

# **A note on the sub-optimality of rank ordering of objects on the basis of the leading principal component factor scores**

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## **Abstract**

This paper demonstrates that if we intend to optimally rank order  $n$  objects (candidates) each of which has  $m$  rank ordered attributes or rank scores have been awarded by  $m$  evaluators, then the overall ordinal ranking of objects by the conventional principal component based factor scores turns out to be suboptimal. Three numerical examples have been provided to show that principal component based rankings do not necessarily maximize the sum of squared correlation coefficients between the individual  $m$  rank scores arrays,  $X(n,m)$ , and overall rank scores array,  $Z(n)$ .

Keywords: Rankings, sub-optimal, optimality, principal component, factor scores, Differential Evolution, global optimization

## A note on the sub-optimality of rank ordering of objects on the basis of the leading principal component factor scores

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**1. The Problem:** Suppose we intend to optimally rank order  $n$  objects (candidates) each of which has  $m$  rank ordered attributes or rank scores have been awarded by  $m$  evaluators. More explicitly, let  $m$  evaluators award ranking scores to  $n$  ( $n > m$ ) candidates and let the array of ranking scores be denoted by  $X$  in  $n$  rows and  $m$  columns. The objective is to summarize  $X$  by a single  $n$ -element column vector of ranking scores,  $Z$ , such that, in some sense,  $Z$  is the optimal representation of  $X$ . Among many possible criteria of representation, the one could follow the principle that the sum of squares of the product moment coefficients of correlation between  $Z$  and  $x_j \in X$  is maximum or, stated symbolically,  $\sum_{j=1}^m r^2(Z, x_j)$  is maximum.

Conventionally, this problem is solved by the principal component analysis (Kendall and Stuart, 1968). It consists of obtaining  $Z = \mathfrak{R}(Y) : Y = Xw$ ;  $\max \sum_{j=1}^m r^2(Y, x_j)$ , where the transformation  $\mathfrak{R}(Y)$  assigns ranking scores to the elements of  $Y$  by any one of the ranking rules such as (a) standard competition ranking or the 1-2-2-4 rule, (b) modified competition ranking or 1-3-3-4 rule, (c) dense ranking or 1-2-2-3 rule, (d) ordinal ranking or 1-2-3-4 rule, (e) fractional ranking or 1-2.5-2.5-4 rule, etc (Wikipedia, 2008). The crux of the problem is, however, that optimality of  $Y$  does not entail optimality of  $Z$  or, stated differently,  $Z = \mathfrak{R}(Y) : Y = Xw$ ;  $\max \sum_{j=1}^m r^2(Y, x_j)$  does not necessarily ensure the optimality of  $Z$  with respect to the constituent variables. If  $Z = \mathfrak{R}(Y) : Y = Xw$ ;  $\max \sum_{j=1}^m r^2(Y, x_j)$  then oftentimes  $Z$  turns out to be only a suboptimal representation of  $X$ .

The procedure of obtaining the ranking scores  $Z = \mathfrak{R}(Y) : Y = Xw$ ;  $\max \sum_{j=1}^m r^2(Y, x_j)$  follows two straightforward steps. First, the Principal component analysis is run on  $X$  to obtain  $Y$ , which is the best linear combination,  $Xw$ , of  $X$ . It lies in (i) computing the inter-correlation matrix,  $R(m, m)$ , from  $X$  such that  $r_{ij} \in R$ ;  $r_{ij} = \text{cov}(x_i, x_j) / \sqrt{\text{var}(x_i) \text{var}(x_j)} \forall i, j$ , (ii) finding the normed eigenvector,  $w$ , of  $R$  associated with its largest eigenvalue,  $\lambda_1$  and (ii) using  $w$  to obtain  $Y = X^* w$ , where  $x_{ij}^* = (x_{ij} - \bar{x}_j) / \sqrt{\text{var}(x_j)}$ . Next,  $Z = \mathfrak{R}(Y)$  is obtained. It is well known that this procedure maximizes  $\sum_{j=1}^m r^2(Y, x_j)$ . The direct maximization of  $\sum_{j=1}^m r^2(Y, x_j)$  is a rare practice.

However, what we seek is not the  $Z$  that maximizes  $\sum_{j=1}^m r^2(Y, x_j)$  but, instead, the  $Z$  that maximizes  $\sum_{j=1}^m r^2(Z, x_j)$ . This is not guaranteed by the conventional Principal Component Analysis. This is what we demonstrate in this paper.

It may also be mentioned that Korhonen (1984) and Korhonen and Siljamaki (1998) were, perhaps, the initial attempts to obtain an ordinal principal component,  $Z$ , that maximizes  $\sum_{j=1}^m r^2(Z, x_j)$ . As mentioned by Li and Li (2004), the authors used the constrained integer programming method of

optimization to obtain  $Z$ . Obviously, an integer programming based method would not function if rank ordering follows fractional ranking and it would become quite complicated if 1-2-2-4, 1-3-3-4 or 1-2-2-3 rule is applied.

**2. The Objective:** Our objective in this paper is to show, by means of a few numerical examples, that the rank ordering of objects (candidates) by  $Z = \mathfrak{R}(Y)$ , where  $Y = Xw$  obtained by maximizing  $\sum_{j=1}^m r^2(Y, x_j)$ , is suboptimal in the sense that it may not maximize  $\sum_{j=1}^m r^2(Z, x_j)$ . The rank ordering of objects by direct maximization of  $\sum_{j=1}^m r^2(Z, x_j)$  is a better alternative. No conventional procedure of Principal Component analysis provides any scope for obtaining such ordinal or rank-ordering principal component. There is no ready-to-use software available to maximize  $\sum_{j=1}^m r^2(Z, x_j)$ .

**3. Materials and Methods:** We simulate ranking score awarded to 30 candidates (objects) by 7 evaluators. The dataset,  $X$ , may also be viewed as the ranking scores obtained by 30 objects on each of their 7 attributes. First,  $X$  is subjected to the Principal Component analysis and the composite score,  $Y = Xw$ , is obtained. This composite score relates to the first principal component associated with the largest eigenvalue of the correlation matrix of  $X$ . Therefore,  $Y$  maximizes  $\sum_{j=1}^m r^2(Y, x_j)$ . Then  $Y$  is rank ordered according to the 1-2-3-4 ordinal ranking rule to obtain  $Z_1$ , the overall ranking scores for the candidates (objects). The inter-correlation matrix,  $R_1$ , is computed for  $[Z_1 \ x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7]$  and, consequently, the measure of representativeness of  $Z_1$ , namely  $F_1 = \sum_{j=1}^7 r^2(Z_1, x_j)$ , is computed. It may be noted that in this scheme  $\sum_{j=1}^m r^2(Y, x_j)$  rather than  $F_1 = \sum_{j=1}^7 r^2(Z_1, x_j)$  is maximized.

In the second scheme,  $F_2 = \sum_{j=1}^7 r^2(Z_2, x_j)$ , an alternative measure of representativeness, is directly maximized, where  $Z_2 = \mathfrak{R}(Y')$ :  $Y' = Xv$ . In finding  $Z_2 = \mathfrak{R}(Y')$  the 1234 ordinal ranking rule is applied. As before, the inter-correlation matrix,  $R_2$ , is computed for  $[Z_2 \ x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7]$ . The method of Differential Evolution has been used for maximization of these alternative measures of representativeness,  $F_1$  and  $F_2$ .

It would be pertinent to provide an introduction to the Differential Evolution (DE) method of optimization. The DE is one of the most recently invented methods of global optimization that has been very successful in optimization of extremely difficult multimodal functions. The DE is a population-based stochastic search method of optimization grown out of the Genetic algorithms. The crucial idea behind DE is a scheme for generating trial parameter vectors. Initially, a population of points ( $p$  in  $d$ -dimensional space) is generated and evaluated (i.e.  $f(p)$  is obtained) for their fitness. Then for each point ( $p_i$ ) three different points ( $p_a$ ,  $p_b$  and  $p_c$ ) are randomly chosen from the population. A new point ( $p_z$ ) is constructed from those three points by adding the weighted difference between two points ( $w(p_b - p_c)$ ) to the third point ( $p_a$ ). Then this new point ( $p_z$ ) is subjected to a crossover with the current point ( $p_i$ ) with a probability of crossover ( $c_r$ ), yielding a candidate point, say  $p_w$ . This point,  $p_w$ , is evaluated and if found better than  $p_i$ , then it replaces  $p_i$  else  $p_i$  remains. Thus we obtain a new vector in which all points are either better than or as good as the current points. This new vector is used for the next iteration. This process makes the differential evaluation scheme completely self-organizing. Operationally, this method consists of three basic steps: (i) generation of (large enough) population with  $N$  individuals [ $u = (u_1, u_2, \dots, u_m)$ ] in the  $m$ -dimensional space, randomly distributed over the entire domain of the function in question and evaluation of the individuals of the so generated by finding  $f(u)$ ; (ii) replacement of this current population by a better fit new population, and (iii) repetition of this replacement until satisfactory results are obtained or certain criteria of termination are met. The crux of the problem lays in replacement of the

current population by a new population that is better fit. In this context, the meaning of ‘better’ is in the Pareto improvement sense. A set  $S_a$  is better than another set  $S_b$  iff : (i) no  $u_i \in S_a$  is inferior to the corresponding member of  $u_i \in S_b$  ; and (ii) at least one member  $u_k \in S_a$  is better than the corresponding member  $u_k \in S_b$ . Thus, every new population is an improvement over the earlier one. To accomplish this, the DE procedure generates a candidate individual to replace each current individual in the population. The candidate individual is obtained by a crossover of the current individual and three other randomly selected individuals from the current population. The crossover itself is probabilistic in nature. Further, if the candidate individual is better fit than the current individual, it takes the place of the current individual, else the current individual stays and passes into the next iteration (Mishra, 2006).

It may further be noted that we have maximized  $\sum_{j=1}^m r^2(Y, x_j)$  as well as  $\sum_{j=1}^7 r^2(Z_2, x_j)$  by DE. We have compared our results of maximization of  $\sum_{j=1}^m r^2(Y, x_j)$  with those obtained from STATISTICA (which has a built in facility to find the principal component-based factor scores and other related measures such as the eigenvalues, factor loadings, etc). Our results are essentially identical to those obtained from STATISTICA. This comparison ensures that our efforts in direct optimization have been perfectly successful. However, STATISTICA does not have a provision to maximize  $\sum_{j=1}^7 r^2(Z_2, x_j)$ . But since in the examples shown below  $F_2$  is larger than  $F_1$ , we conclude that rank ordering of objects by  $Z_1$  is suboptimal.

**Example-1:** The simulated dataset ( $X$ ) on ranking scores of 30 candidates awarded by 7 evaluators, the results obtained by running the principal component algorithm and the overall rankings based on the same ( $Y$  and  $Z_1$ ) and the results of rank order optimization exercise based on our method ( $Y'$  and  $Z_2$ ) are presented in Table-1.1. In table-1.2 are presented the inter-correlation matrix,  $R_1$ , for the variables [ $Z_1$   $x_1$   $x_2$   $x_3$   $x_4$   $x_5$   $x_6$   $x_7$ ]. The last two rows of Table-1.2 are the weight ( $w$ ) vector used to obtain  $Y = Xw$  and factor loadings, that is,  $r(Y, x_j)$ . The sum of squares of factor loadings ( $S_1$ ) = 4.352171. the measure of representativeness of  $Z_1$  that is  $F_1 = \sum_{j=1}^7 r^2(Z_1, x_j) = 4.287558$ .

In Table-1.3 we have presented the inter-correlation matrix,  $R_2$ , for variables [ $Z_2$   $x_1$   $x_2$   $x_3$   $x_4$   $x_5$   $x_6$   $x_7$ ], weights and the factor loadings when the same dataset (as mentioned above) is subjected to the direct maximization of  $\sum_{j=1}^7 r^2(Z_2, x_j)$ . The weights and the factor loadings relate to  $Y' = Xv$  and  $r(Z_2, x_j)$ . The sum of square of factor loadings ( $S_2$ )= 4.287902 and the measure of representativeness of  $Z_2$  that is  $F_2 = \sum_{j=1}^7 r^2(Z_2, x_j)$  also is 4.287902. Since  $F_2 > F_1$ , the sub-optimality of the PC-based  $F_1$  for this dataset is demonstrated. Notably, the candidates #8, #20, #21 and #26 are rank ordered differently by the two methods.

**Example-2:** The simulated data and  $Y$ ,  $Y'$ ,  $Z_1$  and  $Z_2$  for this dataset are presented in Table-2.1. The inter-correlation matrices,  $R_1$  and  $R_2$  and the associated weights and factor loadings also are presented in Tables-2.2 and 2.3. The values of  $F_1$  and  $F_2$  for this dataset are 3.120505 and 3.124149 respectively. This also shows the sub-optimality of the PC-based  $F_1$ . The candidates #8, #16, #18, #21, #23, #27, #28 and #29 are rank ordered differently by the two methods.

**Example-3:** One more simulated dataset and  $Y$ ,  $Y'$ ,  $Z_1$  and  $Z_2$  for this dataset are presented in Table-3.1. The inter-correlation matrices,  $R_1$  and  $R_2$  and the associated weights and factor loadings also are presented in Tables-3.2 and 3.3. The values of  $F_1$  and  $F_2$  for this dataset are 2.424195 and 2.426101

respectively. Once again, it is demonstrated that the PC-based  $F_1$  is sub-optimal. The candidates #8, #13, #15, #19, and #26 are rank ordered differently by the two methods.

**4. Conclusion:** The three numerical examples suffice to demonstrate that the Principal Component based rankings scores,  $Z_1$ , obtained by maximization of  $\sum_{j=1}^m r^2(Y, x_j): Y = XW$  are not the best rank order scores and do not optimally represent the dataset from which they have been derived. Its alternative is to obtain  $Z_2$  by the direct optimization of  $\sum_{j=1}^m r^2(Z_2, x_j) = \sum_{j=1}^m r^2(\Re(Y'), x_j): Y' = XV$ . Although this direct optimization problem is extremely nonlinear and complicated, it can be accomplished by the use of advanced methods such as the Differential Evaluation method of global optimization.

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**Note:** The Fortran Computer program for this exercise may also be obtained from [mishrasknehu@yahoo.com](mailto:mishrasknehu@yahoo.com)  
(The appended program has been run with Norm=2, Entropy=0, Ncor=0)

**Table-1.1: Dataset Relating to Example-1 Showing Sub-optimality of PC-based Rank-ordering of Objects**

Sl. No.	Ranking Scores of 30 candidates awarded by Seven Evaluators							Composite Score (Y) Optimized Results		Rank-Order (Z <sub>2</sub> ) Optimized Results	
	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>	Y	Z <sub>1</sub>	Y'	Z <sub>2</sub>
1	1	10	3	1	1	6	8	11.22414	3	11.31748	3
2	4	9	12	14	11	5	1	21.21812	5	20.12333	5
3	28	18	20	25	27	15	30	61.89764	26	61.72889	26
4	23	29	15	18	30	17	29	60.44481	25	60.21783	25
5	11	19	18	26	20	23	26	54.18963	22	52.19204	22
6	26	27	28	24	29	28	18	67.29609	28	64.74165	28
7	18	25	30	21	16	18	24	57.60993	24	57.02551	24
8	8	16	9	15	15	27	12	37.83836	12	34.81273	11
9	5	21	26	23	23	9	15	46.22697	19	44.92322	19
10	16	17	11	16	14	20	19	42.55673	16	41.42991	16
11	22	15	21	20	17	19	13	47.86480	20	46.33942	20
12	25	12	22	22	19	30	21	56.74565	23	54.07614	23
13	15	23	16	27	10	8	14	43.60476	17	43.97609	17
14	21	4	25	9	22	16	16	42.06427	15	40.35769	15
15	24	26	27	28	13	29	25	65.25874	27	63.76723	27
16	29	24	29	30	21	24	28	70.25956	30	69.24598	30
17	3	8	13	8	3	13	17	24.69261	7	23.98144	7
18	12	30	14	12	12	14	11	39.33102	14	39.07950	14
19	14	1	5	3	2	1	9	13.52068	4	14.54720	4
20	17	5	7	17	8	26	20	37.82410	11	35.84812	12
21	2	3	1	2	4	12	4	10.12451	2	8.71359	1
22	20	28	8	13	25	21	23	51.38784	21	50.56218	21
23	10	7	6	7	9	22	5	24.16558	6	21.65172	6
24	9	14	17	5	18	2	2	24.73667	8	24.30123	8
25	30	20	23	19	6	11	6	43.86090	18	44.38925	18
26	6	2	2	4	7	3	3	10.08933	1	9.77266	2
27	13	13	10	11	28	7	22	38.94057	13	38.59856	13
28	19	6	19	6	5	10	7	27.10184	10	26.86406	10
29	27	22	24	29	26	25	27	68.06344	29	66.42601	29
30	7	11	4	10	24	4	10	26.02995	9	25.28209	9

**Table-1.2: Inter-Correlation Matrix, Weights and Factor Loadings of Composite Score Optimized Overall Ranking Scores for the Dataset in Example-1 (F<sub>1</sub>=4.287558; S<sub>1</sub>= 4.352171)**

	Z <sub>1</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>
Z <sub>1</sub>	1.000000	<b>0.805117</b>	<b>0.781980</b>	<b>0.801112</b>	<b>0.891880</b>	<b>0.690768</b>	<b>0.658287</b>	<b>0.824694</b>
X <sub>1</sub>	<b>0.805117</b>	1.000000	0.474082	0.666741	0.645384	0.451390	0.537709	0.597330
X <sub>2</sub>	<b>0.781980</b>	0.474082	1.000000	0.554616	0.688543	0.552392	0.409121	0.569299
X <sub>3</sub>	<b>0.801112</b>	0.666741	0.554616	1.000000	0.731257	0.438487	0.426919	0.491880
X <sub>4</sub>	<b>0.891880</b>	0.645384	0.688543	0.731257	1.000000	0.526140	0.606229	0.708120
X <sub>5</sub>	<b>0.690768</b>	0.451390	0.552392	0.438487	0.526140	1.000000	0.324583	0.630256
X <sub>6</sub>	<b>0.658287</b>	0.537709	0.409121	0.426919	0.606229	0.324583	1.000000	0.608009
X <sub>7</sub>	<b>0.824694</b>	0.597330	0.569299	0.491880	0.708120	0.630256	0.608009	1.000000
Weights		0.381563	0.369979	0.377237	0.430743	0.337585	0.337890	0.401926
Loadings		0.796012	0.771845	0.786986	0.898611	0.704264	0.704901	0.838493

	Z <sub>2</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>
Z <sub>2</sub>	1.000000	<b>0.810901</b>	<b>0.776641</b>	<b>0.800667</b>	<b>0.893660</b>	<b>0.688988</b>	<b>0.653838</b>	<b>0.827809</b>
X <sub>1</sub>	<b>0.810901</b>	1.000000	0.474082	0.666741	0.645384	0.451390	0.537709	0.597330
X <sub>2</sub>	<b>0.776641</b>	0.474082	1.000000	0.554616	0.688543	0.552392	0.409121	0.569299
X <sub>3</sub>	<b>0.800667</b>	0.666741	0.554616	1.000000	0.731257	0.438487	0.426919	0.491880
X <sub>4</sub>	<b>0.893660</b>	0.645384	0.688543	0.731257	1.000000	0.526140	0.606229	0.708120
X <sub>5</sub>	<b>0.688988</b>	0.451390	0.552392	0.438487	0.526140	1.000000	0.324583	0.630256
X <sub>6</sub>	<b>0.653838</b>	0.537709	0.409121	0.426919	0.606229	0.324583	1.000000	0.608009
X <sub>7</sub>	<b>0.827809</b>	0.597330	0.569299	0.491880	0.708120	0.630256	0.608009	1.000000
Weights		0.441363	0.418945	0.360646	0.411243	0.274724	0.203625	0.462126
Loadings		0.810901	0.776641	0.800667	0.89366	0.688988	0.653838	0.827809

Sl No.	Ranking Scores of 30 candidates awarded by Seven Evaluators							Composite Score (Y) Optimized Results		Rank-Order (Z <sub>2</sub> ) Optimized Results	
	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>	Y	Z <sub>1</sub>	Y'	Z <sub>2</sub>
1	10	5	10	7	5	18	18	27.96401	9	27.66608	9
2	23	25	30	13	25	24	22	61.09793	26	60.93414	26
3	11	4	15	3	3	12	3	20.05915	3	19.25318	3
4	29	7	24	30	29	28	21	65.03329	27	64.65080	27
5	12	13	2	4	6	23	14	27.23433	8	27.55713	8
6	3	1	6	11	11	15	1	18.47398	2	18.61630	2
7	17	27	9	18	1	14	25	39.62596	13	40.64115	13
8	20	10	18	28	17	21	10	47.14993	20	47.27035	19
9	24	19	19	12	23	6	13	43.75459	17	43.78827	17
10	22	21	26	19	30	27	30	66.20587	28	66.35746	28
11	13	12	12	16	24	30	12	44.98232	18	45.39889	18
12	21	24	7	23	10	19	29	48.43576	24	49.56098	24
13	5	2	16	9	15	2	16	25.42514	7	25.15835	7
14	27	18	21	29	18	16	15	54.18113	25	54.42185	25
15	15	23	11	1	4	7	8	24.68224	6	24.91456	6
16	16	15	28	6	20	20	4	41.87725	16	41.18156	15
17	4	16	1	2	2	5	6	11.99989	1	12.74124	1
18	6	26	25	21	14	13	26	47.83244	21	48.71014	22
19	25	9	4	5	8	11	23	32.41507	11	32.13677	11
20	18	11	3	10	21	10	19	34.62240	12	35.12062	12
21	7	30	20	15	27	9	24	47.94087	22	49.26167	23
22	30	22	29	20	26	29	28	69.86155	30	69.59827	30
23	26	8	14	14	9	17	20	41.57229	15	41.00634	14
24	9	20	8	26	13	1	5	29.02329	10	30.49082	10
25	1	6	13	25	12	4	2	23.40574	5	24.09491	5
26	28	17	23	27	28	26	27	66.86960	29	66.99722	29
27	14	29	27	22	19	8	9	46.79227	19	47.55844	20
28	8	28	17	24	7	25	7	41.44998	14	42.63972	16
29	19	14	22	17	16	22	17	48.27078	23	48.05095	21
30	2	3	5	8	22	3	11	20.66025	4	21.15313	4

Table-2.2: Inter-Correlation Matrix, Weights and Factor Loadings of Composite Score Optimized Overall Ranking Scores for the Dataset in Example-2 ( $F_1=3.120505$ ; $S_1= 3.215147$ )								
	$Z_1$	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$
$Z_1$	1.000000	<b>0.726363</b>	<b>0.499889</b>	<b>0.739266</b>	<b>0.636040</b>	<b>0.696997</b>	<b>0.650723</b>	<b>0.694772</b>
$X_1$	<b>0.726363</b>	1.000000	0.189766	0.419355	0.315684	0.393548	0.554616	0.568409
$X_2$	<b>0.499889</b>	0.189766	1.000000	0.388654	0.295217	0.125695	0.126585	0.353949
$X_3$	<b>0.739266</b>	0.419355	0.388654	1.000000	0.439377	0.617798	0.426919	0.276085
$X_4$	<b>0.636040</b>	0.315684	0.295217	0.439377	1.000000	0.417575	0.314794	0.245384
$X_5$	<b>0.696997</b>	0.393548	0.125695	0.617798	0.417575	1.000000	0.348165	0.370857
$X_6$	<b>0.650723</b>	0.554616	0.126585	0.426919	0.314794	0.348165	1.000000	0.409566
$X_7$	<b>0.694772</b>	0.568409	0.353949	0.276085	0.245384	0.370857	0.409566	1.000000
Weights		0.417835	0.262744	0.427355	0.351103	0.398451	0.384622	0.379179
Loadings		0.749214	0.471122	0.766283	0.629557	0.714456	0.689659	0.679899

Table-2.3: Inter-Correlation Matrix, Weights and Factor Loadings of Rank Order Optimized Overall Ranking Scores for the Dataset in Example-2 ( $F_2=3.124149$ ; $S_2=3.124149$ )								
	$Z_2$	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$
$Z_2$	1.000000	<b>0.701001</b>	<b>0.535484</b>	<b>0.740156</b>	<b>0.646719</b>	<b>0.695217</b>	<b>0.640934</b>	<b>0.696997</b>
$X_1$	<b>0.701001</b>	1.000000	0.189766	0.419355	0.315684	0.393548	0.554616	0.568409
$X_2$	<b>0.535484</b>	0.189766	1.000000	0.388654	0.295217	0.125695	0.126585	0.353949
$X_3$	<b>0.740156</b>	0.419355	0.388654	1.000000	0.439377	0.617798	0.426919	0.276085
$X_4$	<b>0.646719</b>	0.315684	0.295217	0.439377	1.000000	0.417575	0.314794	0.245384
$X_5$	<b>0.695217</b>	0.393548	0.125695	0.617798	0.417575	1.000000	0.348165	0.370857
$X_6$	<b>0.640934</b>	0.554616	0.126585	0.426919	0.314794	0.348165	1.000000	0.409566
$X_7$	<b>0.696997</b>	0.568409	0.353949	0.276085	0.245384	0.370857	0.409566	1.000000
Weights		0.383664	0.31504	0.378173	0.385789	0.414933	0.379452	0.381511
Loadings		0.701001	0.535484	0.740156	0.646719	0.695217	0.640934	0.696997

Table-3.1: Dataset Relating to Example-3 Showing Sub-optimality of PC-based Rank-ordering of Objects												
Sl No.	Ranking Scores of 30 candidates awarded by Seven Evaluators							Composite Score (Y) Optimized Results		Rank-Order ( $Z_2$ ) Optimized Results		
	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$	Y	$Z_1$	Y'	$Z_2$	
1	4	5	27	18	2	20	11	32.10776	10	31.59814	10	
2	8	28	30	29	17	29	13	59.10295	26	58.52745	26	
3	2	22	13	3	1	1	20	22.97097	3	21.15144	3	
4	24	23	9	26	30	25	28	62.42732	28	61.62047	28	
5	25	25	22	5	22	6	1	39.50937	18	40.81833	18	
6	27	30	16	24	28	27	30	68.43141	30	67.46975	30	
7	22	2	19	12	16	4	16	32.23785	11	32.42260	11	
8	3	8	20	2	9	21	18	30.81457	6	30.37723	8	
9	18	19	8	21	8	14	7	35.11395	14	34.68187	14	
10	14	6	15	4	15	3	29	30.98154	7	29.98579	7	
11	26	4	4	1	7	7	3	17.61105	1	18.44842	1	
12	21	7	21	14	4	22	10	35.41392	15	35.50598	15	
13	1	20	7	16	5	13	21	31.73604	9	29.76353	6	
14	19	3	2	8	14	2	2	17.95374	2	18.75359	2	

15	10	10	14	11	20	15	22	38.67293	17	38.17462	16
16	13	11	18	22	26	11	23	46.74022	20	46.21905	20
17	20	12	17	20	6	10	8	33.26055	13	33.02942	13
18	16	13	29	27	21	8	17	48.42594	21	48.12730	21
19	12	17	6	7	11	16	14	31.43870	8	30.92993	9
20	6	18	5	28	3	5	9	27.61333	5	26.12312	5
21	29	27	23	30	24	18	27	65.74003	29	64.76105	29
22	30	26	26	9	19	19	19	54.32872	25	54.40519	25
23	15	1	11	17	13	17	15	32.69124	12	32.54366	12
24	28	16	10	23	23	9	26	49.29651	22	48.46761	22
25	5	29	3	19	10	23	25	44.09388	19	41.91719	19
26	9	9	12	6	29	24	6	37.86774	16	39.22546	17
27	7	14	1	15	12	12	4	25.41235	4	25.27022	4
28	17	15	25	25	18	28	12	52.66056	24	52.81574	24
29	23	21	28	13	25	26	24	59.89362	27	59.87255	27
30	11	24	24	10	27	30	5	51.48540	23	52.63350	23

**Table-3.2: Inter-Correlation Matrix, Weights and Factor Loadings of Composite Score Optimized Overall Ranking Scores for the Dataset in Example-3**  
( $F_1=2.424195$ ;  $S_1=2.521362$ )

	Z <sub>1</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>
Z <sub>1</sub>	1.000000	<b>0.473637</b>	<b>0.612013</b>	<b>0.566185</b>	<b>0.562180</b>	<b>0.741046</b>	<b>0.654283</b>	<b>0.459844</b>
X <sub>1</sub>	<b>0.473637</b>	1.000000	0.108788	0.187542	0.132814	0.470078	-0.006007	0.119911
X <sub>2</sub>	<b>0.612013</b>	0.108788	1.000000	0.163070	0.360623	0.303226	0.402447	0.309010
X <sub>3</sub>	<b>0.566185</b>	0.187542	0.163070	1.000000	0.180868	0.296552	0.389544	0.126585
X <sub>4</sub>	<b>0.562180</b>	0.132814	0.360623	0.180868	1.000000	0.200000	0.284983	0.289433
X <sub>5</sub>	<b>0.741046</b>	0.470078	0.303226	0.296552	0.200000	1.000000	0.384650	0.315684
X <sub>6</sub>	<b>0.654283</b>	-0.006007	0.402447	0.389544	0.284983	0.384650	1.000000	0.156396
X <sub>7</sub>	<b>0.459844</b>	0.119911	0.309010	0.126585	0.289433	0.315684	0.156396	1.000000
Weights		0.271198	0.414408	0.345937	0.366429	0.458344	0.416812	0.341998
Loadings		0.430630	0.658031	0.549306	0.581846	0.727796	0.661847	0.543052

**Table-3.3: Inter-Correlation Matrix, Weights and Factor Loadings of Rank Order Optimized Overall Ranking Scores for the Dataset in Example-3**  
( $F_2=2.426101$ ;  $S_2=2.426101$ )

	Z <sub>2</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>
Z <sub>2</sub>	1.000000	<b>0.479867</b>	<b>0.599555</b>	<b>0.576418</b>	<b>0.543493</b>	<b>0.751279</b>	<b>0.666741</b>	<b>0.446941</b>
X <sub>1</sub>	<b>0.479867</b>	1.000000	0.108788	0.187542	0.132814	0.470078	-0.006007	0.119911
X <sub>2</sub>	<b>0.599555</b>	0.108788	1.000000	0.163070	0.360623	0.303226	0.402447	0.309010
X <sub>3</sub>	<b>0.576418</b>	0.187542	0.163070	1.000000	0.180868	0.296552	0.389544	0.126585
X <sub>4</sub>	<b>0.543493</b>	0.132814	0.360623	0.180868	1.000000	0.200000	0.284983	0.289433
X <sub>5</sub>	<b>0.751279</b>	0.470078	0.303226	0.296552	0.200000	1.000000	0.384650	0.315684
X <sub>6</sub>	<b>0.666741</b>	-0.006007	0.402447	0.389544	0.284983	0.384650	1.000000	0.156396
X <sub>7</sub>	<b>0.446941</b>	0.119911	0.309010	0.126585	0.289433	0.315684	0.156396	1.000000
Weights		0.297447	0.386881	0.363218	0.338013	0.508040	0.430645	0.268531
Loadings		0.479867	0.599555	0.576418	0.543493	0.751279	0.666741	0.446941

```

1: C      MAIN PROGRAM : PROVIDES TO USE DIFFERENTIAL EVOLUTION METHOD TO
2: C                      COMPUTE COMPOSITE INDEX INDICES
3: C      BY MAXIMIZING SUM OF (SQUARES, OR ABSOLUTES, OR MINIMUM) OF
4: C      CORRELATION OF THE INDEX WITH THE CONSTITUENT VARIABLES. THE MAX
5: C      SUM OF SQUARES IS THE PRINCIPAL COMPONENT INDEX. IT ALSO PROVIDES
6: C      TO OBTAIN MAXIMUM ENTROPY ABSOLUTE CORRELATION INDICES.
7: C      PRODUCT MOMENT AS WELL AS ABSOLUTE CORRELATION (BRADLEY, 1985) MAY
8: C      BE USED. PROGRAM BY SK MISHRA, DEPT. OF ECONOMICS, NORTH-EASTERN
9: C      HILL UNIVERSITY, SHILLONG (INDIA)
10: C      -----
11: C      ADJUST THE PARAMETERS SUITABLY IN SUBROUTINES DE
12: C      WHEN THE PROGRAM ASKS FOR PARAMETERS, FEED THEM SUITABLY
13: C      -----
14: C      PROGRAM DERPSINDEX
15: C      PARAMETER (NOB=30,MVAR=7)!CHANGE THE PARAMETERS HERE AS NEEDED.
16: C      -----
17: C      NOB=NO. OF CASES AND MVAR=NO. OF VARIABLES
18: C      TO BE ADJUSTED IN SUBROUTINE CORD(M,X,F) ALSO: STATEMENT 931
19: C      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
20: C      COMMON /KFF/KF,NFCALL,FTIT ! FUNCTION CODE, NO. OF CALLS & TITLE
21: C      CHARACTER *30 METHOD(1)
22: C      CHARACTER *70 FTIT
23: C      CHARACTER *40 INFILE,OUTFILE
24: C      COMMON /CORDAT/CDAT (NOB,MVAR) , QIND (NOB) , R (MVAR) , ENTROPY,NORM,NCOR
25: C      COMMON /XBASE/XBAS
26: C      COMMON /RNDM/IU,IV ! RANDOM NUMBER GENERATION (IU = 4-DIGIT SEED)
27: C      COMMON /GETRANK/MRNK
28: C      INTEGER IU,IV
29: C      DIMENSION XX(3,50) , KKF(3) , MM(3) , FMINN(3) , XBAS(1000,50)
30: C      DIMENSION ZDAT (NOB,MVAR+1) , FRANK (NOB) , RMAT (MVAR+1,MVAR+1)
31: C      DIMENSION X(50)! X IS THE DECISION VARIABLE X IN F(X) TO MINIMIZE
32: C      M IS THE DIMENSION OF THE PROBLEM, KF IS TEST FUNCTION CODE AND
33: C      FMIN IS THE MIN VALUE OF F(X) OBTAINED FROM DE OR RPS
34: C      WRITE (*,*) '===== WARNING ===== '
35: C      WRITE (*,*) 'ADJUST PARAMETERS IN SUBROUTINES DE IF NEEDED '
36: C      NOPT=1 ! OPTIMIZATION BY DE METHOD
37: C      WRITE (*,*) '===== '
38: C      METHOD(1)=' : DIFFERENTIAL EVOLUTION'
39: C      INITIALIZATION. THIS XBAS WILL BE USED TO
40: C      INITIALIZE THE POPULATION.
41: C      WRITE (*,*) ' '
42: C      WRITE (*,*) 'FEED RANDOM NUMBER SEED,NORM,ENTROPY,NCOR'
43: C      WRITE (*,*) 'SEED[ANY 4-DIGIT NUMBER]; NORM[1,2,3]; ENTROPY[0,1]; &
44: C      &NCOR[0,1] '
45: C      WRITE (*,*) ' '
46: C      WRITE (*,*) 'NORM(1)=ABSOLUTE;NORM(2)=PCA-EUCLIDEAN;NORM(3)=MAXIMIN'
47: C      WRITE (*,*) 'ENTROPY(0)=MAXIMIZES NORM;ENTROPY(1)=MAXIMIZES ENTROPY'
48: C      WRITE (*,*) 'NCOR(0)=PRODUCT MOMENT; NCOR(1)=ABSOLUTE CORRELATION'
49: C      READ (*,*) IU,NORM,ENTROPY,NCOR
50: C      WRITE (*,*) 'WANT RANK SCORE OPTIMIZATION? YES(1); NO(OTHER THAN 1)'
51: C      READ (*,*) MRNK
52: C      WRITE (*,*) 'INPUT FILE TO READ DATA:YOUR DATA MUST BE IN THIS FILE'
53: C      WRITE (*,*) 'CASES (NOB) IN ROWS ; VARIABLES (MVAR) IN COLUMNS'
54: C      READ (*,*) INFILE
55: C      WRITE (*,*) 'SPECIFY THE OUTPUT FILE TO STORE THE RESULTS'
56: C      READ (*,*) OUTFILE
57: C      OPEN (9, FILE=OUTFILE)
58: C      OPEN (7, FILE=INFILE)
59: C      DO I=1,NOB
60: C      READ (7,*) ,CDA, (CDAT (I, J) , J=1,MVAR)
61: C      ENDDO
62: C      CLOSE (7)
63: C      DO I=1,NOB
64: C      DO J=1,MVAR
65: C      ZDAT (I, J+1)=CDAT (I, J)
66: C      ENDDO
67: C      ENDDO

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68:      WRITE(*,*) 'DATA HAS BEEN READ. WOULD YOU UNITIZE VARIABLES? [YES=1
69: & ELSE NO UNITIZATION] '
70:      WRITE(*,*) 'UNITIZE MEANS TRANSFORMATION FROM X(I,J) TO UNITIZED X'
71:      WRITE(*,*) '[X(I,J)-MIN(X(.,J))]/[MAX(X(.,J))-MIN(X(.,J))]'
72:      READ(*,*) NUN
73:      IF(NUN.EQ.1) THEN
74:      DO J=1,MVAR
75:      CMIN=CDAT(1,J)
76:      CMAX=CDAT(1,J)
77:      DO I=2,NOB
78:      IF(CMIN.GT.CDAT(I,J)) CMIN=CDAT(I,J)
79:      IF(CMAX.LT.CDAT(I,J)) CMAX=CDAT(I,J)
80:      ENDDO
81:      DO I=1,NOB
82:      CDAT(I,J)=(CDAT(I,J)-CMIN)/(CMAX-CMIN)
83:      ENDDO
84:      ENDDO
85:      ENDIF
86: C -----
87:      WRITE(*,*) ' '
88:      WRITE(*,*) 'FEED RANDOM NUMBER SEED [4-DIGIT ODD INTEGER] TO BEGIN'
89:      READ(*,*) IU
90: C THIS XBAS WILL BE USED AS INITIAL X
91:      DO I=1,1000
92:      DO J=1,50
93:      CALL RANDOM(RAND)
94:      XBAS(I,J)=RAND ! RANDOM NUMBER BETWEEN (0, 1)
95:      ENDDO
96:      ENDDO
97: C -----
98: C HOWEVER, THE FIRST ROW OF, THAT IS, XBAS(1,J),J=1,MVAR) MAY BE
99: C SPECIFIED HERE IF THE USER KNOWS IT TO BE OPTIMAL OR NEAR-OPTIMAL
100: C DATA (XBAS(1,J),J=1,MVAR) /DATA1, DATA2, ....., DATAMVAR/
101: C -----
102:      WRITE(*,*) ' *****'
103: C -----
104:      DO I=1,NOPT
105:      IF(I.EQ.1) THEN
106:      WRITE(*,*) '==== WELCOME TO DE/RPS PROGRAM FOR INDEX CONSTRUCTION'
107:      CALL DE(M,X,FMINDE,Q0,Q1) !CALLS DE AND RETURNS OPTIMAL X AND FMIN
108:      IF(KF.EQ.1) THEN
109:      WRITE(9,*) 'DIFFERENTIAL EVALUATION OPTIMIZATION RESULTS'
110:      RSUM1=0.D0
111:      RSUM2=0.D0
112:      DO J=1,MVAR
113:      RSUM1=RSUM1+DABS(R(J))
114:      RSUM2=RSUM2+DABS(R(J))**2
115:      ENDDO
116:      WRITE(9,*) 'CORRELATION OF INDEX WITH CONSTITUENT VARIABLES'
117:      WRITE(9,*) (R(J),J=1,MVAR)
118:      WRITE(9,*) 'SUM OF ABS (R)=' ,RSUM1, '; SUM OF SQUARE(R)=' ,RSUM2
119:      WRITE(9,*) 'THE INDEX OR SCORE OF DIFFERENT CASES'
120:      DO II=1,NOB
121:      WRITE(9,*) QIND(II)
122:      FRANK(II)=QIND(II)
123:      ENDDO
124:      ENDIF
125:      FMIN=FMINDE
126:      ENDIF
127: C -----
128:      DO J=1,M
129:      XX(I,J)=X(J)
130:      ENDDO
131:      KKF(I)=KF
132:      MM(I)=M
133:      FMINN(I)=FMIN
134:      ENDDO

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135:      WRITE (*,*) ' '
136:      WRITE (*,*) ' '
137:      WRITE (*,*) '----- FINAL RESULTS-----'
138:      DO I=1,NOPT
139:      WRITE (*,*) 'FUNCT CODE=',KKF(I), ' FMIN=',FMINN(I), ' : DIM=',MM(I)
140:      WRITE (*,*) 'OPTIMAL DECISION VARIABLES : ',METHOD(I)
141:      WRITE (*,*) 'WEIGHTS ARE AS FOLLOWS -----'
142:      WRITE (9,*) 'WEIGHTS ARE AS FOLLOWS -----'
143:      WRITE (9,*) (XX(I,J),J=1,M)
144:      WRITE (*,*) (XX(I,J),J=1,M)
145:      WRITE (*,*) '/////////////////////////////////////'
146:      ENDDO
147:      WRITE (*,*) 'OPTIMIZATION PROGRAM ENDED'
148:      WRITE (*,*) '*****'
149:      WRITE (*,*) 'MEASURE OF EQUALITY/INEQUALITY'
150:      WRITE (*,*) 'DE: BEFORE AND AFTER OPTIMIZATION = ',Q0,Q1
151:      WRITE (*,*) ' '
152:      WRITE (*,*) 'RESULTS STORED IN FILE= ',OUTFILE
153:      WRITE (*,*) 'OPEN BY MSWORD OR EDIT OR ANY OTHER EDITOR'
154:      WRITE (*,*) ' '
155:      WRITE (*,*) 'NOTE:VECTORS OF CORRELATIONS & INDEX(BOTH TOGETHER) ARE
156: & IDETERMINATE FOR SIGN & MAY BE MULTIPLIED BY (-1) IF NEEDED'
157:      WRITE (*,*) 'THAT IS IF R(J) IS TRANSFORMED TO -R(J) FOR ALL J THEN
158: &THE INDEX(I) TOO IS TRANSFORMED TO -INDEX(I) FOR ALL I'
159:      WRITE (9,*) ' '
160:      WRITE (9,*) 'NOTE: VECTORS OF CORRELATIONS AND INDEX (BOTH TOGETHER)
161: & ARE IDETERMINATE FOR SIGN AND MAY BE MULTIPLIED BY (-1) IF NEEDED'
162:      WRITE (9,*) 'THAT IS IF R(J) IS TRANSFORMED TO -R(J) FOR ALL J THEN
163: &THE INDEX(I) TOO IS TRANSFORMED TO -INDEX(I) FOR ALL I'
164:      CALL DORANK(FRANK,NOB)
165:      DO I=1,NOB
166:      ZDAT(I,1)=FRANK(I)
167:      ENDDO
168:      IF(NCOR.EQ.0) THEN
169:      CALL CORREL(ZDAT,NOB,MVAR+1,RMAT)
170:      ELSE
171:      CALL DOCORA(ZDAT,NOB,MVAR+1,RMAT)
172:      ENDIF
173:      WRITE (9,*) '----- CORRELATION MATRIX -----'
174:      WRITE (*,*) '----- CORRELATION MATRIX -----'
175:      DO I=1,MVAR+1
176:      WRITE (9,1) (RMAT(I,J),J=1,MVAR+1)
177:      WRITE (*,1) (RMAT(I,J),J=1,MVAR+1)
178:      ENDDO
179:      1 FORMAT(8F10.6)
180:      WRITE (9,*) '===== '
181:      WRITE (*,*) '===== '
182:      WRITE (9,*) 'VARIABLES: 1ST IS THE INDEX AND 2ND THE RANK OF INDEX'
183:      WRITE (*,*) 'VARIABLES: 1ST IS THE INDEX AND 2ND THE RANK OF INDEX'
184:      WRITE (9,*) '===== '
185:      WRITE (*,*) '===== '
186:      DO I=1,NOB
187:      IF(MRNK.EQ.1) THEN
188:      QIND(I)=0.D0
189:      DO J=1,MVAR
190:      QIND(I)=QIND(I)+ZDAT(I,J+1)*XX(NOPT,J)
191:      ENDDO
192:      ENDIF
193:      WRITE (9,2) I,QIND(I), (ZDAT(I,J),J=1,MVAR+1)
194:      WRITE (*,2) I,QIND(I), (ZDAT(I,J),J=1,MVAR+1)
195:      ENDDO
196:      2 FORMAT(I3,F12.6,13F5.0)
197:      SR2=0.D0
198:      IF(NORM.LE.2) THEN
199:      DO J=2,MVAR+1
200:      SR2=SR2+DABS(RMAT(1,J))**NORM
201:      ENDDO

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202:      WRITE(9,*) 'SUM OF SQUARE OF CORRELATION R(RANK(INDEX),VAR)=' ,SR2
203:      WRITE(*,*) 'SUM OF SQUARE OF CORRELATION R(RANK(INDEX),VAR)=' ,SR2
204:      ELSE ! GIVES MAXIMAL CORRELATION
205:      DO J=2,MVAR+1
206:      IF(SR2.LT.DABS(RMAT(1,J))) SR2=DABS(RMAT(1,J))
207:      ENDDO
208:      WRITE(9,*) 'MAXIMAL CORRELATION R(RANK(INDEX),VAR)=' ,SR2
209:      WRITE(*,*) 'MAXIMAL CORRELATION R(RANK(INDEX),VAR)=' ,SR2
210:      ENDF
211:      CLOSE(9)
212:      WRITE(*,*) 'THE JOB IS OVER'
213:      END
214:  C
215:      SUBROUTINE DE(M,A,FBEST,G0,G1)
216:  C      PROGRAM: "DIFFERENTIAL EVOLUTION ALGORITHM" OF GLOBAL OPTIMIZATION
217:  C      THIS METHOD WAS PROPOSED BY R. STORN AND K. PRICE IN 1995. REF --
218:  C      "DIFFERENTIAL EVOLUTION - A SIMPLE AND EFFICIENT ADAPTIVE SCHEME
219:  C      FOR GLOBAL OPTIMIZATION OVER CONTINUOUS SPACES" : TECHNICAL REPORT
220:  C      INTERNATIONAL COMPUTER SCIENCE INSTITUTE, BERKLEY, 1995.
221:  C      PROGRAM BY SK MISHRA, DEPT. OF ECONOMICS, NEHU, SHILLONG (INDIA)
222:  C      -----
223:  C      PROGRAM DE
224:      IMPLICIT DOUBLE PRECISION (A-H, O-Z) ! TYPE DECLARATION
225:      PARAMETER(NMAX=500,MMAX=50) ! MAXIMUM DIMENSION PARAMETERS
226:      PARAMETER(RX1=1.0D0, RX2=1.0D0)!TO BE ADJUSTED SUITABLY,IF NEEDED
227:  C      RX1 AND RX2 CONTROL THE SCHEME OF CROSSOVER. (0 <= RX1 <= RX2) <=1
228:  C      RX1 DETERMINES THE UPPER LIMIT OF SCHEME 1 (AND LOWER LIMIT OF
229:  C      SCHEME 2; RX2 IS THE UPPER LIMIT OF SCHEME 2 AND LOWER LIMIT OF
230:  C      SCHEME 3. THUS RX1 = .2 AND RX2 = .8 MEANS 0-20% SCHEME1, 20 TO 80
231:  C      PERCENT SCHEME 2 AND THE REST (80 TO 100 %) SCHEME 3.
232:  C      PARAMETER(NCROSS=2) ! CROSS-OVER SCHEME (NCROSS <=0 OR =1 OR =>2)
233:      PARAMETER(IPRINT=100,EPS=1.D-10)!FOR WATCHING INTERMEDIATE RESULTS
234:  C      IT PRINTS THE INTERMEDIATE RESULTS AFTER EACH IPRINT ITERATION AND
235:  C      EPS DETERMINES ACCURACY FOR TERMINATION. IF EPS= 0, ALL ITERATIONS
236:  C      WOULD BE UNDERGONE EVEN IF NO IMPROVEMENT IN RESULTS IS THERE.
237:  C      ULTIMATELY "DID NOT CONVERGE" IS REOPORTED.
238:      COMMON /RNDM/IU,IV ! RANDOM NUMBER GENERATION (IU = 4-DIGIT SEED)
239:      INTEGER IU,IV ! FOR RANDOM NUMBER GENERATION
240:      COMMON /KFF/KF,NFCALL,FTIT ! FUNCTION CODE, NO. OF CALLS * TITLE
241:      COMMON /XBASE/XBAS
242:      CHARACTER *70 FTIT ! TITLE OF THE FUNCTION
243:  C
244:  C      THE PROGRAM REQUIRES INPUTS FROM THE USER ON THE FOLLOWING -----
245:  C      (1) FUNCTION CODE (KF), (2) NO. OF VARIABLES IN THE FUNCTION (M);
246:  C      (3) N=POPULATION SIZE (SUGGESTED 10 TIMES OF NO. OF VARIABLES, M,
247:  C      FOR SMALLER PROBLEMS N=100 WORKS VERY WELL);
248:  C      (4) PCROS = PROB. OF CROSS-OVER (SUGGESTED : ABOUT 0.85 TO .99);
249:  C      (5) FACT = SCALE (SUGGESTED 0.5 TO .95 OR 1, ETC);
250:  C      (6) ITER = MAXIMUM NUMBER OF ITERATIONS PERMITTED (5000 OR MORE)
251:  C      (7) RANDOM NUMBER SEED (4 DIGITS INTEGER)
252:  C      -----
253:      DIMENSION X(NMAX,MMAX),Y(NMAX,MMAX),A(MMAX),FV(NMAX)
254:      DIMENSION IR(3),XBAS(1000,50)
255:  C
256:  C      ----- SELECT THE FUNCTION TO MINIMIZE AND ITS DIMENSION -----
257:      CALL FSELECT(KF,M,FTIT)
258:  C      SPECIFY OTHER PARAMETERS -----
259:      WRITE(*,*) 'POPULATION SIZE [N] AND NO. OF ITERATIONS [ITER] ?'
260:      WRITE(*,*) 'SUGGESTED : N => 100 OR =>10.M; ITER 10000 OR SO'
261:      READ(*,*) N,ITER
262:      WRITE(*,*) 'CROSSOVER PROBABILITY [PCROS] AND SCALE [FACT] ?'
263:      WRITE(*,*) 'SUGGESTED : PCROS ABOUT 0.9; FACT=.5 OR LARGER BUT <=1'
264:      READ(*,*) PCROS,FACT
265:      WRITE(*,*) 'RANDOM NUMBER SEED ?'
266:      WRITE(*,*) 'A FOUR-DIGIT POSITIVE ODD INTEGER, SAY, 1171'
267:      READ(*,*) IU
268:      NFCALL=0 ! INITIALIZE COUNTER FOR FUNCTION CALLS

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269:         GBEST=1.D30 ! TO BE USED FOR TERMINATION CRITERION
270: C      INITIALIZATION : GENERATE X(N,M) RANDOMLY
271:         DO I=1,N
272:         DO J=1,M
273: C      CALL RANDOM(RAND) ! GENERATES INITION X WITHIN
274: C      X(I,J)=(RAND-.5D00)*2000 ! GENERATES INITION X WITHIN
275: C      RANDOM NUMBERS BETWEEN -RRANGE AND +RRANGE (BOTH EXCLUSIVE)
276: C      X(I,J)=XBAS(I,J) ! TAKES THESE NUMBERS FROM THE MAIN PROGRAM
277:         ENDDO
278:         ENDDO
279:         WRITE(*,*) 'COMPUTING --- PLEASE WAIT '
280:         IPCOUNT=0
281:         DO 100 ITR=1,ITER ! ITERATION BEGINS
282: C      -----
283: C      EVALUATE ALL X FOR THE GIVEN FUNCTION
284:         DO I=1,N
285:         DO J=1,M
286:         A(J)=X(I,J)
287:         ENDDO
288:         CALL FUNC(A,M,F)
289: C      STORE FUNCTION VALUES IN FV VECTOR
290:         FV(I)=F
291:         ENDDO
292:         IF(ITR.EQ.1) CALL GINI(FV,N,G0)
293: C      -----
294: C      FIND THE FITTEST (BEST) INDIVIDUAL AT THIS ITERATION
295:         FBEST=FV(1)
296:         KB=1
297:         DO IB=2,N
298:             IF(FV(IB).LT.FBEST) THEN
299:                 FBEST=FV(IB)
300:                 KB=IB
301:             ENDIF
302:         ENDDO
303: C      BEST FITNESS VALUE = FBEST : INDIVIDUAL X(KB)
304: C      -----
305: C      GENERATE OFFSPRINGS
306:         DO I=1,N ! I LOOP BEGINS
307: C      INITIALIZE CHILDREN IDENTICAL TO PARENTS; THEY WILL CHANGE LATER
308:         DO J=1,M
309:         Y(I,J)=X(I,J)
310:         ENDDO
311: C      SELECT RANDOMLY THREE OTHER INDIVIDUALS
312: 20         DO IRI=1,3 ! IRI LOOP BEGINS
313:             IR(IRI)=0
314:
315:             CALL RANDOM(RAND)
316:             IRJ=INT(RAND*N)+1
317: C      CHECK THAT THESE THREE INDIVIDUALS ARE DISTICT AND OTHER THAN I
318:             IF(IRI.EQ.1.AND.IRJ.NE.I) THEN
319:                 IR(IRI)=IRJ
320:             ENDIF
321:             IF(IRI.EQ.2.AND.IRJ.NE.I.AND.IRJ.NE.IR(1)) THEN
322:                 IR(IRI)=IRJ
323:             ENDIF
324:             IF(IRI.EQ.3.AND.IRJ.NE.I.AND.IRJ.NE.IR(1).AND.IRJ.NE.IR(2)) THEN
325:                 IR(IRI)=IRJ
326:             ENDIF
327:             ENDDO ! IRI LOOP ENDS
328: C      CHECK IF ALL THE THREE IR ARE POSITIVE (INTEGERS)
329:             DO IX=1,3
330:             IF(IR(IX).LE.0) THEN
331:                 GOTO 20 ! IF NOT THEN REGENERATE
332:             ENDIF
333:             ENDDO
334: C      THREE RANDOMLY CHOSEN INDIVIDUALS DIFFERENT FROM I AND DIFFERENT
335: C      FROM EACH OTHER ARE IR(1),IR(2) AND IR(3)

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336: C      ===== RANDOMIZATION OF NCROSS =====
337: C      RANDOMIZES NCROSS
338: NCROSS=0
339: CALL RANDOM(RAND)
340: IF (RAND.GT.RX1) NCROSS=1 ! IF RX1=>1, SCHEME 2 NEVER IMPLEMENTED
341: IF (RAND.GT.RX2) NCROSS=2 ! IF RX2=>1, SCHEME 3 NEVER IMPLEMENTED
342:
343: C      ----- SCHEME 1 -----
344: C      NO CROSS OVER, ONLY REPLACEMENT THAT IS PROBABILISTIC
345: IF (NCROSS.LE.0) THEN
346: DO J=1,M ! J LOOP BEGINS
347: CALL RANDOM(RAND)
348: IF (RAND.LE.PCROS) THEN ! REPLACE IF RAND < PCROS
349: A(J)=X(IR(1),J)+(X(IR(2),J)-X(IR(3),J))*FACT ! CANDIDATE CHILD
350: ENDF
351: ENDDO ! J LOOP ENDS
352: ENDF
353: C      ----- SCHEME 2 -----
354: C      THE STANDARD CROSSOVER SCHEME
355: C      CROSSOVER SCHEME (EXPONENTIAL) SUGGESTED BY KENNETH PRICE IN HIS
356: C      PERSONAL LETTER TO THE AUTHOR (DATED SEPTEMBER 29, 2006)
357: IF (NCROSS.EQ.1) THEN
358: CALL RANDOM(RAND)
359: JR=INT(RAND*M)+1
360: J=JR
361: 2 A(J)=X(IR(1),J)+FACT*(X(IR(2),J)-X(IR(3),J))
362: J=J+1
363: IF (J.GT.M) J=1
364: IF (J.EQ.JR) GOTO 10
365: CALL RANDOM(RAND)
366: IF (PCROS.LE.RAND) GOTO 2
367: 6 A(J)=X(I,J)
368: J=J+1
369: IF (J.GT.M) J=1
370: IF (J.EQ.JR) GOTO 10
371: GOTO 6
372: 10 CONTINUE
373: ENDF
374: C      ----- SCHEME 3 -----
375: C      ESPECIALLY SUITABLE TO NON-DECOMPOSABLE (NON-SEPERABLE) FUNCTIONS
376: C      CROSSOVER SCHEME (NEW) SUGGESTED BY KENNETH PRICE IN HIS
377: C      PERSONAL LETTER TO THE AUTHOR (DATED OCTOBER 18, 2006)
378: IF (NCROSS.GE.2) THEN
379: CALL RANDOM(RAND)
380: IF (RAND.LE.PCROS) THEN
381: CALL NORMAL(RN)
382: DO J=1,M
383: A(J)=X(I,J)+(X(IR(1),J)+X(IR(2),J)-2*X(I,J))*RN
384: ENDDO
385: ELSE
386: DO J=1,M
387: A(J)=X(I,J)+(X(IR(1),J)-X(IR(2),J))*FACT ! FACT ASSUMED TO BE 1
388: ENDDO
389: ENDF
390: ENDF
391: C      -----
392: CALL FUNC(A,M,F) ! EVALUATE THE OFFSPRING
393: IF (F.LT.FV(I)) THEN ! IF BETTER, REPLACE PARENTS BY THE CHILD
394: FV(I)=F
395: DO J=1,M
396: Y(I,J)=A(J)
397: ENDDO
398: ENDF
399: ENDDO ! I LOOP ENDS
400: DO I=1,N
401: DO J=1,M
402: X(I,J)=Y(I,J) ! NEW GENERATION IS A MIX OF BETTER PARENTS AND

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403: C          BETTER CHILDREN
404: ENDDO
405: ENDDO
406: DO J=1,M
407: A(J)=X(KB,J)
408: ENDDO
409: IPCOUNT=IPCOUNT+1
410: IF(IPCOUNT.EQ.IPRINT) THEN
411:
412: WRITE(*,*) (X(KB,J),J=1,M), ' FBEST UPTO NOW = ',FBEST
413: WRITE(*,*) 'TOTAL NUMBER OF FUNCTION CALLS =',NFCALL
414: IF(DABS(FBEST-GBEST).LT.EPS) THEN
415: WRITE(*,*) FTIT
416: WRITE(*,*) 'COMPUTATION OVER'
417: GOTO 999
418: ELSE
419: GBEST=FBEST
420: ENDF
421: IPCOUNT=0
422: ENDF
423: C -----
424: 100 ENDDO ! ITERATION ENDS : GO FOR NEXT ITERATION, IF APPLICABLE
425: C -----
426: WRITE(*,*) 'DID NOT CONVERGE. REDUCE EPS OR RAISE ITER OR DO BOTH'
427: WRITE(*,*) 'INCREASE N, PCROS, OR SCALE FACTOR (FACT)'
428: 999 CALL FUNC(A,M,FBEST)
429: CALL GINI(FV,N,G1)
430: RETURN
431: END
432: C -----
433: SUBROUTINE NORMAL(R)
434: C PROGRAM TO GENERATE N(0,1) FROM RECTANGULAR RANDOM NUMBERS
435: C IT USES BOX-MULLER VARIATE TRANSFORMATION FOR THIS PURPOSE.
436: C -----
437: C ----- BOX-MULLER METHOD BY GEP BOX AND ME MULLER (1958) -----
438: C BOX, G. E. P. AND MULLER, M. E. "A NOTE ON THE GENERATION OF
439: C RANDOM NORMAL DEVIATES." ANN. MATH. STAT. 29, 610-611, 1958.
440: C IF U1 AND U2 ARE UNIFORMLY DISTRIBUTED RANDOM NUMBERS (0,1),
441: C THEN X=[(-2*LN(U1))**.5]*(COS(2*PI*U2) IS N(0,1)
442: C ALSO, X=[(-2*LN(U1))**.5]*(SIN(2*PI*U2) IS N(0,1)
443: C PI = 4*ARCTAN(1.0)= 3.1415926535897932384626433832795
444: C 2*PI = 6.283185307179586476925286766559
445: C -----
446: IMPLICIT DOUBLE PRECISION (A-H,O-Z)
447: COMMON /RNDM/IU,IV
448: INTEGER IU,IV
449: C -----
450: CALL RANDOM(RAND) ! INVOKES RANDOM TO GENERATE UNIFORM RAND [0, 1]
451: U1=RAND ! U1 IS UNIFORMLY DISTRIBUTED [0, 1]
452: CALL RANDOM(RAND) ! INVOKES RANDOM TO GENERATE UNIFORM RAND [0, 1]
453: U2=RAND ! U1 IS UNIFORMLY DISTRIBUTED [0, 1]
454: R=DSQRT(-2.D0*DLOG(U1))
455: R=R*DCOS(U2*6.283185307179586476925286766559D00)
456: C R=R*DCOS(U2*6.28318530718D00)
457: RETURN
458: END
459: C -----
460: C RANDOM NUMBER GENERATOR (UNIFORM BETWEEN 0 AND 1 - BOTH EXCLUSIVE)
461: SUBROUTINE RANDOM(RAND1)
462: DOUBLE PRECISION RAND1
463: COMMON /RNDM/IU,IV
464: INTEGER IU,IV
465: IV=IU*65539
466: IF(IV.LT.0) THEN
467: IV=IV+2147483647+1
468: ENDF
469: RAND=IV

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470:      IU=IV
471:      RAND=RAND*0.4656613E-09
472:      RAND1= DBLE(RAND)
473:      RETURN
474:      END
475: C -----
476:      SUBROUTINE GINI(F,N,G)
477:      PARAMETER (K=1) !K=1 GINI COEFFICIENT; K=2 COEFFICIENT OF VARIATION
478: C THIS PROGRAM COMPUTES MEASURE OF INEQUALITY
479: C IF K =1 GET THE GINI COEFFICIENT. IF K=2 GET COEFF OF VARIATIONE
480:      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
481:      DIMENSION F(*)
482:      S=0.D0
483:      DO I=1,N
484:      S=S+F(I)
485:      ENDDO
486:      S=S/N
487:      H=0.D00
488:      DO I=1,N-1
489:      DO J=I+1,N
490:      H=H+(DABS(F(I)-F(J)))**K
491:      ENDDO
492:      ENDDO
493:      H=(H/(N**2))**(1.D0/K) ! FOR K=1 H IS MEAN DEVIATION;
494: C FOR K=2 H IS STANDARD DEVIATION
495:      WRITE(*,*)'MEASURES OF DISPERSION AND CENTRAL TENDENCY = ',G,S
496:      G=DEXP(-H) ! G IS THE MEASURE OF EQUALITY (NOT GINI OR CV)
497: C G=H/DABS(S) !IF S NOT ZERO, K=1 THEN G=GINI, K=2 G=COEFF VARIATION
498:      RETURN
499:      END
500: C -----
501:      SUBROUTINE FSELECT(KF,M,FTIT)
502: C THE PROGRAM REQUIRES INPUTS FROM THE USER ON THE FOLLOWING -----
503: C (1) FUNCTION CODE (KF), (2) NO. OF VARIABLES IN THE FUNCTION (M);
504:      CHARACTER *70 TIT(100),FTIT
505:      NFN=1
506:      KF=1
507:      WRITE(*,*)'-----'
508:      DATA TIT(1)/'CONSTRUCTION OF INDEX FROM M VARIABLES '/
509: C -----
510:      DO I=1,NFN
511:      WRITE(*,*)TIT(I)
512:      ENDDO
513:      WRITE(*,*)'-----'
514:      WRITE(*,*)'SPECIFY NO. OF VARIABLES [MVAR] HERE ALSO ?'
515:      READ(*,*) M
516:      FTIT=TIT(KF) ! STORE THE NAME OF THE CHOSEN FUNCTION IN FTIT
517:      RETURN
518:      END
519: C -----
520:      SUBROUTINE FUNC(X,M,F)
521: C TEST FUNCTIONS FOR GLOBAL OPTIMIZATION PROGRAM
522:      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
523:      COMMON /RNDM/IU,IV
524:      COMMON /KFF/KF,NFCALL,FTIT
525:      INTEGER IU,IV
526:      DIMENSION X(*)
527:      CHARACTER *70 FTIT
528:      NFCALL=NFCALL+1 ! INCREMENT TO NUMBER OF FUNCTION CALLS
529: C KF IS THE CODE OF THE TEST FUNCTION
530:      IF (KF.EQ.1) THEN
531:      CALL CORD(M,X,F)
532:      RETURN
533:      ENDIF
534: C =====
535:      WRITE(*,*)'FUNCTION NOT DEFINED. PROGRAM ABORTED'
536:      STOP

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537:      END
538: C -----
539:      SUBROUTINE CORD(M,X,F)
540:      PARAMETER (NOB=30,MVAR=7) ! CHANGE THE PARAMETERS HERE AS NEEDED.
541: C -----
542: C      NOB=NO. OF OBSERVATIONS (CASES) & MVAR= NO. OF VARIABLES
543:      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
544:      COMMON /RNDM/IU,IV
545:      COMMON /CORDAT/CDAT (NOB,MVAR) , QIND (NOB) , R (MVAR) , ENTROPY , NORM , NCOR
546:      COMMON /GETRANK/MRNK
547:      INTEGER IU,IV
548:      DIMENSION X (*), Z (NOB,2)
549:      DO I=1,M
550:      IF (X(I) .LT. -1.0D0 .OR. X(I) .GT. 1.0D0) THEN
551:      CALL RANDOM(RAND)
552:      X(I) = (RAND-0.5D0) * 2
553:      ENDIF
554:      ENDDO
555:      XNORM=0.D0
556:      DO J=1,M
557:      XNORM=XNORM+X(J)**2
558:      ENDDO
559:      XNORM=DSQRT(XNORM)
560:      DO J=1,M
561:      X(J) = X(J) / XNORM
562:      ENDDO
563: C      CONSTRUCT INDEX
564:      DO I=1,NOB
565:      QIND(I) = 0.D0
566:      DO J=1,M
567:      QIND(I) = QIND(I) + CDAT(I,J) * X(J)
568:      ENDDO
569:      ENDDO
570: C -----
571:      !FIND THE RANK OF QIND
572:      IF (MRNK.EQ.1) CALL DORANK(QIND,NOB)
573: C -----
574: C      COMPUTE CORRELATIONS
575:      DO I=1,NOB
576:      Z(I,1) = QIND(I)
577:      ENDDO
578:      DO J=1,M
579:      DO I=1,NOB
580:      Z(I,2) = CDAT(I,J)
581:      ENDDO
582:      IF (NCOR.EQ.0) THEN
583:      CALL CORLN(Z,NOB,RHO)
584:      ELSE
585:      CALL CORA(Z,NOB,RHO)
586:      ENDIF
587:      R(J) = RHO
588:      ENDDO
589:      IF (ENTROPY.EQ.0.D0) THEN
590: C ----- MAXIMIN SOLUTION -----
591:      IF (NORM.GT.2) THEN
592:      F = DABS(R(1))
593:      DO J=2,M
594:      IF (F.GT.DABS(R(J))) F = DABS(R(J))
595:      ENDDO
596:      ENDIF
597: C ----- FOR NORM = 1 OR 2 -----
598:      IF (NORM.LE.2) THEN
599:      F = 0.D0
600:      DO J=1,M
601:      F = F + DABS(R(J)) ** NORM
602:      ENDDO
603:      ENDIF

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604: C -----
605: ELSE
606: IF (ENTROPY.NE.0.D0) THEN
607: C ENTROPY MAXIMIZATION
608: ENT=0.0D0
609: DO J=1,M
610: ENT=ENT+DABS (R (J))
611: ENDDO
612: F=ENT*DLOG (ENT)
613: DO J=1,M
614: FX=DABS (R (J))
615: F=F+ (FX/ENT) *DLOG (FX/ENT)
616: ENDDO
617: ENDIFF
618: ENDIFF
619: C -----
620: F=-F
621: RETURN
622: END
623: SUBROUTINE CORLN (Z, NOB, RHO)
624: C NOB = NO. OF CASES
625: IMPLICIT DOUBLE PRECISION (A-H, O-Z)
626: DIMENSION Z (NOB, 2), AV (2), SD (2)
627: DO J=1, 2
628: AV (J)=0.D0
629: SD (J)=0.D0
630: DO I=1, NOB
631: AV (J)=AV (J)+Z (I, J)
632: SD (J)=SD (J)+Z (I, J) **2
633: ENDDO
634: ENDDO
635: DO J=1, 2
636: AV (J)=AV (J)/NOB
637: SD (J)=DSQRT (SD (J)/NOB-AV (J) **2)
638: ENDDO
639: C WRITE (*, *) 'AV AND SD ', AV (1), AV (2), SD (1), SD (2)
640: RHO=0.D0
641: DO I=1, NOB
642: RHO=RHO+ (Z (I, 1)-AV (1)) * (Z (I, 2)-AV (2))
643: ENDDO
644: RHO= (RHO/NOB) / (SD (1) *SD (2))
645: RETURN
646: END
647: C -----
648: SUBROUTINE CORA (Z, N, R)
649: C COMPUTING ABSOLUTE CORRELATION MATRIX
650: IMPLICIT DOUBLE PRECISION (A-H, O-Z)
651: DIMENSION Z (N, 2), X (N), Y (N)
652: C -----
653: C PUT Z INTO X AND Y
654: DO I=1, N
655: X (I)=Z (I, 1)
656: Y (I)=Z (I, 2)
657: ENDDO
658: C ARRANGE X ANY IN AN ASCENDING ORDER
659: DO I=1, N-1
660: DO II=I+1, N
661: IF (X (I).GT.X (II)) THEN
662: TEMP=X (I)
663: X (I)=X (II)
664: X (II)=TEMP
665: ENDIFF
666: IF (Y (I).GT.Y (II)) THEN
667: TEMP=Y (I)
668: Y (I)=Y (II)
669: Y (II)=TEMP
670: ENDIFF

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671:      ENDDO
672:      ENDDO
673: C      FIND MEDIAN
674:      IF (INT(N/2) .EQ. N/2.D0) THEN
675:      XMED=(X(N/2)+X(N/2+1))/2.D0
676:      YMED=(Y(N/2)+Y(N/2+1))/2.D0
677:      ENDIF
678:      IF (INT(N/2) .NE. N/2.D0) THEN
679:      XMED=X(N/2+1)
680:      YMED=Y(N/2+1)
681:      ENDIFF
682: C      SUBTRACT RESPECTIVE MEDIANS FROM X AND Y AND FIND ABS DEVIATIONS
683:      VX=0.D0
684:      VY=0.D0
685:      DO I=1,N
686:      X(I)=X(I)-XMED
687:      Y(I)=Y(I)-YMED
688:      VX=VX+DABS(X(I))
689:      VY=VY+DABS(Y(I))
690:      ENDDO
691: C      SCALE THE VARIABLES X AND Y SUCH THAT VX=VY
692:      IF (VX.EQ.0.D0.OR.VY.EQ.0.D0) THEN
693:      R=0.D0
694:      RETURN
695:      ENDIFF
696:      DO I=1,N
697:      X(I)=X(I)*VY/VX
698:      ENDDO
699: C      COMPUTE CORRELATION COEFFICIENT
700:      VZ=0.D0
701:      R=0.D0
702:      DO I=1,N
703:      VZ=VZ+DABS(X(I))+DABS(Y(I))
704:      R=R+DABS(X(I)+Y(I))-DABS(X(I)-Y(I))
705:      ENDDO
706:      R=R/VZ
707:      RETURN
708:      END
709: C      -----
710:      SUBROUTINE DORANK(X,N) ! N IS THE NUMBER OF OBSERVATIONS
711: C      !THIS PROGRAM RETURNS 1-2-3-4 ORDINAL RANK-ORDER OF A GIVEN VECTOR
712:      PARAMETER (MXD=1000) ! MXD IS MAX DIMENSION FOR TEMPORARY VARIABLES
713:      ! THAT ARE LOCAL AND DO NOT GO TO THE INVOKING PROGRAM
714:      ! X IS THE VARIABLE TO BE SUBSTITUTED BY ITS RANK VALUES
715:      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
716:      DIMENSION X(N),ID(MXD)
717:      ! GENERATE ID
718:      DO I=1,N
719:      ID(I)=I
720:      ENDDO
721:      DO I=1,N-1
722:      DO II=I+1,N
723:      IF (X(I) .GT. X(II)) THEN ! ARRANGE ACCORDING TO ASCENDING ORDER OF X
724:      T=X(I)
725:      X(I)=X(II)
726:      X(II)=T
727:      IT=ID(I)
728:      ID(I)=ID(II)
729:      ID(II)=IT
730:      ENDIFF
731:      ENDDO
732:      ENDDO
733:      DO I=1,N
734:      X(I)=I+0.D0
735:      ENDDO
736:      DO I=1,N-1
737:      DO II=I+1,N

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738:      IF (ID(I) .GT. ID(II)) THEN
739: C      !ARRANGE ACCORDING TO ASCENDNG ORDER OF ID
740:      T=X(I)
741:      X(I)=X(II)
742:      X(II)=T
743:      IT=ID(I)
744:      ID(I)=ID(II)
745:      ID(II)=IT
746:      ENDDO
747:      ENDDO
748:      ENDDO
749:      RETURN
750:      END
751: C -----
752:      SUBROUTINE CORREL (X, N, M, RMAT)
753:      PARAMETER (NMX=30) !DO NOT CHANGE UNLESS NO. OF VARIABLES EXCEED 30
754:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
755:      DIMENSION X (N, M), RMAT (M, M), AV (NMX), SD (NMX)
756:      DO J=1, M
757:      AV (J)=0.D0
758:      SD (J)=0.D0
759:      DO I=1, N
760:      AV (J)=AV (J)+X (I, J)
761:      SD (J)=SD (J)+X (I, J)**2
762:      ENDDO
763:      AV (J)=AV (J)/N
764:      SD (J)=DSQRT (SD (J)/N-AV (J)**2)
765:      ENDDO
766:      DO J=1, M
767:      DO JJ=1, M
768:      RMAT (J, JJ)=0.D0
769:      DO I=1, N
770:      RMAT (J, JJ)=RMAT (J, JJ)+X (I, J)*X (I, JJ)
771:      ENDDO
772:      ENDDO
773:      ENDDO
774:      DO J=1, M
775:      DO JJ=1, M
776:      RMAT (J, JJ)=RMAT (J, JJ)/N-AV (J)*AV (JJ)
777:      RMAT (J, JJ)=RMAT (J, JJ)/(SD (J)*SD (JJ))
778:      ENDDO
779:      ENDDO
780:      RETURN
781:      END
782: C -----
783:      SUBROUTINE DOCORA (ZDAT, N, M, RMAT)
784:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
785:      DIMENSION ZDAT (N, M), RMAT (M, M), Z (N, 2)
786:      DO J=1, M-1
787:      DO JJ=J+1, M
788:      DO I=1, N
789:      Z (I, 1)=ZDAT (I, J)
790:      Z (I, 2)=ZDAT (I, JJ)
791:      ENDDO
792:      CALL CORA (Z, N, R)
793:      RMAT (J, JJ)=R
794:      RMAT (JJ, J)=R
795:      ENDDO
796:      ENDDO
797:      DO J=1, M
798:      RMAT (J, J)=1.D0
799:      ENDDO
800:      RETURN
801:      END

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