SOME THRESHOLD EFFECTS IN GAUGE THEORIES

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This is to certify that the thesis entitled "Some Threshold Effects in Gauge Theories" submitted by Sri Merostar Rani for the fulfilment of the degree of Doctor of Philosophy of the North-Eastern Hill University, Shillong embodies the record of an original investigation carried out by him under my supervision. He has been duly registered and the thesis submitted is worthy of being considered for the award of the Ph.D degree.

This work has not been submitted for any other degree to any other institution.

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(Merostar Rani)

CONTENTS

List of Publications

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The following papers have been produced under this thesis:

1. "*Confronting CERN-LEP data, the proton lifetime and small neutrino masses by threshold effects in SO(10) with $SU(2)_L \times U(1)_{I_{3R}} \times SU(4)_C$ intermediate breaking*", M. Rani and M.K. Parida, Phys. Rev. D49,3704(1994).
2. "*Predictions for proton lifetime in minimal non-supersymmetric SO(10) models: An update*", Dae-Gyu Lee, R.N. Mohapatra, M.K. Parida and M. Rani, Phys. Rev. D51,229(1995).
3. "*Radiative corrections and uncertainties in seesaw formulas for neutrino masses*", M.K. Parida and M. Rani (submitted for publication).

CHAPTER 1

INTRODUCTION AND SCOPE OF THE THESIS

INTRODUCTION AND SCOPE OF THE THESIS

1.1 INTRODUCTION

Elementary particles participate in virtually unlimited number of processes, all of which are the outcome of only four types of fundamental interactions: strong, electromagetic,weak and gravitational.Historically the origin of electromagnetic force has been first proved to be due to local gauge interactions where photon is the massless gauge boson. The origin of strong and electroweak interactions is now proved beyond doubt to be due to underlying gauge interactions also. Whereas the gauge symmetry for the electromagnetic interaction is the local abelian group $U(1)_{em}$, with electric charge as the generator and the corresponding gauge boson is the massless photon, the local gauge group for the electroweak interaction is $SU(2)_L \times U(1)_Y$ and that for the strong interaction is $SU(3)_C$. The electroweak symmetry which is exact at mass scales $\mu > M_Z$, undergoes spontaneous symmetry breaking to $U(1)_{em}$ at $\mu = M_Z$ through the vacuum expectation value of the neutral component of the standard Higgs doublet. As a result, the theory has three massive vector bosons, $W^{\pm}_{\mu}$ and Z_{μ} and the massless photon. The strong interaction symmetry $SU(3)_C$ has eight massless gluons and remains unbroken down to low energies. The gauge symmetry of the Standard Model (SM) is $SU(2)_L \times U(1)_Y \times SU(3)_C$ (G_{213}). The particle representations of the SM are briefly sketched below where the transformation properties under G_{213} have also been shown

Gauge bosons

$$W_\mu^{1,2,3}(3,0,1)+B_\mu(1,0,1)+G_{\mu j}^{i}(1,0,8) \qquad (1.1)$$

where the first, second, and the third correspond to the gauge bosons of $SU(2)_L$, $U(1)_Y$, and $SU(3)_C$ respectively.

Fermions

So far three fermion generations have been known to be light. The μ and the τ family repeat with exactly the same transformation properties under G_{213} as the electron family:

1st generation

$$\begin{pmatrix} u \\ d \end{pmatrix}_L (2,1/3,3), \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L (2,-1,1),$$

$$u_R^{r,y,b}(1,4/3,3), \quad e_R(1,-2,1),$$

$$d_R^{r,y,b}(1,-2/3,3). \qquad (1.2)$$

2nd generation

$$\begin{pmatrix} c \\ s \end{pmatrix}_L (2,1/3,3), \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L (2,-1,1),$$

$$c_R^{r,y,b}(1,4/3,1), \quad \mu_R(1,-2,1),$$

$$s_R^{r,y,b}(1,-2/3,3). \qquad (1.3)$$

3rd generation

$$\begin{pmatrix} t \\ b \end{pmatrix}_L (2,1/3,3), \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L (2,-1,1),$$

$$t_R^{r,y,b}(1,4/3,3), \quad \tau_R(1,-2,1),$$

$$b_R^{r,y,b}(1,-2/3,3). \tag{1.4}$$

Higgs scalars

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad (2,\sqrt{3/5}\ 1/2,1) \tag{1.5}$$

The SM does not have charged Higgs scalars as they are absorbed as longitudinal modes of $W^{\pm}$. Similarly the imaginary part of the neutral component of ϕ^0 provides the longitudinal mode to Z^0. Only the real part of the Higgs scalar ϕ^0 acquires vacuum expectation value

$$\langle\phi\rangle = 1/\sqrt{2}\begin{pmatrix} 0 \\ v \end{pmatrix} \tag{1.6}$$

and appears as the massive physical Higgs scalar. The masses of the gauge bosons and fermions are

$$M_{W^{\pm}} = 80.22 \pm 0.26 \text{ GeV}, \quad M_{Z^0} = 91.187 \pm 0.007 \text{ GeV} \tag{1.7}$$

m_u= 2 to 8 MeV, m_c= 1to 1.6 GeV, m_t= 176 ±24GeV, m_d= 5 to 15 MeV
m_s = 100 to 300 MeV, m_b= 4.1 to 4.5 GeV, m_e= 0.5109 MeV,
m_μ = 105.65 MeV, m_τ= 1.777 GeV (1.8)

The laboratory limits on neutrino masses are

$$m_{\nu_e} < 5.1 \text{ eV}, \; m_{\nu_\mu} < 0.27 \text{ MeV}, \; m_{\nu_\tau} < 31 \text{ MeV} \qquad (1.9)$$

Except for the Higgs bosons, all the particles of the standard model have been experimentally detected while the signature for the top quark was found recently. All fermions have masses $m_f \ll M_Z$, but only the top quark appears to be heavier than M_Z.

The electroweak mixing angle between the W_μ^3 and B_μ gauge bosons giving rise to the massive Z^0 boson and the massless photon is defined as

$$\mathrm{Sin}^2\theta_W(M_Z)=e(M_Z)/g_2(M_Z) \qquad (1.10)$$

where

$$e(M_Z)=\frac{g_1 \; g_2}{\sqrt{(g_1^2+g_2^2)}} \qquad (1.11)$$

and g_1, g_2, and $e(M_Z)$ are the gauge couplings of $SU(2)_L$, $U(1)_Y$ and $U(1)_{em}$ respectively. In addition we denote the $SU(3)_C$ coupling as g_3 and $\alpha_i(M_Z) = g_i^2(M_Z)/4\pi$ (i=1,2,3). While improved experimental and theoretical estimation have determined the electromagnetic finestructure constant at the Z-mass to be

$$\alpha^{-1}(M_Z)=128.8\pm0.1. \tag{1.12}$$

The CERN-LEP measurements[1] have estimated $\sin^2\theta_W(M_Z)$ and $\alpha_3(M_Z)$ more accurately,

$$\begin{aligned}&\sin^2\theta_W(M_Z)=0.2316\pm0.0006\\&\alpha_3(M_Z)=0.12\pm0.007\end{aligned} \tag{1.13}$$

The electric charge relation in SM is

$$Q = T_{3L} + Y/2 , \tag{1.14}$$

where Q = electric charge, T_{3L} = third generation of $SU(2)_L$ and

$$I_Y=\sqrt{3/5}(Y/2) \tag{1.15}$$

is the U(1) generator of SM when embedded in a GUT. Equations (1.14) and (1.15) leads to

$$\alpha^{-1}(M_Z) = \frac{5}{3}\alpha_Y^{-1}(M_Z) + \alpha_{2L}^{-1}(M_Z). \tag{1.16}$$

Also the definition of $\sin^2\theta_W$ can be written as

$$\sin^2\theta_W(M_Z) = \frac{\alpha(M_Z)}{\alpha_{2L}(M_Z)} \qquad (1.17)$$

Using (1.12) and (1.13) in (1.16) and (1.17) yields the gauge couplings of the electroweak theory at $\mu = M_Z$ to be.

$$\alpha_{1Y}(M_Z) = 0.016887 \pm 0.00004$$
$$\alpha_{2L}(M_Z) = 0.03322 \pm 0.00025 \qquad (1.18)$$

Thus, the CERN-LEP measurements[1] have determined the gauge couplings of $SU(2)_L \times U(1)_Y$ with high precision although the accuracy in the determination of the $SU(3)_C$ coupling is still to be improved.

Inspite of its impressive successes, the SM has several limitations. To mention some, it has too many unknown parameters, such as the six quark masses, three lepton masses, 3 mixing angles and a phase parametrising weak CP violation, three gauge couplings and two bosonic mass scales. Further there is a very small parameter, $\theta \cong 10^{-8}$ which describes strong CP violation. It does not explain the origin of parity(P) and CP violations although the weak CP violation has been successfully parametrized through Kobayashi-Maskawa mechanism. To be more explicit the SM does not explain why parity violation is a monopoly of weak interaction alone. There is no understanding of the family replication and the quantization of the electric charge. Although weak and electromagnetic interactions are unified through $SU(2)_L \times U(1)_Y$ for $\mu \geq 100$GeV, this unification is only partial. All the three gauge couplings of the SM are different from one another and the model can never be called a truly unified theory of strong, weak and electromagnetic interactions. Also the gravitational interaction is completely left out of the arena of unification.

These limitations of SM could be suggesting that the model is only the first step towards a more fundamental theory of all the basic forces of nature. There could be a deeper fundamental theory of the four basic interactions hitherto unknown and the SM may be an effective gauge theory for $\mu \geq 100$GeV emerging from such a fundamental theory in one step of spontaneous symmetry breaking or through few steps. The gauge symmetry of such a fundamental theory could be exact for $\mu \gg M_Z$. Extensive theoretical investigations have been carried out to formulate a completely renormalizable gauge theory of the four basic interactions with three light fermion generations but no complete success has been achieved so far.

Grand Unified Theories[2-5] (GUTs) either supersymmetric or non-supersymmetric provide an interesting physics beyond the SM unifying strong, weak and electromagnetic interactions for $\mu \geq M_U \geq 10^{15}$ GeV. Many interesting extensions of SM in the gauge, fermion, and Higgs boson sectors can be embedded in GUTs. A neat prediction of non-SUSY GUTs is the proton decay through $p \longrightarrow e^+\pi^0$

mode whose lifetime is constrained by the experimental lower limit

$$\tau_p(p \longrightarrow e^+\pi^0) \geq 3\times10^{32} \text{ years} \qquad (1.19)$$

The Georgi-Glashow minimal non-SUSY SU(5) model[9] was the first attempt to embed the SM in an underlying grand unified simple gauge group, but it has been ruled out by the recent CERN-LEP data on $\sin^2\theta_W(M_Z)$ and the experimental lower limit on the proton lifetime. On the other hand SUSY SU(5) predicts $\tau_p \cong 10^{34}$ years through the dominant proton decay mode $p \longrightarrow K^+ \bar{\nu}_\mu$. In addition one could also have proton decay in the minimal non-SUSY SU(5) model via color triplet Higgs scalar exchange with mass of 10^{11} GeV and via 6 dimensional operators in SUSY SU(5) model.

Although there are no conclusive proof of neutrinos having masses, a number of physical phenomena point to the massive neutrinos. The current laboratory limits for the three different neutrinos are given in (1.9).

Measurement of neutrino flux from the sun carried out by experiments on earth[6] appear to show a substantial decrease from theoretical expectations by nearly 50%, although such experimental indications are yet to be confirmed universally. Such a deficit in ν_e-flux is explained by hypothesizing ν_e- oscillations to other species where the oscillation amplitude is considerably enhanced by MSW[7] mechanism due to high matter density in the interior of the sun. The explanation of the observed solar neutrino deficit via MSW mechanism requires the following mass differences and mixings:

Nonadiabatic solution

$$\Delta m^2_{ei}=(0.3-1.2)\times10^{-5}eV^2, \qquad Sin^2 2\theta=(0.4-1.2)\times10^{-2} \qquad (1.20)$$

Adiabatic solution

$$\Delta m^2_{ei}=(0.3-3)\times10^{-5}\ eV^2, \qquad Sin^2 2\theta=(0.6-0.9) \qquad (1.21)$$

On the other hand neutrino oscillations in vacuum require much smaller mass differences

Vacuum oscillation

$$\Delta m^2_{ei}=(0.5-1.1)\times10^{-10}eV^2, \qquad sin^2 2\theta=(0.8-1.0) \qquad (1.22)$$

In (1.21)-(1.23), $\Delta m^2_{ei}=(m_{\nu_e}-m_{\nu_i})^2$ where $i=\mu, \tau$.

An apparent decrease in the expected flux of atmospheric ν_μ relative to ν_e, arising from the decays of π and K produced in the atmosphere and from the subsequent muon decays, has been observed in three underground experiments, Kamiokande, IMB and possibly

Soudan[8] 2. This deficit can be ascribed to neutrino oscillations $\nu_\mu - \nu_\tau$. Combining these and other experimental observations[9] lead to the following range of neutrino oscillation parameters

$$\Delta m^2_{\mu\tau} \cong 0.005-0.5\ \mathrm{eV}^2,\ \sin^2 2\theta \cong 0.5 \qquad (1.23)$$

Recent observations of the large scale structure of the universe by the COBE[10] sattelite, when taken together with observation of the amplitude of primordial fluctuations on smaller distance scales, indicate that the dark matter consists of 30% hot dark matter and 60% cold dark matter. Massive neutrinos like ν_τ provide the most plausible hot dark matter candidate if

$$m_{\nu_\tau} \cong 7\mathrm{eV} \qquad (1.24)$$

The SM and SU(5) predict neutrino mass to be zero since they do not have the right handed neutrino in the fermion representation.

SO(10) is one of the most interesting grand unified theories which has the potentialities to provide a variety of physical phenomena beyond the SM. It has rank 5 and contains all the 15 fermions of a generation plus the right-handed neutrino in the single spinorial representation <u>16</u>. The model also contains the left-right symmetric gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times P$ ($g_{2L}=g_{2R}$) ($\equiv G_{2213P}$) and the Pati-Salam group $SU(2)_L \times SU(2)_R$

$\times SU(4)_C \times P$ ($g_{2L} = g_{2R}$) ($\equiv G_{224P}$) containing the quark lepton unification. SO(10) can be interpreted to contain the P(=left-right discrete symmetry) and C (=charge conjugation symmetry). As such it has the potentiality to explain the origins of P and CP violations through spontaneous symmetry breaking . Moreover the proton lifetime prediction of the model is enhanced by the introduction of intermediate symmetries . The mechanism of decoupling the parity and $SU(2)_R$ breaking can be naturally embedded in SO(10) leading to the predictions which can be testified by rare decays and $n-\bar{n}$ oscillations. Using the standard technique of seesaw mechanism, the intermediate scale in SO(10) can be related inversely to the Majorana mass of the left-handed neutrinos over a wider range. In addition the domain wall problem is also solved by the decoupling mechanism. It has also been shown that the second gauge hierarchy in SUSY GUTs is natural in SO(10) when the Higgs representation $\underline{45}$ is used to break the GUT symmetry as it provides a natural mechanism for splitting the weak doublet and color triplet masses in $\underline{10}$ of SO(10).

1.2 SCOPE OF THE THESIS

Model predictions in grand unified theories using one-loop solutions to the renormalization group equations(RGEs) have been investigated in detail. Although threshold effects in gauge theories have been discovered as early as 1980-81[11], only over the last six years detailed calculations of threshold effects have been taken into account to refine SO(10) predictions[12] including two-loop. We exploit threshold effects and examine how far SO(10) models with well known specific intermediate gauge symmetries confront available experimental limit on the proton lifetime for $p \longrightarrow e^+\pi^0$. Like SU(5) the grand desert model emerging from the one step breaking of SO(10)

$$SO(10) \xrightarrow{M_U} SU(2)_L \times U(1)_Y \times SU(3)_C \xrightarrow{M_Z} U(1)_{em} \times SU(3)_C$$

is also ruled out by the CERN-LEP measurements and $\tau_p \longrightarrow e^+\pi^0$, unless certain artificial adjustments of superheavy gauge bosons and Higgs scalar near M_U are carried out[13]. But it has been noted that τ_p and $\sin^2\theta_W(M_Z)$ prediction by SO(10) can be reconciled with all the available data provided the GUT symmetry undergoes SSB to the SM in two steps. In principle, the following gauge groups can occur as intermediate symmetry in two steps breaking of SO(10)

$SU(2)_L \times U(1)_{I_{3R}} \times SU(4)_C$ $(\equiv G_{214})$,

$SU(2)_L \times SU(2)_R \times SU(4)_C \times P$ $(\equiv G_{224P})$ $(g_{2L}=g_{2R})$,

$SU(2)_L \times SU(2)_R \times SU(4)_C$ $(\equiv G_{224})$ $(g_{2L}\neq g_{2R})$,

$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times P$ $(\equiv G_{2213P})$ $(g_{2L}=g_{2R})$,

$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ $(\equiv G_{2213})$ $(g_{2L}\neq g_{2R})$,

$SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L} \times SU(3)_C$ $(\equiv G_{2113})$.

Recently in a detailed two-loop calculation of the SO(10) model, Deshpande, Keith, and Pal[14] have ruled out the model with G_{214} or G_{2113} occuring as single intermediate symmetry. In the third chapter of this thesis we demonstrate how the model with G_{214} intermediate symmetry is consistent with all the available data when threshold effects are taken into account. In fact for certain values of superheavy Higgs scalar masses, we show that it is impossible to rule out the model.

Deshpande et al.[14] have also concluded that G_{224P} is barely acceptable, and any improvement in proton lifetime measurement is very likely to rule out the model. In the fourth chapter of this thesis we carry out a detail analysis of the model and find that when threshold effects are included, the $\sin^2\theta_W(M_Z)$ value from CERN-LEP is consistent with the model predictions and proton lifetime predictions are several orders of magnitude larger than the experimental lower limit.

Including threshold effects due to Higgs scalars near the intermediate and the GUT scales, the SO(10) model predictions on

τ_p are updated with G_{224P}, G_{224}, G_{2213P}, G_{2213} occuring as single intermediate symmetries.

In the fifth chapter we examine radiative corrections on the coefficients of the seesaw formulas for neutrino masses involving Dirac masses of quarks. For the first time we derive uncertainties in the coefficients due to the input parameters. We find that the coefficients in formulas for m_{ν_e} and m_{ν_μ} have large uncertainties compared to their central values. In this chapter we also derive for the first time radiative corrections and uncertainties in the coefficients of seesaw formulas involving Dirac masses for the charged lepton. The uncertainties in these cases are found to be much smaller. The consequences of these radiatively corrected seesaw formulas on neutrino mass predictions are evaluated including threshold uncertainties on the intermediate scale and the right handed neutrino mass.

Summary and conclusions from the investigations carried out in this thesis are presented in chapter 6.

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