Models with natural seesaw mechanism for neutrino masses with identical parity- and $SU(2)_R$-breaking scales

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(Received 17 September 1990)

Chang and Mohapatra have observed that the implementation of the seesaw mechanism explaining small neutrino masses in left-right-symmetric or SO(10) models requires the parity- (P-) and $SU(2)_R$-breaking scales to be widely separated ($M_P \gg M_R$). In this paper we show how the mechanism operates naturally even though the two scales are identical. The gauge group immediately preceding the standard model emerges to be its minimal extension based upon $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ with a second neutral Z_R-boson mass $M_R^0 \simeq 300$–10^5 GeV. An embedding in the partial unification scheme leads to observable rare kaon decays. In the two symmetry-breaking chains investigated in SO(10) with parity broken either at the unification scale ($M_P = M_U$), or at an intermediate scale ($M_P \gtrsim 10^{11}$ GeV), proton decay is predicted with lifetime near the observable limit; but, in the former case, rare kaon decays are also predicted near the observable limit when an intermediate gauge group $SU(2)_L \times U(1)_R \times SU(4)_C$ survives down to the scale $M_C \simeq 10^5$ GeV provided $\sin^2\theta_W \simeq 0.24$. The criterion for naturalness turns out to be wide separation between P- and $U(1)_{B-L}$-breaking scales.

I. INTRODUCTION

Out of several methods proposed to explain small neutrino masses, the seesaw mechanism[1,2] has been widely exploited in partially unified or grand unified theories (GUT's) of strong, weak, and electromagnetic interactions. Recently Chang and Mohapatra[3,4] have made an important observation on the general validity of the mechanism[1,2] as a viable theory for neutrino masses. They found that the implementation of the mechanism in the left-right-symmetric[5] (LRS) model or SO(10) GUT[6] needs a wide separation of parity- (P-) and $SU(2)_R$-breaking scales. Starting with LRS models or GUT's such as SO(10), SU(16), or $SU(8)_L \times SU(8)_R$, it has been demonstrated earlier that a wide separation between P- and $SU(2)_R$-breaking scales can be realized in the presence of suitable Higgs representations,[7] or specific spontaneous symmetry-breaking patterns.[8] In the case when the P- and $SU(2)_R$-breaking scales are identical, the present bound on neutrino masses does not permit the right-handed gauge bosons to be light ($M_{W_R} = M_{Z_R} = M_R = M_P > 10^8$–$10^{10}$ GeV), thus leaving no other testable signatures at lower energies. In the latter situation the mechanism[1,2] has no role in explaining neutrino masses. The main objective of this paper is to demonstrate that the seesaw mechanism is natural in certain models even if the two scales are identical. This is achieved in spontaneous symmetry breaking (SSB) of a LRS gauge group or a GUT to the standard model in several steps such that $SU(2)_R \to U(1)_R$ in the first step. In a subsequent step of SSB, $U(1)_R$ combines with the $U(1)_{B-L}$ present in the intermediate gauge group to form $U(1)_Y$. We provide two examples in SO(10) GUT where the proton lifetime is predicted to be within the observable limit of the second generation experiments. In all cases the neutral current exhibits an admixture of $V+A$ structure corresponding to a low-mass Z_R boson manifesting itself in the SSB of the minimally extended gauge group based upon

$$SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C \quad (\equiv G_{2113}) .$$

That a seesaw formula different from the usual lore exists in the G_{2113} model was first noted in Ref. 9, but the left-handed triplet Δ_L carrying $B-L=2$ was taken to be light for the sake of convenience, which spoils the naturalness of the mechanism. In all models leading to G_{2113} considered in this paper, the condition of minimal fine-tuning of parameters requires all the components of Δ_L to be much heavier than the $U(1)_{B-L}$-breaking scale which renders the mechanism to be natural.

This paper is organized in the following manner. In Sec. II we review the work of Chang and Mohapatra illustrating the naturalness of the seesaw mechanism in gauge models with a wide separation between P- and $SU(2)_R$-breaking scales. In Sec. III we show how the naturalness criterion operates with identical P- and $SU(2)_R$-breaking scales using the LRS model and partial unification scheme. In Secs. IV and V we show how such models can be embedded in two different scenarios of SO(10) grand unification. Section VI is devoted to summary and discussions.

II. NATURAL SEESAW MECHANISM WITH SEPARATE P- AND $SU(2)_R$-BREAKING SCALES

In this section we summarize the work of Chang and Mohapatra[3] establishing the naturalness of the seesaw mechanism in left-right gauge models and GUT's with a wide separation between P- and $SU(2)_R$-breaking scales.[7,8] For convenience we discuss the conventional mechanism in the context of LRS models[5] based upon the gauge group

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times P \quad (\equiv G_{2213P},\ g_{2L}=g_{2R}) ,$$

where the quarks (Q_L, Q_R) and leptons (ψ_L, ψ_R) of each generation, and Higgs scalars $(\phi, \Delta_L, \Delta_R)$, possess the following transformation properties under G_{2213}: $\psi_L(2,1,-1,1)$, $\psi_R(1,2,-1,1)$, $Q_L(2,1,\frac{1}{3},1)$, $Q_R(1,2,\frac{1}{3},1)$, $\phi(2,2,0,1)$, $\Delta_L(3,1,2,1)$, $\Delta_R(1,3,2,1)$. In order to drive the SSB, and implement the seesaw mechanism in the chain

$$G_{2213P} \xrightarrow[M_R]{\langle \Delta_R^0 \rangle = V_R} SU(2)_L \times U(1)_Y \times SU(3)_C (\equiv G_{st}) \xrightarrow[M_W]{\langle \phi^0 \rangle = k} U(1)_{em} \times SU(3)_C \quad (\equiv G_{13}) , \tag{1}$$

the scalars are assigned the following vacuum expectation values (VEV's):

$$\langle \Delta_L \rangle = \begin{bmatrix} 0 & 0 \\ V_L & 0 \end{bmatrix}, \quad \langle \Delta_R \rangle = \begin{bmatrix} 0 & 0 \\ V_R & 0 \end{bmatrix}, \quad \phi = \begin{bmatrix} k & 0 \\ 0 & k' \end{bmatrix} \tag{2}$$

leading to the neutrino mass term in the Lagrangian:

$$\mathcal{L}^{(\nu)}_{\text{mass}} = (\nu^T N^T) \begin{bmatrix} m_{LL} & m_{LR} \\ m_{LR} & m_{RR} \end{bmatrix} \tau_2 \begin{bmatrix} \nu \\ N \end{bmatrix} , \tag{3}$$

where $N = C(\bar{\nu}_R)^T$, $m_{LL} = h_3 V_L$, $m_{RR} = h_3 V_R$, $m_{LR} = h_1 k + h_2 k'$, and h_i's are Yukawa couplings. Under the condition $V_R \gg k \gg V_L, k'$, the generalization of Eq. (2) to three generations leads to small (large) mass eigenvalues for the left- (right-)handed neutrinos $\nu_i(N_i)$:

$$m_{\nu_i} \simeq \frac{(m_i^D)^2}{M_R}, \quad m_{N_i} = M_R, \quad i = 1,2,3 , \tag{4}$$

M_R being the mass of $W_R^{\pm}$ and Z_R bosons where P and $SU(2)_R$ break simultaneously. The Dirac mass in (4) has been chosen to be the charged-lepton mass[2] ($m_1^D = m_e$, $m_2^D = m_\mu$, $m_3^D = m_\tau$) leading to $m_{\nu_e} \simeq 0.2$ eV, $m_{\nu_\mu} \simeq 10$ keV, and $m_{\nu_\tau} \simeq 4$ MeV. SO(10) breaks into G_{2213P} through the Higgs representation **210** at the unification scale (M_U), and the other scalars needed for (1) are contained in the representations **126** and **10**$\subset$SO(10). Also, SO(10) can break directly into G_{st} through the scalar representations **45**, and **126**$\subset$SO(10). Using the quark masses $m_1^D = m_u$, $m_2^D = m_c$, $m_3^D = m_t \simeq 80$ GeV and $M_U = M_R = 10^{15}$ GeV, the Gell-Mann–Ramond–Slansky[1]–type spectrum is $m_{\nu_e} \simeq 10^{-11}$ eV, $m_{\nu_\mu} \simeq 10^{-6}$ eV, and $m_{\nu_\tau} \simeq 10^{-2}$ eV. Such a feature of the mechanism as obtaining small ν_i masses simultaneously with small mixing angles was considered very natural until Chang and Mohapatra[3,4] observed that the presence of the terms

$$V_I = \sum_{i,j=1}^{2} \lambda_{ij} \text{Tr}(\Delta_L^{+} \phi_i \Delta_R \tau_2 \phi_j^{+} \tau_2) \tag{5}$$

in the Higgs potential, where $\phi_1 = \phi$ and $\phi_2 = \tau_2 \phi^* \tau_2$, leads to much larger induced values of $\langle \Delta_L^0 \rangle$ and the left-handed Majorana mass through Fig. 1,

$$m_{LL}^{(I)} = \lambda h_3 \frac{k^2 V_R}{M_\Delta^2} , \tag{6}$$

even though one has $\langle \Delta_L^0 \rangle = 0$ to start with. Here λ is a function of scalar couplings and M_Δ is the mass of Δ_L. Thus, the seesaw mechanism[1,2] meant to explain small neutrino masses holds provided values given by (4) dominate over those in (6), which requires

$$\frac{\lambda V_R^2}{M_\Delta^2} \ll \left[\frac{h_1}{h_3} \right]^2 . \tag{7}$$

For the maximum values of $M_\Delta = M_P = M_R$, obtained for $V_L = 0$ using extended survival hypothesis, the fine-tuning needed to satisfy (7), or $\lambda \ll (h_1/h_3)^2$, is arbitrary since the standard-model Yukawa coupling $h_1 \simeq 10^{-5}$, and there is no reason for h_3 to be small. Without arbitrary fine-tuning, (6) dominates over (4) and the bound on neutrino masses needs $M_\Delta = M_P = M_R \simeq 10^{10}-10^{11}$ GeV consistent with $m_{\nu_i} \simeq 1-10$ eV, $i = 1,2,3$. In such a situation the proposed mechanism[1,2] does not explain neutrino masses.

Introducing P-odd singlets of scalars in LRS, or using D-odd singlets already present in SO(10) (D = a discrete symmetry defined in Ref. 7) in the representations **45** and **210**, P and $SU(2)_R$ breaking were decoupled earlier with the possibility $M_\Delta \simeq M_P \gg M_R$. Then m_{LL}^I in Eq. (6) is made negligible compared to Eq. (4) and the seesaw mechanism provides a natural explanation for neutrino masses even for low values of M_R. A number of symmetry-breaking patterns including two-loop effects have been worked out in SO(10) and found to be consistent with $M_P \gg M_R$. In these new SO(10) models the seesaw mechanism is natural.[7,10] In GUT's of higher rank such as $SU(8)_L \times SU(8)_R$ or SU(16) specific SSB pat-

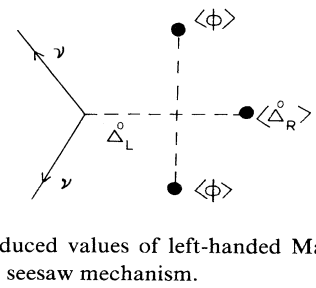

FIG. 1. Induced values of left-handed Majorana mass term that spoils the seesaw mechanism.

terns were found that generate asymmetry in the G_{2213} gauge group or partial unification scheme with $g_{2L} \neq g_{2R}$ (Ref. 8) and wide separation between P and $SU(2)_R$ breakings. Similar arguments permit the seesaw mechanism to be natural in these GUT's.

III. NATURAL SEESAW MECHANISM IN MODELS WITH IDENTICAL P- AND $SU(2)_R$-BREAKING SCALES

In this section we demonstrate how the seesaw mechanism is natural in other gauge models with identical P- and $SU(2)_R$-breaking scales. In popular models LRS is associated with the gauge group G_{2213P}, or

$$SU(2)_L \times SU(2)_R \times SU(4)_C (\equiv G_{224P},\ g_{2L} = g_{2R}) .$$

GUT's such as SO(10), E_6, $SU(8)_L \times SU(8)_R$, or SU(16) contain these as subgroups. $B-L$ forms a diagonal generator of $SU(4)_C$. In the alternate class of models exhibiting a natural seesaw mechanism, although P and $SU(2)_R$ break at the same scale, $SU(2)_R \times U(1)_{B-L}$ or $SU(2)_R \times SU(4)_C$ breaks to $U(1)_Y$ in more than one steps. In the first step $U(1)_{B-L}$ or $SU(4)_C$ must remain unbroken but $SU(2)_R \rightarrow U(1)_R$ to generate wide separation between M_P and M_{B-L}, where M_{B-L} is the breaking scale of $U(1)_{B-L}$. This is achieved through the following chains in the two models:

$$\text{(i)}\quad G_{2213P} \xrightarrow[\langle X^0\rangle \neq 0]{M_P = M_R^+} G_{2113} \xrightarrow[\langle \Delta_R^0 \rangle \neq 0]{M_R^0} G_{st} ,$$

$$\text{(ii a)}\quad G_{224P} \xrightarrow[\langle X^0\rangle \neq 0]{M_P = M_R^+} G_{214} \xrightarrow[\langle \xi^0\rangle \neq 0]{M_C} G_{2113} \xrightarrow[\langle \Delta_R^0 \rangle \neq 0]{M_R^0} G_{st} ,$$

$$\text{(ii b)}\quad G_{224P} \xrightarrow[\langle \sigma^0\rangle \neq 0]{M_P = M_R^+ = M_C} G_{2113} \xrightarrow[\langle \Delta_R^0 \rangle \neq 0]{M_R^0} G_{st} ,$$

where $G_{214} \equiv SU(2)_L \times U(1)_R \times SU(4)_C$. In order to understand the naturalness of the seesaw mechanism it is necessary to know the order of the masses of Higgs scalars occurring as left-handed and right-handed triplets which carry $B-L=2$. In case (i) the neutral component of the right-handed Higgs scalar triplet with $B-L=0$ transforming as X (1,3,0,1) under G_{2213P} gets a VEV at the scale $M_R^+ \gg M_R^0$ to yield the minimally extended gauge group G_{2113}. In the second step the neutral component of the right-handed Higgs scalar triplet carrying $B-L=2$ and transforming as Δ_R^0 $(1,-1,2,1)$ under G_{2113} gets a VEV to break $G_{2113} \rightarrow G_{st}$ at the scale $M_R^0 \gg M_W$. In this case $W_R^\pm$ gauge-boson masses are of order M_R^+ whereas the right-handed neutral gauge-boson mass $\simeq M_R^0$. It may be noted that in the first step both P and $SU(2)_R$ break at the same scale ($\mu = M_P = M_R^+$) but $U(1)_R \times U(1)_{B-L}$ remain unbroken, which in turn break to form $U(1)_Y$ at the lower scale $M_R^0 = M_{B-L} \ll M_R^+ = M_P$. In (ii a) $G_{224P} \rightarrow G_{214}$ by the VEV of the neutral component of the Higgs scalar transforming as $X(1,3,1)$ under G_{224}. In the second stage $G_{214} \rightarrow G_{2113}$ due to the VEV of the neutral omponent of the Higgs scalar transforming as $\xi(1,1,15)$ under G_{224} with $M_R^+ \gg M_C$. In the case (ii b) the SSB $G_{224P} \rightarrow G_{2113}$ is achieved by the neutral component of the Higgs scalar transforming as $\sigma(1,3,15)$ under G_{224}. The symmetry breaking $G_{2113} \rightarrow G_{st}$ proceeds in the same manner as in case (i) through the VEV $\langle \Delta_R^0 \rangle \neq 0$. In the cases (i) and (ii) the final stage of SSB is achieved by the standard doublet of Higgs scalars. Thus compared to the seesaw mechanism envisaged in Sec. II new types of Higgs scalars are needed in certain cases to drive the SSB in the models. In the light Higgs-scalar sector there are two neutral particles: the standard Higgs scalar with mass $M_{\phi^0} \simeq M_W$ and the neutral component of the right-handed triplet with mass $M_{\Delta_R^0} \simeq M_R^0$. Since LRS is maintained at scales $\mu \gg M_R^+$, the Higgs sector must be left-right symmetric for such values of μ. Using extended survival hypothesis, the charged components $\Delta_R^\pm$, Δ_R^{++}, and Δ_R^{--} in the triplet $\Delta_R(1,3,2,1)$ under G_{2213} in case (i) acquire masses of order M_R^+. The left-handed counterpart of Δ_R, i.e., $\Delta_L(3,1,2,1)$ under G_{2213} does not contribute to the SSB at any stage. Its role is to maintain LRS and, according to extended survival hypothesis, masses of all the components in Δ_L is of the order M_R^+. In the cases (ii a) and (ii b) the right-handed triplet is contained in the G_{224} representation $\Delta_R(1,3,10)$, whereas the left-handed triplet is contained in $\Delta_L(3,1,10)$. Only the neutral component of Δ_R^0 acquires a mass $\simeq M_R^0$ but all other components of Δ_R and Δ_L have masses of order M_R^+. In the case (ii a) all other components of $\xi(1,1,15)$ under G_{224} have masses $\simeq M_R^+$ except the neutral component which acquires a mass $M_\xi^0 \simeq M_C$. All the components of $X(1,3,1)$ under G_{224} have masses $\simeq M_R^+$. In the case (ii b) all the components of $\sigma(1,3,15)$ under G_{224} have masses $M_R^+ = M_C$.

Now using the seesaw mechanism and adding the induced mass term due to Fig. 1, we obtain, for the neutrino mass of ith generation,

$$m_{\nu_i} = \frac{\lambda h_3^{(i)}}{g^3} \frac{M_W^2 M_R^0}{M_R^{+2}} - \frac{(m_i^D)^2}{M_R^0}, \quad i = e,\mu,\tau , \tag{8}$$

where the first (second) term is the induced (seesaw mechanism) contribution and g is the appropriate gauge coupling. The dominance of the second term over the first, desired by the naturalness criterion, requires

$$m_i^D \gg \frac{M_W M_R^0}{M_R^+} ,$$

or

$$R m_i^D \gg M_W, \quad i = e,\mu,\tau , \tag{9}$$

where $R = M_R^+ / M_R^0$ and we have used $\lambda h_3^{(i)}/g^3 \simeq 1$, $i = e,\mu,\tau$. Inequality (9) is our new naturalness condition in order that the seesaw mechanism provides a meaningful theory for Majorana neutrino masses. If we use the charged-lepton masses for m_i^D, then $R \gg 10^5$ for the first generation. This automatically guarantees naturalness for the second and third generations since $m_\tau \gg m_\mu \gg m_e$. If $V+A$ structure of neutral currents

TABLE I. Some predictions of the partial unification model $G_{224P} \xrightarrow[M_R^+]{} G_{214} \xrightarrow[M_c]{} G_{2113}$ with $M_R^0 = 1$ TeV as described in the text.

M_c (GeV)	$M_R^+ = M_P$ (GeV)	$\sin^2\theta_W$
10^5	5×10^{16}	0.230
	10^{17}	0.225
10^6	8×10^{16}	0.220
	2×10^{16}	0.230
	8×10^{15}	0.235

TABLE II. Some predictions of the partial unification scheme $G_{224P} \xrightarrow[M_P]{} G_{2113} \xrightarrow[M_R^0]{} G_{213}$.

M_R^0 (GeV)	$M_P = M_C = M_R^+$ (GeV)	$\sin^2\theta_W$
10^3	8×10^{12}	0.235
	1.4×10^{13}	0.230
10^5	10^{13}	0.235
	1.6×10^{13}	0.230

are desirable at low energies along with the detection of Z_R at the supercolliders, it is necessary that $M_R^0 \simeq 1$ TeV, which implies that the mechanism is natural if $M_R^+ = M_P \gg 10^8$ GeV. A very interesting feature of the new class of models specified in (i) and (ii) is the naturalness of the mechanism with the minimally extended gauge group G_{2113} at lower energies. Using renormalization-group equations (RGE's) it is easy to satisfy the condition $M_R^+ \gg M_R^0$ in the case (i) and the constraint arising out of $K_L - K_S$ mass difference as observed in Ref. 9. In fact RGE's do not constrain $M_P = M_R^+$ as there are three unknown gauge coupling constants $g_{2L} = g_{2R}$, g_{BL}, and g_{3C} for $\mu \geq M_R^+ = M_P$. But in cases (ii a) and (ii b) there are two unknown gauge coupling constants, $g_{2L} = g_{2R}$ and g_{4C}, for $\mu \geq M_R^+ = M_P$ one of which can be eliminated using the fine-structure constant matching at $\mu = M_W$. For the case (ii a) the relation between $\sin^2\theta_W$ and the mass scales can be expressed including one-loop corrections as

$$\sin^2\theta_W = \frac{1}{2} - \frac{\alpha}{3\alpha_S} - \frac{8\alpha}{3\pi}\ln\frac{M_P}{M_C} - \frac{23\alpha}{6\pi}\ln\frac{M_C}{M_R^0} - \frac{11\alpha}{3\pi}\ln\frac{M_R^0}{M_W} , \tag{10}$$

where $\alpha \equiv \alpha(M_W) = e^2(M_W)/4\pi$ and $\alpha_S = g_3^2(M_W)/4\pi$. The corresponding equation for case (ii b) is obtained from Eq. (10) by using $M_R^+ = M_P = M_C$. Solutions to the RGE's in cases (ii a) and (ii b) have been obtained with the QCD parameter $\Lambda_{\overline{\rm MS}} \simeq 0.2$ GeV, where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme, $\sin^2\theta_W \simeq 0.22 - 0.24$ and for values of $M_R^0 \simeq (3\times10^2 - 10^5)$ GeV, some of which are presented in Tables I and II. For the case (ii a) we found 7×10^{13} GeV $\leq M_R^+ = M_P \leq 2\times10^{17}$ GeV for 10^{10} GeV $\gg M_C \gg 10^5$ GeV. In this case, in addition to predicting the low-energy gauge group to be G_{2113} beyond the standard model, the rare kaon decays are also predicted to be observable, corresponding to $M_C \simeq 10^5$ GeV. In the case (ii b) the solutions are consistent: $M_P = M_C = M_R^+ \simeq 10^{12}$ GeV $- 5\times10^{13}$ GeV with a low-mass Z_R boson. The parameter $R = M_R^+/M_R^0 \geq 10^7$ in both cases and is found to guarantee the naturalness condition. The neutrino mass spectrum for lower values of $M_R^0 \simeq 300$ GeV $-$ 1 TeV is of the type eV-keV-MeV for the three generations. In such cases m_{ν_μ} and m_{ν_τ} would violate the cosmological bound. One method of evading the cosmological bound is to make ν_μ and ν_τ unstable against Majoron emission as discussed in Sec. VI. In the next section we examine embeddings of models (i) and (ii) in SO(10) grand unification.

IV. IMPLEMENTATION IN SO(10) WITH G_{2213P} AS AN INTERMEDIATE SYMMETRY

In this section we show how the new seesaw mechanism operates in an SO(10) scenario where G_{2213P} and G_{2113} occur as the two intermediate symmetries. The well-known problem in such a GUT scenario is the presence of undesirable domain walls[11] and inadequate baryon asymmetry[12] of the Universe if $M_P = M_R^+ \ll 10^{11}$ GeV. On the other hand if $M_P = M_R^+ \geq 10^{11}$ GeV the baryon asymmetry is compatible with the observed value and the domain walls created in the early Universe might have been removed by inflation. Our analyses in this paper demonstrate that the RGE's permit such solutions. We discuss the embeddings of these groups in SO(10) and find solutions to the unification mass (M_U), $\sin^2\theta_W$, and intermediate scales including renormalization effects on gauge coupling constants up to two loops and superheavy Higgs scalar effects.[13–15] The case (i) mentioned in Sec. III can be embedded in SO(10) grand unification as follows:

$$\mathrm{SO(10)} \xrightarrow[M_U]{\mathbf{210}} G_{2213P} \xrightarrow[M_P=M_R^+]{\mathbf{45}} G_{2113} \xrightarrow[M_R^c=M_{B-L}]{\mathbf{126}} G_{st} \xrightarrow[M_W]{\mathbf{10}} G_{13} , \tag{11}$$

where the Higgs scalars mentioned in (i) are contained in the respective SO(10) representations: $X \subset \mathbf{45}$, $\Delta_R \subset \mathbf{126}$, $\phi \subset \mathbf{10}$. In addition, the GUT symmetry breaks down to G_{2213P} when the neutral component of the Higgs scalar transforming as (1,1,0,15) under G_{2213P} and contained in **210** acquires VEV $\simeq M_U$. In order to make GUT predictions using the effective gauge theory approach,[13,15] the superheavy components in different Higgs representations needed for SSB in case (i) are noted below along with their masses and transformation properties under G_{2213}:[15]

$$\begin{aligned}
\mathbf{10} &\supset M_{H_1}(1,1,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},3)+M_{H_2}(1,1,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{3})\ ,\\
\mathbf{126} &\supset M'_{H_1}(3,1,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{3})+M'_{H_2}(3,1,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},6)+M'_{H_3}(1,3,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},3)\\
&\quad +M'_{H_4}(1,3,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},6)+M'_{H_5}(1,1,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},3)+M'_{H_6}(1,1,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{3})\\
&\quad +M'_{H_7}(2,2,0,1)+M'_{H_8}(2,2,-\sqrt{\tfrac{3}{2}}\tfrac{2}{3},3)+M'_{H_9}(2,2,\sqrt{\tfrac{3}{2}}\tfrac{2}{3},\bar{3})+M'_{H_{10}}(2,2,0,8)\ ,\\
\mathbf{45} &\supset M_{S_1}(1,1,-\sqrt{\tfrac{3}{2}}\tfrac{2}{3},3)+M_{S_2}(1,1,\sqrt{\tfrac{3}{2}}\tfrac{2}{3},\bar{3})+M_{S_3}(1,1,0,8)+M_{S_4}(2,2,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},3)+M_{S_5}(2,2,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{3})\ ,\\
\mathbf{210} &\supset M'_{S_1}(3,1,-\sqrt{\tfrac{3}{2}}\tfrac{2}{3},3)+M'_{S_2}(3,1,\sqrt{\tfrac{3}{2}}\tfrac{2}{3},\bar{3})+M'_{S_3}(3,1,0,8)\\
&\quad +M'_{S_4}(1,3,-\sqrt{\tfrac{3}{2}}\tfrac{2}{3},3)+M'_{S_5}(1,3,\sqrt{\tfrac{3}{2}}\tfrac{2}{3},3)+M'_{S_6}(1,3,0,8)+M'_{S_7}(2,2,\sqrt{\tfrac{3}{2}},1)+M'_{S_8}(2,2,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},3)\\
&\quad +M'_{S_9}(2,2,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},6)+M'_{S_{10}}(2,2,\sqrt{\tfrac{3}{2}},\bar{1})+M'_{S_{11}}(2,2,\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{3})+M'_{S_{12}}(2,2,-\sqrt{\tfrac{3}{2}}\tfrac{1}{3},\bar{6})\ .
\end{aligned} \tag{12}$$

If the component masses are taken to be arbitrarily nondegenerate, the model loses its predictive power on the proton lifetime (τ_p) and $\sin^2\theta_W$. We examine their impact on GUT predictions by assuming the masses to be (a) degenerate, (b) nondegenerate but not arbitrary as they are constrained by a Coleman-Weinberg-type condition in that a nondegeneracy factor up to 10 might be generated among different component masses in a single GUT representation.[14] In all cases τ_p ($p \to e^+\pi^0$) is predicted near the observable limit. In order to constrain masses under condition (b), we maximize τ_p using the RGE for $\ln(M_U/M_W)$, which leads to

$$\begin{aligned}
&M'_{H_1}=M'_{H_7}=M'_{H_8}=M'_{H_9}=M_{S_4}=M'_{S_1}=M'_{S_2}=M'_{S_7}=M'_{S_8}=M'_{S_{10}}=M'_{S_{11}}=M_{S_5}=M^{(+)}\ ,\\
&M_{H_1}=M_{H_2}=M'_{H_2}=M'_{H_5}=M'_{H_6}=M'_{H_{10}}=M'_{S_3}=M'_{S_9}=M'_{S_{12}}=M_{S_1}=M_{S_2}=M_{S_3}=M^{(-)}\ .
\end{aligned} \tag{13}$$

Using a minimal number of Higgs scalars and three fermion generations we have computed the one- and two-loop coefficients in the equations for $\ln(M_U/M_W)$ and $\sin^2\theta_W$ given below:

$$\begin{aligned}
\ln\frac{M_U}{M_W} &= \frac{3\pi}{29}\left[\frac{1}{\alpha}-\frac{8}{3\alpha_s}\right]-\tfrac{11}{58}\ln\frac{M_R^+}{M_W}+\tfrac{1}{29}\ln\frac{M_R^0}{M_W}-\tfrac{3}{116}\big(-27\ln x_{2L}^U-27\ln x_{2R}^U+\tfrac{64}{7}\ln x_{BL}^U-\tfrac{96}{7}\ln x_{3C}^U\\
&\quad +\tfrac{32}{9}\ln x_{BL}^+ +2\ln x_{1R}^+ +\tfrac{10}{19}\ln x_{2L}^+ -\tfrac{96}{7}\ln x_{3C}^+ +\tfrac{46}{41}\ln x_y^0+\tfrac{10}{19}\ln x_{2L}^0-\tfrac{96}{7}\ln x_{3C}^0\big)+\tfrac{1}{29}\left[21\ln\frac{M^{(+)}}{M_U}-28\ln\frac{M^{(-)}}{M_U}\right]\ ,
\end{aligned} \tag{14}$$

$$\begin{aligned}
\sin^2\theta_W &= \tfrac{15}{58}+\frac{9\alpha}{29\alpha_s}-\frac{92\alpha}{87\pi}\ln\frac{M_R^+}{M_W}+\frac{5\alpha}{58\pi}\ln\frac{M_R^0}{m_W}\\
&\quad -\frac{\alpha}{29\pi}\big(\tfrac{1017}{16}\ln x_{2L}^U-\tfrac{897}{16}\ln x_{2R}^U+\tfrac{103}{28}\ln x_{BL}^U+\tfrac{191}{14}\ln x_{3C}^U+\tfrac{91}{18}\ln x_{BL}^+\\
&\quad +\tfrac{193}{36}\ln x_{1R}^+ +\tfrac{307}{76}\ln x_{2L}^+ +\tfrac{191}{14}\ln x_{3C}^+ +\tfrac{403}{164}\ln x_y^0+\tfrac{307}{76}\ln x_{2L}^0+\tfrac{191}{14}\ln x_{3C}^0\big)\\
&\quad -\frac{\alpha}{174\pi}\left[33\ln\frac{M^{(+)}}{M_U}-44\ln\frac{M^{(-)}}{M_U}\right]\ ,
\end{aligned} \tag{15}$$

where

$$x_i^U=\frac{\alpha_i(M_U)}{\alpha_i(M_R^+)},\quad x_i^+=\frac{\alpha_i(M_R^+)}{\alpha_i(M_R^0)},\quad x_i^0=\frac{\alpha_i(M_R^0)}{\alpha_i(M_W)},\quad \text{and } \alpha_i(\mu)=\frac{g_i^2(\mu)}{4\pi}\ .$$

Using an iterative convergence procedure that ensures fine-structure constant matching[15] at $\mu=M_W$ we have computed M_U, τ_p, and $\sin^2\theta_W$ as a function of M_R^+ for the degenerate and nondegenerate cases as shown in Figs. 2 and 3, respectively, while keeping Z_R light ($M_R^0 \simeq 1$ TeV), where $\eta^{(\pm)}=\ln M^{(\pm)}/M_U$. Some interesting solutions are summarized in Table III.

At the one-loop level, neglecting superheavy-scalar effects, the model predicts $M_U \simeq 10^{15}$ GeV and $\sin^2\theta_W \simeq 0.225$ for $\Lambda_{\overline{\rm MS}} \simeq 250$ MeV and $M_P = M_R^+ \simeq 10^{11}$ GeV. This is consistent with $(\tau_p)_{\max} \simeq 10^{35}$ yr, where we have included an uncertainty factor of $10^{\pm 3}$ in τ_p arising

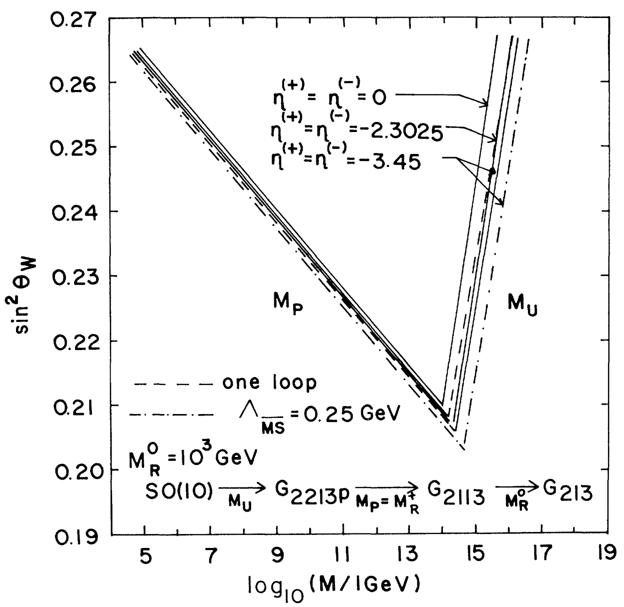

FIG. 2. Predictions of the symmetry-breaking pattern $SO(10)\rightarrow G_{2213P}\rightarrow G_{2113}$ as described in the text with $M_{Z_R}\simeq 1$ TeV with and without degenerate superheavy-Higgs-scalar contributions. The dot-dashed curve is for $\Lambda_{\overline{MS}}\simeq 0.250$ GeV, others are for $\Lambda_{\overline{MS}}=0.160$ GeV.

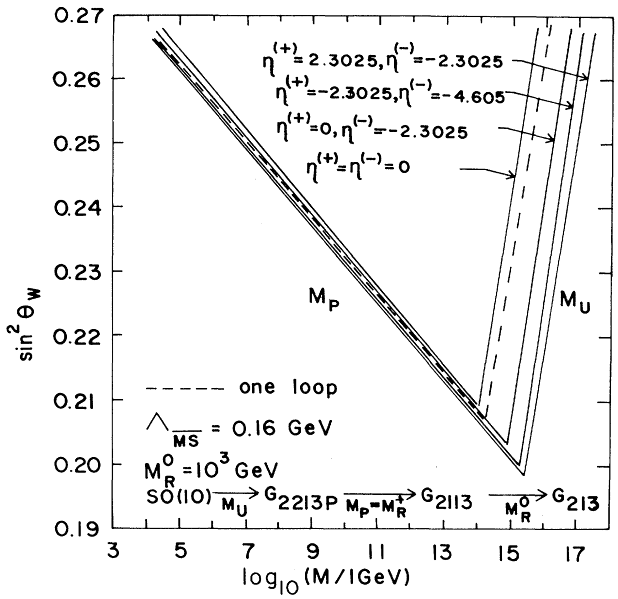

FIG. 3. Same as Fig. 2, but for nondegenerate superheavy scalar masses under a Coleman-Weinberg-type constraint, and for $\Lambda_{\overline{MS}}=0.160$ GeV.

out of uncertainties in the estimation of the proton-decay matrix elements, branching ratios, and $\Lambda_{\overline{MS}}$.[16,17] Including contributions up to two loops and no superheavy-Higgs-scalar effects, τ_p decreases by two orders corresponding to the curve $\eta^{(+)}=\eta^{(-)}=0$ in Fig. 2 for which $\Lambda_{\overline{MS}}=160$ MeV. Including the superheavy Higgs scalars lighter than M_U by a factor 10 (50) increases the two-loop computation of τ_p by 2 (3) orders for $\Lambda_{\overline{MS}}=160$ MeV, and the decrease in $\sin^2\theta_W$ is only 0.0015. Allowing the possibility of $\Lambda_{\overline{MS}}\simeq 250$ MeV and the superheavy scalars lighter by a factor 50 from M_U, we find $(\tau_p)_{\max}=10^{34}-10^{36}$ yr, with $M_P=M_R^+\geq 10^{11}$ GeV and $\sin^2\theta_W=0.220-0.227$ as shown in Fig. 2 and Table III. Increasing M_R^0 from 1 TeV to 100 TeV does not have a significant impact on the GUT predictions. In the case of nondegenerate superheavy components, restricting $M_P=M_R^+>10^{11}$ GeV and $\sin^2\theta_W\simeq 0.22-0.23$, τ_p is found to increase over the one-loop predictions by nearly 2 orders if $M^{(+)}=M_U$ and $M^{(-)}=M_U/10$. In this case $\tau_p\simeq 10^{32\pm 3}-10^{34\pm 3}$ yr, with $M_R^+=M_P=10^{11}-10^{12}$ GeV and $\sin^2\theta_W\simeq 0.22-0.225$. For larger values of nondegeneracy factor, τ_p could be larger as shown in Fig. 3. The allowed values of the low mass of the Z_R boson (300 GeV – 1 TeV) are consistent with the eV-keV-MeV type of mass spectrum for the neutrinos of the three generations when we choose $m_1^D=m_e$, $m_2^D=m_\mu$, and $m_3^D=m_\tau$, as a

TABLE III. Some predictions of the model $SO(10)\xrightarrow{M_U}G_{2213P}\xrightarrow{M_P}G_{2113}$ on $\sin^2\theta_W$ and τ_p with $M_R^0=1$ TeV, $\Lambda_{\overline{MS}}=0.16$ GeV, and different values of the parity violating scale (M_P), including superheavy-Higgs-scalar effects.

$\eta^{(+)}$	$\eta^{(-)}$	$M_R^+=M_P$ (GeV)	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_p (yr)
−2.30	−2.30	8×10^8	2×10^{15}	0.240	36.1	$1.2\times 10^{32\pm 3}$
		5.6×10^9	1.4×10^{15}	0.235	35.7	$3\times 10^{31\pm 3}$
		2.5×10^{11}	7×10^{14}	0.225	32.0	$1.5\times 10^{30\pm 3}$
−3.45	−3.45	4.5×10^9	2×10^{15}	0.235	32.4	$10^{32\pm 3}$
		2×10^{11}	1.2×10^{15}	0.226	31.6	$10^{31\pm 3}$
−2.3	−4.6	1.6×10^{11}	2.4×10^{15}	0.230	31.1	$2\times 10^{32\pm 3}$
		10^{12}	1.8×10^{15}	0.225	30.8	$10^{32\pm 3}$
		6.3×10^{12}	1.3×10^{15}	0.221	30.4	$1.4\times 10^{31\pm 3}$

consequence of natural seesaw mechanism.

One difficulty in having masses of the order of keV and MeV for ν_μ and ν_τ is the violation of the cosmological bound. The difficulty is removed by making them unstable with respect to decay into ν_e by the emission of a Majoron,[18] which is obtained by introducing an additional global $U(1)_l$ (l = lepton number) symmetry in the theory and breaking it spontaneously at a scale $M \gg M_R^0$. The RGE's also permit solutions with larger values of $M_R^0 = M_{Z_R} \simeq 10^5 - 10^6$ GeV. When $M_P = M_R^+ \simeq 10^{11} - 10^{12}$ GeV, $R \simeq 10^6 - 10^7$ for such larger values of M_R^0, which satisfies the naturalness criterion. In this case $m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau} \simeq 1-10$ eV and there is no conflict with the cosmological bound. The weak-interaction phenomenology with such large Z_R mass is indistinguishable from the standard-model predictions.

V. IMPLEMENTATION IN SO(10) WITH G_{214} AS AN INTERMEDIATE SYMMETRY

In this section we show how the seesaw mechanism can be implemented naturally with G_{214} as one of the two intermediate symmetries occurring in an SO(10) scenario. Compared to the case discussed in Sec. IV, this has the novel feature that both $SU(2)_R$ and P break at the GUT scale that is experimentally constrained as $M_U \geq 10^{15}$ GeV. The question of domain-wall problem does not arise in this case; in addition, the proton decay rate could be close to the observable limit if RGE's permit $M_U \simeq 10^{15}$ GeV. For case (ii b) we have found the unification mass too low to be allowed by proton-lifetime measurements unless additional fine-tuning is permitted. On the other hand, case (ii a) is promising in the context of the GUT scenario:

$$\mathrm{SO}(10) \xrightarrow[M_U]{54+45_1} G_{214} \xrightarrow[M_C]{45_2} G_{2113} \xrightarrow[M_R^0]{126} G_{213} \ . \tag{16}$$

The Higgs scalars mentioned in Sec. III for case (ii a) are contained in various SO(10) representations: $X(1,3,1) \subset \mathbf{45}_1$, $\xi(1,1,15) \subset \mathbf{45}_2$, $\Delta_R(1,3,\overline{10}) \subset \mathbf{126}$, $\phi(2,2,1) \subset \mathbf{10}$ where the transformation properties mentioned are under G_{224}. Note that both **54** and $\mathbf{45}_1$ are needed for the SSB at $\mu \sim M_U$. The masses of superheavy components of different Higgs representations needed for SSB in the case (16) are noted below with their transformation properties under G_{214}:

$$\begin{aligned}
&\mathbf{10} \supset M_{H_1}(2,-\tfrac{1}{2},1) + M_{H_2}(1,0,6) \ , \\
&\mathbf{126} \supset M'_{H_1}(1,0,6) + M'_{H_2}(3,0,\overline{10}) + M'_{H_3}(1,0,10) + M'_{H_4}(1,-1,10) + M'_{H_5}(2,\tfrac{1}{2},15) + M'_{H_6}(2,-\tfrac{1}{2},15) \ , \\
&\mathbf{45}_1 \supset M'_{S_1}(3,0,1) + M'_{S_2}(1,0,15) \ , \\
&\mathbf{45}_2 \supset M_{S_1}(3,0,1) + M_{S_2}(1,1,1) + M_{S_3}(1,0,1) + M_{S_4}(1,-1,1) + M_{S_5}(2,\tfrac{1}{2},6) + M_{S_6}(2,-\tfrac{1}{2},6) \ , \\
&\mathbf{54} \supset M''_{S_1}(3,1,1) + M''_{S_2}(3,0,1) + M''_{S_3}(3,-1,1) + M''_{S_4}(1,0,20) + M''_{S_5}(2,\tfrac{1}{2},6) + M''_{S_6}(2,-\tfrac{1}{2},6) \ .
\end{aligned} \tag{17}$$

Maximization of τ_p leads to the following constraint on the superheavy-component masses:

$$\begin{aligned}
&M_{H_1} = M'_{H_2} = M'_{H_4} = M_{S_1} = M_{S_2} = M_{S_4} = M_{S_5} = M_{S_6} = M'_{S_1} = M''_{S_1} = M''_{S_2} = M''_{S_3} = M''_{S_5} = M''_{S_6} = M^{(+)} \ , \\
&M_{H_2} = M'_{H_1} = M'_{H_3} = M'_{H_5} = M'_{H_6} = M'_{S_2} = M''_{S_4} = M^{(-)} \ .
\end{aligned} \tag{18}$$

Using three generations of fermions with masses $\mu < M_W$, minimal number of Higgs scalars at various stages of SSB, and the super heavy-Higgs-scalar effects near $\mu \simeq M_U$ we compute $\ln M_U/M_W$ and $\sin^2\theta_W$ up to two loops as

$$\begin{aligned}
\ln\frac{M_U}{M_W} = {} & \frac{6\pi}{67}\left[\frac{1}{\alpha} - \frac{8}{3\alpha_s}\right] - \frac{2}{67}\ln\frac{M_c}{M_W} + \frac{2}{67}\ln\frac{M_R^0}{M_W} \\
& - \tfrac{3}{134}\left(\tfrac{10}{19}\ln x_{2L}^U + \tfrac{34}{15}\ln x_{1R}^U - \tfrac{224}{9}\ln x_{4c}^U + \tfrac{10}{19}\ln x_{2L}^C + 2\ln x_{1R}^C + \tfrac{32}{9}\ln x_{BL}^C - \tfrac{96}{7}\ln x_{3c}^C + \tfrac{46}{41}\ln x_y^0 + \tfrac{10}{19}\ln x_{2L}^0 - \tfrac{96}{7}\ln x_{3c}^0\right) \\
& + \frac{1}{67}\left[28\ln\frac{M^{(+)}}{M_U} - 29\ln\frac{M^{(-)}}{M_U}\right] \ ,
\end{aligned} \tag{19}$$

$$\begin{aligned}
\sin^2\theta_W = {} & \frac{35}{134} + \frac{61\alpha}{201\alpha_s} - \frac{437\alpha}{402\pi}\ln\frac{M_c}{M_W} + \frac{35\alpha}{402\pi}\ln\frac{M_R^c}{M_W} \\
& - \frac{\alpha}{67\pi}\left(\tfrac{711}{76}\ln x_{2L}^U + \tfrac{4473}{128}\ln x_{1R}^U - \tfrac{13187}{44}\ln x_{4c}^U + \tfrac{711}{76}\ln x_{2L}^C + \tfrac{449}{36}\ln x_{1R}^C \right. \\
& \left. + \tfrac{71}{6}\ln x_{Bl}^C + \tfrac{433}{14}\ln x_{3c}^C + \tfrac{939}{164}\ln x_y^0 + \tfrac{711}{76}\ln x_{2L}^0 + \tfrac{433}{14}\ln x_{3c}^0\right) + \frac{\alpha}{804\pi}\left[-1633\ln\frac{M^{(+)}}{M_U} + 928\ln\frac{M^{(-)}}{M_U}\right] \ ,
\end{aligned} \tag{20}$$

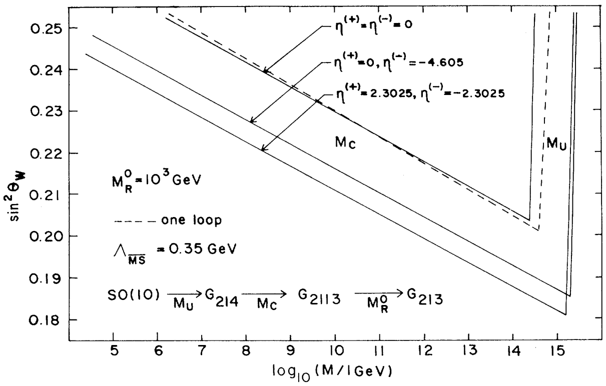

FIG. 4. Predictions of the symmetry breaking pattern $SO(10)\rightarrow G_{214}\rightarrow G_{2113}$ as described in the text including superheavy-Higgs-scalar effects for $M_{Z_R}=1$ TeV and $\Lambda_{\overline{MS}}=0.16$ GeV.

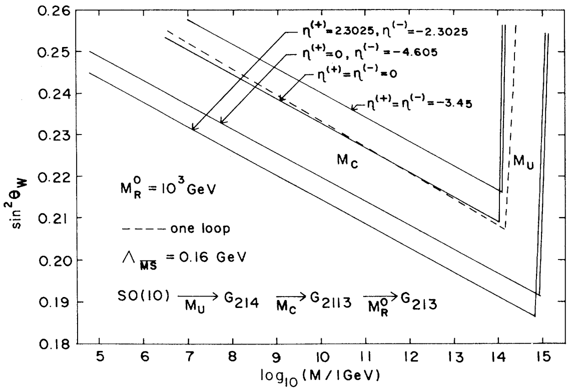

FIG. 5. Same as Fig. 4 but for $\Lambda_{\overline{MS}}=0.35$ GeV.

where

$$x_i^U=\frac{\alpha_i(M_U)}{\alpha_i(M_c)},\quad x_i^C=\frac{\alpha_i(M_c)}{\alpha_i(M_R^0)},\quad \text{and}\quad x_i^0=\frac{\alpha_i(M_R^0)}{\alpha_i(M_W)}\ .$$

Following the iterative convergence approach to solve two-loop renormalization group equations and using plausible values of superheavy component masses, our solutions for the intermediate scale and M_U for $M_R^0\simeq 1$ TeV are shown in Figs. 4 and 5 for $\Lambda_{\overline{MS}}\simeq 0.160$ GeV, and 0.350 GeV, respectively (Ref. 17) where $\eta^{(\pm)}=\ln(M^{(\pm)}/M_U)$. Some of the interesting solutions are also presented in Table IV. At the one-loop level with $\Lambda_{\overline{MS}}=0.350$ GeV the predicted value of τ_p is found to be very close to the observed experimental limit for $M_C=10^{11}$ GeV and $\sin^2\theta_W\simeq 0.235$, but τ_p is found to be 1–2 orders less than the experimental limit for $\Lambda_{\overline{MS}}=0.160$ GeV. When superheavy-Higgs-scalar effects are included in two-loop calculations, we find $\tau_p\simeq 10^{32\pm3}-10^{34\pm3}$ yr, $\sin^2\theta_W\simeq 0.230$, $M_C\simeq 10^7$ GeV with $\Lambda_{\overline{MS}}\simeq 160$ MeV if the heavier (lighter) components differ by a factor 10 from the unification mass. For larger values of $\Lambda_{\overline{MS}}$ or nondegeneracy factors, τ_p is found to increase further. We find that this SO(10) model permits observable rare kaon decays corresponding to $M_c\simeq 10^5$ GeV provided $\sin^2\theta_W\simeq 0.24$. In all allowed solutions in this model $M_R^+=M_U=M_P\simeq 10^{15}$ GeV. With $M_R^0\simeq 1$ TeV, $R\simeq 10^{12}$, and the naturalness criterion is easily satisfied. As in Sec. II, the low-mass Z_R boson yields the neutrino-mass spectrum as eV-keV-MeV for the three generations. The violation of the cosmological bound by the ν_μ and ν_τ masses is avoided by making these neutrinos unstable against Majoron emission through the introduction of an additional global lepton-number symmetry $U(1)_1$ (Ref. 18). But the RGE's also permit $M_R^0\simeq 10^5-10^6$ GeV as the Z_R-boson mass for which $m_{\nu_p}<m_{\nu_\mu}<m_{\nu_\tau}\simeq 1-10$ eV as a consequence of the natural seesaw mechanism with $R=10^9-10^{10}$, and this is consistent with the cosmological bound with stable neutrinos. In this case the predicted weak-interaction phenomenology at low energy cannot be distinguished from the standard-model predictions.

TABLE IV. Some predictions of the model $SO(10)\xrightarrow{M_U} G_{214}\xrightarrow{M_C} G_{2113}$ for two values of $\Lambda_{\overline{MS}}$ on $\sin^2\theta_W$ and τ_p with $M_R^0=1$ TeV and different M_C including superheavy-Higgs scalar-effects as described in the text.

$\Lambda_{\overline{MS}}$ (GeV)	$\eta^{(+)}$	$\eta^{(-)}$	M_C (GeV)	M_U (GeV)	$\sin^2\theta_W$	α_G^{-1}	τ_p (yr)
0.16	2.3	−2.3	4×10^5	10^{15}	0.240	48.3	$1.7\times10^{31\pm3}$
			3×10^6	10^{15}	0.235	47.6	$1.4\times10^{31\pm3}$
	0	−4.6	3×10^6	1.3×10^{15}	0.240	42.5	$2.5\times10^{31\pm3}$
			1.6×10^8	1.1×10^{15}	0.230	40.8	$1.5\times10^{31\pm3}$
0.35	2.3	−2.3	10^5	2.4×10^{15}	0.24	48.8	$4.3\times10^{32\pm3}$
			5.6×10^6	2.2×10^{15}	0.230	47.4	$2.9\times10^{32\pm3}$
	0	−4.6	5.6×10^6	2.7×10^{15}	0.235	41.7	$5\times10^{32\pm3}$
			4×10^7	2.5×10^{15}	0.230	41.0	$3.8\times10^{32\pm3}$

VI. SUMMARY AND DISCUSSION

In this paper we have suggested the new possibility that the seesaw mechanism for neutrino masses could be natural in the context of the left-right-symmetric gauge group, partial unification scheme, and GUT's even if the scales of P and $SU(2)_R$ breakings are identical. In these models, the P-breaking scale is the same as the $W_R^{\pm}$ gauge-boson mass ($M_P = M_R^+$) and the $U(1)_{B-L}$-breaking scale is the same as the Z_R-boson mass (M_R^0). The criterion which guarantees naturalness has been derived and is found to depend upon the largeness of the ratio $R = M_R^+ / M_R^0 \gg 10^5$. At the critical value of the ratio $R \simeq 10^5$, the induced and seesaw mechanism contributions are comparable, but for larger values of R the induced neutrino mass becomes smaller.

In the LRS model based upon the gauge group G_{2213P}, it is very easy to implement the mechanism as there is not much restriction on $M_P = M_R^+$. In the partial-unification scheme with one intermediate symmetry G_{2113}, the RGE permits $M_P = M_R^+ \simeq M_c = 10^{12} - 5\times 10^{13}$ GeV with $M_R^0 = M_{Z_R} = 300$ GeV-10^6 GeV [case (ii b)]. However, with two intermediate symmetries G_{214} and G_{2113} [case (ii a)] the solutions allow $M_P = M_R^+ \simeq 7\times 10^{13} - 10^{17}$ GeV for 10^{10} GeV $> M_C > 10^5$ GeV, predicting rare kaon decays to be observable by low-energy experiments besides a low-mass Z_R boson.

In the SO(10) model, implementation of the natural seesaw mechanism has been found to be possible with parity (P) surviving down to an intermediate scale $M_P = M_R^+ \simeq 10^{11}$ GeV or broken at the GUT scale $M_P = M_U = M_R^+ \geq 10^{15}$ GeV. With G_{2213P} and G_{2113} intermediate symmetries, RGE's up to two loops, with superheavy-Higgs-scalar masses lighter than M_U by a factor of 10–50, are found to allow the intermediate P-breaking scale $M_P \simeq 10^{11} - 10^{12}$ GeV, observable proton decay by the second generation of experiments with $\tau_p \simeq 10^{33} - 10^{35}$ yr, and a low-mass Z_R boson ($M_R^0 \simeq 300 - 10^3$ GeV). In this case there is the possibility that the domain walls created in the early Universe might have been removed by inflation. In this context it is to be noted that the large P-violating scale can be associated with the breaking of Peccei-Quinn symmetry invoked to solve the strong CP problem and can be generated by the principle of geometric hierarchy from $M_{Pl} \simeq 10^{19}$ GeV and $M_{Z_R} \simeq 10^3$ GeV, or M_W. Further, it has been observed that while embedding a LRS gauge group as an intermediate symmetry in SO(10), the generation of an adequate baryon asymmetry of the Universe needs such a large P-violating scale.[12] In the other interesting SO(10) scenario with G_{214} and G_{2113} as the two intermediate symmetries, superheavy-Higgs-scalar masses differing by a factor 10 (lighter or heavier) from the unification mass allow $(\tau_p)_{\max} \simeq 10^{35}$ yr with the possibility of observable rare kaon decays and a low-mass Z_R boson. In the two SO(10) models discussed here G_{2113} is allowed to be the gauge symmetry beyond the standard model with the permitted values of a Z_R-boson mass varying over a wider range: $300-10^5$ GeV.

The weak-interaction phenomenology at low energy does permit a low-mass Z_R boson ($M_R^0 \simeq 300$ GeV-1 TeV) in the G_{2113} model which yields fits to the neutral- and charged-current data similar to the standard-model predictions.[9,19] When such values of M_R^0 are used in the natural seesaw mechanism, the neutrino masses are of the order eV, keV, and MeV for the first-, second-, and third-generation neutrinos, respectively, out of which the latter two violate the cosmological bound. The cosmological bound can still be respected with low-Z_R masses by making ν_μ and ν_τ unstable with respect to the emission of a Majoron which is a massless scalar carrying 2 units of lepton number, and it is created when an additional global symmetry $U(1)_l$ (l =lepton number), attached to the models, breaks spontaneously.[18] With the other allowed possibility, $M_R^0 \simeq 10^5$ GeV, $m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau} \simeq 1 - 10$ eV, there is no violation of the cosmological bound. The weak-interaction phenomenology at lower energies is then indistinguishable from the standard-model predictions within the available experimental accuracies. However, one novel feature in the partial-unification scheme and SO(10) model with G_{214} and G_{2213} intermediate symmetries is the prediction of observable rare kaon decays such as $K_L \to \mu \bar{e}$. The analyses carried out here in SO(10) can be easily implemented in other GUT's such as SO($2N$) ($N > 5$), E_6, and SU(16) with similar predictions. However, in $SU(8)_L \times SU(8)_R$ while all other low-energy predictions are similar, it is possible to have a more stable proton since the gauge-boson-mediated interaction corresponding to the proton decay is absent.

Finally from the investigations carried out in this paper we conclude that scenarios different from those discussed by Chang and Mohapatra in Ref. 3 and worked out earlier[7,8] do exist in LRS models, partial unification schemes, and GUT's in which the seesaw mechanism can provide a natural explanation for small Majorana neutrino masses even if the P- and $SU(2)_R$-breaking scales are identical.

ACKNOWLEDGMENTS

One of us (M.K.P) thanks Professor C. K. Majumdar, Professor Partha Ghose and Professor D. P. Roy for invitation to the CP-Violation Meeting and High Energy Physics Workshop at the S. N. Bose National Centre, Calcutta, where a part of this work was reported.

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