Journal of
Computational and Theoretical Nanoscience
Vol. 2, 1–6, 2005

Electronic Contribution to the Elastic Constants in Strained Layer Quantum Dot Superlattices of Non-Parabolic Semiconductors with Graded Interfaces

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Most papers in the area of semiconductor superlattices (SLs) are based on the assumption that the interface between the layers are sharply defined with the zero thickness so as to be devoid of any interface effects. The advanced experimental techniques may produce SL with physical interface between the two materials crystallographically abrupt, but the bonding environment of the atoms adjoining these interface will change at least on an atomic scale as the potential form changes from a well (barrier) to a barrier (well), an intermediate potential region exists for the charge carriers. Thus, the influence of the finite thickness of the interface on the carrier dispersion law becomes very important. In this paper we study the electronic contribution to the elastic constants in strained layer quantum dot superlattices of non-parabolic semiconductors with graded structures and compare the same with that of the constituent materials, by formulating the appropriate dispersion laws. It is found, taking InSb/GaSb quantum dot strained superlattices of non-parabolic semiconductors as an example, that the carrier contribution to the second- and third-order elastic constants oscillates with the electronic concentration together with the fact that the nature of oscillations are totally dispersion relation dependent. We have also suggested an experimental method for determining the electronic contribution to the elastic constants in quantum-confined materials having arbitrary carrier energy spectra. In addition, the well-known results for bulk specimens of wide-gap stress free materials have been obtained as special cases from our generalized formulation under certain limiting conditions.

Keywords: Elastic Constants, Strained Layer Quantum Dot Superlattices, Graded Interfaces, Dispersion Relation, Suggestion for the Experimental Determination of the Elastic Constants.

RESEARCH ARTICLE

1. INTRODUCTION

With the advent of molecular beam epitaxy, organometallic chemical vapor phase epitaxy, fine line lithography and other experimental techniques, it has become possible to fabricate Superlattices (SLs) of non-parabolic semiconductors composed of alternate layers of two different materials with controlled thickness, many of which are currently under study due to their new physical properties.[1,2] The SL, as originally proposed by Esaki and Tsu,[3] has found wide applications in many device structures, such as photo detectors,[4] avalanche photo diodes,[5] transistors,[6] tunneling devices,[7] light emitters,[8] etc.

The most extensively studied SL is consisting of alternate layers of GaAs and $Ga_{1-x}Al_xAs$. In addition to the SLs with usual structure, the SLs with more complex structures such as PbTe/SnTe,[9] II-VI,[10] strained layer,[11] and HgTe/CdTe,[12] etc have also been proposed. These complex

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1546-198X/2005/2/001/006/$17.00+.25 doi:10.1166/jctn.2005.113

structures have provided additional degrees of freedom through band gap engineering, so that the more parameters could be available for obtaining of the desired electronic and optical properties. It is worth remarking that the most papers in this vital area of modern nanostructured electronics are based on the assumption that the interface between the layers is sharply defined with the zero thickness so as to be devoid of any interface effects: the SL potential distribution may be considered as a one dimensional array of rectangular potential wells. The advanced experimental techniques may produce SL with physical interface between the two materials crystallographically abrupt, but the bonding environment of the atoms adjoining these interface will change at least on an atomic scale as the potential form changes from a well (barrier) to a barrier (well), an intermediate potential region exists for the charge carriers. Thus, the influence of the finite thickness of the interface on the carrier energy spectra becomes very important since the dispersion relation of the carriers governs all types of electron transport and optoelectronic properties. Although the strained layer quantum dot SLs of non parabolic semiconductors arc of current interest for both scientific and device purposes, nevertheless, it appears from the literature that the electronic contribution to the elastic constants for such heterostructures has yet to be investigated for the more practical case which occurs from the consideration of the finite width of the interface.

In this context, we wish to note that the theory for determining the carrier contribution to the elastic constants in ultra thin films of p-type Si already exists.[13] It has been shown that the carrier contribution to the second and the third order elastic constants depend on the density-of-states function.[14] Sreedhar and Gupta[14] formulated the same for bulk specimens of non-parabolic compounds whose energy band structures are defined by the two-band model of Kane. It has therefore different values in various materials and varies with the electron concentration, with the thickness of ultra thin films and with the temperature for semiconductors and their heterostructures having various carrier energy spectra. The nature of these variations has been investigated by Ghatak and co-workers[13, 15, 16] and a few others.[14, 17] Some of the significant features, which have emerged from these studies, are:

(a) The carrier contribution of the elastic constants increases monotonically with electron concentrations in bulk materials.
(b) The nature of the variations is significantly affected by the presence of the band non-parabolicity.
(c) The said contribution has relatively large values in ultra thin films in the presence of size quantization.

The above characteristics are considered as theoretical predictions, and no experimental results are available to the knowledge of the authors in support of these predictions for the present generalized systems. Therefore, it would be of much interest to study the carrier contribution to the elastic constants for the present case and to suggest an experimental method of determining them for materials having arbitrary band structures. In quantum dot SLs with graded interfaces, the carrier contribution to the elastic constants will be rather significant since the Fermi energy for the present case exhibits composite oscillations with the electron concentration as compared with that of the constituent materials.

In the following, we shall investigate the second- and third-order elastic constants (ΔC_{44} and ΔC_{456}) for the present system. The corresponding results for bulk specimens of stress free wide-gap materials have also been obtained from our generalized formulation for the purpose of assessing the influence of strain and interface width respectively. As already remarked we have also suggested the experimental methods for determining such contribution for materials having arbitrary dispersion laws. We shall study the doping dependence of ΔC_{44} and ΔC_{456} taking strained quantum dot InAs/GaSb SL as an example for the purpose of numerical computations.

2. THEORETICAL BACKGROUND

The dispersion relation of the conduction electrons in bulk specimens of stressed non-parabolic semiconductors can be expressed,[18] in the absence of any quantization, as

$$[k_x/a_i^*(E)]^2 + [k_y/b_i^*(E)]^2 + [k_z/f_i^*(E)]^2 = 1 \qquad (1)$$

where $[a_i^*(E)]^2 = L_i^*(E)[A_i^*(E) + (1/2)D_i^*(E)]^{-1}$,

$$i = 1, 2, L_i^*(E) = [3E^2 + Eq_i - R_i],$$
$$q_i = [(3E_{gi}/2e_i^2) - (3\varepsilon_i c_i^c/e_i^2)]$$

where E_{gi} is the band gap, e_i are the momentum matrix elements, c_i^c is the conduction band deformation potential constant, ε_i is the trace of the strain tensor $\hat{\varepsilon}_i$ which is given by

$$\hat{\varepsilon}_i = \begin{vmatrix} \varepsilon_{xxi} & \varepsilon_{xyi} & 0 \\ \varepsilon_{xyi} & \varepsilon_{yyi} & 0 \\ 0 & 0 & \varepsilon_{zzi} \end{vmatrix} \qquad (2)$$

ε_{xxi}, ε_{xyi}, ε_{yyi}, and ε_{zzi} are the various elements of $\hat{\varepsilon}_i$,

$$R_i = [e_i^{-2}(g_i)^2(\varepsilon_{xyi})^2 + (3/2)e_i^{-2}c_i^c\varepsilon_i(E_{gi} - c_i^c\varepsilon_i)]$$

g_i is a constant describing the strain interaction between the conduction and valence bands,

$$A_i^*(E) = (E + G_i^x)(E + H_i)^{-1},$$
$$G_i^x = \left[E_{gi} - c_i^c\varepsilon_i + 3\bar{b}_{0i}\varepsilon_{xxi} - \bar{b}_{0i}\varepsilon_i(a_{0i} + c_i^c)\varepsilon_i\right],$$
$$a_{0i} = -(x_i + 2_{y_i})/3, \quad \bar{b}_{0i} = (x_{i-}\, y_i)/3,$$
$$d_{0i} = 2Z_i/\sqrt{3}, x_i, y_i$$

and Z_i are the matrix elements of the strain projection operator,

$$H_i = [E_{gi} + c_i^c \varepsilon_i], \quad D_i^*(E) = \rho_i (E + H_i)^{-1},$$

$$\rho_i = \left[d_{0i} \varepsilon_{xyi} (3)^{1/2}\right]$$

$$[b_i^*(E)]^2 = L_I^*(E)\left[A_I^*(E) - (1/2) D_i^*(E)\right]^{-1},$$

$$[f_i^*(E)]^2 = L_i^*(E)/C_i^*(E), \ \ C_i^*(E) = (E + G_i^z)(E + H_i)^{-1}$$

and

$$G_i^z = \left[E_{gi} - c_i^c \varepsilon_i - (a_{0i} + c_i^c)\varepsilon_i + 3\bar{b}_{0i}\varepsilon_{zzi} - \bar{b}_{0i}\varepsilon_i\right]$$

Therefore the dispersion law of the electrons in strained layer SLs of non-parabolic semiconductors with graded interfaces can be written, by extending the method as given in Ref. [19], as

$$\cos(L_0 k) = [\phi(E)/2] \tag{3}$$

where $L_0 (= L_1 + L_2)$ is the period length, L_1 and L_2 are the widths of the barrier and well respectively,

$$\begin{aligned}
\phi(E) = \Bigg\{ & 2\cosh[\beta(E)]\cos[\gamma(E)] \\
& + T(E)\sinh[\beta(E)]\sin[\gamma(E)] \\
& + \Delta_0 \Bigg\{ \left[\frac{K_0^2(E)}{K'(E)} - 3K'(E)\right] \cosh[\beta(E)]\sin[\gamma(E)] \\
& + \left(3K_0(E) - \frac{[K'(E)]^2}{K_0(E)}\right) \sinh[\beta(E)]\cos[\gamma(E)] \Bigg\} \\
& + \Delta_0 \Bigg\{ \{2\{K_0(E)\}^2 - 2\{K'(E)\}^2\} \\
& \times \cosh[\beta(E)]\cos[\gamma(E)] \\
& + \frac{1}{12}\left\{\frac{5K_0^3(E)}{K'(E)} + \frac{5[K'(E)]^3}{K_0(E)} - 34K'(E)K_0(E)\right\} \\
& \times \sinh[\beta(E)]\sin[\gamma(E)] \Bigg\} \Bigg\},
\end{aligned}$$

$$T(E) = \left[\frac{K_0(E)}{K'(E)} - \frac{K'(E)}{K_0(E)}\right], \quad \beta(E) = K_0(E)(L_1 - \Delta_0)$$

Δ_0 is the interface width,

$$\gamma(E) = K'(E)(L_2 - \Delta_0),$$

$$\begin{aligned}
K'(E) = \Bigg\{ & (E + G_1^z)^{-1}(E + H_1)(3E^2 + Eq_1 - R_1) \\
& - (E + G_1^z)^{-1}\left(E + G_1^x + \frac{1}{2}\rho_1\right) k_x^2 \\
& - (E + G_1^z)^{-1}\left(E + G_1^x - \frac{1}{2}\rho_1\right) k_x^2 \Bigg\}^{1/2},
\end{aligned}$$

$$\begin{aligned}
K_0(E) = \Bigg\{ & (V_0 - E - H_2)(E_1 + G_2^x)^{-1}(3E_1^2 + E_1 q_2 - R_2) \\
& + (E_1 + G_2^z)^{-1}\left(E_1 G_2^x + \frac{1}{2}\rho_2\right) k_x^2 \\
& + (E_1 + G_2^z)^{-1}\left[E_1 + G_2^x - \frac{1}{2}\rho_2\right] k_y^2 \Bigg\}^{1/2}
\end{aligned}$$

$E_1 = E - V_0$, and $V_0 (= E_{g2} - E_{g1})$ is the potential barrier encountered by the electron.

Therefore the electron energy spectrum in strained layer quantum dot SLs of non-parabolic semiconductors with graded interfaces can be written as

$$(n_z \pi/d_z)^2 = \left[-\theta_0^2 + (\delta(E')/L_0^2)\right] \tag{4}$$

where $n_z (= 1, 2, 3, \ldots)$ is the size quantum number along z-direction, d_z is the width along z direction, $\theta_0^2 = [(n_x \pi/d_x)^2 + (n_y \pi/d_y)^2]$, n_x and n_y are size quantum numbers along x and y directions, d_x and d_y are the widths along x and y directions, $\delta(E') = \{\cos^{-1}[\frac{1}{2}\psi(E')]\}^2$

$$\begin{aligned}
\psi(E') = \Bigg\{ & 2\cosh[\beta(E')]\cos[\gamma(E')] \\
& + T(E')\sinh[\beta(E')]\sin[\gamma(E')] \\
& + \Delta_0 \Bigg\{ \left[\frac{K_0^2(E)}{K'(E)} - 3K'(E')\right] \cosh[\beta(E)]\sin[\gamma(E')] \\
& + \left(3K_0(E) - \frac{[K'(E')]^2}{K_0(E')}\right) \sinh[\beta(E')]\cos[\gamma(E')] \Bigg\} \\
& + \Delta_0 \Bigg\{ \{2\{K_0(E')\}^2 - 2\{K'(E')\}^2\} \\
& \times \cosh[\beta(E')]\cos[\gamma(E')] \\
& + \frac{1}{12}\left\{\frac{5K_0^3(E')}{K'(E')} + \frac{5[K'(E')]^3}{K_0(E')} - 34K'(E')K_0(E')\right\} \\
& \times \sinh[\beta(E')]\sin[\gamma(E')] \Bigg\} \Bigg\}
\end{aligned}$$

$$\beta(E') = K_0(E')(d_x - \Delta_0),$$

$$\begin{aligned}
K_0(E') = \Bigg\{ & (V_0 - E' - H_2)(E_1 + G_2^z)^{-1}(3E_1^2 + E_1 q_2 - R_2) \\
& + (E_1 + G_2^z)^{-1}\left(E_1 + G_2^x + \frac{1}{2}\rho_2\right)\left(\frac{n_x \pi}{d_x}\right)^2 \\
& + (E_1 + G_2^z)^{-1}\left[E_1 + G_2^x - \frac{1}{2}\rho_2\right]\left(\frac{n_y \pi}{d_y}\right)^2 \Bigg\}^{1/2}
\end{aligned}$$

$E_1 = E' - V_0$ and $V_0 (= E_{g2} - E_{g1})$ is the potential barrier encountered by the electron.

$$\gamma(E') = K'(E')(d_y - \Delta_0),$$

$$K'(E') = \Bigg\{ (E'+G_1^z)^{-1}(E'+H_1)(3E'^2+Eq_1-R_1) - (E'+G_1^z)^{-1}\left(E'+G_1^x+\frac{1}{2}\rho_1\right)\left(\frac{n_x\pi}{d_x}\right)^2 - (E'+G_1^z)^{-1}\left(E'+G_1^x-\frac{1}{2}\rho_1\right)\left(\frac{n_y\pi}{d_y}\right)^2\Bigg\}^{1/2}$$

$T(E') = \left[\frac{K_0(E')}{K'(E')} - \frac{K'(E')}{K_0(E')}\right]$ and E' is the totally quantized energy.

Considering only the lowest mini band, since in an actual SL only the lowest miniband is significantly populated at low temperatures, where the quantum effects become prominent, the electron statistics can be written as

$$n_0 = \frac{1}{d_x d_y d_z}\sum_{n_x=1}^{n_{x\max}}\sum_{n_y=1}^{n_{y\max}}\sum_{n_z=1}^{n_{z\max}}\frac{A(1+A\cos\lambda)}{(1+A^2+2A\cos\lambda)} \tag{5}$$

in which, $A = \exp[(E_F - E')/k_BT]$, E' is the root of Eq. (4), E_F is the Fermi energy in the present case, k_B is Boltzmann constant, T is temperature, $\lambda = \Gamma/k_BT$, $\Gamma = \pi \cdot k_B \cdot T_D$ and T_D is the Dingle temperature. The carrier contribution to the second- and third-order elastic constants can, respectively, be expressed[16] as

$$\Delta C_{44} = -(G_0^2/9)\frac{\partial n_0}{\partial E_F} \tag{6}$$

$$\Delta C_{456} = (G_0^3/27)\frac{\partial^2 n_0}{\partial E_F^2} \tag{7}$$

where G_0 is the deformation potential.

Thus, combining Eqs. (5), (6), and (7), we get

$$\Delta C_{44} = \left(-\frac{G_0^2}{9d_xd_yd_zk_BT}\right)\sum_{n=0}^{n_{\max}}\sum_{L=1}^{L_{\max}} \times\left[\frac{A(1-A^2+3A\cos\lambda+3A^2\cos^2\lambda)}{(1+A^2+2A\cos\lambda)^2}\right] \tag{8}$$

$$\begin{aligned}\Delta C_{456} = {} & \left(\frac{G_0^3}{27d_xd_yd_z(k_BT)^2}\right) \\ & \times\sum_{n=0}^{n_{\max}}\sum_{L=1}^{L_{\max}}\big[A\big[(1+A^2+2A\cos\lambda)^{-2} \\ & \times(1-A^2+3A\cos\lambda+3A^2\cos^2\lambda) \\ & -4A(1+A^2+2A\cos\lambda)^{-3}(A+\cos\lambda) \\ & \times(1-A^2+3A\cos\lambda+3A^2\cos^2\lambda) \\ & +A(1+A^2+2A\cos\lambda)^{-2} \\ & \times(-2A+3\cos\lambda+6A\cos^2\lambda)\big]\big]\end{aligned} \tag{9}$$

The Eq. (8) and (9) represent the expressions for ΔC_{44} and ΔC_{456} for the present system. The expression for n_0 for the constituent non-parabolic materials will be given by Eq. (5) where E' has to be determined from

$$[n_x\pi/(d_xa_i^*(E'))]^2 + [n_y\pi/(d_yb_i^*(E'))]^2 + [n_z\pi/(d_zf_i^*(E'))]^2 = 1 \tag{10}$$

In the absence of stress and under the substitution $e_i^2 = 3\hbar^2E_{gi}/4m_i^*$ where m_i^* is the effective electron mass at the edge of the conduction band, the Eq. (1) assumes the form[20]

$$E(1+\alpha\cdot E) = \hbar^2k^2/2m_i^* \tag{11}$$

where $\alpha = 1/E_{gi}$ and $\hbar$ is Dirac's constant.

The Eq. (11) is known in the literature as the two band model of Kane.[20] Besides, this particular equation is being extensively used to study the electronic properties of nanostructured III-V compounds, ternary and quaternary alloys, and is a special case of Eq. (1). The expressions of n_0, ΔC_{44}, and ΔC_{456} can, respectively, be written as

$$n_0 = N_c[F_{1/2}(\eta) + (15\alpha\cdot k_BT/4)F_{3/2}(\eta)] \tag{12}$$

$$\Delta C_{44} = -\big(G_0^2N_c/9K_BT\big)[F_{-1/2}(\eta) + (15\alpha\cdot k_BT/4)F_{1/2}(\eta)] \tag{13}$$

and $$\Delta C_{456} = \big(G_0^3N_c/27K_B^2T^2\big) \times[F_{-3/2}(\eta) + (15\alpha\cdot k_BT/4)F_{-1/2}(\eta)] \tag{14}$$

where $N_c \equiv 2(2\pi m_i^*K_BT/h^2)^{3/2}$, $\eta \equiv E_F/K_BT$ and $F_t(\eta)$ is the one parameter Fermi-Dirac integral of order t[21] which can be defined as

$$F_t(\eta) = (\Gamma(t+1))^{-1}\int_0^\infty x^t[1+\exp(x-\eta)]^{-1} \quad \text{for } t > -1 \tag{15}$$

or for all t, analytically continued as a complex contour integral around the negative x-axis as

$$F_t(\eta) = (\Gamma(-t)/2\pi\sqrt{-1})^{-1}\int_\infty^0 x^t[1+\exp(x-\eta)]^{-1} \tag{16}$$

For specimens of stress free wide gap semiconductors, the expressions for n_0, ΔC_{44} and ΔC_{456} can, respectively be written as

$$n_0 = N_cF_{1/2}(\eta) \tag{17}$$

$$\Delta C_{44} = -\big(G_0^2N_c/9K_BT\big)F_{-1/2}(\eta) \tag{18}$$

and $$\Delta C_{456} = \big(G_0^3N_c/27K_B^2T^2\big)F_{-3/2}(\eta) \tag{19}$$

It may be noted that Eqs. (17) to (19) are well-known in the literature.[15]

SUGGESTION FOR EXPERIMENTAL DETERMINATIONS OF THE ELECTRONIC CONTRIBUTION TO THE ELASTIC CONSTANTS FOR MATERIALS HAVING ARBITRARY DISPERSION LAWS

It is well-known that in the presence of a classically large magnetic field, the density-of-state function remains unchanged and the thermoelectric power is independent of scattering mechanisms.[22,23] The magnitude of the thermoelectric power (T_0) in the present case can be written as[22]

$$T_0 = \left(\frac{1}{eTn_0}\right)\int_{-\infty}^{\infty}(E-E_F)R(E)\left[-\frac{\partial f}{\partial E}\right]dE \qquad (20)$$

where $R(E)$ is the total number of states and f is the distribution function. Following Tsidilkoviski,[23] Eq. (20) can be written as

$$T_0 = \left(\pi^2 K_B^2 T/3en_0\right)\left(\frac{\partial n_0}{\partial E_F}\right) \qquad (21)$$

Using Eqs. (6), (7), and (20), we get

$$\Delta C_{44} = -\left(G_0^2 eT_0 n_0/3\pi^2 K_B^2 T^2\right) \qquad (22)$$

$$\Delta C_{456} = \left(n_0 e G_0^3 T_0^2/3\pi^4 K_B^3 T\right)\left(1+\frac{n_0}{T_0}\frac{\partial T_0}{\partial n_0}\right) \qquad (23)$$

Thus, we can summarize the whole mathematical background in the following way. From the expression of carrier statistics in strained layer quantum dot superlattices of non-parabolic semiconductors with graded structure by incorporating all the system parameters, we have formulated the generalized expressions for ΔC_{44} and ΔC_{456} respectively. The expressions of ΔC_{44} and ΔC_{456} for quantum dots of the constituent materials form a special case of our analysis where E' is the only dispersion relation dependent quantity. From our generalized formulation, we have obtained the well-known expressions of n_0, ΔC_{44}, and ΔC_{456} in bulk specimens of wide gap stress free degenerate materials.[15] This fact is the indirect mathematical test of our generalized analysis. In addition, we have suggested an experimental method for determining the ΔC_{44} and ΔC_{456} for materials having arbitrary dispersion laws.

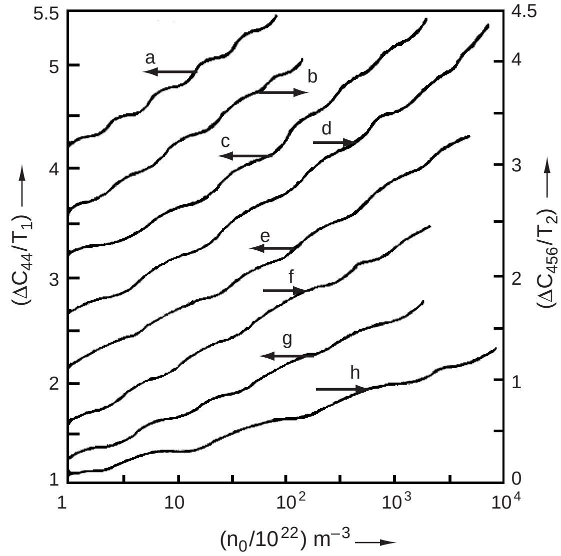

Fig. 1. Plots of the normalized ΔC_{44} and ΔC_{456} versus n_0 in stressed quantum dot InAs/GaSb SL with $\Delta \neq 0$ as shown by curves a and b. The curves c and d exhibit the same dependence in absence of stress. The curves e, f, g, and h exhibit the dependence of ΔC_{44} and ΔC_{456} on n_0 for quantum dots of InAs/GaSb respectively.

3. RESULTS AND DISCUSSION

Using the Eqs. (5), (8), and (9) for the strained layer InAs/GaSb quantum dot SL together with the parameters[18]

$$m_i^* = 0.023m_0,\ E_{g1}=0.41\text{ eV},\ e_1=10\times10^{-11}\text{ eV}m,$$
$$g_1 = 2\text{ eV},\ c_1^c=20\text{ eV},\ (S_{44})_1=0.3\times10^{-3}\ KBar^{-1},$$
$$d_{01} = -4.4\text{ eV},\quad (a_{01}+c_1^c)=8\text{ eV},\quad \bar{b}_{01}=-1.8\text{ eV},$$
$$(S_{11})_1 = 0.09\times10^{-3}\ KBar^{-1},$$
$$(S_{12})_1 = 0.48\times10^{-3}\ KBar^{-1},\quad \sigma=4\ KBar$$

for InAs and

$$m_2^* = 0.048m_0,\ E_{g2}=0.81\text{ eV},\ e_2=14\times10^{-11}\text{ eV}m,$$
$$g_2 = 4\text{ eV},\quad c_2^c=30\text{ eV},\quad d_{02}=-6\text{ eV},$$
$$(S_{44})_2 = 0.6\times10^{-3}\ KBar^{-1},\quad (a_{02}+c_2^c)=10\text{ eV},$$
$$\bar{b}_{02} = -4\text{ eV},\quad (S_{11})_2=0.71\times10^{-3}\ KBar^{-1},$$
$$(S_{12})_2 = 0.46\times10^{-3}\ KBar^{-1}\quad \text{for GaSb}$$
$$\Delta_0 = 5\text{ Å},\quad L_0=120\text{ Å},\quad L_1=60\text{ Å},\quad T_D=9.4\text{ K},$$
$$d_x = d_y=d_z=60\text{ Å and } T=4.2\text{ K}$$

we have plotted in Figure 1 the normalized ΔC_{44} ($\Delta C_{44}/T_1$, $T_1 = -(G_0^2 n_0/9K_B T)$) and the normalized ΔC_{456} ($\Delta C_{456}/T_2$, $T_2 = (G_0^3 n_0/27K_B^2 T^2)$) versus the electron concentration in stressed quantum dot InAs/GaSb SL with Δ_0 (the finite width of the interface) $\neq 0$ as shown by curves a and b. The curves c and d of Figure 1 exhibit the same dependence in the absence of stress. Besides the curves e, f, g, and h show the dependences of ΔC_{44} and ΔC_{456} on n_0 for quantum dots of InAs and GaSb respectively. It appears from Figure 1 that both ΔC_{44} and ΔC_{456} increase with increasing n_0 in different oscillatory manners for all types of materials as considered here. The natures of oscillations are totally determined by the respective dispersion realation. The combined influences of the strain and the finite width of the interface enhance the numerical values of both ΔC_{44} and ΔC_{456} respectively. The numerical values of ΔC_{44} and ΔC_{456} are greatest for the present system and the least for the quantum dot of GaSb respectively.

It may be noted that our experimental suggestions for the determination of ΔC_{44} and ΔC_{456} are valid for materials having arbitrary dispersion relations. Since the experimental curve of n_0 versus T_0 is not available in the literature to the best our knowledge for the present system, we cannot compare our theoretical formulation with the proposed experiment although the generalized analysis as presented in this context would be useful and the Eqs. (22) and (23) are important in probing the band structures of different materials.

We wish to note in view of large changes of the elastic constants with n_0, detailed experimental work on second- and third-order elastic constants as functions of n_0 would be interesting for the present class of quantum confined materials. It may be suggested that the experiments on the velocity of sound involving the shear mode as function of n_0 may exhibit the carrier contribution to the elastic constants for materials having arbitrary carrier energy spectra. It is worth noting that the above statement again suggests the experimental determinations of ΔC_{44} and ΔC_{456}, besides the suggested method of determining them as given by Eqs. (22) and (23) respectively. It may be noted that our study covers different materials having various electron dispersion laws and the formulations of ΔC_{44} and ΔC_{456} are based on the dispersion relations in such compounds.

It is worth remarking that influence of energy band models on ΔC_{44} and ΔC_{456} in various types of materials can also be assessed from our present work. We have not considered other types of compounds or external physical variables for numerical computations in order to keep the presentation brief. With different sets of energy band parameters, we shall get different numerical values of ΔC_{44} and ΔC_{456} although the nature of variation of the said elastic constants with respect to carrier statistics as shown here would be similar for the other types materials. We have not considered the many body, the hot electron and the allied SL effects in this simplified theoretical formalism due to the absence of proper analytical techniques for including them for the present generalized systems. The inclusion of these effects would increase the accuracy of the results although our suggestion for the experimental determination of ΔC_{44} and ΔC_{456} is independent of incorporating the said effects and the qualitative features of ΔC_{44} and ΔC_{456} would not change in the presence of the aforementioned effects. The formulation as presented in this paper is general and E' is the only band structure dependent quantity as already stated. Finally, it may be noted that the basic aim of the present paper is not solely to investigate that ΔC_{44} and ΔC_{456} but also to suggest the experimental determination of them for materials having arbitrary dispersion laws which, in turn, is again in dimension independent.

Acknowledgment: The senior author K. P. Ghatak is grateful for funding to AICTE for the award of R&D project (having the sanction number F. No. /8020/RID/R&D-146, 2001–2002 dated 04.03.2002) under which this research paper has been completed.

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Received: 13 February 2005. Accepted 1 March 2005.