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Neutralino mass matrix in the nonminimal supersymmetric Standard Model

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Abstract. We study in detail the neutralino mass matrix in the minimal extension of the minimal supersymmetric Standard Model. The model contains a Higgs singlet besides the two Higgs doublets of the minimal model. A number of limiting cases are considered wherein the neutralino mass spectrum can be analytically obtained. Based on these analytical results several useful approximation formulas for the neutralino masses are obtained. Using the constraints from the renormalization group analysis of the model, we obtain the spectrum numerically and study the dependence of the neutralino masses on the parameters of the model. We compare the results of the approximation formulas with the numerical results, and thereby establish the applicability of the these formulae in the different regions of the parameter space.

1 Introduction

Supersymmetry is at present the only known framework [1] in which the large hierarchy between the Planck (or GUT) scale and the electroweak scale, introduced by the vacuum expectation value of an elementary Higgs scalar [2] in the Standard Model, is natural. In supersymmetric theories [3], all particles in the standard model are accompanied by their superpartners. Furthermore, in order to give masses to all quarks and leptons, and to cancel triangle gauge anamolies, at least two Higgs doublets $H_1=(H_1^0, H_1^-)$ and $H_2=(H_2^+, H_2^0)$, with opposite hypercharge ($Y(H_1)=-1$, $Y(H_2)=+1$), are required in the minimal version of the supersymmetric Standard Model(MSSM). The superpotential of the MSSM is [3]

$$W=h_U Q_L U_L^c H_2 + h_D Q_L D_L^c H_1 + h_E L E_L^c H_1 + \mu H_1 H_2, \tag{1.1}$$

where we have suppressed the gauge and generation indices. Notice that the first three terms are the analogues to the Yukawa couplings of the Standard Model and the last term is the Higgs mixing term which is necessary if we want to have electroweak symmetry breaking without having an unacceptable electroweak axion in the model. Softly broken supersymmetry ensures that the radiative corrections to μ are under control so that a small $\mu \sim O(m_W)$ is technically natural. However, softly broken supersymmetry does not explain why μ should be so small to start with. The simplest possibility through which one can generate the bilinear term dynamically is to include an additional gauge singlet Higgs superfield N [4] in the minimal SUSY model. If the superpotential contains a trilinear term $\lambda H_1 H_2 N$, and if the Higgs component of N develops a vacuum expectation value $\langle N \rangle \equiv x$, a bilinear term $\lambda x H_1 H_2$ is generated so that we can identify $\mu=\lambda x$. When supersymmetry is softly broken, we would expect $x=O(M_W)$, and hence $\mu=O(m_W)$, thereby providing a dynamical mechanism for a small μ. There are alternative mechanisms for generating a bilinear term in Higgs fields that have been proposed. These include spontaneous breaking [5] of a global symmetry which forbids the bilinear term at the tree level. The breaking scale should be large enough ($\approx 10^9 - 10^{12}$ GeV) in order for the associated Goldstone boson to be phenomenologically acceptable. Once supersymmetry is broken, radiative corrections could generate a bilinear term with a mass of the order of the supersymmetry breaking scale. This could happen even if the global symmetry is explicitly broken [6]. Another method to generate a bilinear term at the tree level is to have non-renormalizable terms in the superpotential involving gauge singlet field [5]. When this gauge singlet field obtains a vacuum expectation value at intermediate scale a bilinear term of the form $\mu H_1 H_2$ results in the superpotential. Another alternative [7] is to have a purely trilinear superpotential and to generate a bilinear term by coupling to a hidden sector of a supergravity model via a particular Kahler potential containing a mass scale $M \approx 10^{11} - 10^{12}$ GeV which breaks local supersymmetry. However, the Higgs singlet mechanism of generating a bilinear term in the superpotential remains the most appealing one and is present [8] in superstring models with low energy $N=1$ supersymmetry*.

The superpotential of the model with two Higgs doublets and a Higgs singlet N can be written as [10]

$$W=h_U Q_L U_L^c H_2+h_D Q_L D_L^c H_1+h_E L E_L^c H_1 + \lambda H_1 H_2 N-\frac{1}{3} k N^3, \tag{1.2}$$

where the gauge and family indices are implicit, and the sign of the N^3 term is a matter of convenience. Note that if $k=0$, then the Lagrangian corresponding to (1.2) has a global symmetry $N \rightarrow N e^{i\theta}$, $H_1 H_2 \rightarrow H_1 H_2 e^{-i\theta}$ which is spontaneously broken by the vacuum expectation values of the Higgs fields, giving rise to an unacceptable axion. In order to avoid this, we introduce the additional term proportional to k which explicitly breaks this symmetry. We note that the superpotential (1.2) does not contain any bilinear terms and is, therefore, scale independent. This means that if bilinear terms are absent at the tree level, they cannot be generated by renormalization because of the nonrenormalization properties of supersymmetry [11]. A superpotential which is purely trilinear contains only dimensionless couplings and the electroweak scale appears only in the form of soft supersymmetry breaking terms. Thus, the minimal extension of the minimal supersymmetric Standard Model with an additional singlet as embodied in (1.2) is an appealing alternative to the MSSM, and could be an appropriate model for testing the general assumptions of low energy supersymmetry.

Recently much attention [12–18] has been devoted to the study of the nonminimal supersymmetric Standard Model (NMSSM) (1.2). These studies have concentrated on the Higgs sector and the radiative corrections to the Higgs boson masses in the NMSSM. After the Higgs mechanism takes place, of the ten Higgs degrees of freedom, three are absorbed in giving masses to the gauge bosons. This leaves a total of seven physical Higgs bosons. In supersymmetric theories the gauge and Higgs bosons are accompanied by their fermionic partners, the gauginos and higgsinos. Once electroweak symmetry is broken, the gauginos and higgsinos with the same spin, electric charge and color mix to form mass eigenstates (charginos and neutralinos). After mixing, in the NMSSM, there are two chargino $\tilde{\chi}_1^{\pm}$, $\tilde{\chi}_2^{\pm}$, and five neutralino, $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$, $\tilde{\chi}_4^0$, $\tilde{\chi}_5^0$, states. In supersymmetric models with R-parity conservation the lightest SUSY particle (LSP) is expected to be the lightest neutralino, and, being stable, is expected to be the end product of any process involving SUSY particles in the final state, leading to the characteristic missing transverse momentum signal. It is, therefore, of great importance to study the neutralino properties in supersymmetric models. A considerable amount of effort has been devoted to the study of neutralino spectrum in the minimal supersymmetric Standard Model [19–24].

In this paper we make a systematic analysis of the neutralino sector of the nonminimal supersymmetric model represented by the superpotential (1.2). The reasons are twofold. Firstly, as discussed above, the Higgs bilinear term in the superpotential can be generated dynamically in a model with a singlet. Secondly, the minimal supersymmetric standard model makes definite predictions about the spectrum of Higgs bosons, including radiative corrections [25]. These predictions about Higgs masses and couplings can be tested experimentally. If these predictions are not borne out, then it would be natural to go to the nonminimal supersymmetric model.

In Sect. 2 we set up the neutralino mass matrix for the model with the superpotential (1.2) and the corresponding soft breaking terms, and describe our notations and conventions. In Sect. 3 we discuss those cases where analytic solutions can be obtained. Since the number of parameters describing the neutralino mass matrix in the present case is larger as compared to the minimal model, some constraints are required in order to discuss numerical solutions in a meaningful manner. We impose constraints which follow from the scalar sector and the renormalization group analysis of the model. In Sect. 4 we present a systematic discussion and numerical results for the general dependence of the neutralino mass eigenvalues on the parameters of the model after imposing these constraints. Section 5 presents several approximations to the eigenvalues of the neutralino mass matrix which may be relevant for the energy of the forthcoming experiments. We also compare these approximations with the exact results, thereby establishing their range of validity. Section 6 presents a summary and some concluding remarks.

2 The neutralino mass matrix

The neutralino mass matrix arises from the interaction between gauge and matter multiplets as well as the last two terms in the superpotential (1.2) when the Higgs fields obtain vacuum expectation values. In addition there are supersymmetry breaking gaugino masses, M_1, M_2, M_3 associated with the $U(1)$, $SU(2)$ and $SU(3)$ subgroups of the standard model, respectively. It is a standard practice to reduce the parameter freedom by assuming that M_2 and M_1 are related to the gaugino mass parameter M_3. This requires that the three mass scales are equal at some grand unification scale. At the electroweak scale all three mass parameters can be expressed in terms of one of them, which is chosen to be $M_2 \equiv M$ [3]. Thus the other two mass scales are

$$M_1=\frac{3}{5} M'=\tan^2\theta_W M, \quad M_3=\tilde{M}_g=(\alpha_3/\alpha_2) M, \tag{2.1}$$

in the standard notation. We shall use these relations in what follows. In the minimal model, the neutralino sector consists of two gauginos, $\tilde{W}^3$ and $\tilde{B}$, the partners of the third component of the $SU(2)_L$ gauge boson and the $U(1)$ gauge boson, respectively, and the two higgsinos $\tilde{H}_1$ and $\tilde{H}_2$, the fermionic components of the two Higgs doublets, with the result that the neutralino mass matrix is a 4×4 matrix. In the nonminimal model (1.2) there is an additional gauge singlet fermion $\tilde{N}$ resulting in a 5×5 mass matrix. We shall choose the following basis for the

*For a recent discussion of the μ problem, see [9] gaugino-higgsino system of our model:

$$\psi_j^0=(-i\lambda_\gamma,\ -i\lambda_Z, \psi_H^a, \psi_H^b, \psi_N),\quad j=1,2,3,4,5, \tag{2.2a}$$

and λ_γ and λ_Z are the two component spinors of the photino and zino, respectively, and

$$\psi_H^a=\psi_{H_1}^1\sin\theta_V-\psi_{H_2}^2\cos\theta_V,\ \psi_H^b=\psi_{H_1}^1\cos\theta_V+\psi_{H_2}^2\sin\theta_V \tag{2.2b}$$

are the higgsino states with $\psi_{H_1}^1$, $\psi_{H_2}^2$, ψ_N the two component spinors of the neutral higgsinos $\tilde{H}_1^0$, $\tilde{H}_2^0$ and $\tilde{N}$, respectively. Here*

$$\langle H_1^0\rangle=v_1/\sqrt{2},\ \langle H_2^0\rangle=v_2/\sqrt{2},\ \tan\theta_V=v_1/v_2, \tag{2.2c}$$

The mass term in the Langrangian has the form

$$\mathscr{L}_M=-\frac{1}{2}M_Z\psi_i^0 Y_{ij}\psi_j^0+\text{h.c.}, \tag{2.3a}$$

where the mass matrix [26]

$$Y=\begin{bmatrix} \Lambda(\alpha\cos^2\theta_W+\sin^2\theta_W) & \Lambda(1-\alpha)\sin\theta_W\cos\theta_W & 0 & 0 & 0 \\ \Lambda(1-\alpha)\sin\theta_W\cos\theta_W & \Lambda(\alpha\sin^2\theta_W+\cos^2\theta_W) & 1 & 0 & 0 \\ 0 & 1 & -\nu\sin 2\theta_V & -\nu\cos 2\theta_V & 0 \\ 0 & 0 & -\nu\cos 2\theta_V & \nu\sin 2\theta_V & \gamma \\ 0 & 0 & 0 & \gamma & -\delta \end{bmatrix}, \tag{2.3b}$$

with

$$\Lambda=\frac{M}{M_Z},\ \alpha=\frac{M'}{M},\ \nu=\frac{\lambda x}{M_Z},\ \gamma=\frac{\lambda(v_1^2+v_2^2)}{\sqrt{2}M_Z},\ \delta=\frac{2kx}{M_Z}, \tag{2.3c}$$

where we have taken out a factor of M_Z so that we deal with dimensionless quantities only. Neglecting CP violation, Y is a real symmetric matrix which can be diagonalized by a 5×5 unitary matrix N:

$$N_{im}N_{kn}Y_{mn}=\xi_i\delta_{ik},\quad \chi_i^0=N_{ij}\psi_j^0, \tag{2.4}$$

where $\xi_i=m_i/M_Z$, with m_i being the mass eigenvalue of the neutralino eigenstate χ_i^0. Since Y is a real symmetric matrix, we can take N_{im} to be real orthogonal matrix. Some of the mass eigenvalues may be negative. These can be made positive by an appropriate choice of phases in N_{im}, but we shall not do that here. The sign of m_i is related to the CP quantum number of χ_i^0 [27]. The eigenvalues ξ_i of (2.3b) are the solutions of the eigenvalue equation

$$\begin{aligned}&(\xi-\Lambda)(\xi-\Lambda\alpha)[(\xi+\delta)(\xi^2-\nu^2)-\gamma^2(\xi+\nu\sin 2\theta_V)]\\ &\quad-(\xi-\Lambda\alpha\cos^2\theta_W-\Lambda\sin^2\theta_W)[(\xi+\delta)\\ &\qquad\times(\xi-\nu\sin 2\theta_V)-\gamma^2]=0.\end{aligned} \tag{2.5}$$

*The ratio v_2/v_1 is generally denoted as $\tan\beta$ in the context of minimal supersymmetric Standard Model. Here we use a different notation because we denote by β one of the mixing angles in a later section

Once we obtain the eigenvalues ξ_i, the eigenstates of the neutralino mass matrix can be written as

$$\chi_i^0=\frac{1}{N_i}\begin{bmatrix} \Lambda(1-\alpha)\sin\theta_W\cos\theta_W[(\nu^2-\xi_i^2)(\xi_i+\nu)+\gamma^2(\xi_i+\nu\sin 2\theta_V)] \\ [\xi_i-\Lambda(\alpha\cos^2\theta_W+\sin^2\theta_W)][(\nu^2-\xi_i^2)(\xi_i+\delta)+\gamma^2(\xi_i+\nu\sin 2\theta_V)] \\ [\xi_i-\Lambda(\alpha\cos^2\theta_W+\sin^2\theta_W)][(\nu\sin 2\theta_V-\xi_i)(\xi_i+\delta)+\gamma^2] \\ [\xi_i-\Lambda(\alpha\cos^2\theta_W+\sin^2\theta_W)](\xi_i+\delta)\nu\cos 2\theta_V \\ [\xi_i-\Lambda(\alpha\cos^2\theta_W+\sin^2\theta_W)]\gamma\nu\cos 2\theta_V \end{bmatrix} \tag{2.6}$$

in the chosen basis. Here N_i is the appropriate normalization factor. The four component Majorana eigenstates $\tilde{\chi}_i^0$ of neutralinos are defined as usual in terms χ_i^0 and $\bar{\chi}_i^0$. The neutralino components given in (2.6) are elements of the transformation matrix N which diagonalizes the mass matrix Y. These will determine the couplings of the neutralino's to the other states in the model.

3 Special limiting cases

The eigenvalue equation (2.5) cannot be solved analytically to obtain the mass eigenvalues of the neutralino mass matrix. In the absence of analytic solutions we have to depend on the numerical solutions of the eigenvalue equation. Before doing so it is, however, instructive to study certain special cases where the eigenvalue equation (2.5) factorizes so that one or more eigenvalues can be obtained analytically. Apart from serving as a basis for various approximation schemes, such solutions also allow us to identify the regions of the parameter space where one or more light neutralino states exist. Since the lightest neutralino state is the end product of all the SUSY processes, as well as a candidate dark matter [28], it is of considerable importance to identify the regions of parameter space where such a state might exist in the nonminimal supersymmetric model. In this section we shall identify those cases where it is possible to obtain complete analytical solutions for the neutralino mass eigenvalue problem.

3.1 Complete solutions

(a) $\alpha=1$, $\sin 2\theta_V=1$: In this case we obtain the neutralino states

$$\chi_1^0=\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix},\quad \chi_2^0=\begin{bmatrix}0\\\cos\phi\\\sin\phi\\0\\0\end{bmatrix},\quad \chi_3^0=\begin{bmatrix}0\\\sin\phi\\-\cos\phi\\0\\0\end{bmatrix},$$

$$\chi_4^0=\begin{bmatrix}0\\0\\0\\\cos\beta\\\sin\beta\end{bmatrix},\quad \chi_5^0=\begin{bmatrix}0\\0\\0\\\sin\beta\\-\cos\beta\end{bmatrix}, \tag{3.1a}$$

where the mixing angles are given by

$$\sin\varphi=\frac{1}{\sqrt{2}}\left[1-\frac{\Lambda+v}{\sqrt{(\Lambda+v)^2+4}}\right]^{1/2},$$

$$\sin\beta=\frac{1}{\sqrt{2}}\left[1+\frac{v+\delta}{\sqrt{(v+\delta)^2+4\gamma^2}}\right]^{1/2}. \tag{3.1b}$$

The corresponding masses are

$$\xi_1^0=\Lambda,$$

$$\xi_2^0=\frac{(\Lambda-v)+\sqrt{(\Lambda+v)^2+4}}{2},$$

$$\xi_3^0=\frac{(\Lambda-v)-\sqrt{(\Lambda+v)^2+4}}{2},$$

$$\xi_4^0=\frac{(v-\delta)-\sqrt{(v+\delta)^2+4\gamma^2}}{2},$$

$$\xi_5^0=\frac{(v-\delta)+\sqrt{(v+\delta)^2+4\gamma^2}}{2}. \tag{3.1c}$$

This case essentially corresponds to the case discussed by Nath et al. in [3, 4].

b) $\sin^2\theta_W=0$, $\sin 2\theta_V=1$: The eigenstates are given by

$$\chi_1^0=\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix},\quad \chi_2^0=\begin{bmatrix}0\\\cos\phi\\\sin\phi\\0\\0\end{bmatrix},\quad \chi_3^0=\begin{bmatrix}0\\\sin\phi\\-\cos\phi\\0\\0\end{bmatrix},$$

$$\chi_4^0=\begin{bmatrix}0\\0\\0\\\cos\beta\\\sin\beta\end{bmatrix},\quad \chi_5^0=\begin{bmatrix}0\\0\\0\\\sin\beta\\-\cos\beta\end{bmatrix}, \tag{3.2}$$

with mixing angles as given in (3.1b). The corresponding neutralino masses are the same as given in (3.1c) except that in the present case $\xi_1^0=\Lambda\alpha$.

c) $\sin^2\theta_W=0$, $v=0$: In this case we have the same eigenstates and neutralino masses as in the case (b) with $v=0$.

d) $\Lambda=0$, $\sin 2\theta_V=1$: Here also the neutralino states and masses are same as in the case (b) with $\Lambda=0$.

e) $\alpha=1, v=0$: In this case the neutralino states and masses are given as in the case (a) with $v=0$.

f) If the condition

$$\Lambda\alpha v=-(\alpha\cos^2\theta_W+\sin^2\theta_W)\frac{(v\delta\sin 2\theta_V+\gamma^2)}{(v\delta+\gamma^2\sin 2\theta_V)} \tag{3.3}$$

is satisfied, then there exists a massless state. In this case complete solutions are possible for $\sin 2\theta_V=1$. The eigenstates are

$$\chi_1^0=\frac{1}{\sqrt{\Lambda\alpha(\Lambda\alpha-v)}}\begin{bmatrix}-(1-\alpha)\sin\theta_W\cos\theta_W\\(\alpha\cos^2\theta_W+\sin^2\theta_W)\\-\Lambda\alpha\\0\\0\end{bmatrix},$$

$$\chi_2^0=\begin{bmatrix}0\\0\\0\\\sin\beta\\-\cos\beta\end{bmatrix},\quad \chi_3^0=\begin{bmatrix}0\\0\\0\\\cos\beta\\\sin\beta\end{bmatrix},$$

$$\chi_i^0=\frac{1}{N_i^0}\begin{bmatrix}\Lambda(1-\alpha)\sin\theta_W\cos\theta_W\\(\xi_i^0+\Lambda^2\alpha v)\\(\xi_i^0-\Lambda)(\xi_i^0-\Lambda\alpha)\\0\\0\end{bmatrix}, \tag{3.4a}$$

where

$$N_i=[(\xi_i^0-\Lambda)^2\sin^2\theta_W+(\xi_i^0-\Lambda\alpha)^2\cos^2\theta_W$$
$$+(\xi_i^0-\Lambda)^2(\xi_i^0-\Lambda\alpha)^2]^{1/2},\quad i=4,5,$$

is the normalization factor. The masses are given by

$$\xi_1^0=0,\quad \xi_{2,3}^0=\frac{(v-\delta)\pm\sqrt{(v+\delta)^2+4\gamma^2}}{2},$$

$$\xi_{4,5}^0=\frac{(\Lambda+\Lambda\alpha-v)\pm\sqrt{(\Lambda-\Lambda\alpha-v)^2+4(1+\Lambda\alpha)}}{2}. \tag{3.4b}$$

Here the mixing angle β is the same as in (3.1b).
We note that all the cases (a) to (f) that we have discussed so far are analogous to the corresponding cases which

occur in the minimal model [22]. In the limit, $\gamma\to 0$, $\sin\beta\to 1$, four of the eigenvalues coincide with those in the minimal model. This is because, in general, in the limit $\gamma\to 0$, the mass matrix (2.3b) decouples into a 4×4 matrix and the last diagonal entry $-\delta$ becomes an eigenvalue by itself. There is another case which, though not quite similar to the corresponding case in the minimal model, can be considered to be analogous to that case. We now come to this limiting case [29].

g) $\Lambda\gg 1$ and/or $v\gg 1$: In this case complete solutions exist only for $\sin 2\theta_V=1$. This last condition is not required for the analogous situation in the minimal model. In this limit the states are

$$\chi_1^0=\begin{bmatrix}\sin\theta_W\\ \cos\theta_W\\ 0\\ 0\\ 0\end{bmatrix},\ \chi_2^0=\begin{bmatrix}\cos\theta_W\\ -\sin\theta_W\\ 0\\ 0\\ 0\end{bmatrix},\ \chi_3^0=\begin{bmatrix}0\\ 0\\ 1\\ 0\\ 0\end{bmatrix},$$

$$\chi_4^0=\begin{bmatrix}0\\ 0\\ 0\\ \cos\beta\\ \sin\beta\end{bmatrix},\ \chi_5^0=\begin{bmatrix}0\\ 0\\ 0\\ \sin\beta\\ -\cos\beta\end{bmatrix}, \tag{3.5a}$$

with eigenvalues

$$\xi_1^0=\Lambda,\ \xi_2^0=\Lambda\alpha,\ \xi_3^0=-v,\ \xi_{4,5}^0=\frac{(v-\delta)\mp\sqrt{(v+\delta)^2+4\gamma^2}}{2}, \tag{3.5b}$$

and the mixing angle β is the same as in (3.1b). Note that the states χ_1^0 and χ_2^0 are $\tilde{W}^3$ and $\tilde{B}$ states, respectively, whereas χ_3^0 is a pure doublet higgsino state. The states χ_4^0 and χ_5^0 are a mixture of doublet and singlet higgsino states.

We now come to those cases which are specific to the non-minimal model. Since the vacuum expectation value of the singlet, x, does not break any gauge symmetry, x/v_1 or x/v_2 is not constrained by neutral current experiments. We are, therefore, led to consider two classes of limiting cases: (i) $x\gg v_1, v_2$ and (ii) $x\ll v_1, v_2$. In the first case we may have either λ and k fixed or λx and kx fixed [10].

h) The limit $x\gg v_1$, v_2 with λ and k fixed: In this case, we can take $v, \delta\gg\gamma$. The mass matrix then decouples with the result that one of the eigenstates is the singlet higgsino state

$$\chi_4^0=\begin{bmatrix}0\\ 0\\ 0\\ 0\\ 1\end{bmatrix},\quad \xi_4^0=-\delta. \tag{3.6a}$$

The remaining four neutralino states are, in general, a mixture of gaugino and doublet higgsino states only. These cannot be obtained analytically. However, if we consider this limiting case together with $\sin 2\theta_V=1$ and $\sin^2\theta_W=0$ then the states and their masses can be obtained analytically. These are given by

$$\chi_1^0=\begin{bmatrix}1\\ 0\\ 0\\ 0\\ 0\end{bmatrix},\ \chi_2^0=\begin{bmatrix}0\\ \cos\phi\\ \sin\phi\\ 0\\ 0\end{bmatrix},\ \chi_3^0=\begin{bmatrix}0\\ \sin\phi\\ -\cos\phi\\ 0\\ 0\end{bmatrix},$$

$$\chi_5^0=\begin{bmatrix}0\\ 0\\ 0\\ 1\\ 0\end{bmatrix}, \tag{3.6b}$$

with masses

$$\xi_1^0=\Lambda\alpha,\quad \xi_{2,3}^0=\frac{(\Lambda-v)\pm\sqrt{(\Lambda+v)^2+4}}{2},\quad \xi_5^0=v, \tag{3.6c}$$

in the lowest order. There will be corrections to these results which can be obtained in a perturbation expansion in the parameters v_1/x and v_2/x. It is interesting to note that the state χ_4^0, is mainly a singlet Higgsino and, like its scalar [10] counterpart, it tends to decouple from the non-singlet fields, thus making it virtually impossible to observe in the large x limit.

i) The limit $x\gg v_1$, v_2 with λx and kx fixed: We have seen above that in the large x limit with λ and k fixed the neutralino mass matrix decouples into a singlet part and a non-singlet part. The mass of the singlet, however, does not remain finite. The same is true of one of the doublet neutralinos, ξ_5^0. If, one the other hand, we consider the limit of the non-minimal model in which $x\to\infty$, λ and $k\to 0$ with λx and kx fixed (this amounts to taking the limit $\gamma\to 0$), then again the mass matrix of the neutralinos decouples into a singlet state and the mass matrix for the $SU(2)$ doublet states. The singlet state is the same as (3.6a) in the lowest approximation, and its mass unlike the previous large x limit with λ and k fixed, remains finite. The 4×4 mass matrix of doublet neutralinos in the lowest approximation reduces to the mass matrix for neutralinos in the minimal model and can be analyzed in an analogous manner. In particular, in the limit $\sin^2\theta_W=0$, the eigenstates and masses are as given in (3.6b) and (3.6c) with ξ_5^0 finite, and agrees with the results obtained for the minimal model in this limit [22]. We note that these results for the neutralino mass matrix in the limit of large x with λx and kx fixed parallel the corresponding results for the scalar mass matrix [10], wherein also in the same limit the results of the minimal model are recovered.

j) The limit of $x\ll v_1$, v_2: This limit is typical of the result that emerges from renormalization group analysis [10] of the model and has been studied for the Higgs sector of the model. In the present case this limit corresponds to taking $v, \delta \to 0$ in the lowest approximation. Then the mass matrix splits into a 2×2 matrix whose eigenvectors are a mixture of doublet higgsino and the singlet higgsino,

$$\chi_4^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, \quad \chi_5^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}, \tag{3.7a}$$

with eigenvalues

$$\xi_{4,5}^0 = \pm \gamma, \tag{3.7b}$$

and a 3×3 matrix which cannot be diagonalized analytically. However, in the limit $\sin^2 \theta_W = 0$, the 3×3 matrix can also be diagonalized analytically with eigenstates and masses given by

$$\chi_1^0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \chi_2^0 = \begin{bmatrix} 0 \\ \cos\phi \\ \sin\phi \\ 0 \\ 0 \end{bmatrix}, \quad \chi_3^0 = \begin{bmatrix} 0 \\ \sin\phi \\ -\cos\phi \\ 0 \\ 0 \end{bmatrix}, \tag{3.7c}$$

$$\xi_1^0 = \Lambda \alpha, \quad \xi_{2,3}^0 = \frac{\Lambda \pm \sqrt{\Lambda^2 + 4}}{2}, \tag{3.7d}$$

$$\sin\phi = \frac{1}{\sqrt{2}} \left[1 - \frac{\Lambda}{\sqrt{\Lambda^2 + 4}} \right]^{1/2}. \tag{3.7e}$$

Here χ_1^0 is a pure photino, whereas $\chi_{2,3}^0$ are mixtures of the zino and the doublet higgsino. This limit will serve as the basis of one of the approximation schemes for the neutralino masses that we shall construct in Sect. 5.

4 Numerical results

Having discussed various special cases where partial or complete analytical solutions are possible, we come to the numerical results for the eigenstates and mass eigenvalues of the neutralino mass matrix. Since the number of parameters on which the neutralino mass matrix depends is large, some restrictions on parameters will have to be placed in order to do meaningful calculations. We shall take the renormalization group equations as a guiding principle to restrict the parameter space and to motivate specific choice of the parameters. The relevant renormalization group equations are [30]

$$\frac{d\lambda}{dt} = \frac{1}{8\pi^2} \left[2\lambda^2 + k^2 + \frac{3}{2} h_t^2 - \frac{3}{2} g^2 - \frac{1}{2} g'^2 \right] \lambda,$$

$$\frac{dk}{dt} = \frac{3}{8\pi^2} [\lambda^2 + k^2] k,$$

$$\frac{dh_t}{dt} = \frac{1}{8\pi^2} \left[\frac{1}{2} \lambda^2 + 3h_t^2 - \frac{8}{3} g_3^2 - \frac{3}{2} g^2 - \frac{13}{18} g'^2 \right] h_t, \tag{4.1}$$

where h_t is the top quark Yukawa coupling in (1.2), and where the contributions from lighter fermions have been neglected. Here g_3 and g' are the $SU(3)$ and $U(1)$ gauge coupling constants, respectively. The requirement of no explicit CP violation in the scalar sector [10] gives the condition

$$\lambda k^* \varepsilon R. \tag{4.2}$$

One can define the phase of the fields H_2 and N so that one can take λ, k to be real. Through an electroweak gauge transformation it is possible to choose v_2 to be real and positive. Furthermore, the phase of the vev's v_1 and x enters the scalar potential and allows for the possibility of spontaneous CP violation. It is sufficient to choose

$$\lambda k \varepsilon R^+ \tag{4.3}$$

in order to forbid spontaneous CP violation. If (4.3) holds then one can choose to work in the vacuum with all the three VEV's real and positive [31]. We shall make this choice in this paper.

The parameters that determine the neutralino states and their masses are Λ, α, v, $\tan\theta_V$, γ, and δ. The renormalization group equations (4.1) favor two classes of values for λ and k, and hence v and δ. It has been demonstrated [10] that there are infrared fixed points for λ and k. If the parameters λ and k have values of order 1 or larger at the GUT scale, then at low energies their values will be near these fixed point values:

$$\lambda \sim 0.87, \quad k \sim 0.63. \tag{4.4}$$

The second class of values is obtained by imposing special boundary conditions at the unification scale and result in values of λ and k which are considerably lower than those given in (4.4). The values in (4.4), thus, serve as a upper limit for the corresponding parameters and are less restrictive compared to the second class of values in their prediction of mass spectra. We shall choose to work with the values given in (4.4), which implies

$$\frac{v}{\delta} \sim 0.70, \quad \gamma \sim 1.40, \tag{4.5}$$

$$\alpha \sim 0.47, \tag{4.6}$$

where we have used (2.1) in arriving at (4.6). Thus, in a renormalization group inspired model we have only three independent parameters which we shall take to be Λ, v and $\tan\theta_V$. We recall that the value of $\tan\theta_V$ is to a large extent determined by the top quark mass. It is known that all the models based on low energy supersymmetry and radiative breaking of electroweak symmetry that have been studied so far lead to values of $\tan\theta_V$ that are less than unity [32]. We shall, therefore, consider values of $\tan\theta_V$ which are smaller than unity.

The spectrum of the particle masses and the pattern of electroweak symmetry breaking in the renormalization group inspired model depends on the boundary conditions chosen at the GUT scale. If the supersymmetry breaking at the grand-unification scale is induced by a universal gaugino mass term $M' = M = M_3 = M_U \neq 0$, while the other soft supersymmetry breaking terms including the trilinear terms and the scalar masses are generated by radiative corrections, then M_U must be chosen to be positive in order to generate positive trilinear couplings at the electroweak scale as is required to avoid explicit CP violation in the scalar sector [10]. Although we shall not use any detailed results following from these boundary conditions, we shall use the general result that follows from this type of boundary condition*. If we recall from our earlier discussion that x can be chosen to be positive with λ given by (4.4), then the effective μ ($\equiv \lambda x$) parameter corresponding to that of the minimal SUSY model is positive. Therefore the renormalization group equations with the above boundary conditions imply that the gaugino mass has the same sign as the effective μ parameter in the non-minimal model. This must be contrasted with the situation that obtains in the minimal model where [33], in general, the μ parameter and the gaugino mass, can have opposite sign. Thus, in the renormalization group inspired nonminimal model both v and A have positive sign.

Having fixed our parameter space, we now present the numerical results for the neutralino mass eigenvalues. For phenomenology it is important to find the values of A and v for which the lowest neutralino mass eigenstate has an eigenvalue equal to a fixed value. Figure 1 presents a contour plot of the mass eigenvalue of the lightest neutralino state as a function of A and v following from the eigenvalue equation (2.5) for $\sin^2\theta_W = 0.23$, $\alpha = 0.47$, $\gamma = 1.40$ and $\tan\theta_V = 0.5$. Notice that the eigenvalues are positive. The situation here is qualitatively similar to that in the minimal model [22], although the constant mass contours in the non-minimal model are nearly straight lines (for the chosen values of the parameters). We shall discuss this aspect when we study the dependence of the neutralino mass on A and v separately.

We now turn to a systematic analysis of the dependence of the eigenvalues on A, v and $\tan\theta_V$ with the parameters γ and δ fixed by the renormalization group equation constraints (4.4) and (4.5). With only three independent parameters, the model is now similar to the minimal model (this will not be true in general) with the additional constraint that A and v are both positive. The A dependence of the mass spectrum is obtained from the eigenvalue equation (2.5) and can be written as

$$A_{\pm} = \frac{(1+\alpha)\xi}{2\alpha} - \frac{(\alpha\cos^2\theta_W + \sin^2\theta_W)}{2\alpha}\left(\frac{X_2}{X_1}\right) \pm \frac{1}{2\alpha}\left(\frac{X_2}{X_1}\right)\left\{\left[(1-\alpha)\left(\frac{X_1}{X_2}\right) + (\alpha\cos^2\theta_W - \sin^2\theta_W)\right]^2 + (2\alpha\sin\theta_W\cos\theta_W)^2\right\}^{1/2}, \tag{4.7}$$

where

$$X_1 = [(\xi+\delta)(\xi^2 - v^2) - \gamma^2(\xi + v\sin 2\theta_V)],$$

$$X_2 = [(\xi+\delta)(\xi - v\sin 2\theta_V) - \gamma^2]. \tag{4.8}$$

*Other boundary conditions with either the scalar mass or the trilinear coupling being nonzero, with the remaining soft breaking terms being zero, lead to unacceptable phenomenology

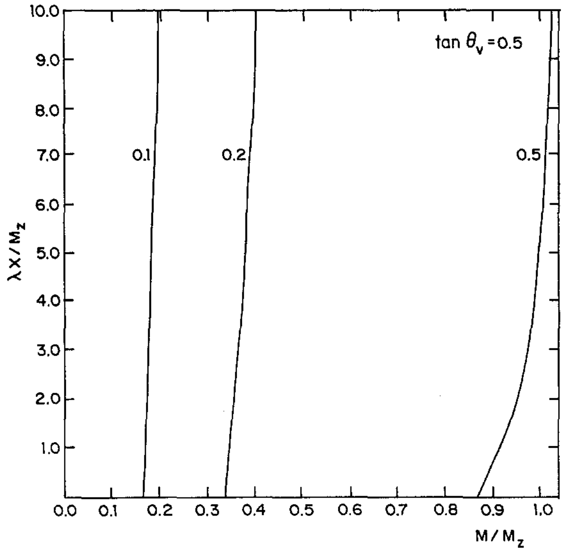

Fig. 1. Mass eigenvalues of ξ of the lightest neutralino as a function of A and v. The contour plots are for eigenvalues of 0.1, 0.2 and 0.5 in units of M_Z, respectively, and for $\tan\theta_V = 0.5$. In this and all other figures the neutralino masses are in units of M_Z and the parameter values are fixed to be $\sin^2\theta_W = 0.23$, $\alpha = 0.47$ and $\gamma = 1.40$

The A dependence of the eigenvalues is shown in Fig. 2 for two different values of v and $\tan\theta_V$. We note that there will be crossing points for the eigenvalues at $\sin 2\theta_V = 1$ (not shown), which is split for $\sin 2\theta_V \neq 1$. The mass eigenvalues depend sensitively on θ_V only in the neighbourhood of the crossing points. Two of the eigenvalues are essentially constant throughout the domain of A, their values depending on v and $\tan\theta_V$, whereas a third eigenvalue is constant for values of A which depend on v and $\tan\theta_V$. For $A > 4-5$ the neutralino masses rapidly approach their asymptotic values given by (3.5b).

If we solve the eigenvalue equation (2.5) for v we obtain

$$v_{\pm} = \left[\frac{X_4}{X_3} - \frac{\gamma^2}{(\xi+\delta)}\right]\sin\theta_V\cos\theta_V \pm \frac{1}{2X_3(\xi+\delta)} [4(\xi+\delta)^2(X_3\xi - X_4\cos^2\theta_V)(X_3\xi - X_4\sin^2\theta_V) - 2X_3\gamma^2(\xi+\delta)(X_4\sin^2 2\theta_V + 2(X_3\xi - X_4)) + X_3^2\sin^2 2\theta_V\gamma^4]^{1/2}, \tag{4.9}$$

where

$$X_3 = (\xi-\alpha)(\xi - A\alpha),$$

$$X_4 = [\xi - A(\alpha\cos^2\theta_W + \sin^2\theta_W)]. \tag{4.10}$$

The v dependence of the eigenvalues is shown in Fig. 3 for two different values of $\tan\theta_V$ and A. We note that for $A \lesssim 1.0$, one of the eigenvalues is nearly constant for the entire range of v for the two values of $\tan\theta_V$ that are shown (this is a general feature for a wide range of $\tan\theta_V$ values). This eigenvalue is the lowest mass eigenvalue for

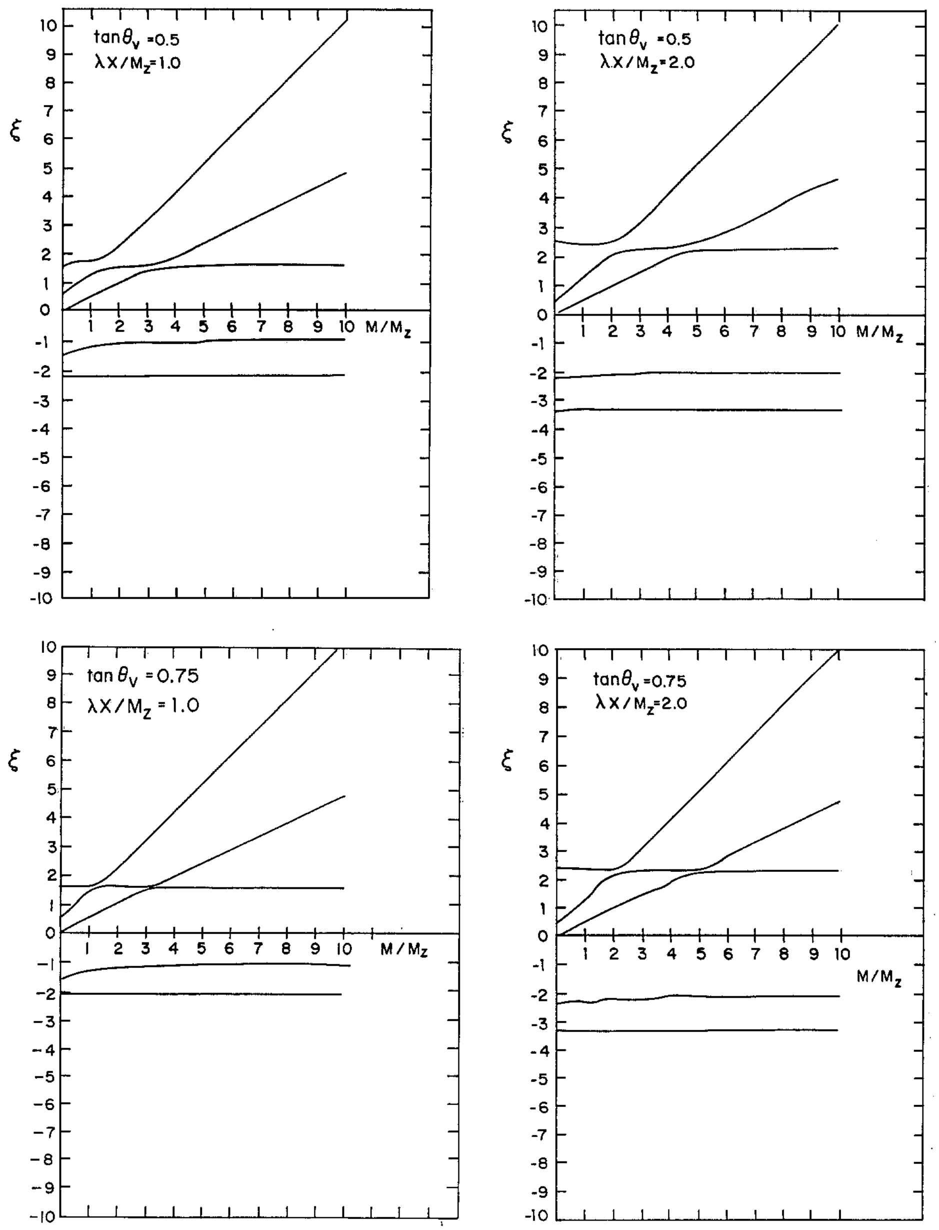

Fig. 2a–d. Eigenvalues ξ of the neutralino mass matrix as a function of $\Lambda = M/M_Z$ for different values of $\nu = \lambda x/M_Z$ and $\tan\theta_V$. **a** $\nu = 1.0$, $\tan\theta_V = 0.5$; **b** $\nu = 2.0$, $\tan\theta_V = 0.5$; **c** $\nu = 1.0$, $\tan\theta_V = 0.75$; **d** $\nu = 2.0$, $\tan\theta_V = 0.75$. Only the positive Λ domain is plotted here, following the constraints of the renormalization group equations. There will be crossing of the eigenvalues for $\sin 2\theta_V = 1$ which is not shown here but which is apparent from the graphs for $\tan\theta_V = 0.75$

the values of parameters that have been studied. This is a reflection of the constant mass contours of Fig. 1, where we noticed that the constant mass curves were nearly straight lines. However, for values of $\Lambda \gtrsim 1.0$ this is not necessarily the case as can be seen from Fig. 3b and Fig. 3d. It s not difficult to see the reasons for this behaviour. To be specific we take $\sin 2\theta_V \sim 1.0$, in which case the eigenvalue equation approximately factorizes and the eigenvalues can be written as ($\sin^2\theta_W = 0.23$)

$$\xi_1 \sim 0.5\Lambda, \quad \xi_{2,3} \sim \frac{(\Lambda-\nu) \pm \sqrt{(\Lambda+\nu)^2+4}}{2},$$

$$\xi_{4,5} \sim \frac{(\nu-\delta) \mp \sqrt{(\nu+\delta)^2+4\gamma^2}}{2}. \tag{4.11}$$

For $\Lambda \leq 1.0$, ξ_1 is the smallest eigenvalue for all values of ν with γ and δ fixed by the renormalization group equations to be equal to the values given by (4.5). Thus the smallest eigenvalue forms a contour in the $\Lambda-\nu$ plane which is approximately a straight line. For values of $\Lambda \geq 1.0$, eigenvalue other than ξ_1 is the smallest one, so that the constant mass contour is no longer a straight line.

Finally, we come to θ_V dependence of the neutralino masses. Again, this can be obtained from the eigenvalue equation (2.5), and in Fig. 4 we plot the neutralino masses as a function of $\tan\theta_V (\equiv v_1/v_2)$ for two different values of ν and Λ. Although the analysis of the scalar potential has restricted $\tan\theta_V$ to be positive, for completeness we have shown the eigenvalues for negative values of $\tan\theta_V$ as well. The dependence on $\tan\theta_V$ is rather weak for positive values of $\tan\theta_V$. For example, for $\nu = \Lambda = 1.0$, the largest variation in an eigenvalue is from $\xi \sim -0.8$ at $\tan\theta_V = 0$ to $\xi \sim -1.4$ at $\tan\theta_V = 1.0$, whereas the smallest change in an eigenvalue is from $\xi \sim 0.52$ at $\tan\theta_V = 0$ to $\xi \sim 0.54$ at $\tan\theta_V = 1.0$. Similar conclusions hold for other values of

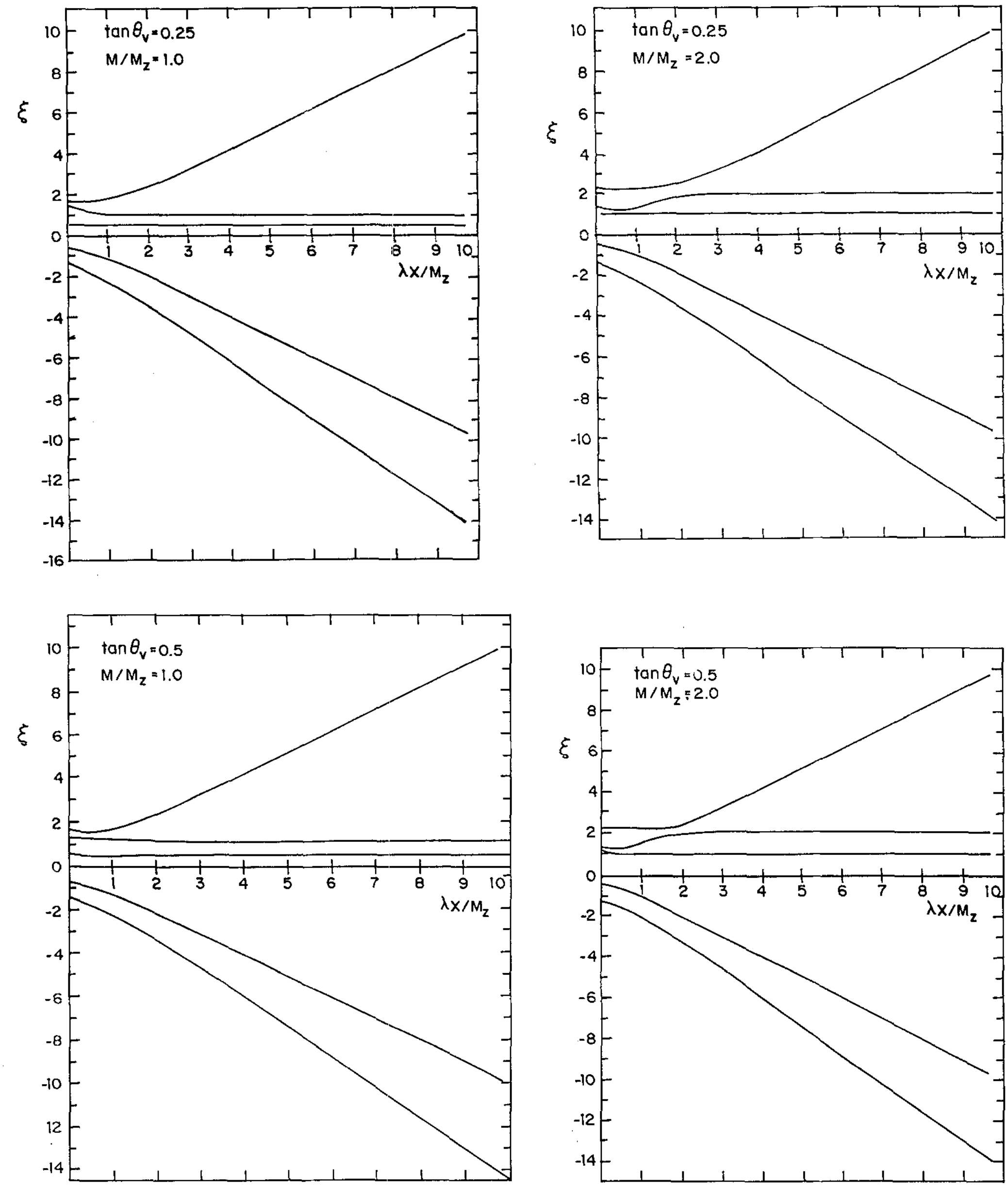

Fig. 3a–d. Neutralino masses ξ as a function of $v = \lambda x/M_Z$ for different values of $A = M/M_Z$ and $\tan\theta_V$, **a** $A = 1.0$, $\tan\theta_V = 0.25$; **b** $A = 2.0$, $\tan\theta_V = 0.25$; **c** $A = 1.0$, $\tan\theta_V = 0.5$; **d** $A = 2.0$, $\tan\theta = 0.5$. In the curves for $A = 1$, the graph for the smallest eigenvalue is almost a straight line. See text for details. Following the constraints of the renormalization group equations, only positive v region is shown here

A and v. It is also clear that the variation of the largest eigenvalue (in magnitude) with $\tan\theta_V$ is weak. We also note that the behaviour of the eigenvalues as a function of $\tan\theta_V$ is qualitatively similar for fixed values of A, independent of the value of v.

5 Approximate solutions

Having studied both the analytical solutions in special limiting cases as well as the exact numerical results, we now discuss, for the sake of completeness, several approximation schemes [34] which may be useful in different domains of the parameter space. The approximation formulae that we shall present are based on applying perturbation theory to the exact analytical results that we obtained in Sect. 3.1.

5.1 Expansion in $\sin^2\theta_W$ *and* $\sin 2(\theta_V - \pi/4)$

This approximation scheme, which is based on the analytical solution (b) of Sect. 3.1 for $\sin^2\theta_W = 0$ and $\sin 2\theta_V = 1$, is analogous to the corresponding scheme for the minimal model [22]. This expansion is applicable for a large range of A, v and $\tan\theta_V$ values, $A \leq 10$, $v \leq 10$ and $0.1 \leq \tan\theta_V \leq 1$, respectively. The mass matrix Y, (2.3b), can be written as

$$Y = Y_0 + A(1-\alpha)\sin^2\theta_W \Sigma'_3 + A(1-\alpha)\sin\theta_W\cos\theta_W \Sigma'_1$$
$$+ 2v\sin^2\varepsilon\Sigma_3 + 2v\sin\varepsilon\cos\varepsilon\Sigma_1, \quad (5.1)$$

where Y_0 is the mass matrix for $\sin^2\theta_W = 0$, $\sin 2\theta_V = 1$, and $\varepsilon = \theta_V - \pi/4$. The 5×5 matrices Σ_i and Σ'_i are given by

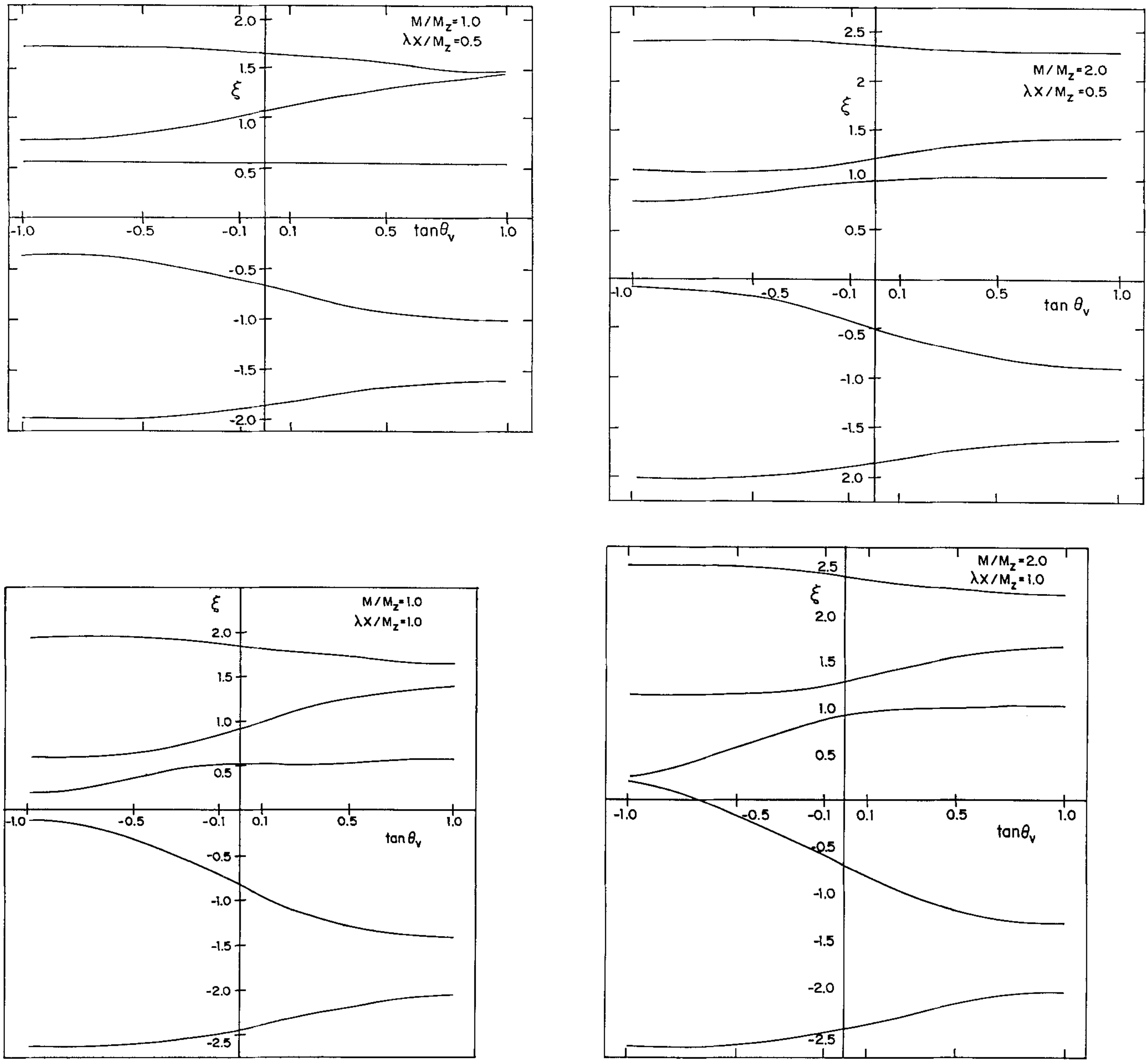

Fig. 4a–d. Neutralino mass eigenvalues ξ as a function of $\tan\theta_V$ for different values of Λ and v. **a** $\Lambda = 1.0$, $v = 0.5$; **b** $\Lambda = 2.0$, $v = 0.5$; **c** $\Lambda = 1.0$, $v = 1.0$; **d** $\Lambda = 2.0$, $v = 1.0$. Note that both positive and negative values of $\tan\theta_V$ are plotted. All other parameters remain the same as in previous figs

$$\Sigma_i=\begin{bmatrix} 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \sigma_i & & 0\\ 0 & 0 & & & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\Sigma_i'=\begin{bmatrix} \sigma_i & & 0 & 0 & 0\\ & & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}, \qquad (5.2)$$

with σ_i the Pauli matrices. Perturbation theory applied to (5.1) gives

$$\xi_1=\xi_1^0+\Lambda(1-\alpha)\sin^2\theta_W+\frac{[\Lambda(1-\alpha)\sin\theta_W\cos\theta_W]^2\cos^2\phi}{\xi_1^0-\xi_2^0}+\frac{[(\Lambda(1-\alpha)\sin\theta_W\cos\theta_W]^2\sin^2\phi}{\xi_1^0-\xi_3^0},$$

$$\xi_2=\xi_1^0-\Lambda(1-\alpha)\sin^2\theta_W\cos^2\phi+2v\sin^2\varepsilon\sin^2\phi+\frac{[\Lambda(1-\alpha)\sin\theta_W\cos\theta_W]^2\cos^2\phi}{\xi_2^0-\xi_1^0}+\frac{[2v\sin\varepsilon\sin\phi\cos\beta]^2}{\xi_2^0-\xi_4^0}+\frac{[2v\sin\varepsilon\sin\phi\sin\beta]^2}{\xi_2^0-\xi_5^0},$$

$$\xi_3=\xi_3^0-\Lambda(1-\alpha)\sin^2\theta_W\sin^2\phi+2v\sin^2\varepsilon\cos^2\phi$$
$$+\frac{[\Lambda(1-\alpha)\sin\theta_W\sin\phi]^2}{\xi_3^0-\xi_1^0}$$
$$+\frac{[2v\sin\varepsilon\cos\phi\cos\beta]^2}{\xi_3^0-\xi_4^0}+\frac{[2v\sin\varepsilon\cos\phi\sin\beta]^2}{\xi_3^0-\xi_5^0}, \quad (5.3)$$
$$\xi_4=\xi_4^0-2v\sin^2\varepsilon\cos^2\beta++\frac{[2v\sin\varepsilon\sin\phi\cos\beta]^2}{\xi_4^0-\xi_2^0}$$
$$+\frac{[2v\sin\varepsilon\cos\phi\cos\beta]^2}{\xi_4^0-\xi_3^0},$$
$$\xi_5=\xi_5^0-2v\sin^2\varepsilon\sin^2\beta+\frac{[2v\sin\varepsilon\sin\phi\sin\beta]^2}{\xi_5^0-\xi_2^0}$$
$$+\frac{[2v\sin\varepsilon\cos\phi\sin\beta]^2}{\xi_5^0-\xi_3^0},$$

where ξ_i^0 are the eigenvalues of Y_0 and are given by $\xi_1^0=\Lambda\alpha$, and ξ_i^0 $(i=2,3,4,5)$ as in (3.1c), with ϕ and β given by (3.1b). These eigenvalues are plotted in Fig. 5a as a function of v for $\Lambda=1.0$ and $\tan\theta_V=0.4$ $(\sin 2\theta_V\sim 0.7)$, together with the exact results for the same set of parameters, It is obvious that (5.3) is an excellent approximation for $v\lesssim 10$ and $\tan\theta_V<1$. This covers almost the entire phenomenologically interesting range of parameters.

5.2 *Expansion in* v *and* $\sin^2\theta_W$

Since v and δ are related through the renormalization group equation constraint (4.5), this is effectively an expansion in v, δ and $\sin^2\theta_W$. The limit of small v and δ is interesting, because it is a result which emerges from a renormalization group analysis of the non-minimal model [10]. If we expand about this limit, we will get approximation formulae for $v<1$ which are valid for all values values of $\tan\theta_V$. Starting from the exact solution (3.7) of Sect. 3.1, and expanding in v, δ and $\sin^2\theta_W$ we get the eigenvalues

$$\xi_1=\xi_1^0+\Lambda(1-\alpha)\sin^2\theta_W+\frac{[\Lambda(1-\alpha)\sin\theta_W\cos\phi]^2}{\xi_1^0-\xi_2^0}$$
$$+\frac{[\Lambda(1-\alpha)\sin\theta_W\sin\phi]^2}{\xi_1^0-\xi_3^0},$$
$$\xi_2=\xi_2^0-\Lambda(1-\alpha)\sin^2\theta_W\cos^2\phi+\frac{[\Lambda(1-\alpha)\sin\theta_W\cos\phi]^2}{\xi_2^0-\xi_1^0}$$
$$-v\sin 2\theta_V\sin^2\phi+\frac{[v\sin 2\theta_V\sin\phi\cos\phi]^2}{\xi_2^0-\xi_3^0}$$
$$+\frac{[v\cos 2\theta_V\sin\phi]^2}{2(\xi_2^0-\xi_4^0)}\left[1+\frac{(\xi_2^0-\xi_4^0)}{(\xi_2^0-\xi_5^0)}\right],$$
$$\xi_3=\xi_3^0-\Lambda(1-\alpha)\sin^2\theta_W\sin^2\phi-v\sin 2\theta_V\cos^2\phi$$

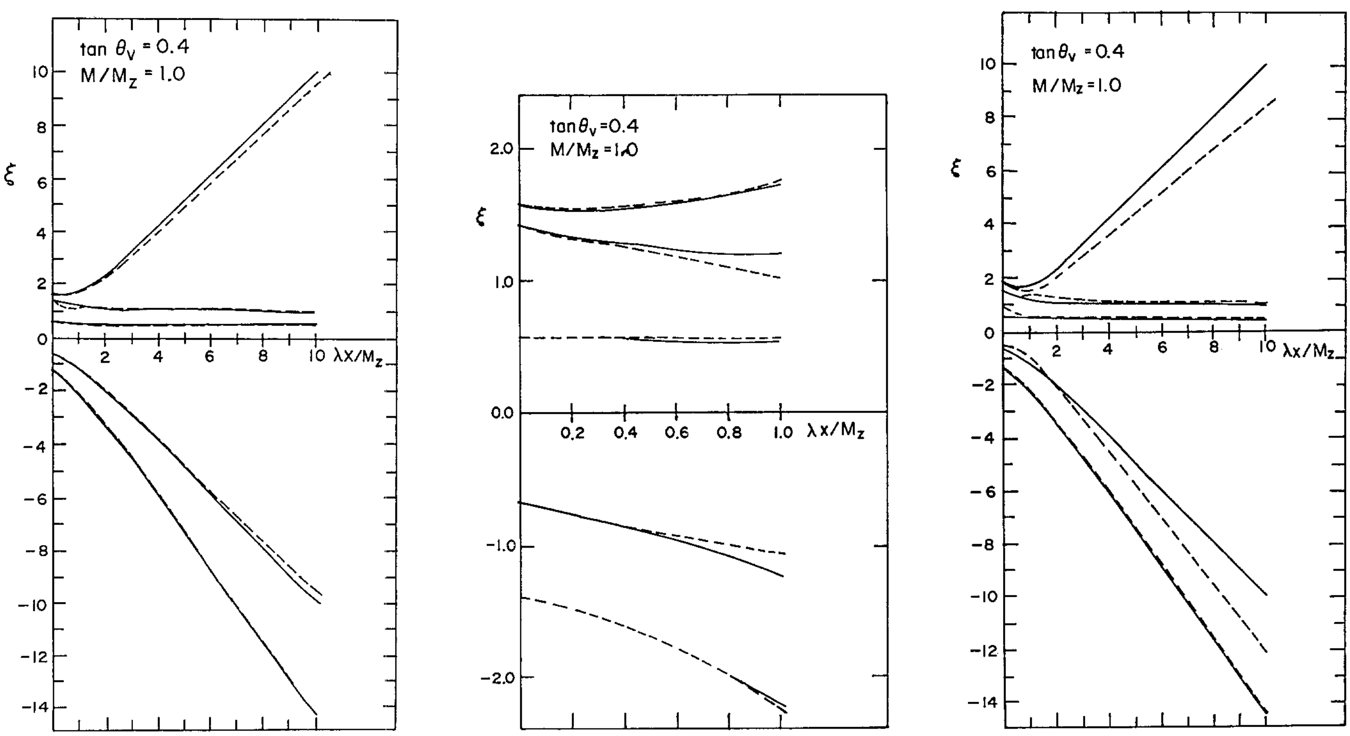

Fig. 5a–c. Approximation formulae for the neutralino mass eigenvalues as a function of v for fixed value of $\Lambda=1.0$ and $\tan\theta_V=0.4$. All other parameters are fixed according to the renormalization group constraints as done throughout this paper, and $\sin^2\theta_W=0.23$. **a** Approximation formulae (5.3) represented as dashed lines with exact solutions represented as solid lines. **b** Neutralino masses as obtained from the approximation formulae (5.4), represented as dashed lines, as a function of v. Solid lines represent exact solutions. This approximation is valid for all values of $\tan\theta_V$. **c** Approximation formulae (5.5) (dashed lines) for neutralino masses plotted as a function of v. Solid lines represent exact results. The approximation becomes better for larger values of v at values of $\Lambda\geq 2$

$$+\frac{[\Lambda(1-\alpha)\sin\theta_W\cos\theta_W\sin\phi]^2}{\xi_3^0-\xi_1^0}$$

$$+\frac{[v\sin 2\theta_V\cos\phi\sin\phi]^2}{\xi_3^0-\xi_2^0}+\frac{[v\cos 2\theta_V\cos\phi]^2}{2(\xi_3^0-\xi_4^0)}$$

$$\times\left[1+\frac{(\xi_3^0-\xi_4^0)}{(\xi_3^0-\xi_5^0)}\right],$$

$$\xi_4=\xi_4^0+\frac{v\sin 2\theta_V}{2}+\frac{[v\sin 2\theta_V]^2}{4(\xi_4^0-\xi_5^0)}+\frac{[v\cos 2\theta_V\sin\phi]^2}{2(\xi_4^0-\xi_2^0)}$$

$$+\frac{[v\cos 2\theta_V\cos\phi]^2}{2(\xi_4^0-\xi_3^0)}-\frac{\delta}{2}+\frac{\delta^2}{4(\xi_4^0-\xi_5^0)},$$

$$\xi_5=\xi_5^0+\frac{v\sin 2\theta_V}{2}+\frac{[v\sin 2\theta_V]^2}{4(\xi_5^0-\xi_4^0)}+\frac{[v\cos 2\theta_V\sin\phi]^2}{2(\xi_5^0-\xi_2^0)}$$

$$+\frac{[v\cos 2\theta_V\cos\phi]^2}{2(\xi_5^0-\xi_3^0)}-\frac{\delta}{2}+\frac{\delta^2}{4(\xi_5^0-\xi_4^0)},$$

where ξ_i^0 are the eigenvalues in the limit of v, δ, $\sin^2\theta_W\to 0$, and are given in (3.7b) and (3.7d). Note that only ξ_4 and ξ_5 depend on δ. These eigenvalues are plotted in Fig. 5b, together with the exact results, as a function of v for $\tan\theta_V=0.4$ and $\Lambda=1.0$. From the figure we see that the approximation is of a good quality for $v<1.0$. This approximation is in fact valid for all values of $\tan\theta_V$.

5.3. Expansion in $1/\Lambda$ or $1/v$ and $\sin 2(\theta_V-\pi/4)$

We have seen in Sect. 3.1 that complete analytical solutions can be obtained for $\Lambda\gg 1$ and/or $v\gg 1$ with $\sin 2\theta_V=1$. Taking solution (3.5b) as the zeroth approximation, perturbation theory gives,

$$\xi_1=\xi_1^0+\frac{\cos^2\theta_W}{\xi_1^0-\xi_3^0},\quad \xi_2=\xi_2^0+\frac{\sin^2\theta_W}{\xi_2^0-\xi_3^0},$$

$$\xi_3=\xi_3^0+2v\sin^2\varepsilon+\frac{\cos^2\theta_W}{\xi_3^0-\xi_1^0}+\frac{\sin^2\theta_W}{\xi_3^0-\xi_2^0}$$

$$+\frac{[v\sin 2\varepsilon\cos\beta]^2}{\xi_3^0-\xi_4^0}+\frac{[v\sin 2\varepsilon\sin\beta]^2}{\xi_3^0-\xi_5^0}, \tag{5.5}$$

$$\xi_4=\xi_4^0-2v\sin^2\varepsilon\cos^2\beta+\frac{[v\sin 2\varepsilon\cos\beta]^2}{\xi_4^0-\xi_3^0}$$

$$+\frac{[v\sin^2\varepsilon\sin 2\beta]^2}{\xi_4^0-\xi_5^0},$$

$$\xi_5=\xi_5^0-2v\sin^2\varepsilon\sin^2\beta+\frac{[v\sin 2\varepsilon\sin\beta]^2}{\xi_5^0-\xi_3^0}$$

$$+\frac{[v\sin^2\varepsilon\sin 2\beta]^2}{\xi_5^0-\xi_4^0},$$

where ξ_i^0 are the limiting values given in (3.5b), and $\varepsilon=\theta_V-\pi/4$. The result (5.5) is shown in Fig. 5c where we have plotted the eigenvalues as a function of v and $\tan\theta_V$ for $\Lambda=1.0$. We see from the figure that the approximation is fairly good even for low values of Λ when v is small. The approximation becomes better for larger values of v for large $\Lambda\geq 2$.

6 Summary and concluding remarks

We have studied the neutralino mass matrix of the non-minimal supersymmetric standard model with two Higgs doublets and a Higgs singlet. This model is a natural extension of the minimal model where the bilinear term involving Higgs doublets is replaced by a trilinear term in the superpotential, thereby avoiding the problem of having to explain a small mass parameter in the model. The spectrum of the neutral gauge-Higgs fermions of the model now contains a gauge singlet fermion in addition to the usual gauginos and doublet Higgsinos. We have studied exact numerical solutions for the mass eigenvalues with some of the parameters constrained by the renormalization group equations. We have also studied some special values of the parameters for which exact analytical solutions can be obtained. Based on these solutions we have build up approximate formulae through a perturbation expansion which covers a wide region of the parameter space relevant for phenomenology. The formulae (5.3) for the eigenvalues serve as a good approximation for a large range of Λ and v values for $\tan\theta_V<1$. The approximation formulae (5.4) are valid for all values of $\tan\theta_V$. The result (5.4) is a good approximation for small values of v (and hence δ, $v<1$). Similarly, the result (5.5) is a good approximation for larger values of $\Lambda\geq 2$ with $\tan\theta_V<1$. We could have built an approximation scheme based on (3.3), but we found that the analytical formulae are quite complicated and perhaps may not serve a useful phenomenological purpose.

It is important to point out that the approximation formulae (5.3), (5.4) and (5.5) are valid when the corresponding eigenvalues ξ_i^0 are non-degenerate. In case of degeneracy one must apply degenerate perturbation theory. In our numerical analysis, with the parameter space that we have considered, we have not actually come across a degeneracy. To illustrate this point we consider the solution (b) of Sect. 3.1. We note that ξ_2^0 and ξ_3^0, and ξ_4^0 and ξ_5^0 are never degenerate for any physical values of the parameters. ξ_1^0 and ξ_2^0 can be degenerate only for negative values of v, which we have not considered here. Similar remarks apply to ξ_1^0 and ξ_3^0. Since ξ_4^0 is always negative in the region of parameter space allowed by the renormalization group analysis, and ξ_1^0 is positive, the two cannot be degenerate. ξ_1^0 and ξ_5^0 are degenerate only in the region $\Lambda\gtrsim 3$. ξ_3^0 is always negative for positive values of Λ, v and hence cannot be degenerate with ξ_5^0 which is always positive for positive v. Similar remarks apply to the eigenvalues ξ_3^0 and ξ_4^0, and ξ_2^0 and ξ_5^0.

If the future data rules out the minimal supersymmetric model, then in the context of supersymmetry the non-minimal model could be a viable alternative. We have seen that in the context of renormalization group analysis, the effective number of parameters describing the neutralino sector is three, the same as in the minimal model. It will, therefore, be interesting to see whether there are distinctive signatures of the model in the neutralino sector in the context of present and new colliders. This question is under study and will be reported elsewhere [35].

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