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# IMPACT OF SETTLEMENT SIZE AND PATTERN ON THE LOCATIONAL SYSTEM OF FACILITY DISTRIBUTION

**Surendra Singh**
*North-Eastern Hill University*
*Shillong, India*

**Abstract.** Reviewing concerned literature on facility locations, it is valid to say that location-allocation models basicly produce the solution of optimal partitioning problems of space. Generating data of facility locations (m-centroids) and their aggregate travel distances (R) at each stage of procedure, a distance-location relationship of the location system R(m) has been established and empirically tested. It is found that this relationship is always inverse and it is formally hyperbolic in its nature. There is a fast reduction of aggregate as well as proportionate distances at early stages and slow reduction at latter stages of locational system. Consequently, optimal solutions recorded 5.70 Km aggregated optimal distance at 9 centroids in non-random distribution, and 10.91 km at 10 centroids in cluster distribution of unweighted point patterns in the central parts of West Bengal plains of India.

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## 1. INTRODUCTION

During the last two decades, there have been numerous studies on the application and modification in the location-allocation procedures for solving the problems of optimisation of the complex nature of spatially oriented elements of facility distribution (Scott 1971, 1975, Lea 1978, Leonardi 1981). Recent surveys of facility location literature highlight the validity of locational problems which have been mainly classified into three broad areas of locational systems by developing accordingly the corresponding workable models. They are: continuous models in which a set of points in the plane is considered, discrete models that are based on a set of points of finite numbers, and network models which are developed and applied for a set of points on a topological graph representing a transport system (Hansen, et. al. 1987). However, there are many problems of locational system optimisation with relation to its objective functions, which have been dealt in three ways:

(a) Minisum Euclidean distance facility location problems on a plane. This type of problems has been tackled from mid-17th century and logically put forward by Weber in the early 20th century. They were analyzed by Eells in 1930 who noted that minisum aggregate travel is spatial median (Wesolowsky 1993). In the recent past, the minisum Euclidean distance technique was extended in the form of a location-allocation model for a larger set of point data for which Kuhn & Kuenne (1962) and Cooper (1963) developed it in algebraic form and proposed an algorithm. Later on, the algorithm was used for locational choices of facility distribution in a competitive or non-competitive environment and for a variety of locational problems of dynamic nature in order to increase the number of facilities over time (Neft 1966, Presscot & Visscher 1977, Ghosh & Craig 1984, Singh 1990, Tansel & Yesilkokcen 1993).

(b) Optimisation of multi-facility locations. In this case p-hub median problems, as analogous to a p-median ones, have been applied to deal with transportation and communication facilities. They minimise movement costs by applying integer programming formulations (O'Kelly 1986, Campbell 1990, 1993, 1994). Now, Some solutions of multi-facility optimisation problems are also available on the basis of exact optimality conditions rather than heuristic ones which are derived by developing the model of multi- dimentional flow on the problem graph (Lefebvre et. al. 1991, Plastria 1992).

(c) Network optimisation problems which are solved by considering a topological graph representing transport and communication systems and their dynamic aspects such as changing shape and configuration of the network over time (Hakimi 1965, Peeters & Thomas 1993, Tansel 1993).

One area that has so far received limited attention is the measurement of spatial efficiency which is closely related to the impact of physical components of locational system optimisation. The physical components of locational systems are mainly three: (i) the point pattern as a distance factor of a locational system, (ii) the population size of settlement points as a demand factor which has been used either in the form of spatial competition of demand or in a direct way considering the population as the weight of the settlement points, and (iii) the mode of conveyance and efficient transport network as a factor of travel cost reduction. In fact, point pattern and population size affect directly the point configuration and alter the choice of locational decisions by changing the locus of the system. In point distribution, the points may either attract or repulse each another. The processes of attraction, which are operated by some types of diffusion and competition of distributive facilities, form a clustered pattern, whereas processes of repulsion would likely produce regular or non-random patterns (Haggett, et. al. 1977, Boots & Getis 1988). There are various methods for measuring point pattern. Some are stochastic methods based on the probability theory and others are distance-dependent. Since distance is an important element of a locational system, the Nearest Neighbour Distance (NND) method is appropriate for measuring the degree of point dispersion. RN value, which is the ratio between mean NND, Do which is observed from point distribution and represents the expected value of average NND, De which is 0.5 $\sqrt{A/N}$ (where A is the total area and N is the total number of points in the study area), is an ideal measurement of point pattern (Clark & Evans 1954). Higher the value of RN (around 2.00) lesser will be the degree of compactness of point pattern which denotes uniform point pattern and vice-versa. The RN value around 1.00 indicates the randomness of the point distribution. On the other hand, population size of the points is an indicative attribute of the availability and intensity of distributive facilities. There is a positive relationship between population size and number of facilities available on the settlements. That is, as the population size increases so does the intensity and a sufficient number of facilities will be available thereon. These processes of point distribution can be studied and the optimal solution of locational system can be achieved by establishing its location-distance relationship.

Thus, the purpose of the present paper is to establish the distance-location relationship through applying a location-allocation model and to compare its coefficients with the RN values of various point patterns. It would help to understand the salient features of locational systems working with their different point-pattern conditions.

## 2. DISTANCE-LOCATION RELATIONSHIP AND OPTIMAL PARTITIONING

Location-allocation models basically produce the solution of optimal partitioning problems of space. Denote the number of central facilities to be located by m (known as centroids of the system) and the total number of the points that are fixed in the locational system to be served by these m centroids by n. This problem is then reduced to the problem of findings an optimal m-fold partitioning of the set of n points. If it is blocked out by the lines, then these regions necessarily form a set of non-overlapping convex polygons for finding out the optimal aggregate distance of the locational system.

There are many computer programmes developed for this purpose to get the exact or heuristic solution of a location-allocation problem. The work of the Department of Geography, University of Iowa, is recognized in connection with the compilation of almost all algorithms developed for non-competitive environmental conditions (Rushton, Goodchild & Ostresh 1973). An application of the Multi-Source Location-Allocation Programme, for a heuristic solution of centroid selection and point configuration, was proposed by Ostresh (1973), where the total distance of the m-centroid system and its point configuration have been calculated for different point patterns. Furthermore, the locus of the system is studied by establishing a distance-location relationship, R(m). This relationship is always inverse, though not necesarily linear. The total distance of the system, R, reduces when its number of centroids, m, is increased. This fact can be confirmed by explaining theoretically two extreme conditions of facility distribution of the locational system. They are:

(i) If a facility distributed in an area is to be performed by only one-centroid, a case of complete concentration of facility distribution where $m = 1$ and all settlements are assigned to only one cnetroid, then the total travel distance of the system, R1, will be maximum, and

(ii) If a facility is to be performed by each and every settlement point of the area, a case of complete unification of facility distribution over space where $m = n$, then total travel distance of the system, Rn, must be equal to zero.

Undoubtedly, these conditions are not applicable to real-world situations because of unabiquous nature of facility distribution, but they are helpful for understanding its degree of spatial primacy and locational behaviour. In fact, the distance-location relationship, R(m), is formally hyperbolic (Amedeo 1969, Singh 1990) and its degree of concavity is comparable with the RN scale of point patterns. An algebraic expression of the form R(m) is:

$R(m) = R1\ (m^{-a} - n^{-a}), \qquad \text{s.t. } 0 < a < 1,$ ...(1)

where a, R1 and n are constants, R and m are variables as defined above in the text.

The system's optimality must be at the point where rate of change of Rth variable will be equal to the rate of change of the m-centroid variable. It is defined as

$dR/dm = 1,$ ... ... ...(2)

where the optimal number of centroids, m*, as well as of the aggregate distance, R*, must be

$m^* = - (aR1)^{1/(1+a)},$ and ... ...(3)

$$R^* = R1\ [\{(aR1)^{1/(1+a)}\}^{-a}\ -n^{-a}] \qquad \ldots(4)$$

## 3. INTERPRETATION

The calculations of total distances of the locational system with respect to its number of centroids for unweighted and weighted point patterns (unweighted for observing the effects of settlement patterns and population weight for showing the population effects on locational system of facility distribution) have been performed successfully for four different sample areas of 56.25 sq. km. each, keeping in mind that these areas are the proper representatives of various point patterns viz., uniform (i.e. hypothetical), random, non-random and cluster. The empirical data of settlement size and pattern have been collected from the parts of two toposheets (No. 780-4 and 780-2) of 1:50,000 scale which include the central parts of West Bengal plains of India. For illustrative purposes, the maps and tables have been restricted to random and non-random distributions only because uniform and cluster point patterns, which are the extreme cases of either sides of the distribution, are rarely found in these areas. The salient features of the m-centroid locational system highlight the following main points in connection to its internal componental relationship and point configuration.

(a) In case of the unweighted point patterns of random and non-random distributions, the aggregate distances and configurations of point allocation seem to be different at various levels of the m-centroid system. Aggregate distance is reduced by 22.5 percent from 133.1 km. in one-centroid systems to 103.1 km. in two-centroid systems, by 21.6 per cent from two- to three-centroid systems and 15.8 per cent from three- to four-centroid systems in random distributions, while these proportionate reductions of distances are higher in non-random distributions: 33.4 per cent at two-centroid systems, 25.0 per cent at three and 9.0 per cent only at four-centroid systems. It seems to be a trend of distance reduction at four- or five-centroid systems of weighted as well as unweighted point patterns (Table- 1 & 2).

(b) Since the aggregate distance of the system is a hyperbolic function of the number of centroids, R(m), and the constant of this function a (which denotes the degree of concavity of system's locii) is the non-random distribution (RN = 1.0923) a fast reduction is recorded of aggregated total as well as proportionate distances at early stages of increasing centroids and slow reduction at its latter stages because of its higher values for weighted and unweighted point patterns. Consequently, the optimal solution of m-centroid systems as calculated by equations 3 and 4 is feasible for nine centroids at which the optimal aggregate distance recorded is 70 km. in non-random and 6.41 km. in random distributions of unweighted point patterns (Table- 3).

(c) There is no significant difference in the configuration of the points of centroid systems of weighted and unweighted point patterns of random as well as non-random distributions. However, the optimal arrangements and horizontal features of point configuration have been marked differentiated at each level of optimal partitioning between these two separate distributions (Figs.- 1 & 2).

(d) The differences of constant a between unweighted and weighted point patterns have been calculated to show the effects of population size on locational systems of facility distribution. Obviously, higher differential values indicate greater influence of population size on the locational system and vice-versa. For instance, the differential value is higher (0.808) in random distributions, lesser (0.7729) in non-random and least (0.1408) in cluster distributions, though it increases from unweighted to weighted point patterns (Table- 3). Consequently, in random distribution of point patterns, the population size has greater influence on the locational systems. Undoubtedly, population size of the settlements, which is positively related to the intensity of facility availability in the locational system, determines the location of centroids. In addition, the effects of point dispersion can not be ignored in a locational system. For instance, clear cut groupings with less aggregate travel distance can be observed in the clustered point patterns for the optimal solution of facility distribution for which point dispersion is a more influential attribute than the population size and hence, unweighted point patterns have lesser degree of concavity (-.5133) than weighted ones (-.6541).

At the end, it has been concluded that the degree of concavity and the maximum aggregate distance of the hyperbolic form of the m-centroid system are the main determinants of its optimal solution which are directly influenced by point patterns and the population size of the locational system. Conclusively, the uniform point pattern exhibits a lower degree of concavity of system's locii and hence its optimality shows a diversified facility distribution, whereas the above is not true in the case of random as well as non-random distributions.

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Table- 1: Emerging Features of Locational System in Random and Non-Random Settlement Patterns (unweighted).

| Optimal Centroid System | RANDOM PATTERN | | | | NON-RANDOM PATTERN | | | |
|---|---|---|---|---|---|---|---|---|
| | n | d | dR | | n | d | dR | |
| | | | Total | % | | | Total | % |
| One-Centroid System: | | | | | | | | |
| I-group | 49 | 133.1 | — | — | 49 | 159.7 | — | — |
| R1 | — | 133.1 | — | — | — | 159.7 | — | — |
| Two-Centroid System: | | | | | | | | |
| I-group | 23 | 60.0 | — | — | 26 | 53.7 | — | — |
| II-group | 26 | 43.1 | — | — | 23 | 52.6 | — | — |
| R2 | — | 103.1 | 30.0 | 22.5 | — | 106.4 | 53.3 | 33.4 |
| Three-Centroid System: | | | | | | | | |
| I-group | 16 | 30.7 | — | — | 23 | 40.1 | — | — |
| II-group | 17 | 28.6 | — | — | 13 | 19.4 | — | — |
| III-group | 16 | 21.5 | — | — | 13 | 20.3 | — | — |
| R3 | — | 80.8 | 22.3 | 21.6 | — | 79.8 | 26.6 | 25.0 |
| Four-Centroid System: | | | | | | | | |
| I-group | 10 | 15.2 | — | — | 15 | 22.0 | — | — |
| II-group | 11 | 16.8 | — | — | 11 | 15.0 | — | — |
| III-group | 14 | 20.3 | — | — | 10 | 15.0 | — | — |
| IV-group | 14 | 17.5 | — | — | 13 | 20.6 | — | — |
| R4 | — | 69.8 | 11.0 | 15.8 | — | 72.6 | 7.2 | 9.0 |
| Five-Centroid System: | | | | | | | | |
| R5 | — | 61.2 | 8.6 | 12.3 | — | 66.2 | 6.4 | 8.8 |

N.B.: For abbreviations, See foot note of Table- 2.

Table- 2: Emerging Features of Locational System in Random and Non-Random Settlement Patterns with Population Weight.

| Optimal Centroid Systems | RANDOM PATTERN |  |  |  | NON-RANDOM PATTERN |  |  |  |
|---|---|---|---|---|---|---|---|---|
|  | n | d | dRp Total | % | n | d | dRp Total | % |
| One-Centroid System: |  |  |  |  |  |  |  |  |
| I-group | 49 | 6781.4 | — | — | 49 | 8444.4 | — | — |
| Rp1 | — | 6781.4 | — | — | — | 8444.4 | — | — |
| Two-Centroid system: |  |  |  |  |  |  |  |  |
| I-group | 18 | 2093.2 | — | — | 26 | 3000.1 | — | — |
| II-group | 31 | 2811.3 | — | — | 23 | 2341.1 | — | — |
| Rp2 | — | 4904.4 | 1877.0 | 27.7 | — | 5341.2 | 3103.2 | 36.7 |
| Three-Centroid System: |  |  |  |  |  |  |  |  |
| I-group | 20 | 1307.0 | — | — | 15 | 859.8 | — | — |
| II-group | 18 | 1384.0 | — | — | 11 | 550.2 | — | — |
| III-group | 11 | 1189.0 | — | — | 23 | 2341.1 | — | — |
| Rp3 | — | 3880.0 | 1024.4 | 20.9 | — | 3751.1 | 1590.1 | 29.8 |
| Four-Centroil System: |  |  |  |  |  |  |  |  |
| I-group | 10 | 630.0 | — | — | 15 | 859.8 | — | — |
| II-group | 16 | 1218.0 | — | — | 14 | 1480.6 | — | — |
| III-group | 10 | 987.2 | — | — | 11 | 550.2 | — | — |
| IV-group | 12 | 777.0 | — | — | 9 | 340.9 | — | — |
| Rp4 | — | 3612.2 | 267.8 | 6.9 | — | 3231.5 | 519.6 | 13.8 |
| Five-Centroid System: |  |  |  |  |  |  |  |  |
| Rp5 | — | 3094.3 | 517.9 | 14.3 | — | 2930.3 | 301.2 | 9.3 |

Abbreviations:

n = number of settlements in each group,

d = total distance in each group,

Ri = aggregate distance (in km.) of the centroid system,

Rpi = aggregate distance with population weight of the centroid system,

dR = reduction of distance in the centroid system from pre-centroid system, and

dRp = reduction of weighted distance in the centroid system from pre-centroid system.

Table- 3: Constants and Optimal Solutions of Locational Systems Identified in Various Settlement Patterns.

| Sl. No. | Items | | SETTLEMENT PATTERNS | | | |
|---|---|---|---|---|---|---|
| | | | Uniform | Random | Non-Random | Cluster |
| (a) | Distribution of Settlement Patterns: | | | | | |
| 1. | RN Value | | 2.1428 | 1.4869 | 1.0923 | 0.4023 |
| 2. | Total Points | | 49 | 49 | 49 | 49 |
| (b) | Unweighted Settlement Patterns: | | | | | |
| 3. | Constants | R | 129.9145 | 133.1500 | 159.7054 | 64.2032 |
| | | a | −0.4860 | −1.2961 | −1.4656 | −0.5133 |
| 4. | Solution | m* | 16 | 9 | 9 | 10 |
| | | R* | 13.9016 | 6.4141 | 5.7004 | 10.9131 |
| (c) | Weighted Settlement Patterns: | | | | | |
| 5. | Constants | Rp | — | 6781.0000 | 8444.0000 | 2063.0000 |
| | | a | — | −0.4881 | −0.6927 | −0.6541 |
| 6. | Solution | mp* | — | 231.9001 | 168.0709 | 78.0513 |
| | | Rp* | — | 450.2705 | 239.6903 | 718.8315 |

N.B.: Optimal solutions for weighted system are drawn by giving the population weight to the point patterns.