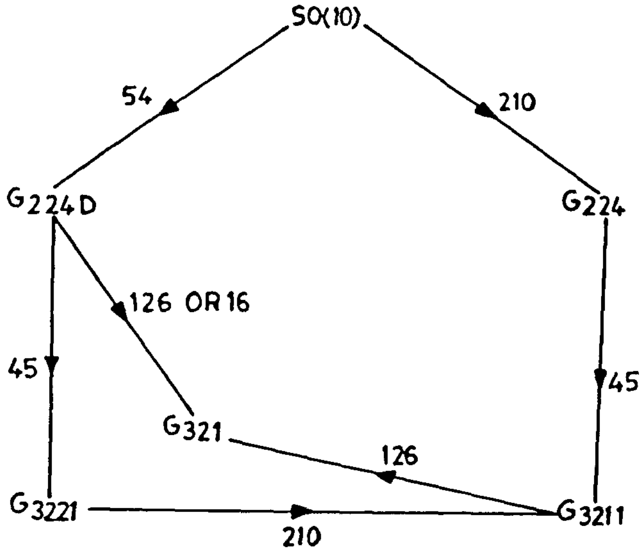
$45 \supset (3,1,1) + (1,3,1) + (1,1,15) + (2,2,6)$

When G_{224} further decomposes to $G_{2213}(SU(2)_L X SU(2)_R X U'(1) X SU(3)_C)$ the gauge bosons may similarly be expressed in terms of their $SU(2)_L \times SU(2)_R \times SU(3)_C$ transformations as

$$45 = (3,1,1)+(1,3,1)+(1,1,8)+(1,1,1)+(1,1,3)+(1,1,\bar{3})+(2,2,\bar{3})+(2,2,3)$$

The terms in the R.H.S. of the above expression represent $W_L^{\pm,0}$, $W_R^{\pm,0} G^{\alpha}_{\beta}$, B', $X_S, \bar{X}_S, \begin{pmatrix} X_\alpha \bar{Y}'_\alpha \\ Y'_\alpha \bar{X}'_\alpha \end{pmatrix}, \begin{pmatrix} X'^\alpha \bar{Y}^\alpha \\ Y'^\alpha \bar{X}^\alpha \end{pmatrix}$.

The possible symmetry breaking chains of $SO(10)$ are shown below where the dimensionality of Higgs boson responsible for each stage of symmetry breaking have also been listed.



So, there are many possible routes, each with its characteristic set of Higgs and Higgs potential. The gauge bosons contained in the quotient group $SO(10)/SU(2)_L \times SU(2)_R \times SU(4)_C$ become massive at the first stage of symmetry breaking and contribute to baryon nonconserving processes such as proton decay.

The allowed fermion masses in $SO(10)$ transform as the product of two 16s

$$\{16\}X\{16\} = \{10\} \oplus \{120\} \oplus \{126\}$$

The fermion masses may only be generated by spontaneous symmetry breaking through Yukawa couplings to Higgs in the 10, 120 or 126 representations. The ϕ_{10} and ϕ_{126} couplings are symmetric in family indices while the ϕ_{120}

couplings are antisymmetric. With only a 10 the minimal $SU(5)$ relations $m_d = m_e, m_s = m_\mu$ and $m_b = m_\tau$ are obtained.

With a 10 and 126 representation it is possible, by imposing discrete or continuous global symmetries, to achieve the form of the mass matrices

$$H^d = \begin{pmatrix} 0 & A' & 0 \\ A & C & 0 \\ 0 & 0 & B \end{pmatrix}, H^e = \begin{pmatrix} 0 & A' & 0 \\ A & -3C & 0 \\ 0 & 0 & B \end{pmatrix}$$

which gives the relations

$$m_b = m_\tau = B, m_e m_\mu = m_d m_s$$

with $\mathrm{A}{=}\mathrm{A}' since the \phi_{10}$ and ϕ_{120} couplings are family symmetric. Many other combinations are possible leading to acceptable fermion masses and to successful mass predictions but only with the use of additional global symmetries.

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CHAPTER 4

CHAPTER IV

LOW-MASS RIGHT-HANDED GAUGE BOSONS, PROTON DECAY AND OTHER OBSERVABLE PREDICTIONS IN $SU(8)_L \times SU(8)_R$ AND $SU(16)$

IV.1 INTRODUCTION

Precision measurements at the CERN -LEP leading to the determination of standard model gauge couplings and their extrapolation to higher scales have led to the revival of interests in grand unified theories (GUT's) [1-3]. Considerable attention has been paid to the study of implications of quark-lepton unification through $SU(4)_C$ -gauge hypothesis. If $SU(4)_C$ occurs as a part of left-right gauge group such as $SU(2)_L \times SU(2)_R \times SU(4)_C (\equiv G_{224})$, it can undergo spontaneous symmetry breaking to the standard model (SM) gauge group, directly or by more than one step, such that weak interaction phenomenology at lower energies remains close to that of the SM. In addition, rare kaon decays mediated by $SU(4)_C$ gauge bosons might be observable by low-energy experiments in near future, if the relevant symmetry breaking scale is not too high. If neutrinos are Majorana particles, a G_{224} breaking scale near $10^6 - 10^7 GeV$ might lead to observable proton decays, $H - H$ and neutron-antineutron oscillations. Also, a G_{224} breaking scale near $10^9 - 10^{11} GeV$ yields small neutrino masses necessary to understand solar-neutrino puzzle by MSW mechanism or dark matter of theuniverse. Further, the new physics associated with V + A interactions in left-right gauge models can be experimentally tested provided the right -handed gauge bosons $(W_R^{\pm}, Z_R)$ resulting from the spontaneous breaking of $SU(2)_L \times SU(2)_R \times SU(3)^C \times U(1)_{B-L}$ ($\equiv G_{2213}$) [5] symmetry are reasonable light. ($M_R < 1TeV$).

In all conventional methods of spontaneous breaking of left- right symmetric gauge structures, such as $SU(2)_L \times SU(2)_R \times SU(4)_C \times P(\equiv G_{224P}$, P = Parity = the left-right discrete symmetry, $g_{2L} = g_{2R}) \subset SO(10), SU(8)_L \times SU(8)_R$ or $SU(16)$, be it via a single step or via multiple steps to the standard gauge group, the scales of parity breaking (M_p) and $SU(2)_R$- breakings (M_R) are identical ($M_P = M_R$). This has the consequence that the effective gauge-coupling constants of $SU(2)_L$ and $SU(2)_R$ are equal to each other at low energies, barring small radiative corrections ($g_{2L} = g_{2R}$). Because of this, the GUTs like $SO(10), SU(8)_L \times SU(8)_R$ and $SU(16)$ require $\sin^2\theta_W(M_Z)$, to be too large ($\simeq 0.26 - 0.28$) compared to the observed value, $sin^2\theta_W = 0.2315 \pm 0.0002$ But, two different methods of decoupling of the $SU(2)_R$- breaking scale from the parity-breaking scale have been proposed leading to $g_{2L}(\mu) > g_{2R}(\mu)$ for $\mu < M_P$ with $M_R << M_P$ [7-8,10].

In $SO(10)$ or $SU(16)$, the eight left-handed fermions (ψ_L) and their charge conjugates $(\psi^c)_L$ are in the same spinorial representation $\underline{16}$. There exists an element of gauge transformation, called D, which takes $\psi_L \longrightarrow (\psi^c)_L$ and vice versa. The gauge transformation D acts like parity (P). The decoupling mechanism is implemented in the first step by giving vaccum expectation values (VEVs) to the D-odd (or P-odd) neutral components of scalars which are singlets under G_{2213} or G_{224} leading to spontaneous breaking of P and not $SU(2)_R$ which is broken in the usual manner at a lower scale.In this approach there is left-right assymetry in the Higgs sector only for $\mu < M_P$. The lowest value of G_{2213} breaking scale obtained in this method in $SO(10)$ has been found to be $M_R \simeq 10^7 GeV$. [7-9]

The alternative method [10] of decoupling parity operates within all GUTs containing $SU(2)_L \times SU(4)_L^C \times SU(2)_R \times SU(4)_R^C$ possessing chiral $SU(4)$ - colour in addition to the flavour subgroup $SU(2)_L \times SU(2)_R$. Therefore, it operates within $SU(8)_L \times SU(8)_R, SU(16)$ and also in $[SU(4)]^4$, but not in $SO(10)$. The decoupling is implemented by following specific symmetry breaking patterns where, after the first step, an assymetry in the left-handed and right-handed gauge-boson -sectors are created. For example, the symmetry breaking $SU(16), SU(8)_L \times SU(8)_R \longrightarrow SU(8)_L \times SU(2)_R \times SU(4)_R^C (\equiv G_{824})$ yields the symmetry having different left and right handed gauge groups. The subsequent breakings of either $G_{824} \longrightarrow G_{224}$ or $G_{824} \longrightarrow G_{2213}$ yields the left right gauge groups with $g_{2L}(\mu) > g_{2R}(\mu)$; the inequality in the two couplings being due to the asymmetry in the Higgs scalar as well as the gauge boson sectors.

In SU(16) and $SU(8)_L \times SU(8)_R$ [11,12] there are triangle anamolies whose cancellation requires the introduction of the mirror family (Ψ_L^m, Ψ_R^m) corresponding to every standard family of quarks and leptons (Ψ_L, Ψ_R) and the imposition of mirror symmetry. $\Psi_L \longrightarrow \Psi_R^m, \Psi_R \longrightarrow \Psi_L^m$

In order to achieve the standard model phenomenology at low energies, when attempt is made to keep only the three standard families light and the mirror masses near M_U, the standard fermions also get bare Dirac masses of the order M_U. This is avoided by imposing additional discrete symmetry,

$\Psi_L^m \longleftrightarrow -\Psi_L^m, \Psi_R^m \longleftrightarrow -\Psi_R^m$

The purpose of this chapter is that the alternate decoupling mechanism proposed by Parida and Pati [10] is capable of achieving M_R and M_C as low as $10^{2.8\pm0.7} GeV$ and $10^{5.6\pm0.6} GeV$, respectively, consistent with the CERN - LEP data. An approximate one-loop analysis in $SU(8)_L \times SU(8)_R$ and SU(16) was made in ref [10] using the old input data on $\sin^2\theta_W, \alpha_S(M_W)$ and $\alpha^{-1}(M_W)$. In this chapter we carry out more accurate analysis including two-loop effects and using the CERN-LEP measurements and the improved estimation of $\alpha^{-1}(M_Z)$ [13]

$$sin^2\theta_W(M_Z) = 0.2315 \pm 0.0002$$

$$\alpha_S(M_Z) = 0.119 \pm 0.004$$

$$\alpha^{-1}(M_Z) = 128.9 \pm 0.1 \tag{4.1}$$

We examine the unification of gauge couplings by plotting the couplings constant trajectories as a function of mass scale (μ) and following an improved procedure which simultaneously uses analytic formulas for mass scales derived here and the renormalisation group equations (RGEs). While qualitative estimates of the proton lifetimes and matter-antimatter oscillations were made in ref [10],quantitative evaluations including important uncertainties due to input parameters and the Higgs- scalar masses have also been carried out here. The uncertainties on the predicted values of mass scales arising out of the input parameters are evaluated and are noted to play a crucial role in bringing the $\Delta(B-L) = -2$ proton decay rates within the accessible range of the superkamio-kande experiments, although central values are 4-5 orders larger than the present accessible limit. The uncertainties due to the Higgs-scalar masses near the G_{224} -breaking are found to widen the range of proton life -time further, and predicted values of $n - \bar{n}$ and $H - \bar{H}$ oscillation mixing times.

This chapter is organised in the following manner. In Sec.IV.2 we obtain analytic formulas for mass scales in the minimal chains. In Sec. IV.3 we analyse the implications of G_{224}- breaking scale, $M_C = 10^6$ GeV. In Sec. IV.4 we discuss how the effect of including the Higgs-scalar multiplet $\xi(2,2,15)$ mecures the problem of bad fermion mass relation and all Higgs scalars, mediating $\Delta(B-L) = -2$ proton decay, naturally acquire the desired masses

$M_C \simeq 10^{5.6} Gev$ without the necessity of invoking a special mechanism as in $SO(10)$ model [6]. A brief summary of the chapter is given in Sec. IV.5.

IV.2. MASS SCALES IN THE MINIMAL CHAINS

In the alternative method of decoupling the parity and $SU(2)_R$- breakings, the assymetry $g_{2L}(\mu) > g_{2R}(\mu)$ for $\mu < M_U$ is created by a specific Higgs mechanism where SU(16) or $SU(8)_L \times SU(8)_R$ breaks spontaneously to $SU(8)_L \times SU(2)_R \times SU(4)_R^C (\equiv G_{824})$ at the GUT scale such that the residual symmetry $G_{824} \supset G_{224}$ or G_{2213},as its subgroup. The $SU(8)_L \times SU(8)_R$ Higgs representation (1,330) containing the G_{824}-singlet achieves this breaking.

In the second stage, the popular left-right gauge groups G_{224} with $g_{2L} \neq g_{2R}$ is obtained by the spontaneous symmetry breaking of G_{824} through the G_{88} - Higgs representation $(36, 36)$ containing the G_{224}- singlet. However, to obtain G_{2213} from G_{824}, two of the G_{88}-representations, $(36, 36)$ and $(1, 63)$, are needed. Whereas the representation $(36, 36)$ contains the G_{224}- singlet, the representation $(1, 63)$ contains the G_{224}- submultiplet $(1, 1, 15)$ whose vaccum expectation value in the G_{2213}-singlet direction yields the desired left-right gauge group. In the third step the spontaneous symmetry breaking of G_{224} or G_{2213} leads to the standard gauge group G_{213} by using the usual G_{224} -submultiplet $\Delta_R(1, 3, 10) \subset (1, 36)$ of G_{88}. Finally the low energy gauge group is obtained through the vaccum expectation value of the standard

Higgs doublet contained in the G_{88} representation $(8,\bar{8})$.

Thus, in order to obtain the asymmetric gauge groups G_{224} or G_{2213} with $g_{2L} \neq g_{2R}$, the new mechanism needs spontaneous symmetry breaking to the standard gauge group in at least two steps as illustrated in the minimal models (A) and (B),

$$(A) SU(16), SU(8)_L \times SU(8)_R \xrightarrow{M_U} G_{824} \xrightarrow{M_8} G_{224} \xrightarrow{M_C} G_{213} \xrightarrow{M_Z} U(1)_{em} \times SU(3)_C$$

$$(B) SU(16), SU(8)_L \times SU(8)_R \xrightarrow{M_U} G_{824} \xrightarrow{M_8} G_{2213} \xrightarrow{M_C} G_{213} \xrightarrow{M_Z} U(1)_{em} \times SU(3)_C$$

Instead of $SU(8)_L \times SU(8)_R$, it is possible to break SU(16) directly to G_{824} since the G_{88}-Higgs multiplets shown in models (A) and (B) are contained in the corresponding $SU(16)$ representations.

For the unbroken gauge symmetry G_i with coupling constants $g_i(\mu)$ and $\alpha_i(\mu) = \frac{g_i^2}{4\pi}$ in the mass range $M_1 \leq \mu \leq M_2$,the renormalisation group equations (RGEs) are

$$\frac{1}{\alpha_i(M_1)} = \frac{1}{\alpha_i(M_2)} + \frac{a_i}{2\pi} ln \frac{M_2}{M_1} + \frac{1}{4\pi} P_i(M_1, M_2) - \frac{C_i}{12\pi} \qquad (4.2a)$$

where

$$P_i(M_1, M_2) = \sum_j B_{ij} ln \frac{\alpha_j(M_2)}{\alpha_j(M_1)}.$$

The second (third) term in the R.H.S. of (4.2) is the one (two)-loop contribution. In the last term in (4.2a) C_i is the mass independent matching function corrections at the guage-symmetry breaking scales (boundaries). They are obtained by summing over all effective gauge theory (EGT) representations of massive gauge bosons assumed to be degenerate at M_2. Even if mass dependent terms in the threshold effect at M_2 are neglected, as in the present chapter, the C_i terms are always present. The values of C_i at each boundary is given below for the two models. For the minimal choice of Higgs scalars necessary to implement the spontaneous symmetry breaking for $\mu \leq M_8$ the loop co-efficients have been derived earlier [8]. They are given by,

$\underline{M}odel(A)$

For the G_{213} group $i, j = 2L, 1Y, 3C$

$a_{2L} = -\frac{19}{6}, a_{1Y} = \frac{41}{10}, a_{3C} = -7$

$C^c_{2L} = 0, C^c_Y = \frac{14}{5}, C^c_{3C} = 1$

where the superscript C in C^c_i stands for the boundary at M_C. The two-loop coefficients in (4.2a) are given by the matrix,

$$B_{ij} = \begin{pmatrix} \frac{-35}{19} & \frac{9}{41} & \frac{-12}{7} \\ \frac{-81}{95} & \frac{199}{205} & \frac{-44}{35} \\ \frac{-27}{19} & \frac{11}{41} & \frac{26}{7} \end{pmatrix}$$

To the R.H.S. of (4.2a) and in the presence of the SM symmetry, we also add the threshold effect arising at two-loop level due to the heavy top-quark Yukawa coupling for $m_t = 175.6 \pm 5.5\text{GeV}$,

$$P_i^{(t)} = \frac{h_t^2}{8\pi^2} b_i(t) ln \frac{M_I}{m_t}, i = 1Y, 2L, 3C \tag{4.2b}$$

where

$b_{1Y}^t = \frac{17}{10}, b_{2L}^t = \frac{3}{2}, b_{3C}^t = 2.$

In eqn(4.2b) h_t represents the top quark Yukawa coupling which is known to have approached $h_t \simeq 1$ and M_2 represents the intermediate scale M_R or M_C.The term in (4.2b) is present in all models discussed in this paper.

For the G_{224} group $i, j = 2L, 2R, 4C$

$a_{2L} = -3, a_{2R} = \frac{11}{3}, a_{4C} = -\frac{23}{3}$

$C_{2L}^8 = 30, C_{4L}^8 = 12, C_{4R}^8 = 0, C_{2R}^8 = 0$

$$B_{ij} = \begin{pmatrix} \frac{-8}{3} & -\frac{9}{26} & \frac{-135}{24} \\ -1 & \frac{584}{11} & \frac{-2295}{46} \\ \frac{-3}{2} & \frac{459}{22} & -\frac{643}{46} \end{pmatrix}$$

<u>*Model(B)*</u>

For the G_{213} group the one and two-loop coefficients are the same as in model(A).

$C_{2L}^R = 0, C_{3C}^R = 0, C_Y^R = \frac{6}{5}$

For the G_{2213} group $i, j = 2L, 2R, 1B-L, 3C$

$a_{2L} = -3, a_{2R} = \frac{-7}{3}, a_{B-L} = \frac{11}{2}, a_{3C} = -7$

$C^8_{2L} = 30, C^8_{3L} = 13, C^8_{(B-L)_L} = 16, C^8_{(B-L)_R} = 4,\ C^8_{3R} = 1, C^8_{2R} = 0$

$$B_{ij} = \begin{pmatrix} -\frac{37}{8} & -3 & -\frac{12}{7} & \frac{3}{11} \\ \frac{-9}{4} & -\frac{31}{2} & \frac{-12}{7} & \frac{27}{11} \\ -\frac{27}{16} & -\frac{9}{4} & \frac{26}{7} & \frac{1}{11} \\ -\frac{27}{16} & -\frac{729}{4} & -\frac{4}{7} & \frac{61}{11} \end{pmatrix}$$

For $\mu \geq M_8$, the new coefficients are given below for the models(A) and (B). In each chain at first the formulas for $sin^2\theta_W(M_Z)$ and $\frac{\alpha(M_Z)}{\alpha_S(M_Z)}$ are obtained in terms of mass scales (M_U, M_C) or (M_U, M_R) is expressed analytically in terms of $sin^2\theta_W(M_Z), \alpha(M_Z), \alpha_S(M_Z)$ and $ln\frac{M_8}{M_Z}$.

Model (A)

For the G_{824} group $i, j = 8L, 2R, 4R$

$a_{8L} = -14, a_{2R} = \frac{26}{3}, a_{4R} = 7$

$C^U_{8L} = 0, C^U_{2R} = 30, C^U_{4R} = 12$

where the superscript U in C^U_i stands for the boundary at M_U.

$$B_{ij} = \begin{pmatrix} -\frac{281}{4} & \frac{1845}{104} & \frac{435}{7} \\ -45 & \frac{904}{13} & \frac{660}{7} \\ \frac{-225}{2} & \frac{198}{13} & 136 \end{pmatrix}$$

leading to

$$ln\frac{M_U}{M_Z}=\frac{\pi}{2035\alpha}\left(154\frac{\alpha}{\alpha_s}-114\sin^2\theta_W-15\right)$$

$$+\frac{2421}{2035}ln\frac{M_8}{M_Z}+\frac{1}{8140}\left(25P_Y^C+129P_{2L}^C-154P_{3C}^C+15P_{2R}^8-144P_{4C}^8+129P_{2L}^8\right)$$

$$+\frac{1}{8140}\left(15P_{2R}^U-144P_{4R}^U+228P_{8L}^U\right)-\frac{13}{407} \tag{4.3}$$

$$ln\frac{M_C}{M_Z}=\frac{2\pi}{2035\alpha}\left(315-792\frac{\alpha}{\alpha_s}-48\sin^2\theta_W\right)$$

$$-\frac{4816}{2035}ln\frac{M_8}{M_Z}-\frac{1}{4070}\left(525P_Y^C+267P_{2L}^C-792P_{3C}^C+315P_{2R}^8-582P_{4C}^8+267P_{2L}^8\right)$$

$$-\frac{1}{4070}\left(315P_{2R}^U-582P_{4R}^U-96P_{8L}^U\right)-\frac{81}{2035} \tag{4.4}$$

<u>$Model(B)$</u>

$a_{8L}=-14, a_{2R}=\frac{26}{3}, a_{4R}=7$

$C_{8L}^U, C_{2R}^U=30, C_{4R}^U$ are the same as in model(A).

$$B_{ij}=\begin{pmatrix} -\frac{281}{4} & \frac{1845}{104} & \frac{435}{7} \\ -45 & \frac{904}{13} & \frac{660}{7} \\ \frac{-225}{2} & \frac{198}{13} & 136 \end{pmatrix}$$

leading to

$$ln\frac{M_U}{M_Z}=\frac{\pi}{1688\alpha}\left(104\frac{\alpha}{\alpha_s}-96\sin^2\theta_W-3\right)$$

$$+\frac{3811}{3376}ln\frac{M_8}{M_Z}+\frac{1}{6752}\left(125P_Y^R+21P_{2L}^R-104P_{3C}^R+75P_{2R}^8-104P_{3C}^8-21P_{2L}^8\right)$$

$$+\frac{1}{6752}\left(50P_{BL}^8+3P_{2R}^U-102P_{4R}^U+192P_{8L}^U\right)+\frac{1035}{10128} \qquad (4.5)$$

$$ln\frac{M_R}{M_Z}=\frac{\pi}{844\alpha}\left(315-792\frac{\alpha}{\alpha_s}-48\sin^2\theta_W\right)$$

$$-\frac{5163}{1688}ln\frac{M_8}{M_Z}-\frac{1}{3376}\left(525P_Y^R+267P_{2L}^R-792P_{3C}^R+315P_{2R}^8-792P_{3C}^8+267P_{2L}^8\right)$$

$$-\frac{1}{3376}\left(210P_{BL}^8+315P_{2R}^U-582P_{4R}^U-96P_{8L}^U\right)+\frac{703}{1688} \qquad (4.4)$$

In (4.3) - (4.6) we use the notation $P_i^b = P_i(M_a, M_b)$ between two successible mass scales $M_b > M_a$.

In models with two intermediate scales one of the mass scales (M_8) is not determined in terms of the input parameters, but the consistency with the unification scheme is examined by imposing $M_U > M_8 > M_C$ or M_R. In addition to estimating values of mass scales, one important advantage of the formulas (4.3) - (4.6) is that uncertainties in M_U and M_C or M_R arising out of the input parameters of equation (4.1) are deriveable in a straight - forward manner using the dominant one-loop contributions.

The improved method of solutions for the mass scales consists in exploiting the analytic formulas as well as the renormalisation group equations (RGEs), together. For example, in Model (A), the first step begins by dropping the two-loop contributions in (4.3) and (4.4) and obtaining an approximate set of values for (M_U, M_8, M_C). This set is used in (4.2a) alongwith (4.2b) to obtain $\alpha_i^{-1}(\mu)$ as a function of μ by iterative convergence procedure. The values of $\alpha_i(M_U), \alpha_i(M_8)$ and $\alpha_i(M_C)$ thus obtained are used to calculate all P_i-functions and the two-loop contributions in (4.3) and (4.4) resulting in improved values of M_U and M_C in the beginning of the second step while M_8 remains fixed throughout. The second step and subsequent steps are repeated until the values of the mass scales and $\alpha_i^{-1}(\mu)$ obtained in two successive steps converge. The process is repeated for another set of (M_U, M_8, M_C). The values of $\alpha_i(\mu)$ and mass scales of model (B) are obtained in the same manner as model (A). For every set of solutions in both the chains, the ratio σ is obtained using

$$\sigma = \frac{\alpha_{2L}(M_I)}{\alpha_{2R}(M_I)}, M_I = M_C or M_R \tag{4.7}$$

As noted in Sec.I, $\sigma \simeq 1$ in the conventional methods of left-right symmetry breaking, where the scales of parity and $SU(2)_R-$ breakings are identical. A substantial deviation of σ from unity signifies left-right assymetry $(g_{2L} \neq g_{2R})$ in G_{224} or G_{2213} and the possibility of a lower $SU(2)_R$- breaking scale. In the present model $g_{2L}(\mu)$ receives renormalisation from the full $SU(8)_L$ gauge group as μ decreases from M_U to M_8, whereas g_{2R} is renormalised by $SU(2)_R \times SU(4)_R^C$. For values of $\mu < M_8$, the assymetry in the Higgs sector also increases the difference between g_{2L} and g_{2R}. These lead to

Table IV.1 Predictions on mass scales and the ratio of gauge coupling constants σ in model(A) where the uncertainty factor in $M_U(M_C)$ is $10^{\pm 0.03}(10^{\pm 0.34})$ due to the input parameters.α_G is the $SU(8)_L \times SU(8)_R$ GUT coupling constant.

M_C(GeV)	M_8(GeV)	M_U(GeV)	σ	α_G^{-1}
$1.3X10^6$	$3.0X10^{18}$	$8.0X10^{18}$	2.3	13.4
$2.7X10^8$	$3.2X10^{17}$	$5.5X10^{17}$	2.25	12.7
$2.5X10^{11}$	$1.8X10^{16}$	$1.8X10^{16}$	1.8	11.4

Table IV.2 Same as Table IV.I but for model(B) where the uncertainty factor in $M_U(M_R)$ is $10^{\pm 0.03}(10^{\pm 0.4})$.

M_R(GeV)	M_8(GeV)	M_U(GeV)	σ	α_G^{-1}
$6.0X10^2$	$5.5X10^{18}$	$9.6X10^{18}$	1.24	13.0
$6.7X10^3$	$2.5X10^{18}$	$4.0X10^{18}$	1.22	12.5
$1.1X10^5$	$1.0X10^{18}$	$1.4X10^{18}$	1.26	12.5
$1.26X10^8$	$1.0X10^{17}$	$1.0X10^{17}$	1.28	11.6

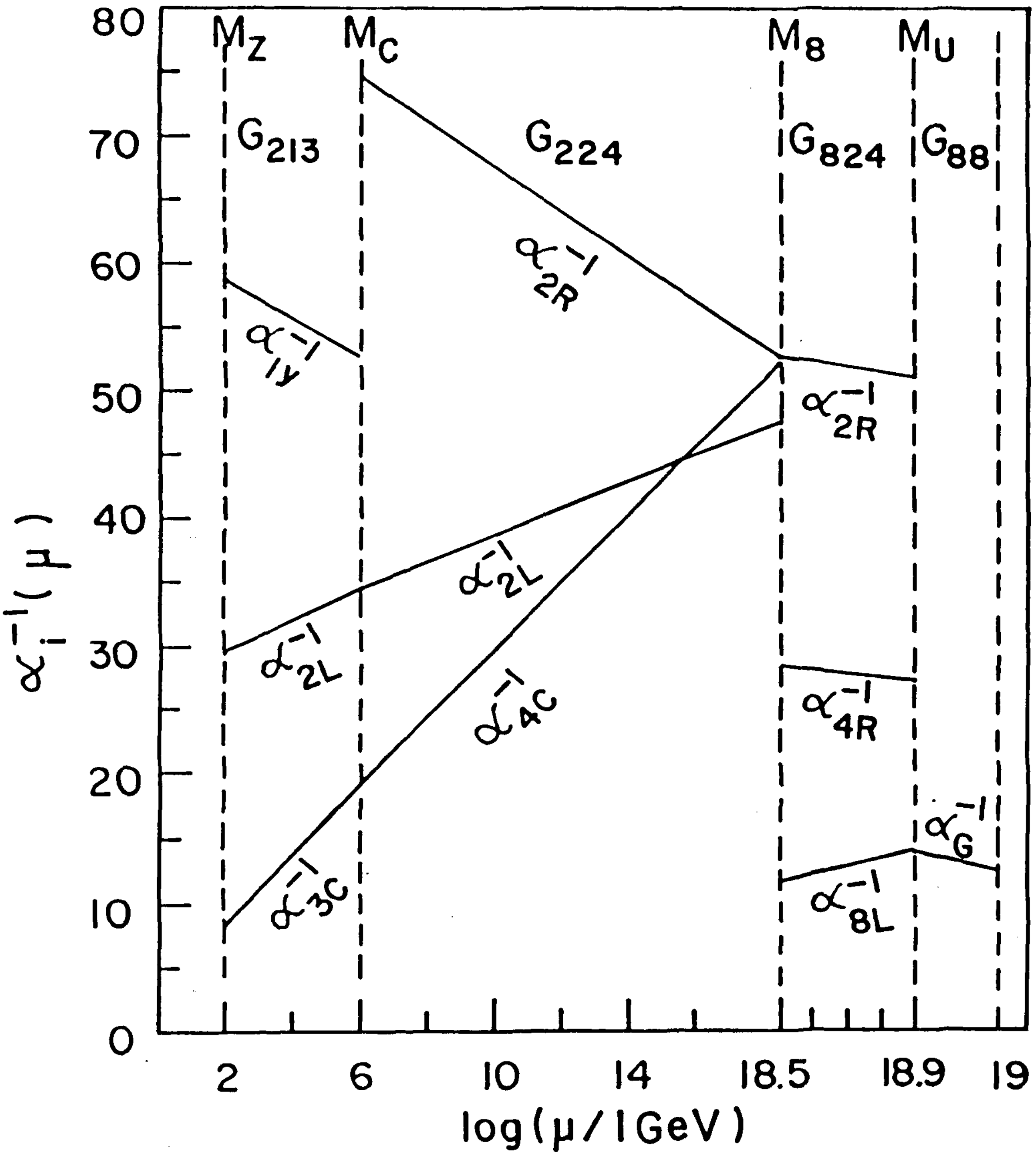

Fig.1. Coupling-constant trajectories $\alpha_i^{-1}(\mu)$ as a function of the mass scale μ in model A with two intermediate gauge groups. The scale for $\log\mu =$ 18.5 – 19 has been enlarged to guide the eye. α_G^{-1} shows the evolution of the GUT coupling constant.

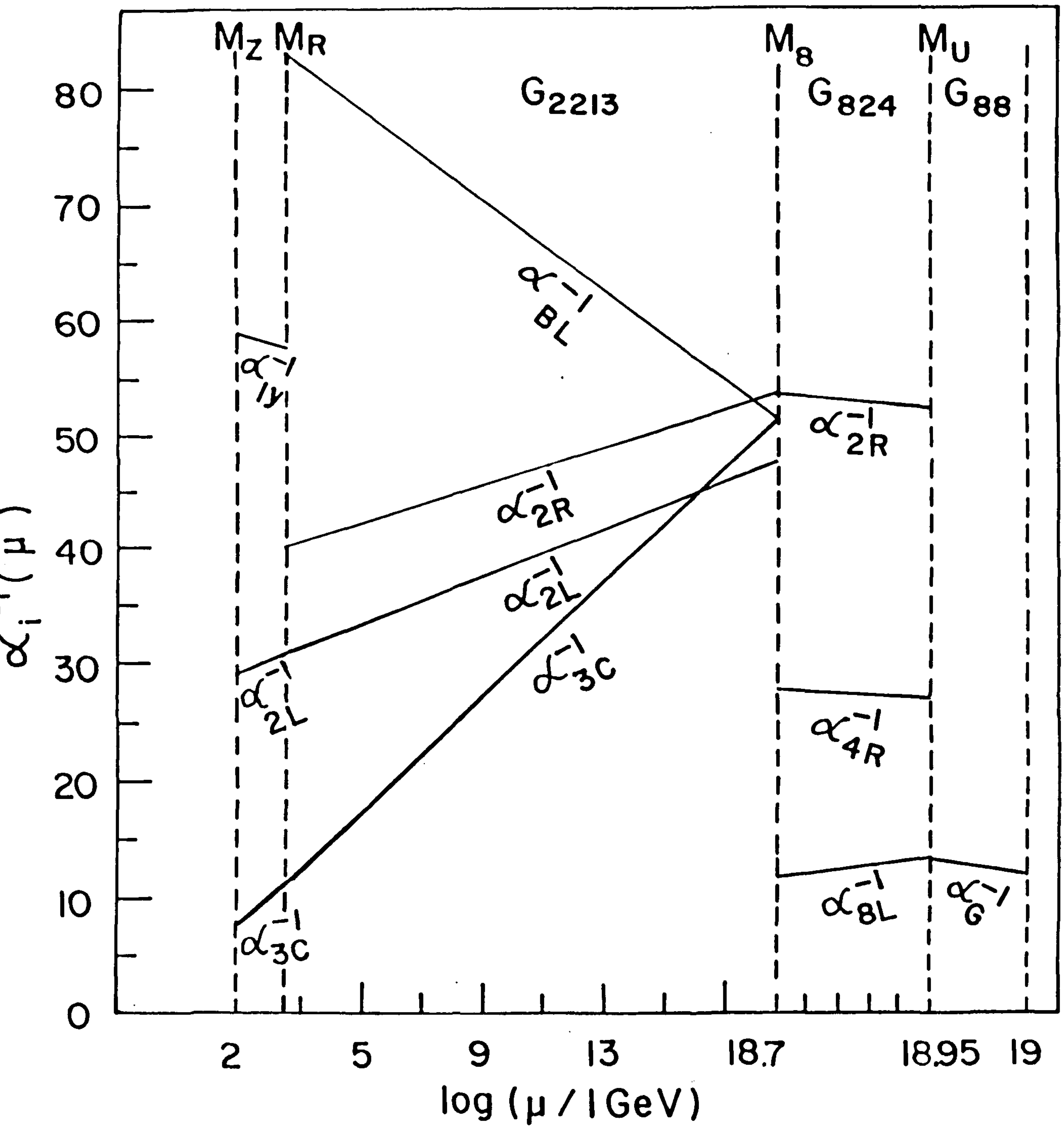

Fig.2. Same as Fig.1 but for model B.

values of $\sigma \simeq 1.8 - 2.3$ as shown in Table IV.1 and it is this assymetry which allows G_{224}-breaking scale $M_C \ll M_P = M_U$.

The solutions for mass scales with the corresponding σ values are presented in Tables IV.1 and IV.2 for models (A) and (B), respectively. The unification of the gauge couplings in $SU(8)_L \times SU(8)_R$ in the cases of the two models are presented in Figs.1 and 2 where α_G represents the GUT coupling constant.

Compared to $SO(10)$, where the lowest value of G_{2213}- breaking scale is achieved $M_R = 10^7$ GeV with two intermediate gauge symmetries, the alternate mechanism in the present case achieves much lower values of $M_R = 10^{2.8 \pm 0.4}$GeV in model (B). The low mass $W_R^{\pm}$ and Z_R gauge bosons and the Higgs scalars Δ_R^0, Δ_R^+ and Δ_R^{++} associated with this scale lead to experimentally detectable V+A currents in neutrinoless double beta decay, muonium-antimuonium transitions, $K_L - K_S$ mass difference, electric dipole moments of the neutron and electron 4-5 orders larger than the standard model and neutrino mass spectrum of the eV-KeV-MeV type [15]. Such low-mass right-handed guage bosons can be detected at future accelerator energies. So far this appears to be the lowest value of M_R predicted in the context of any nonSUSY GUT consistent with the CERN-LEP data and minimal finetuning of parameters.

IV.3 PREDICTIONS ON RARE DECAYS, $n - n$ AND $H - H$ OSCILLATIONS.

The analytic expression for the branching ratios in $K_L \longrightarrow \mu^{\pm} e^{\mp}$ is given by [15],

$$B\left(K_L \longrightarrow \mu^{\pm} e^{\mp}\right) = \frac{\Gamma\left(K_L \longrightarrow \mu^{\pm} e^{\mp}\right)}{\Gamma(K^{+} \longrightarrow \mu^{+} \nu_{\mu})}$$

$$= \frac{4\pi^2 m_k^4 \alpha_s^2(M_c) R}{G_F^2 \sin^2 \theta_C m_\mu^2 (m_s + m_d)^2 M_c^4}$$

$$= 6X10^{13}.R.\frac{\alpha_s^2(M_c)}{M_C (GeV)^4}$$

where

$$R = \left(\frac{\alpha_s(\mu = 1GeV)}{\alpha_s(m_b)}\right)^{24/25} \left(\frac{\alpha_s(m_b)}{\alpha_s(m_t)}\right)^{24/23} \left(\frac{\alpha_s(m_t)}{\alpha_s(M_c)}\right)^{24/21} \tag{4.8}$$

Here m_K is the K^o- meson mass ; m_μ, m_s and m_d are the masses of μ, s-quark and d-quark, respectively, at low energies ($\mu = 1GeV$); R is the renormalisation factor for the quark masses, $\sin\theta_c \simeq 0.22, G_F = 1.166 \times 10^{-5}\text{GeV}^{-2}$ and $\alpha_S(\mu)$ is the $SU(3)_C$-coupling at μ. We use $m_t = 175.6 \pm 5.5\text{GeV}$ [13].

The processes $n - \bar{n}$ and $H - \bar{H}$ oscillations [15-17] are dominated by the exchanges of the G_{213}-submultiplets of Higgs -scalars contained in the G_{224}-multiplet $\Delta_R(1,3,10) \subset (1,36)$ of $SU(8)_L \times SU(8)_R$, or $\underline{1}36$ of $SU(16)$. Following the analogy of the standard-model-Higgs-boson whose mass, theoretically, could be a factor $10(1/10)$ above (below) M_W all the components of $\Delta_R(1,3,10)$ mediating $n-\bar{n}$ and $H-\bar{H}$ oscillations are now of the order $10^{\pm 1}M_C$. We take the masses to be $10^{\pm \log\beta}M_C$ with $\beta = 1-10$. The amplitude for the two processes are given by,

$$G_{n-\bar{n}} = \frac{\lambda h_\Delta^3 \langle \Delta_R^0 \rangle}{M_\Delta^6}$$

$$G_{H-\bar{H}} = \frac{\lambda h_\Delta^4}{M_\Delta^8}$$

Here $\lambda(h_\Delta)$ is the Higgs-quartic (Higgs-Yukawa) coupling of Δ_R and all the Higgs scalar masses exchanged are assumed to be degenerate. These lead to the cannonical values of mixing times,

$$\tau_{n-\bar{n}}(secs) = \frac{0.954\times 10^{-21}}{\lambda h_\Delta^3}\frac{M_\Delta^6}{\langle \Delta_R^0 \rangle}$$

$$\tau_{H-\bar{H}}(yrs) = 3.5\times 10^{-19}\frac{M_\Delta^8}{\lambda h_\Delta^4} \tag{4.9}$$

In eqn.(4.9) all Higgs scalar masses and the VEV $\langle \Delta_R^0 \rangle$ are in GeV. The mechanism showing rare kaon decays and $n-\bar{n}$ oscillations are represented schematacially in Fig. 3 and Fig.4 respectively.

Using $m_t = 175.6$GeV and the lowest allowed value of $M_C = 10^{6\pm0.34}$GeV in model(A) in equations (4.8) and (4.9), the predictions for the branching ratios in $K_L \longrightarrow \mu^\pm e^\mp$ and the mixing times for $n-\bar{n}$ and $H-\bar{H}$ oscillations

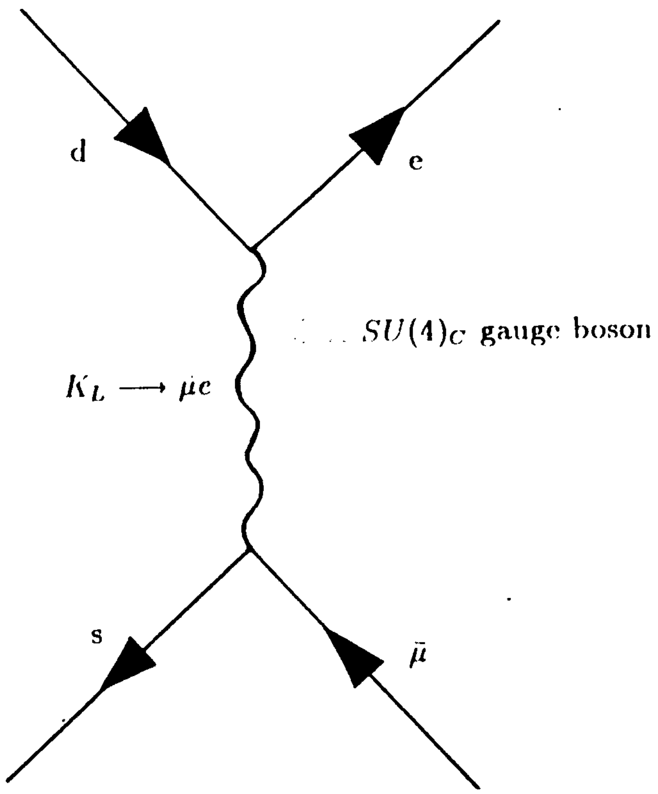

Fig.3. Mechanism showing $K_L \longrightarrow \mu e$ mediated by $SU(4)_C$ gauge bosons

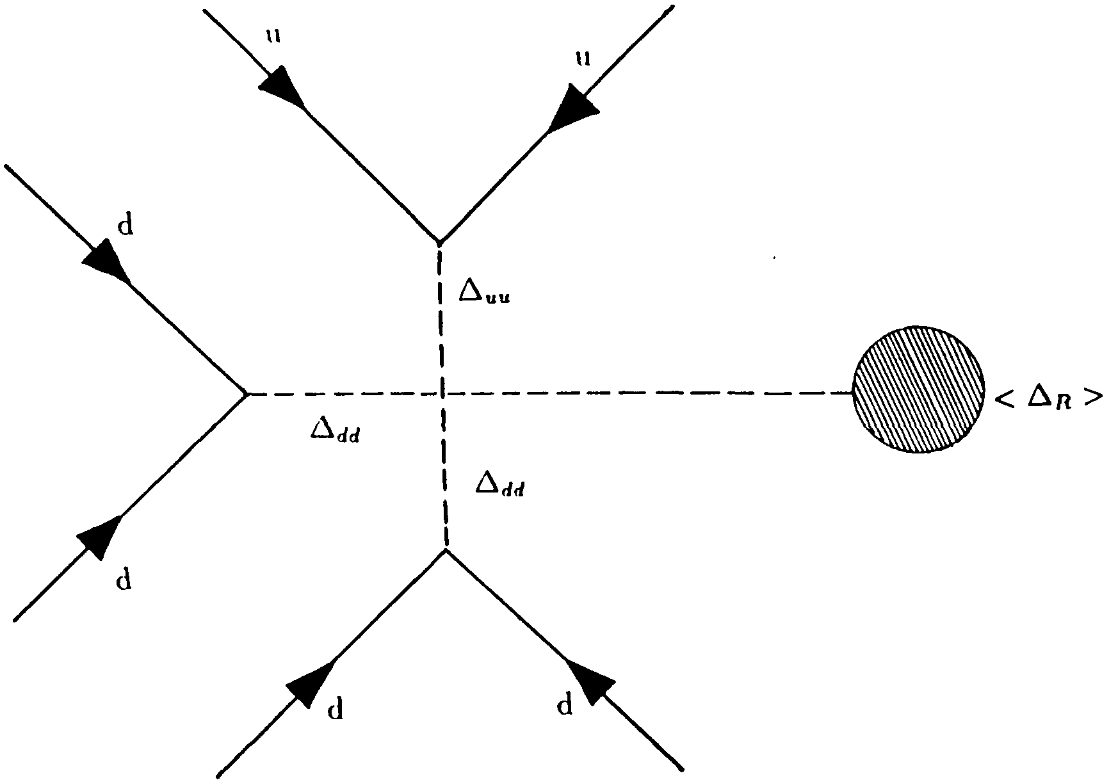

Fig.4. The tree graph that induces the six-fermion $\Delta B=2$ vertex leading to $n\bar{n}$ oscillation.

are

$$B(K_L^0 \longrightarrow \mu^{\pm}e^{\mp}) = 10^{-11.2\pm1.36}$$

$$\tau_{n-\bar{n}} = 10^{8.9\pm1.7\pm5log\beta}secs$$

$$\tau_{H-\bar{H}} = 10^{29.5\pm2.72\pm8log\beta}yrs \tag{4.10}$$

The ongoing experiments have already probed into the limit of the branching ratio [18],

$$B(K_L^0 \longrightarrow \mu^{\pm}e^{\mp})_{expmt} \leq 10^{-10.4} \tag{4.11}$$

In (4.10) the first (second) uncertainties are due to the input parameters (Higgs scalar masses). It is clear that the central value of the branching ratio predicted is nearly one order less than the current experimental limit, but the uncertainty in the input parameters alone makes it consistent with the limit.The second uncertainty widens the ranges of the predicted values which can be readily computed from (4.10). For example, with $\beta = 5$, $(\tau_{n-n}) = 10^{14.1}secs$ and $(\tau_{H-H})_{max} = 10^{37.8}yrs$. This is for the first time that uncertainties of both types are taken into account in Higgs mediated processes. The gauge boson mediated rare decay is affected by the uncertainty in M_C only.

IV.4 $\Delta(B-L)=-2$ PROTON DECAY

The unification mass in both the chains (A) and (B) are high, $M_U \geq 10^{18}$ GeV. Although in $SU(8)_L \times SU(8)_R$ model, there is no gauge-boson-mediated proton decay corresponding to $\Delta(B-L)=0$. But Higgs mediated proton decay corresponding to $\Delta(B-L)=-2, \Delta(B+L)=0$ and $\Delta F=-2$ (F = Fermion number) could be experimentally detectable provided the colour triplet (ξ_3) and colour octet (ξ_8) components of $\xi(2,2,15) \subset 126$ of $SO(10)$ are light ($m_{\xi_3} \simeq m_{\xi_8} \simeq 10^2 GeV$) [6]. An attractive feature of the new model A' is that the vacuum expectation value (VEV) of the neutral colour-singlet (ξ_0) component of $\xi(2,2,15)$ combined with the standard Higgs scalar VEV cures the bad fermion mass relation in $SO(10)$. Since ξ_3 and ξ_8 acquire masses of the order of $M_c \simeq 10^{13}-10^{14} GeV$ by extended survival hypothesis [16], a special mechanism has been devised in $SO(10)$ [6] in the presence of the Higgs representation $\underline{945} \subset SO(10)$ to keep these masses light. With the addition of $\xi(2,2,15) \subset (8,\bar{8})$ to model A in $SU(8)_L \times SU(8)_R$ or $SU(16)$ the masses of ξ_3 and ξ_8 in model A' can be easily of the order 10^5-10^6 GeV naturally [16] without requiring the special mechanism. With the colour-singlet component in $\xi(2,2,15)$, transforming as $\xi^0(2,1,1)$ under G_{213}, acquiring vaccum expectation value $\simeq 10^2$GeV, the loop coefficients for model A' are

$\underline{M_Z \leq \mu \leq M_C}$

$a_{2L} = -3, a_{1Y} = \frac{21}{5}, a_{3C} = -7$

$$B_{ij} = \begin{pmatrix} \frac{-11}{9} & \frac{2}{7} & \frac{-12}{7} \\ \frac{-6}{5} & \frac{104}{105} & \frac{-44}{35} \\ \frac{-3}{2} & \frac{11}{42} & \frac{26}{7} \end{pmatrix}$$

In addition to the SM particle content, the model has the contribution due to one extra $SU(2)_L$ Higgs doublet transforming as $\xi^0(2,1,1)$ under G_{213} for $\mu = M_Z - M_C$.

$\underline{M_C \leq \mu \leq M_8}$

$a_{2L} = 2, a_{2R} = \frac{26}{3}, a_{4C} = \frac{-7}{3}$

$$B_{ij} = \begin{pmatrix} \frac{73}{2} & \frac{3609}{676} & \frac{-27585}{238} \\ 24 & \frac{779}{26} & \frac{-3735}{14} \\ \frac{105}{4} & \frac{747}{52} & \frac{-2435}{14} \end{pmatrix}$$

In this mass range, in addition to the Higgs triplet $\Delta_R(1,3,\bar{10})$, the full multiplet $\xi(2,2,15)$ also contributes to the one and two-loop beta functions.

$\underline{M_8 \leq \mu \leq M_U}$

$a_{8L} = -\frac{38}{3}, a_{2R} = 14, a_{4R} = \frac{29}{3}$

$$B_{ij} = \begin{pmatrix} \frac{-6407}{76} & \frac{663}{56} & \frac{1395}{29} \\ \frac{-1323}{19} & 48 & \frac{2340}{29} \\ \frac{-5103}{38} & \frac{78}{7} & \frac{3100}{29} \end{pmatrix}$$

The formula for mass scales are

$$ln\frac{M_U}{M_Z} = \frac{\pi}{1163\alpha}\left(74\frac{\alpha}{\alpha_s} - 66\sin^2\theta_W - 3\right)$$

$$+\frac{1329}{1163}ln\frac{M_8}{M_Z} + \frac{1}{4652}\left(5P_Y^C + 69P_{2L}^C - 74P_{3C}^C + 3P_{2R}^8 + 74P_{2L}^8 - 72P_{4C}^8\right)$$

$$+\frac{1}{4652}\left(3P_{2R}^U - 72P_{4R}^U + 132P_{8L}^U\right) - \frac{249}{6978} \qquad (4.13)$$

$$ln\frac{M_C}{M_Z} = \frac{\pi}{1163\alpha}\left(315 - 792\frac{\alpha}{\alpha_s} - 48\sin^2\theta_W\right)$$

$$-\frac{2405}{1163}ln\frac{M_8}{M_Z} - \frac{1}{4652}\left(525P_Y^C + 267P_{2L}^C - 792P_{3C}^C + 315P_{2R}^8 + 267P_{2L}^8 - 582P_{4C}^8\right)$$

$$-\frac{1}{4652}\left(315P_{2R}^U - 582P_{4R}^U - 96P_{8L}^U\right) + \frac{2085}{2326} \qquad (4.14)$$

Following the procedure already described, we compute values of mass scales, $\alpha_i^{-1}(\mu)$ and presented in Table IV.3. The coupling-constant trajectories are shown in Fig.5 for the lowest allowed value of M_C in the model $A', M_C = 10^{5.64\pm0.3}GeV$

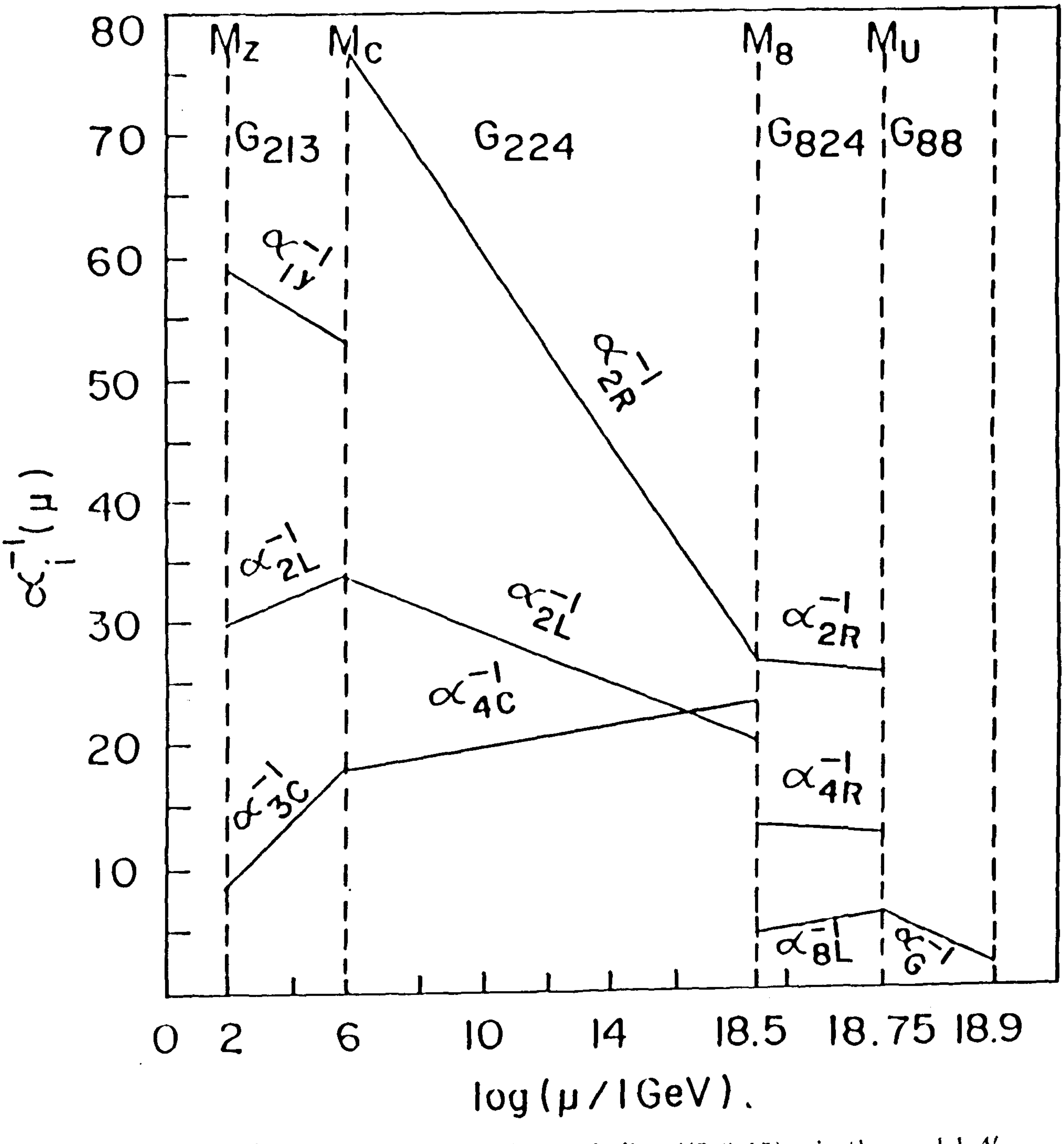

Fig.5. Same as Fig.1. but including $\xi(2,2,15)$as in the model A'.

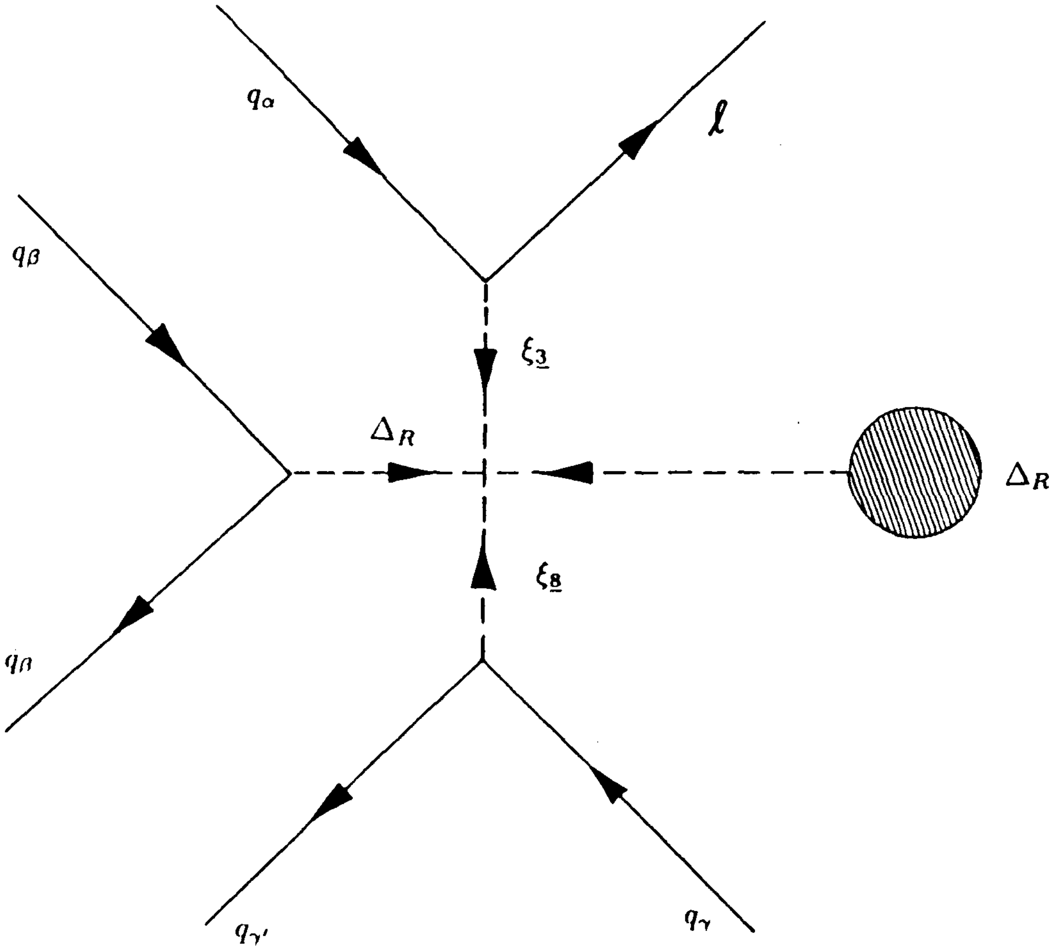

Fig.6. Mechanism for $3q \longrightarrow lq\bar{q}$

It is necessary to emphasize that the use of VEV of the neutral component in $\xi(2,2,15)$ at the electroweak scale is not needed purely from the ground of spontaneous symmetry breaking (SSB) as the SM doublet contained in the bidoublet $\phi(2,2,1) \subset (8,\bar{8})$ of $SU(8)_L \times SU(8)_R$ derives the SSB of the standard gauge symmetry. Rather this is motivated to cure the bad fermion mass relation in the minimal model. But once $\xi(2,2,15)$ is used to improve the fermion mass relation, all other components in $\xi(2,2,15)$ acquire mass of the order of the G_{224} breaking scale by extended survival hypothesis. Thus it is more natural than the SO(10) model [6], to have $M_{\xi_3} \simeq M_{\xi_8} \simeq M_{\Delta_R} \simeq M_C$ in the present model (A') Taking the masses of ξ_3 and ξ_8 and the diquark-Higgs scalars in $\Delta_R(1,3,\bar{10})$ in the allowed range of $10^{\pm log\beta} M_C$ and denoting the Higgs-Yukawa couplings of $\Delta_R(1,3,10)$ and $\xi(2,2,15)$ as h_Δ and h_ξ, respectively, with λ as the Higgs-quartric coupling, the amplitude for $3q \longrightarrow lq\bar{q}$ has the canonical strength

$$A(3q \longrightarrow lq\bar{q}) = \frac{\lambda h_\Delta h_\xi^2 \langle \Delta_R^0 \rangle}{M_\xi^4 M_\Delta^2}$$

$$= 10^{-35.5 \pm 1.5 \pm 6 log\beta} \mathrm{GeV}^{-5}$$

where we have assumed $M_{\xi_3} = M_{\xi_8} = M_\xi = M_C$, and $g_{2R}(M_C)\langle \Delta_R^0 \rangle = M_C, \lambda = h_\Delta = 1, h_\xi = 10^{-4}$ for the first generation of quarks.Mechanism for $3q \rightarrow lq\bar{q}$ is shown in Fig.6. This leads to the lifetime predictions

$$\tau_p = 10^{37.6\pm3.0\pm12 log\beta} yrs \tag{4.17}$$

in the Higgs mediated nucleon decay modes :
$n \longrightarrow e^- + (\pi^+\pi), p \longrightarrow \nu + (e^+\nu), n \longrightarrow (e^- or \mu^-) + (e^+\nu), n \longrightarrow (e^- or \mu^-) + e^+ e + (\pi^+ or K^+) etc,$

In (4.16)-(4.17), the second (first) uncertainty is due to those in the Higgs scalar masses (input parameters). We emphasize that even if the central value of τ_P is 4-5 orders larger, the uncertainty in the input parameters along with Higgs-scalar masses brings the lifetime well within the accessible range of superkamiokande measurements. However, because of large uncertainty in the predicted value of the Higgs-scalar masses it is difficult to rule out the model on the basis of proton lifetime measurements.

The prediction for rare decays, $n - \bar{n}$ and $H - \bar{H}$ oscillations are [15-17],

$$B(K_L^0 \longrightarrow \mu^\pm e^\mp) = 10^{-10.42\pm1.2}$$

$$\tau_{n-\bar{n}} = 10^{7.2\pm1.5\pm5 log\beta} secs$$

$$\tau_{H-\bar{H}} = 10^{26.7\pm2.4\pm8 log\beta} yrs \tag{4.18}$$

The see-saw formula [20] including radiative corrections [21-23], is given by,

$$m_{\nu_i} = C_{\nu_i}\frac{m_{q_i}^2}{M_C}, i = 1, 2, 3$$

where

Table IV.3 Same as model(A) but including $\xi(2,2,15)$in case of model(A') where the uncertainty factor in $M_U(M_R)$ is $10^{\pm 0.03}(10^{\pm 0.3})$.

M_R(GeV)	M_8(GeV)	M_U(GeV)	σ	α_G^{-1}
$4.4X10^5$	$3.0X10^{18}$	$6.3X10^{18}$	2.35	6.4
$1.8X10^7$	$5.0X10^{17}$	$8.2X10^{17}$	2.21	7.0
$1.7X10^{10}$	$1.8X10^{16}$	$1.8X10^{16}$	1.85	8.2

$m_{q_1} = m_u, m_{q_2} = m_c, m_{q_3} = m_t, C_{\nu_1} = 0.03, C_{\nu_2} = 0.03, C_{\nu_3} = 0.22$. It is to be noted that radiative corrections including the effect of the top-quark Yukawa coupling has been carried out in ref[23] in nonSUSY models and also in SUSYGUTs [24].Using the see-saw formula and $M_C = 10^{5.64}$GeV the predictions for Majorana neutrino masses are

$$m_{\nu_e} = 10^{-3\pm0.3}\text{eV}$$

$$m_{\nu_\mu} = 10^{2.2\pm0.3}\text{eV}$$

$$m_{\nu_\tau} = 10^{1.2\pm0.3}\text{MeV}$$

As in model A, the cosmological bound on neutrino masses requires that at least ν_τ is made unstable, for example, by the introduction of an additional global lepton number symmetry[19]. The uncertainty in the predicted values increase with values of β. For example with $\beta = 5, (\tau_{n-\bar{n}})_{max} = 10^{12.2} secs, (\tau_{H-\bar{H}})_{max} = 10^{34.7} yrs, (\tau_p)_{max} = 10^{48.99} yrs$. Thus although consistency of the model predictions can be checked if such Higgs mediated processes are found experimentally, it is extremely difficult to rule out such models in near future.

IV.5 SUMMARY AND CONCLUSION

We have examined the predictions of $SU(16)$ and $SU(8)_L \times SU(8)_R$ GUTs under the constraints of the CERN-LEP data using the alternative method of decoupling parity and $SU(2)_R$- breakings [10] in the minimal chains. We find that right-handed gauge boson masses as low as $10^{2.8\pm0.4} GeV$ are allowed in one of the models whereas in $SO(10)$ models with two intermediate symmetries they have high values, $M_R \geq 10^7 GeV$. The present model appears to predict the lowest value of $W_R^{\pm}$ and Z_R gauge bosons among all the non SUSY GUTs, investigated so far, consistent with the CERN-LEP data and minimal finetuning of parameters. Besides manifesting in a number of physical processes such as neutrinoless double beta decay, muonium -antimuonium transitions and contributing to $K_L - K_S$ mass difference, electric dipole moments of the electron and the neutron, and Majorana neutrino mass spectrum of the type eV-KeV-MeV for the three generations, such low mass $W_R^{\pm}$ and Z_R bosons could be detected at accelerator energies.

The symmetry breaking scale for $SU(2)_L \times SU(2)_R \times SU(4)_C$ gauge symmetry is permitted to be $M_C \simeq 10^{6\pm0.3} GeV$ in the minimal Model (Λ) leading to experimentally testable predictions on rare kaon decays, $n - \bar{n}$ and $H - \bar{H}$ oscillations. Including the Higgs-scalar multiplet $\xi(2,2,15)$ as in Model (A'), cures the problem of bad fermion mass relation ; at the same time all Higgs-scalar mediating $\Delta(B - L) = -2$ proton decay naturally acquire the lighter masses $M_C \simeq 10^{5.6\pm0.3} GeV$ without the necessity of invoking a special mechanism as in $SO(10)$ model.[6]. Although the central value of the proton lifetime is noted to be 4-5 orders larger than the current experimentally accessible limit, the uncertainty in the input values of $\alpha_S(M_Z), sin^2\theta_W(M_Z)$

and $\alpha^{-1}(M_Z)$ along with Higgs scalar masses makes these predictions consistent with the limit . However, when uncertainties due to the Higgs boson masses mediating the proton decay and matter-antimatter oscillations are taken into account, the predicted values cover a wider range. Although the relevant experiments might test the consistency of the models, it is extremely difficult to rule out them on the basis of matter-antimatter oscillations or Higgs mediated proton decay experiments.

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CHAPTER 5

CHAPTER V

$SO(10)$ WITH $SU(2)_L \times SU(2)_R \times SU(4)_C$
GAUGE SYMMETRY

V.1 INTRODUCTION

Grand unified theories based upon SUSY SU(5), SO(10), and non SUSY SO(10) with intermediate symmetries and those inspired by superstrings have been the subject of considerable interest over recent years. One of the major motivations in following SUSY SO(10) grand unified theory is its potentiality to explain fermion masses and mixings [1] and, in particular, neutrino masses over a wide range of values via simple see saw mechanism [2], or with specific textures in mass matrices [3]. The observed cosmological baryon assymetry of the universe can be also explained by triggering baryogenesis via leptogenesis, if right-handed Majorana neutrinos are superheavy [4]. Apart from the interesting possibility that a massive ν_τ ($m_{\nu_\tau} \simeq 2 - 10eV$) is a promising candidate for hot dark matter of the universe, experimental hints on solar neutrino deficit could be explained through matter enhanced MSW effects[5] via see-saw prediction of left-handed neutrino masses provided, the right -handed neutrinos have masses in the range of $M_N \simeq 10^{10} - 10^{13}$ Gev [6]. This might be realised in single step breaking of SUSY SO(10) if the Yukawa coupling of $\underline{126}$ to matter spinors is adjusted to be small, or via dim-4 nonrenormalisable couplings between matter multiplets and Higgs fields belonging to $\underline{16}$. Prospects of solar neutrino oscillation in supergrand desert model with right-handed Majorana neutrino masses at intermediate scales have been discussed in ref [6]. But the most attractive possibility is to relate M_N to an intermediate scale (M_I) corresponding to the spontaneous symmetry breaking of the intermediate gauge group such as $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C (\equiv G_{2213})$ or $SU(2)_L \times SU(2)_R \times SU(4)_C (\equiv G_{224})$ [7] without having the necessity to adjust the Majorana type ν_R - Yukawa coupling to very small values. Such an intermediate scale also solves the strong CP problem by Pecci -Quinn mechanism [8]. Recently, although the existence of G_{2213} intermediate gauge symmetry with decoupled parity and $SU(2)_R$ - breaking ($g_{2L} \neq g_{2R}$) [9] has been established in a series of papers [10-12], the intermediate gauge symmetry G_{224} ($g_{2L} \neq g_{2R}$) has been ruled out [11] when the neutral component of the G_{224} submultiplet $\Delta_R(1,3,\overline{10}) \subset \underline{126}$ of $SO(10)$ is used to break the intermediate gauge symmetry [11]. This is due to the fact that the contributions of $\Delta_R \oplus \bar{\Delta}_R \subset \underline{126} \oplus \underline{\overline{126}}$ upsets the solutions of RGEs making it impossible to achieve $M_I \ll M_U$. More recently it has been shown that G_{224} intermediate symmetry can be achieved if two sets of $\underline{16} + \underline{\overline{16}}$ are used, instead of a set of

$\underline{126} + \overline{126}$,in the presence of an additional lighter G_{224} submultiplet $\sigma(1,1,6)$ near the intermediate scale [19].The role of the $\sigma(1,1,6)$ has been noted to remove other lighter degrees of freedom from the model[19]. But we emphasize here that the solutions of ref.[19] are valid only at one-loop level; when two-loop contributions are included the intermediate scale (M_I) exceeds the GUT scale (M_U), thus ruling out the model at two-loop level.However it has been shown that the $G_{224P}(g_{2L} = g_{2R})$ intermediate gauge symmetry with unbroken left-right discrete symmetry ($\equiv Parity(P)$) can survive down to the intermediate scale of $M_I \simeq 10^{12} - 10^{13}$ GeV provided the model permit light Higgs supermultiplet near the TeV scale [13]. In this model the part of two-loop corrections from higher scales ($\mu > M_I$) cancels out. G_{224} is the maximal subgroup of SO(10) which contains the quark- lepton unification of Pati-Salam and has one gauge-coupling constant less as compared to G_{2213}. All the gauge couplings of G_{224} are determined through the CERN-LEP data and the intermediate -scale matching conditions,

$$\alpha_{4C}(M_I) = \alpha_{3C}(M_I)$$

$$\frac{1}{\alpha_Y(M_I)} = \frac{3}{5}\frac{1}{\alpha_{2R}(M_I)} + \frac{2}{5}\frac{1}{\alpha_{4C}(M_I)} \tag{5.1}$$

The see-saw formulas, for neutrino masses where up-quark masses appear instead of the Dirac -neutrino masses [2], emerge more naturally at the intermediate scale due to the presence of quark-lepton symmetry in G_{224}.

The purpose of this Chapter is to show how the $G_{224}(g_{2L} \neq g_{2R})$ intermediate guage symmetry, with parity broken at the GUT scale, is allowed to survive naturally down to the desired intermediate scale by the inclusion of two-loop [14] and threshold effects [15] in SUSY SO(10). In contrast to the model of ref.[19],which is ruled out at two-loop level, we find that this is the only G_{224} intermediate guage symmetry with $g_{2L} \neq g_{2R}$ that is permitted in SUSY $SO(10)$. To achieve $SO(10)$ breaking to G_{224}, we use the Higgs representation $\underline{54}$ in addition to $\underline{210}$ [16] and break G_{224} by $\underline{16} \oplus \overline{\underline{16}}$, instead of $\underline{126} \oplus \overline{\underline{126}}$, to avoid large one-loop contributions of the triplets in the latter upsetting solutions to RGEs.

In Section V.2 we derive analytic formulas for the mass scales. In Section V.3 threshold effects and solutions to mass scales are obtained (a) using effective mass parameters,(b) by assigning specific, plausible and reasonable values to the unknown masses of the superheavy scalars and their superpartners.A brief summary with conclusion is provided in Section V.4.

V.2 ANALYTIC FORMULAS FOR MASS SCALES

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In this section we derive analytic formulas for the unification mass M_U and the intermediate scale M_I including one-loop, two-loop[14] and threshold contributions[15]. We consider the following model using the mechanism of decoupling parity and $SU(2)_R$ - breakings [9],

$$SO(10) \times \mathrm{SUSY} \xrightarrow{M_U} G_{224} \times \mathrm{SUSY} \xrightarrow{M_I}$$

$$G_{213} \times \mathrm{SUSY} \xrightarrow{M_Z} U(1)_{em} \times SU(3)_C \tag{5.2}$$

where G_{213} is the standard gauge symmetry $SU(2)_L \times U(1)_Y \times SU(3)_C$. In the first step of (5.2), the combined effect of $\underline{54}$ and $\underline{210}$, containing G_{224} singlets, break D parity and SO(10) without breaking G_{224}. In the second step, we use (i) one (ii) two sets of $16 \oplus \underline{16}$. The right-handed doublets $(1,2,4) \oplus (1,2,\bar{4})$ contained in $(\underline{16} \oplus \overline{\underline{16}})$ are kept lighter having masses near M_I whereas the left-handed counterparts $(2,1,4) \oplus (2,1,\bar{4})$ acquire masses near M_U. In the third step of (5.2) we use a representation $\underline{10}$ containing the u- and d- type Higgs doublets to break the symmetry to $U(1)_{em} \times SU(3)_c$. The renormalisation group equations in the presence of the two gauge symmetries G_{213} and G_{224} can be written as

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_i(M_I)} + \frac{a_i}{2\pi} ln \frac{M_I}{M_Z} + \frac{1}{4\pi} P_i - \Delta_i; i = 1Y, 2L, 3C \tag{5.3}$$

$$\frac{1}{\alpha_i(M_I)} = \frac{1}{\alpha_i(M_U)} + \frac{a_i'}{2\pi} ln \frac{M_U}{M_I} + \frac{1}{4\pi} P_i' - \Delta_i'; i = 2L, 2R, 4C \tag{5.4}$$

where the second (third) terms in the R.H.S. of (5.3) -(5.4) represent one-loop (two-loop) contributions with

$$P_i = \sum_j B_{ij} ln \frac{\alpha_j(M_I)}{\alpha_j(M_Z)}$$

$$P'_i = \sum_j B'_{ij} ln \frac{\alpha_j(M_U)}{\alpha_j(M_I)}$$

$$B_{ij} = \frac{b_{ij}}{a_j}, B'_{ij} = \frac{b'_{ij}}{a'_j} \tag{5.5}$$

Here $a_j(b_{ij})$ and $a'_j(b'_{ij})$ are the one-loop (two-loop) β - function coefficients in the two mass ranges and their values are given below. The terms Δ_i and Δ'_i in the R.H.S. of (5.3)-(5.4) represent threshold effects at $\mu = M_Z, M_I$ and M_U with

$$\Delta_i = \Delta_i^Z + \Delta_i^I$$

The functions Δ_i^Z includes threshold effects at $\mu = M_Z$ due to the top quark-Yukawa coupling and masses of Higgs scalars and superpartners in SUSY standard model different from M_Z, but Δ_i^I represents, threshold effects due to the Higgs scalars and superpartners having masses near M_I. Δ'_i takes into account threshold effects due to Higgs scalars and their superpartners having masses near M_U. Such scalars are contained in $\underline{54}, \underline{210}, \underline{16} \oplus \overline{\underline{16}}$ and $\underline{10}$. Although one set of $\underline{16} \oplus \overline{\underline{16}} \subset SO(10)$ is sufficient to break the intermediate guage symmetry to the standard SUSY gauge theory, we also investigate the effects of two sets of such spinorial representations to achieve a desired one-loop solution. Expressions for Δ_i and Δ'_i are given in Sec V.3. Using suitable combinations of gauge couplings and equations (5.3) - (5.4), we obtain the following analytic formulaes for mass scales M_I and M_U,

$$ln(\frac{M_I}{M_Z}) = \frac{L_S A_U - L_\theta B_U}{D} + \frac{K_\theta B_U - J_\theta A_U}{D} + \frac{J_\Delta A_U - K_\Delta B_U}{D} \tag{5.6}$$

$$ln(\frac{M_U}{M_Z}) = \frac{L_\theta B_I - L_S A_I}{D} + \frac{J_\theta A_I - K_\theta B_I}{D} + \frac{K_\Delta B_I - J_\Delta A_I}{D} \tag{5.7}$$

where

$$\begin{aligned}
D &= A_U B_I - A_I B_U \\
L_\theta &= \frac{16\pi}{\alpha(M_Z)}\left(\frac{3}{8} - \sin^2\theta_W\right) \\
L_S &= \frac{16\pi}{\alpha(M_Z)}\left(\frac{3}{8} - \frac{\alpha(M_Z)}{\alpha_S(M_Z)}\right) \\
A_U &= 3a'_{2R} + 2a'_{4C} - 5a'_{2L} \\
A_I &= 5a_{1Y} - 5a_{2L} - 3a'_{2R} - 2a'_{4C} + 5a'_{2L} \\
B_U &= 3a'_{2R} + 3a'_{2L} - 6a'_{4C} \\
B_I &= 5a_{1Y} + 3a_{2L} - 8a_{3C} - 3a'_{2R} - 3a'_{2L} + 6a'_{4C} \\
J_\theta &= \frac{1}{2}(3P'_{2R} + 3P'_{2L} - 6P'_{4C} + 5P_{1Y} + 3P_{2L} - 8P_{3C}) \\
K_\theta &= \frac{1}{2}(3P'_{2R} + 2P'_{4C} - 5P'_{2L} + 5P_{1Y} - 5P_{2L}) \\
J_\Delta &= 2\pi(3\Delta'_{2R} + 3\Delta'_{2L} - 6\Delta'_{4C} + 5\Delta_{1Y} + 3\Delta_{2L} - 8\Delta_{3C})
\end{aligned} \tag{5.8}$$

$$K_\Delta = 2\pi\left(3\Delta'_{2R} - 5\Delta'_{2L} + 2\Delta'_{4C} + 5\Delta_{1Y} - 5\Delta_{2L}\right) \tag{5.9}$$

An attractive feature of the analytic formulas given in (5.6)-(5.7) is that, in the R.H.S. contributions due to every loop order or threshold effects are seperated out. For example, the first, second and the third terms in the R.H.S. of (5.6) - (5.7) represent, analytically one-loop, two-loop and threshold corrections, respectively.

The one-loop and two-loop β -function co-efficients for the MSSM[14] are

$$a_i = \begin{pmatrix} 1 \\ 33/5 \\ -3 \end{pmatrix}$$

$$B_{ij} = \frac{b_{ij}}{a_j} = \begin{pmatrix} 25 & 3/11 & -8 \\ 27/5 & 199/65 & -88/15 \\ 9 & 1/3 & -14/3 \end{pmatrix}, i,j = 2L, 1Y, 3C \tag{5.10}$$

In the presence of $G_{224} \times SUSY$ intermediate symmetry in the mass range $\mu = M_I - M_U$, we use the contributions from the Higgs scalars and their superpartners contained in the representations 10 and

i) one set of $\underline{16} \oplus \overline{\underline{16}}$ (i.e.$n_{16} = 1$)

(ii) two sets of $\underline{16} \oplus \overline{\underline{16}}$(i.e.$n_{16} = 2$). The components which have masses near M_I are the G_{224} -submultiplets $\phi(2,2,1)$ and for i) $n_{16} = 1$ one set of $(1,2,4)\oplus(1,2,\bar{4})$ (ii) $n_{16} = 2$, two sets of $(1,2,4)\oplus(1,2,\bar{4})$. Other components of the SO(10) representations such as $(1,1,6) \subset \underline{10}$ and $(2,1,4) \oplus (2,1,\bar{4}) \subset (\underline{16} \oplus \overline{\underline{16}})$ $2\{(2,1,4) \oplus (2,1,\bar{4})\} \subset 2(\underline{16} \oplus \overline{\underline{16}})$ have masses near M_U. Following the standard procedure and including contributions of gauge bosons, fermions, Higgs scalars and their superpartners, the one-loop and two-loop co-efficients for the G_{224} symmetry for both the cases are computed as

Case (i)$\underline{n_{16} = 1}$ (i.e. with one set of $\underline{16} \oplus \overline{16}$).

$$a'_i = \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$$

$$B'_{ij} = \frac{b'_{ij}}{a'_j} = \begin{pmatrix} 25 & 3/5 & -45/4 \\ 3 & 11/5 & -15/2 \\ 9 & 6/5 & 37/8 \end{pmatrix}, i,j = 2L, 2R, 4C \tag{5.10}$$

Using the values of the co-efficients from eqs. (5.10) - (5.11) in (5.8) - (5.9), we obtain

$$A_U = 2, A_I = 26, B_U = 42, B_I = 18, D = -1056 \tag{5.12}$$

Case (ii)$\underline{n_{16} = 2}$ (i.e. with two sets of $16 \oplus \overline{16}$).

$$a'_i = \begin{pmatrix} 1 \\ 9 \\ -2 \end{pmatrix}$$

$$B'_{ij} = \frac{b'_{ij}}{a'_j} = \begin{pmatrix} 25 & 1/3 & -45/2 \\ 3 & 13/3 & -30 \\ 9 & 4/3 & -25/4 \end{pmatrix}, i,j = 2L, 2R, 4C \tag{5.13}$$

Using the values of the co-efficients from eqs. (5.10) and (5.13) in (5.8) - (5.9) we obtain

$$A_U = 18, A_I = 10, B_U = 42, B_I = 18, D = -96. \tag{5.14}$$

In the next section we derive expressions for threshold effects and present solutions to the mass scales.

V.3. LOWERING THE INTERMEDIATE SCALE BY THRESHOLD EFFECTS.

Including only one-loop and two-loop contributions, the expressions for M_I and M_U are given by the first and the second terms, respectively in the R.H.S. of eqs. (5.6) - (5.7). Using eqs. (5.8) and (5.12) or (5.14), the one-loop and two - loop contributions and the mass scales are computed for $n_{16} = 1$ and $n_{16} = 2$ as,

$n_{16} = 1$

$$\left(\frac{L_S A_U - L_\theta B_U}{D}\right)_{oneloop} = \frac{\pi}{\alpha}\left(\frac{5}{22} + \frac{1}{33}\frac{\alpha}{\alpha_S} - \frac{7}{11}sin^2\theta_W\right)$$

$$\left(\frac{L_\theta B_I - L_S A_I}{D}\right)_{oneloop} = \frac{\pi}{\alpha}\left(\frac{1}{22} - \frac{13}{33}\frac{\alpha}{\alpha_S} + \frac{3}{11}sin^2\theta_W\right)$$

$$\left(\frac{K_\theta B_U - J_\theta A_U}{D}\right)_{twoloop} = \frac{1}{264}\left(27P'_{2L} - 15P'_{2R} - 12P'_{4C} + 27P_{2L} - 25P_{1Y} - 2P_{3C}\right)$$

$$\left(\frac{J_\theta A_I - K_\theta B_I}{D}\right)_{twoloop} = \frac{1}{264}\left(48P'_{4C} - 6P'_{2R} - 42P'_{2L} + 52P_{3C} - 10P_{1Y} - 42P_{2L}\right)$$

(5.15)

$$ln(\frac{M_I}{M_Z}) = \frac{\pi}{\alpha}\left(\frac{5}{22} + \frac{1}{33}\frac{\alpha}{\alpha_S} - \frac{7}{11}sin^2\theta_W\right)$$

$$+\frac{1}{264}(27P'_{2L}-15P'_{2R}-12P'_{4C}+27P_{2L}-25P_{1Y}-2P_{3C}) \qquad (5.16a)$$

$$ln(\frac{M_U}{M_Z})=\frac{\pi}{\alpha}\left(\frac{1}{22}-\frac{13}{33}\frac{\alpha}{\alpha_S}+\frac{3}{11}sin^2\theta_W\right)$$

$$+\frac{1}{264}(48P'_{4C}-6P'_{2R}-42P'_{2L}+52P_{3C}-10P_{1Y}-42P_{2L}) \qquad (5.16b)$$

$\underline{n_{16}=2}$

$$\left(\frac{L_SA_U-L_\theta B_U}{D}\right)_{oneloop}=\frac{\pi}{\alpha}\left(\frac{3}{2}+3\frac{\alpha}{\alpha_S}-7sin^2\theta_W\right)$$

$$\left(\frac{L_\theta B_I-L_SA_I}{D}\right)_{oneloop}=\frac{\pi}{\alpha}\left(3sin^2\theta_W-\frac{1}{2}-\frac{5}{3}\frac{\alpha}{\alpha_S}\right)$$

$$\left(\frac{K_\theta B_U-J_\theta A_U}{D}\right)_{twoloop}=\frac{1}{8}(11P'_{2L}-3P'_{2R}-8P'_{4C}+11P_{2L}-5P_{1Y}-6P_{3C})$$

$$\left(\frac{J_\theta A_I-K_\theta B_I}{D}\right)_{twoloop}=\frac{1}{48}(24P'_{4C}+6P'_{2R}-30P'_{2L}+20P_{3C}-35P_{1Y}+15P_{2L})$$

(5.17)

$$ln(\frac{M_I}{M_Z})=\frac{\pi}{\alpha}\left(\frac{3}{2}+3\frac{\alpha}{\alpha_S}-7sin^2\theta_W\right)$$

$$+\frac{1}{8}(11P'_{2L}-3P'_{2R}-8P'_{4C}+11P_{2L}-5P_{1Y}-6P_{3C}) \qquad (5.18a)$$

$$ln(\frac{M_U}{M_Z})=\frac{\pi}{\alpha}\left(3sin^2\theta_W-\frac{1}{2}-\frac{5}{3}\frac{\alpha}{\alpha_S}\right)$$

$$+\frac{1}{48}\left(24P'_{4C}+6P'_{2R}-30P'_{2L}+20P_{3C}-35P_{1Y}+15P_{2L}\right) \qquad (5.18b)$$

For numerical analysis we use the following input parameters[17]

$$\alpha^{-1}(M_Z) = 128.9 \pm 0.1$$

$$\alpha_{3C} = 0.119 \pm 0.004$$

$$\sin^2\theta_W = 0.2315 \pm 0.0002$$

$$M_Z = 91.187\text{Gev} \qquad (5.19)$$

While solving for M_I and M_U, using eqs. (5.16a) - (5.16b) or eqs. (5.18a) - (5.18b) by including only one-loop contributions and ignoring two-loop effects, we obtain for $n_{16} = 1, M_I = 1.76 \times 10^{16} GeV$, $M_U = 2.72 \times 10^{16} GeV$ and for $n_{16} = 2, M_I = 8.30 \times 10^{14} GeV, M_U = 1.01 \times 10^{17} GeV$. Including two-loop effects and ignoring threshold effects we found that the value of M_I increases to (i) $3.09 \times 10^{16} GeV (n_{16} = 1)$ (ii) $4.26 \times 10^{18} GeV (n_{16} = 2)$while the value of M_U decreases to (i) $1.541 \times 10^{16} GeV$ for $n_{16} = 1$, and to (ii) $1.71 \times 10^{15} GeV$ for $n_{16} = 2$. Thus at two-loop level we have $M_I > M_U$, leading to the conclusion that in SUSY $SO(10)$ the G_{224} intermediate symmetry is ruled out. We have verified that even in the model of ref. [19], when two-loop effects are included, the G_{224} breaking scale exceeds the gauge coupling unification scale. But, as one important result of this Chapter, we show that when threshold effects near M_U, M_I and M_Z [13] are included, along with one-loop and two-loop effects, the model yields M_I substantially lower than M_U, which itself is consistent with string unification scale. The threshold effects at M_Z have already been computed [15]. For calculating these effects at M_I we also follow the method of effective mass parameters of ref.[15] where threshold effects due to the superpartners and superheavy masses near the GUT scale have been computed using these parameters. As can be found from this ref.[15], the method of effective mass parameters which parametrizes the heavy or superheavy masses near a symmetry breaking scale, is a very convenient method to compute such effects. At first, we separate J_Δ and K_Δ into three different parts,

$$J_\Delta = J_\Delta^U + J_\Delta^I + J_\Delta^Z$$

$$K_\Delta = K_\Delta^U + K_\Delta^I + K_\Delta^Z \tag{5.20}$$

where

$$J_\Delta^U = 2\pi \left(3\Delta'_{2L} + 3\Delta'_{2R} - 6\Delta'_{4C}\right)$$

$$K_\Delta^U = 2\pi \left(3\Delta'_{2R} + 2\Delta'_{4C} - 5\Delta'_{2L}\right)$$

$$J_\Delta^i = 2\pi \left(5\Delta^i_{1Y} + 3\Delta^i_{2L} - 8\Delta^i_{3C}\right)$$

$$K_\Delta^i = 2\pi \left(5\Delta^i_{1Y} - 5\Delta^i_{2L}\right), i = I, Z \tag{5.21}$$

The expression for Δ_i^Z is given by [15],

$$\Delta_i^Z = \Delta_i^{Conversion} + \Delta_i^{Yukawa} + \Delta_i^{SUSY}, i = 1Y, 2L, 3C \tag{5.22}$$

$$\Delta_i^{Conversion} = -\frac{C_2(G_i)}{12\pi}$$

where

$C_2(G_i)$ is the quadratic Casimir operator for the adjoint representation, with $C_2(G_i) = N[0]$ for $G_i = SU(N)[U(1)]$.

In eq.(5.22)

$$\Delta_i^{Yukawa} = b_i^{top}\frac{h_t^2}{16\pi^2}t$$

$$b_i^{top} = \begin{pmatrix} 26/5 \\ 6 \\ 4 \end{pmatrix} \tag{5.24}$$

In the present case

$$t = \frac{1}{2\pi} ln\frac{M_I}{M_Z}$$

In terms of effective mass parameters ($M_i, i = 1,2,3$) near M_Z threshold [15], the superpartner contribution in (5.22) are

$$\Delta_{1Y}^{SUSY} = \frac{5}{4\pi} ln\frac{M_1}{M_Z}$$

$$\Delta_{2L}^{SUSY} = \frac{25}{12\pi} ln\frac{M_2}{M_Z}$$

$$\Delta_{3C}^{SUSY} = \frac{2}{\pi} ln\frac{M_3}{M_Z}$$

For the sake of convenience we use $M_1 = M_2 = M_3 = 6M_Z$ [15] The threshold effect at the M_Z boundary is taken to be the same as ref [15]. We compute threshold effects on M_I and M_U in two different ways depending upon our choices on masses near M_I and M_U :

(A) by assuming two sets of effective mass parameters near M_I and M_U.

(B) by assigning specific, plausible and reasonable values to the unknown masses of the superheavy scalars and their superpartners.

(A) Threshold Effects with Effective Mass Parameters.

The superheavy components contained in one and two sets of $\underline{16} \oplus \overline{\underline{16}}$ which have masses near M_I are given in Tables V.1 and V.2 respectively. The corresponding threshold effects can be expressed in terms of the effective mass parameters (M_i') as

$$\Delta_i^I = \frac{b_i'}{2\pi} ln \frac{M_i'}{M_I}, i = 1Y, 2L, 3C$$

$$b_i' = \sum_\alpha b_i'^{(\alpha)} \tag{5.26}$$

where α includes Higgs scalar components and their superpartners near M_I. The superheavy components in the representations under G_{224}, contained in $\underline{54}$, $\underline{210}$, $\underline{16} \oplus \overline{\underline{16}}$ and $\underline{10}$ which have masses near the GUT scale are shown in Table V.3. The expression for the threshold effects Δ_i' is given by

Table V.1 The heavy Higgs content of the $SO(10)$ model with G_{224} intermediate symmetry. The G_{213} sub-multiplets become massive when G_{224} is broken. In the extreme right column of the Table are threshold contributions b'_i of the different multiplets for $n_{16} = 1$ (i.e. for one set of $\underline{16} \oplus \overline{16}$).

$SO(10)$ representation	G_{213} multiplet	$b'_{2L}, b'_{1Y}, b'_{3C}$
$\underline{16}$	$(1, \frac{1}{3}, \bar{3})$	$(0, \frac{1}{5}, \frac{1}{2})$
	$(1, -1, 1)$	$(0, \frac{3}{5}, 0)$
$\overline{16}$	$(1, 0, 1)$	$(0, 0, 0)$
	$(1, \frac{-1}{3}, 3)$	$(0, \frac{1}{5}, \frac{1}{2})$
	$(1, \frac{-2}{3}, 3)$	$(0, \frac{4}{5}, \frac{1}{2})$

Table V.2 Same as Table V.1 but for $n_{16} = 2$ (i.e. with two sets of $\underline{16}\oplus\overline{\underline{16}}$.

$SO(10)$ *representation*	G_{213} multiplet	$b'_{2L}, b'_{1Y}, b'_{3C}$
$\underline{16}$	$(1, \frac{1}{3}, \bar{3})$	$(0, \frac{1}{5}, \frac{1}{2})$
	$(1, -1, 1)$	$(0, \frac{3}{5}, 0)$
$\overline{\underline{16}}$	$(1, 0, 1)$	$(0, 0, 0)$
	$(1, \frac{-1}{3}, 3)$	$(0, \frac{1}{5}, \frac{1}{2})$
	$(1, \frac{-2}{3}, 3)$	$(0, \frac{4}{5}, \frac{1}{2})$
$\underline{16}$	$(1, 1, 1)$	$(0, \frac{3}{5}, 0)$
	$(1, 0, 1)$	$(0, 0, 0)$
	$(1, \frac{1}{3}, \bar{3})$	$(0, \frac{1}{5}, \frac{1}{2})$
	$(1, \frac{-2}{3}, \bar{3})$	$(0, \frac{4}{5}, \frac{1}{2})$
$\overline{\underline{16}}$	$(1, 0, 1)$	$(0, 0, 0)$
	$(1, -1, 1)$	$(0, \frac{3}{5}, 0)$
	$(1, \frac{-1}{3}, 3)$	$(0, \frac{1}{5}, \frac{1}{2})$
	$(1, \frac{-2}{3}, 3)$	$(0, \frac{4}{5}, \frac{1}{2})$

Table V.3 Same as Table V.1 and Table V.2, but here the G_{224} sub-multiplets acquire mass when $SO(10)$ is broken. Also listed in the extreme right column of the Table are the threshold contributions b''_i of different multiplets.

$SO(10) representation$	G_{224} multiplet	$b''_{2L}, b''_{2R}, b''_{4C}$
$\underline{210}$	$(2,2,10)$	$(10,10,12)$
	$(2,2,\bar{10})$	$(10,10,12)$
	$(1,1,15)$	$(0,0,4)$
	$(1,3,15)$	$(0,30,12)$
	$(3,1,15)$	$(30,0,12)$
$\underline{54}$	$(3,3,1)$	$(6,6,0)$
	$(1,1,20)$	$(0,0,\frac{13}{2})$
	$(2,2,6)$	$(6,6,4)$
$\underline{16}$	$(2,1,4)$	$(2,0,1)$
$\underline{\bar{16}}$	$(2,1,\bar{4})$	$(2,0,1)$
$\underline{10}$	$(1,1,6)$	$(0,0,1)$

$$\Delta_i^U = \frac{b_i''}{2\pi} ln \frac{M_i''}{M_U}, i = 2L, 2R, 4C$$

$$b_i'' = \sum_\alpha b_i''^{(\alpha)} \tag{5.27}$$

where α includes Higgs scalar components and their superpartners near M_U and M_i'' are the effective mass parameters at $\mu = M_U$. Thus, in the SUSY $SO(10)$ model with G_{224} intermediate symmetry we have three sets of effective mass parameters which parametrize the corresponding masses at the three symmetry breaking scales :

(i) $M_1, M_2, M_3 \equiv M_{1Y}, M_{2L}, M_{3C}, at \mu = M_Z$

(ii) $M_1', M_2', M_3' \equiv M_{1Y}', M_{2L}', M_{3C}'$, at $\mu = M_I$

(iii) $M_{2L}', M_{2R}', M_{4C}'$, at $\mu = M_U$

Using eqs. (5.26) - (5.27) in eqs. (5.20)- (5.21) we obtain

$\underline{n_{16} = 1}$

$$J_\Delta = 198 ln \frac{M_{2L}''}{M_U} + 186 ln \frac{M_{2R}''}{M_U} - 393 ln \frac{M_{4C}''}{M_U} + 9 ln \frac{M_{1Y}'}{M_I} - 12 ln \frac{M_{3C}'}{M_I} + \frac{25}{2} ln \frac{M_1}{M_Z}$$
$$+ \frac{25}{2} ln \frac{M_2}{M_Z} - 32 ln \frac{M_3}{M_Z} + \frac{3}{4\pi^2} ln \frac{M_I}{M_Z} + 3 \tag{5.28a}$$

$$K_\Delta = 186 ln \frac{M_{2R}''}{M_U} + 131 ln \frac{M_{4C}''}{M_U} - 310 ln \frac{M_{2L}''}{M_U} + 9 ln \frac{M_{1Y}'}{M_I} - 12 ln \frac{M_{3C}'}{M_I} + \frac{25}{2} ln \frac{M_1}{M_Z}$$

$$-\frac{125}{6}ln\frac{M_2}{M_Z}-\frac{1}{4\pi^2}ln\frac{M_I}{M_Z}+\frac{5}{3} \tag{5.28b}$$

Using eqs. (5.28a) - (5.28b) in eqs (5.6) - (5.9) we then obtain the formulas for threshold effects M_I and M_U in terms of the effective mass parameters M_i' and M_i'',

$$\Delta ln\frac{M_I}{M_Z}\equiv\frac{(J_\Delta A_U-K_\Delta B_U)}{D}$$

$$=\frac{1}{22}\left(155ln\frac{M''_{2R}}{M_U}-297ln\frac{M''_{2L}}{M_U}+131ln\frac{M''_{4C}}{M_U}\right)$$

$$+\frac{1}{44}\left(15ln\frac{M'_{1Y}}{M_I}+ln\frac{M'_{3C}}{M_I}\right)-0.548018546 \tag{5.29}$$

$$\Delta ln\frac{M_U}{M_Z}\equiv\frac{(K_\Delta B_I-J_\Delta A_I)}{D}$$

$$=\frac{1}{22}\left(31ln\frac{M''_{2R}}{M_U}+231ln\frac{M''_{2L}}{M_U}-262ln\frac{M''_{4C}}{M_U}\right)$$

$$+\frac{1}{44}\left(3ln\frac{M'_{1Y}}{M_I}-13ln\frac{M'_{3C}}{M_I}\right)+0.068197446 \tag{5.30}$$

$\underline{n_{16}=2}$

$$J_\Delta=174ln\frac{M''_{2L}}{M_U}+150ln\frac{M''_{2R}}{M_U}-342ln\frac{M''_{4C}}{M_U}+25ln\frac{M'_{1Y}}{M_I}-28ln\frac{M'_{3C}}{M_I}+\frac{25}{2}ln\frac{M_1}{M_Z}$$

$$+\frac{25}{2}ln\frac{M_2}{M_Z}-32ln\frac{M_3}{M_Z}+\frac{3}{4\pi^2}ln\frac{M_I}{M_Z}+3 \tag{5.31a}$$

$$K_\Delta = 150 ln \frac{M''_{2R}}{M_U} + 114 ln \frac{M''_{4C}}{M_U} - 290 ln \frac{M''_{2L}}{M_U} + 25 ln \frac{M'_{1Y}}{M_I} + \frac{25}{2} ln \frac{M_1}{M_Z}$$

$$-\frac{125}{6} ln \frac{M_2}{M_Z} - \frac{1}{4\pi^2} ln \frac{M_I}{M_Z} + \frac{5}{3} \tag{5.31b}$$

Using eqs. (5.31a) - (5.31b) in eqs (5.6) - (5.9) we then obtain the formulas for threshold effects M_I and M_U in terms of the effective mass parameters M'_i and M''_i,

$$\Delta ln \frac{M_I}{M_Z} \equiv \frac{(J_\Delta A_U - K_\Delta B_U)}{D}$$

$$= \left(\frac{75}{2} ln \frac{M''_{2R}}{M_U} - \frac{319}{2} ln \frac{M''_{2L}}{M_U} + 144 ln \frac{M''_{4C}}{M_U}\right) + \left(\frac{25}{4} ln \frac{M'_{1Y}}{M_I} + \frac{21}{4} ln \frac{M'_{3C}}{M_I}\right) - 4.89 \tag{5.32}$$

$$\Delta ln \frac{M_U}{M_Z} \equiv \frac{(K_\Delta B_I - J_\Delta A_I)}{D}$$

$$= \left(\frac{-25}{2} ln \frac{M''_{2R}}{M_U} + \frac{145}{2} ln \frac{M''_{2L}}{M_U} - 57 ln \frac{M''_{4C}}{M_U}\right) + \left(\frac{-25}{12} ln \frac{M'_{1Y}}{M_I} - \frac{35}{12} ln \frac{M'_{3C}}{M_I}\right) + 1.93 \tag{5.33}$$

The last term in eqs. (5.29) - (5.30) and (5.32) - (5.33) denote threshold contributions at $\mu = M_Z$ corresponding to a choice of ref[15] $M_1 = M_2 = M_3 = 6M_Z$. It is clear that threshold effects on the two mass scales, M_U and M_I can be estimated once $M'_i(i = 1, 2, 3)$ and $M''_j(j = 2L, 2R, 4C)$ are known. In any model the superheavy masses near any particular symmetry breaking scale are parametrized in terms of the corresponding effective mass parameters. In the present model there are three such relations corresponding to the three symmetry breaking scales i.e.$\mu = M_{SUSY} = M_Z, \mu = M_I$ and $\mu = M_U$,

$$\Delta_i^Z = \sum_\alpha \frac{b_i^\alpha}{2\pi} ln \frac{M_\alpha}{M_Z} = \frac{b_i}{2\pi} ln \frac{M_i}{M_Z}, i = 1,2,3; \mu = M_Z \tag{5.34}$$

$$\Delta_i^I = \sum_\alpha \frac{b_i'^\alpha}{2\pi} ln \frac{M_\alpha'}{M_I} = \frac{b_i'}{2\pi} ln \frac{M_i'}{M_I}, i = 1,2,3; \mu = M_I \tag{5.35}$$

$$\Delta_i^U = \sum_\alpha \frac{b_i''^\alpha}{2\pi} ln \frac{M_i''}{M_U} = \frac{b_i''}{2\pi} ln \frac{M_i''}{M_U}, i = 2L, 2R, 4C; \mu = M_U \tag{5.36}$$

where α refers to the actual G_{213} submultiplet near $\mu = M_Z, M_I$ or the G_{224} submultiplet near $\mu = M_U$ and M_α, M_α' or $M"_\alpha$ refer to the actual component masses. The coefficients b_i, b_i' and b_i'' have been defined in (5.25) - (5.27) [15]. The numbers b_i^α refer to the one loop coefficients of the multiplet α under the gauge subgroup, $U(1)_Y, SU(2)_L, SU(2)_R, SU(3)_C, SU(4)_C$ etc. The relation (5.34) has been utilised in ref [15] to compute only one set of values of M_1, M_2, M_3 in MSSM from the model predictions on M_α. But since such predictions are also model dependent, several other assumed values of effective mass parameters have been utilised for computation. At the MSSM unification scale, $M_U \simeq 2 \times 10^{16} GeV$, the values of effective mass parameters in the $SU(5)$ model have been assumed for computing threshold effects. In the present case, in the absence of actual values of component massesin the model, we make quite reasonable assumptions on $M_i' and M_i''$ for computation. In our analysis the effective mass parameters M_i' or M_i'' are taken to vary between 1/5 - 5 times the relevant scale of symmetry breaking i.e. $M_I or M_U$. For example, with one set of $\underline{16} \oplus \overline{\underline{16}}$, when $M'_{1Y} = M'_{3C} = M_I, M''_{2R} = M_U, M''_{2L} = 3M_U, M''_{4C} = 2M_U$ we obtain, $M_I = 4.00 \times 10^{11}$GeVand$M_U = 4.39 \times 10^{17}$GeV. But,with two sets of $\underline{16} \oplus \overline{\underline{16}}$, when $M'_{1Y} = M'_{3C} = 3M_I, M"_{2R} = 2M_U, M"_{2L} = 3M_U$, the values of M_I and M_U are found to be 3.73×10^{11}GeV,and 2.09×10^{17}GeV respectively. More important is the result in the degenerate case, $M'_{1Y} = M'_{3C} = \frac{1}{2}M_I, M''_{2R} = M''_{2L} = M''_{4C} = 2M_U$ in Table V.5 for which we have obtained $M_I = 4.3 \times 10^{10}$GeV and $M_U = 3 \times 10^{18}$GeV for $n_{16} = 2$. We have checked using (5.35) and (5.36) that such effective mass parameters result when the superheavy components in Tables V.1 - V. 3 have degenerate masses near the respective scales. Different results on intermediate scale M_I and unification mass M_U which are obtained as solutions of RGEs including threshold effects as a function of effective mass parameters for both the cases are presented in Tables V.4 and V.5 respectively. We find $M_I \simeq 10^{10} - 10^{13}$GeV for quite reasonable choice of the mass parameters. It is interesting to note that some of the GUT scales are close to the Planck scale or the string unification scale. The solutions given here are by no means exhaustive, but indicate that the

Table V.4 Predictions on mass scales M_I and M_U including threshold effects with effective mass parameters for $n_{16} = 1$.

M'_{1Y}	M'_{3C}	M''_{2L}	M''_{2R}	M''_{4C}	M_I(GeV)	M_U(GeV)
$\frac{1}{2}M_I$	$\frac{1}{2}M_I$	$3M_U$	$\frac{1}{2}M_U$	$2M_U$	2.36×10^9	1.93×10^{17}
$\frac{1}{3}M_I$	$\frac{1}{3}M_I$	$2M_U$	$\frac{1}{2}M_U$	$1.5M_U$	8.74×10^{10}	9.23×10^{16}
M_I	M_I	$3M_U$	M_U	$2M_U$	4.00×10^{11}	4.39×10^{17}
$\frac{1}{2}M_I$	$\frac{1}{2}M_I$	$3M_U$	M_U	$2.5M_U$	1.18×10^{12}	3.60×10^{16}
M_I	M_I	$2M_U$	M_U	$1.5M_U$	1.72×10^{13}	1.91×10^{17}

Table V.5 Same as TableV.4 but for $n_{16} = 2$.

M'_{1Y}	M'_{3C}	M''_{2L}	M''_{2R}	M''_{4C}	M_I(GeV)	M_U(GeV)
$\frac{1}{2}M_I$	$\frac{1}{2}M_I$	$2M_U$	$2M_U$	$2M_U$	4.3×10^{10}	3.03×10^{18}
$2.3M_I$	$2.3M_I$	$3M_U$	$2M_U$	$3M_U$	1.75×10^{10}	7.9×10^{17}
$3M_I$	$3M_I$	$3M_U$	$2M_U$	$3M_U$	3.73×10^{11}	2.09×10^{17}
$3.5M_I$	$3.5M_I$	$3M_U$	$2M_U$	$3M_U$	3.19×10^{12}	9.68×10^{16}
$4M_I$	$4M_I$	$3M_U$	$2M_U$	$3M_U$	1.02×10^{13}	4.96×10^{16}

intermediate scale can be achieved in a natural way via threshold effects by following the method of effective mass parameters [15].

B. THRESHOLD EFFECTS WITH SUPERHEAVY MASSES

We now present our results on threshold effects with specific but reasonable values on the masses of superheavy components of Higgs scalar and their superpartners corresponding to the representations $\underline{210}$,$\underline{54}$, $\underline{10}$ and $\underline{16} \oplus \overline{\underline{16}}(for n_{16} = 1, n_{16} = 2)$ [9].

The expression for threshold effect in terms of actual superheavy component masses M'_α which have masses near M_Z is given by the L.H.S. of eq (5.35),

$$\Delta_i^I = \sum_\alpha \frac{b_i'^\alpha}{2\pi} ln \frac{M'_\alpha}{M_I}, i = 1Y, 2L, 3C \qquad (5.37)$$

The superheavy components α contained in one and two sets of $\underline{16} \oplus \overline{\underline{16}}$ which have masses near M_I and those of $\underline{210}$, $\underline{54}$, $\underline{16} \oplus \overline{\underline{16}}$ and $\underline{10}$ which have masses near M_U are shown in Table V.1 - V.3.

The expression for the threshold effects at $\mu = M_U$ is given by the L.H.S of eq. (5.36),

$$\Delta_i' = \sum_\alpha \frac{b_i^{''(\alpha)}}{2\pi} ln \frac{M''_\alpha}{M_U}, i = 2L, 2R, 4C \qquad (5.38)$$

While computing threshold effects using (5.37) and (5.38) and Tables V.1 - V.3 , we have assumed all the multiplets belonging to an $SO(10)$ representation 'H' to have the same degenerate mass M_H. For example all the superheavy components in $\underline{210}$ given in Table V. 3 near $\mu = M_U$ have been subjected to the following degeneracy condition,

$$M''(2,2,10) = M''(2,2,10) = M''(1,3,15) = M''(3,1,15) = M_{(210)}$$

Similarly for $\underline{54}$ and $\underline{16} \oplus \overline{16}$

$$M"(3,3,1) = M"(1,1,20) = ... = M_{54}$$

$$M"(2,1,4) = M"(2,1,\bar{4}) == M_{16}$$

All the heavy masses near $\mu = M_I$ are assumed to have the same mass $M' = M_R$. Using eqs. (5.37) and 5.38) in eqs. (5.20) - (5.21), and denoting $\eta_H = ln\left(\frac{M_H}{M_U}\right)$, $H = \underline{210}, \underline{54}, \underline{16}, \underline{10}$ and $\eta_R = ln\left(\frac{M_R}{M_I}\right)$, we obtain

$\underline{n_{16} = 1}$.

$$J_\Delta = -12\eta_{210}+9\eta_{54}-6\eta_{10}-3\eta_R+3+\frac{25}{2}ln\frac{M_1}{M_Z}+\frac{25}{2}ln\frac{M_2}{M_Z}-32ln\frac{M_3}{M_Z}+\frac{3}{4\pi^2}ln\frac{M_I}{M_Z} \tag{5.39a}$$

$$K_\Delta = 4\eta_{210}-3\eta_{54}-16\eta_{16}+2\eta_{10}+25\eta_R+\frac{5}{3}+\frac{25}{2}ln\frac{M_1}{M_Z}-\frac{125}{6}ln\frac{M_2}{M_Z}-\frac{1}{4\pi^2}ln\frac{M_I}{M_Z} \tag{5.39b}$$

$\underline{n_{16} = 2}$

$$J_\Delta = -12\eta_{210}+9\eta_{54}-6\eta_{10}-3\eta_R+3+\frac{25}{2}ln\frac{M_1}{M_Z}+\frac{25}{2}ln\frac{M_2}{M_Z}-32ln\frac{M_3}{M_Z}+\frac{3}{4\pi^2}ln\frac{M_U}{M_Z} \quad (5.40a)$$

$$K_\Delta = 4\eta_{210}-3\eta_{54}-32\eta_{16}+2\eta_{10}+25\eta_R+\frac{5}{3}+\frac{25}{2}ln\frac{M_1}{M_Z}-\frac{125}{6}ln\frac{M_2}{M_Z}-\frac{1}{4\pi^2}ln\frac{M_I}{M_Z} \quad (5.40b)$$

Using eqs. (5.39a) - (5.39b) in eqs. (5.6) - (5.9) we then obtain the formulas for threshold effect on mass scales M_I and M_U:

$\underline{n_{16} = 1}$

$$\Delta ln\frac{M_I}{M_Z} \equiv \frac{(J_\Delta A_U - K_\Delta B_U)}{D}$$

$$= \frac{1}{22}(4\eta_{210} - 3\eta_{54} - 14\eta_{16} + 2\eta_{10}) + \frac{4}{11}\eta_R - 0.548018546 \quad (5.41a)$$

$$\Delta ln\frac{M_U}{M_Z} \equiv \frac{(K_\Delta B_I - J_\Delta A_I)}{D}$$

$$= \frac{1}{11}(3\eta_{54} + 3\eta_{16} - 4\eta_{210} - 2\eta_{10}) - \frac{5}{22}\eta_R + 0.068197446 \quad (5.41b)$$

$\underline{n_{16} = 2}$

$$\Delta ln\frac{M_I}{M_Z} \equiv \frac{(J_\Delta A_U - K_\Delta B_U)}{D}$$

$$= (4\eta_{210} - 3\eta_{54} - 14\eta_{16} + 2\eta_{10}) + 11.5\eta_R - 4.89 \qquad (5.41a)$$

$$\Delta ln \frac{M_U}{M_Z} \equiv \frac{(K_\Delta B_I - J_\Delta A_I)}{D}$$

$$= \left(\frac{3}{2}\eta_{54} + 6\eta_{16} - 2\eta_{210} - \eta_{10}\right) - 5\eta_R + 1.93 \qquad (5.42b)$$

The last term in eqs. (5.41) - (5.42) denote threshold contributions at $\mu = M_Z$. As before the Higgs masses are allowed to vary between 1/5 and 5 times the scale of the relevant symmetry breaking i.e. M_I or M_U. For example, when $M_i' = M_R = \frac{1}{2}M_I, M_{210} = M_{10} = \frac{1}{5}M_U$, $M_{54} = M_{16} = 5M_U$ for $n_{16} = 1$ we obtain $M_I = 2.576 \times 10^{15}$GeV and $M_U = 1.04 \times 10^{17} GeV$. But, for $n_{16} = 2$ when $M_i' = M_R = \frac{1}{2}M_I$, $M_{210} = M_{54} = M_{16} = M_{10} = 1.5M_U$ we found interesting solutions with $M_I = 1.46 \times 10^{12}$GeV and $M_U = 2.34 \times 10^{18}$GeV. Different results on intermediate scale M_I and unification mass M_U which are obtained as solutions of RGEs including threshold effects as a function of superheavy masses are presented in Table V.6 for $n_{16} = 2$. In this case we find, that $M_I \simeq 10^{11} - 10^{13} GeV$ for reasonable choices of superheavy masses are easily allowed.

V.4 SUMMARY AND CONCLUSION

While investigating the possibility of G_{224} intermediate gauge symmetry in $SO(10)$, we avoided the representations $\underline{126} \oplus \overline{\underline{126}}$ for intermediate symme-

Table V.6 Predictions on mass scales M_I and M_U including threshold effect with superheavy masses for $n_{16} = 2$

M_R	M_{210}	M_{54}	M_{16}	M_{10}	M_I(GeV)	M_U(GeV)
$\frac{1}{2}M_I$	$1.5M_U$	$1.5M_U$	$1.5M_U$	$1.5M_U$	1.46×10^{12}	2.34×10^{18}
$\frac{1}{2}M_I$	$2M_U$	$2M_U$	$2M_U$	$2M_U$	3.46×10^{11}	8.56×10^{18}
$\frac{1}{2}M_I$	$3M_U$	$3M_U$	$2M_U$	$2M_U$	5.91×10^{12}	6.99×10^{18}
$\frac{1}{2}M_I$	$0.5M_U$	$0.5M_U$	$0.5M_U$	$0.5M_U$	3.54×10^{14}	1.67×10^{16}

try breaking and generating Majorana neutrino masses because of their well-known difficulties against arriving at acceptable values of $M_I \ll M_U$ [11]. As the mechanism of generating Majorana neutrino masses are now well known via the representations $\underline{16} \oplus \overline{\underline{16}}$ and through couplings with $SO(10)$- singlet fields in the superpotential [12], we have utilised one and two sets of them in addition to the representations $\underline{210}$, $\underline{54}$ and $\underline{10}$ needed for spontaneous symmetry breakings at the GUT and the electroweak scales, respectively. At the two-loop level, we have noted that even after using $\underline{16} \oplus \overline{\underline{16}}$ instead of $\underline{126} \oplus \overline{\underline{126}}$, the RGEs do not permit G_{224} intermediate breaking scale M_I lower than M_U, in this model and also in ref.[19].We have found that at the two-loop level, when threshold effects due to superheavy components contained in the relevant $SO(10)$ representations are included, the RGEs permit G_{224} breaking intermediate scales $M_I \simeq 10^{10} - 10^{13}$GeV with high unification scales, $M_U \simeq 10^{17} - 10^{18}$GeV, for certain allowed solutions. The generation of right-handed Majorana neutrino masses and the implementation of seesaw mechanism is carried out by the introduction of $SO(10)$ - singlet following the mechanism of Lee and Mohapatra [12] through purely renormalisable interactions.We thus conclude, in contrast to earlier observations [11], that $SU(2)_L \times SU(2)_R \times SU(4)_C(g_{2L} \neq g_{2R})$ is allowed as an intermediate gauge symmetry in supersymmetric $SO(10)$ model in a natural manner. Even the use of a number of light Higgs supermultiplets at the intermediate scale is not needed to achieve the intermediate scale. Using quadratic or linear seesaw formulas and renormalisation effects [18], it is possible to obtain neutrino masses necessary for ν_τ as a hot dark matter candidate and solution to the solar neutrino puzzle by MSW mechanism in these models.

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CHAPTER 6

CHAPTER VI

SUMMARY AND CONCLUSION

In this concluding chapter, the results of the present investigation are briefly summarised.

The first chapter contains a capsule review of elementary particle phenomenology and introduces some of the issues relating to the grand unification theory. In the second chapter we discuss the non-SUSY and SUSY standard model coupled with a brief appriasal about its successes and failures. The limitations of the standard model in describing only low energy manifestations of an underlying unified theory has led to the development of Grand Unified theories (GUTs). A concise revision of the Grand Unified theories is given in Chapter III, wherein, the basic structures of the groups viz.$SU(5), SU(8)_L \times SU(8)_R$ and $SO(10)$ are also mentioned. The new research investigations carried out under the ambit of the present study are presented in Chapters IV and V.

Quark-lepton unification through the Pati-Salam gauge group $SU(2)_L \times SU(2)_R \times SU(4)_C (\equiv G_{224})$ which forms a subgroup of a number of GUTs permits experimentally observable rare kaon decays and new modes of proton decay provided the symmetry breaking scale $M_C = 10^6$ GeV. In addition,

$n-\bar{n}$ and $H-\bar{H}$ oscillations can be predicted with definite rates to be detected in future. Furthermore, the new physics associated with V+A interactions in left-right gauge models can be experimentally tested provided the right-handed gauge bosons $(W_R^{\pm}, Z_R)$ resulting from the spontaneous breaking of $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C (\equiv G_{2213})$ symmetry are resonably light. The conventional method of left-right symmetry fails to acheive $M_R \simeq 1TeV$ or $M_C = 10^5 - 10^6$ GeV, and yields a high G_{2213} breaking scale unless there are a number of intermediate gauge symmetries. We consider an alternative method of decoupling the parity and $SU(2)_R$-breakings proposed by Parida and Pati which requires chiral $SU(4)$ colour for its implementation and as such can operate within $SU(8)_L \times SU(8)_R, SU(16)$ and $[SU(4)]^4$ but not in $SO(10)$. We examined the predictions of $SU(16)$ and $SU(8)_L \times SU(8)_R$ GUTs under the constraints of CERN-LEP data using the alternative method, in the minimal chains. We found that right-handed gauge-boson masses corresponding to G_{2213}-breaking scale as low as 1 TeV, are allowed in one of the modelswhereas the other chain permits the $SU(2)_L \times SU(2)_R \times SU(4)_C$ breaking scale $M_C \simeq 10^6$ GeV leading to $n-\bar{n}$ oscillations and the $\Delta(B-L) = -2$ nucleon decay modes possibly accessible to the ongoing experiments and observable branching ratios for rare-kaon decay modes.

In recent analysis, the existence of $SU(2)_L \times SU(2)_R \times SU(4)_C (g_{2L} \neq g_{2R})$ intermediate gauge symmetry has been ruled out in the minimal SUSY $SO(10)$ model at one-loop level.But it is allowed with $g_{2L} \neq g_{2R}$ at one-loop level if there are additional degrees of freedom near the intermediate scale ($M_I = 10^{12}$GeV). But at two-loop level such a model appears to be ruled out. With $g_{2L} = g_{2R}$, the model is allowed with masses of pseudogoldstone components in $16_H + \overline{16}_H$ near the Tev scale.In chapterV we show how the assymetric gauge group is allowed in SUSY $SO(10)$ with an intermediate scale $M_I = 10^{10} - 10^{13}$ Gev and high unification scales $M_U = 10^{17} - 10^{18}$ GeV by including two-loop and threshold effects, but without any light degrees of freedom. The generation of right-handed Majorana neutrino masses and the implementation of see-saw mechanism is carried out by the introduction of $SO(10)$-singlet fermions following the mechanism of Lee and Mohapatra through purely renormalisable interactions.