

2-D Electron Density Profile of the Ionosphere Using the Modified Spherical Harmonic Analysis

V.B.S. SRILATHA INDIRA DUTT AND G. SASIBHUSHANA RAO

Abstract

The advent of Global Positioning System (GPS) has revolutionized the field of navigation particularly in the field of civil aviation sector. The GPS is a reliable, all weather satellite-based radio navigation system that provides accurate three dimensional (3D) navigation solution i.e., position, velocity and timing information, up to 10^{-6} seconds anywhere on or above the earth's surface. But the accuracy of the navigation solution is affected by several factors such as the satellite-receiver geometry configuration, refraction of the GPS signal in Ionosphere and Troposphere, multi path and solar activity. Out of all, the refraction of the GPS signals in ionosphere that introduces a delay in the ranging information, significantly influences the navigation solution accuracy. The effect of the ionospheric delay on navigation solution accuracy can be estimated precisely by constructing the electron density profile of the ionosphere. In this paper, electron density profile of the ionosphere is constructed using the tomographic technique at 11:15:00 Hrs of 22nd July 2009 using the data due to the dual frequency GPS receiver located at IISc, Bangalore.

Key words: Global Positioning System, electron density profile, ionosphere, troposphere.

Introduction

Ionospheric delay, as a pronounced error in GPS positioning and navigation especially in the high geomagnetic activity period, has long been extensively

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studied [1]. The speed of propagation of GPS signal in the ionosphere depends upon the number of free electrons in its path, defined as the total electron content (TEC): the number of free electrons present in a tube of 1m^2 cross section extending from the receiver to the satellite[2]. The path length through the ionosphere is shortest in the zenith direction; therefore TEC is lower in the vertical direction. TEC is measured in the units of TEC Units (TECU), defined as 10^{16} electrons/ m^2 . TEC typically varies between 1 and 150 TECU. TEC is represented as

$$\text{TEC} = \int_p N(s) ds \quad (1)$$

Where $N(S)$ is the electron density and 'p' is the propagation path between the source and the detector.

The ionosphere delay should be corrected in order to get a precise positioning and navigation results using GPS. For the ionospheric correction, typically two methods exist in GPS applications. One is the use of dual-frequency GPS data to eliminate the ionospheric effects taking advantage of the ionospheric dispersive feature. The other way is to employ a regional or global network of GPS reference receivers to model the ionosphere and then GPS users use the model to correct the ionospheric delay[3]. The advantage of the latter method is obvious because it can provide ionospheric correction service to numerous GPS users within the coverage area in a cost-effective mode.

There are various two dimensional (2D) ionospheric modeling techniques such as GRID model, Spherical Harmonic model (SHA) and Modified Spherical Harmonic Model (MSHA), etc.

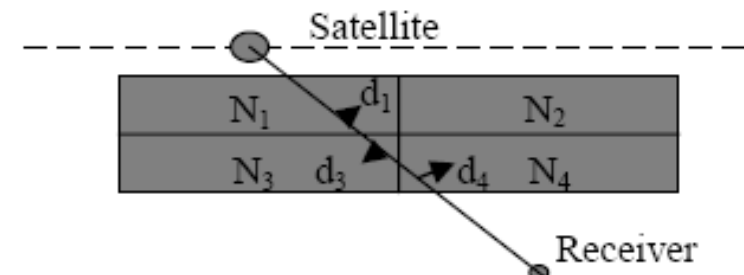


Figure 1 Sample Ionospheric Tomography System

In this paper, a two dimensional modelling of the ionosphere based on MSHA model using the dual frequency GPS receiver measurements is presented.

In the above system, N_k indicates electron density in the pixel; d_k indicates the length of ray occupied by the pixel. For these parameters, k takes a value between 1 and 4, and TEC value for the ray can be given as

$$\text{TEC} = c (d1 \times N1 + d3 \times N3 + d4 \times N4) + \text{err} \quad (2)$$

In Fig. 1, c is an estimate constant and err is the error term. CIT method is based on this basic concept.

Spherical Harmonic Model

The aim of the spherical harmonics model in 2D tomography is to characterize the horizontal ionospheric profile. The spherical harmonics are the angular portion of an orthogonal set of solutions and represented in a system of spherical coordinates. The spherical harmonics functions contain the product of latitude dependent associated Legendre functions and the sum of the longitude dependent sine and cosine terms. The spherical harmonics expansion fit together exactly with identical shapes on the sphere. The number of tiles is determined by the degree and order of the expansion. The degree corresponds to the number of wavelengths on both sides of the latitudinal space and the order corresponds to number of wave lengths on the both sides of the longitudes[4]. The mathematical expression of vertical TEC using spherical harmonics is

$$\text{TEC}(\theta, \lambda) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_{nm}[\cos(\theta)] \{C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda)\} \quad (3)$$

Where, θ is the geographical latitude of an IPP

λ is the geographical longitude of an IPP

n, m are integer degree and order of Legendre function, respectively

C_{nm}, S_{nm} are unknown spherical harmonics coefficients

$P_{nm}[\cos(\theta)]$ are normalized associated legendry functions.

The degree and order of the Legendre function defines the resolution of the model. For sparsely spaced observations, it is necessary to employ a low order fit.

A spherical harmonic expansion is a two-dimensional Fourier series defined on a sphere and comprises the product of latitude-dependent associated Legendre functions and the sum of longitude-dependent sine and cosine terms (the Fourier series). The spherical harmonic expansion tessellates the sphere into a number of rectangular tiles with the boundaries between adjacent tiles representing the roots of the orthonormal Fourier functions. The number of tiles is determined by the degree and order of the expansion, with higher degree and order implying a larger number of tiles. Within the context of this study, the degree corresponds to the number of wavelengths spanning the latitudinal space and the order, the number of wavelengths spanning longitude. An example of a 15x10 degree/order SHM is given in Fig. 2.

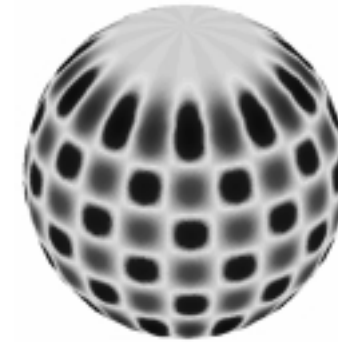


Figure 2 Representation of spherical harmonic orthogonal basis functions of degree 15 and order 10

Modified Spherical Harmonic Analysis

The modified spherical harmonic (MSHA) methodology introduced in this study utilizes the IPPs Sun-fixed longitude, λ_s , i.e., the longitude expressed relative to the Sun's mean geographic longitude[5]. Sun-fixed longitude conveniently encapsulates the IPPs time and longitudinal variation in a single angular observation spanning 360° over a twenty-four hour revolution and is calculated from the geographical longitude, λ , by

$$\lambda_s = (180^\circ - \Omega_e \cdot t) - \lambda \quad (4)$$

Where, Ω_e represents the Earth rotation rate and t the time elapsed since midnight UT.

An n -degree, zero-order spherical harmonic expansion is a zonal harmonic, which is an associated Legendre function P_{nm} , reduced to the Legendre

function $P_n = P_n0$. Zonal harmonics are latitudinal dependent functions defined along a longitude meridian and, in the context of global modeling, have a terrestrial wavelength, λ , of $360^\circ/n$ degrees ($2\pi R/n$ km with R the Earth radius). For example, a 15-degree Legendre function has a terrestrial wavelength of 24° (~ 2672 km).

A new approach introduced in this study involves scaling the regional IPP co-latitude from a spherical angle to a hemisphere prior to applying the conventional spherical harmonic model. The scaling function is similar to that introduced by De Santis (1991), and involves translating and scaling the IPP co-latitude by employing the minimum co-latitude of the IPPs, ϕ_0 , and the spherical angle, θ , of the observations:

$$\phi' = 90 / \theta (\phi - \phi_0) \tag{5}$$

The IPP co-latitude, f , is subsequently replaced by the scaled co-latitude, f' , defined on a hemisphere $[0^\circ, 90^\circ]$ and used in the conventional spherical harmonic model and henceforth referred to as the modified spherical harmonic (MSHA) model. This transformation is valid as the Legendre polynomials form a set of orthogonal functions on $[0^\circ, 180^\circ]$, but may be used as two sets of orthogonal functions on $[0^\circ, 90^\circ]$ to fit any general functions defined in this

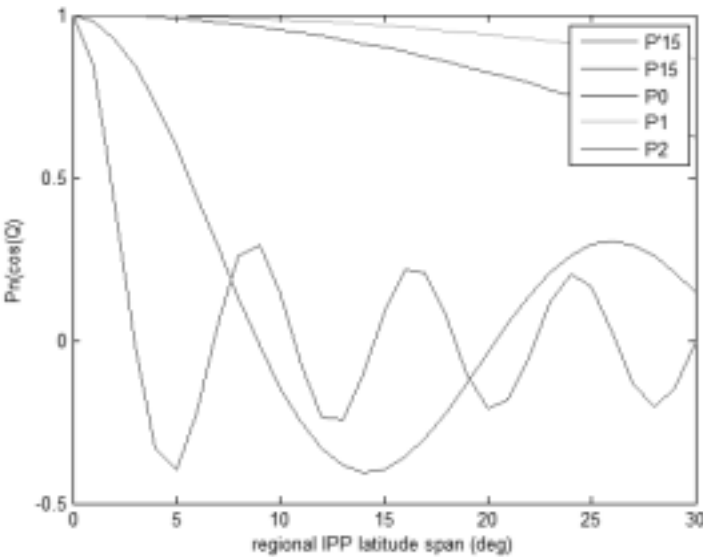


Figure 3 Variation of Legendre function for degrees 0, 1, 2 and 15.

The variation of Legendre function over the regional IPP latitude span for the degrees 0, 1, 2, 15 is shown in Fig. 3. It is clear from the above figure that Ionospheric parameters variation can be better observed with higher order Legendre function.

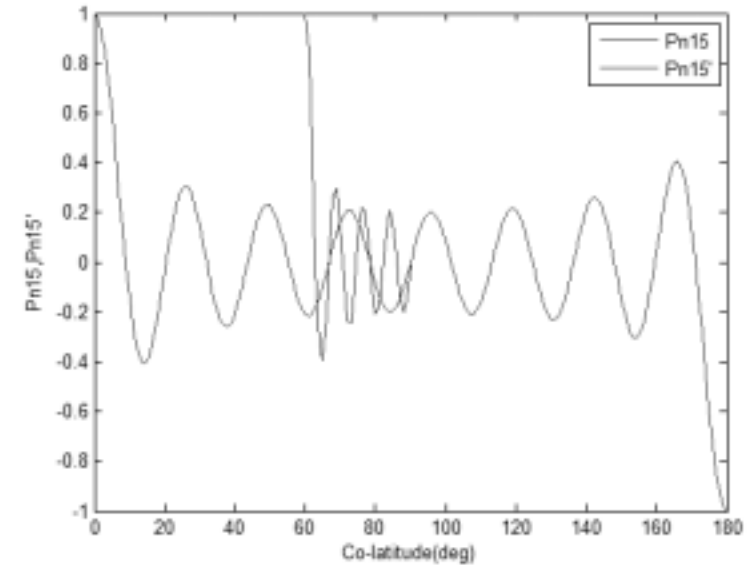


Figure 4 Comparison of P_n and P_n' for $n = 15$.

The Legendre functions P_n, P_n' constructed from a spherical co-latitude range $[0^\circ, 180^\circ]$ and from the modified spherical harmonics latitude range $[0^\circ, 90^\circ]$ are shown in Fig. 4. It is clear from the figure that the Ionospheric parameters variation can be better observed with modified spherical harmonic Legendre function.

Results

The dual frequency GPS receiver data of the IISc, Bangalore (IGS Station Lat/Long: $13.02^\circ / 77.57^\circ$) is collected corresponding to a typical day (22nd July 2009). The data is sampled at a sampling rate of 30 seconds for the analysis. The Converter software tool of Novatel make is used to convert receiver specific data format into Receiver Independent Exchange (RINEX) format. Data is processed to see the visibility of the satellites over the entire day. This data is analyzed to investigate the variations in Ionospheric

parameters such as Total Electron Content, electron density etc. The details of visible Satellites such as Azimuth Angle in Degrees, Elevation Angle in Degrees, IPP latitude, IPP longitude, Slant TEC Using code range measurements, VTEC, Slant factor and Ionospheric delay are listed in Table 1. From these values it can be observed that at 6:30 hrs local time (+5.30 of GPS time gives the local time) the VTEC value is 3.360001 and at 13:30 hrs (i.e., at 8:00 GPS hrs) the VTEC value is 39.19802. Hence, the VTEC is maximum during the mid of the day and is minimum in the early hours.

Table 1 Details of the Satellites in view on 22nd July 2009

SVPRN	GPS Time in Hours	Azimuth Angle in Degrees	Elevation Angle in Degrees	IPP latitude	IPP longitude	Slant TEC Using code range measurements	VTEC	Slant Factor	Ionospheric delay
31	1:00	190.60924	55.722	4.326933	106.323	6.6257	3.360001	1.97193	0.0546
31	2:00	246.52054	86.395	13.36117	81.15816	3.370914	3.370912	1.000001	0.0547
31	3:00	6.55703	60.692	41.75323	85.64127	8.501041	7.308372	1.163192	0.1187
31	4:00	19.72022	35.078	49.03064	120.3109	12.8800	7.327092	1.75787	0.1190
31	5:00	35.52147	16.006	29.82283	15.8563	30.1772	12.76273	2.364485	0.2072
7	6:00	277.65881	16.735	39.84741	146.298	34.79429	30.32253	1.147473	0.4924
7	7:00	305.37782	22.379	41.93916	36.3646	31.61473	15.15909	2.085529	0.2461
28	8:00	242.93328	21.872	22.64532	18.08999	41.34381	39.19802	1.05468	0.6365
28	9:00	272.40423	32.687	21.78501	122.7464	34.51822	33.13861	1.041632	0.5381
28	10:00	305.95204	35.635	5.947078	26.12532	35.19412	31.44878	1.119093	0.5106
28	11:00	337.72235	35.66	13.0836	22.78825	28.17814	25.46825	1.106403	0.4135
28	12:00	8.35625	37.537	11.65291	122.4935	37.20276	13.14213	2.830802	0.2134
28	13:00	41.80672	41.448	15.09191	36.89625	26.35037	16.54012	1.593119	0.2686
28	14:00	80.55884	41.703	34.23964	33.44526	22.77098	17.92295	1.270494	0.2910
28	15:00	115.75369	31.171	37.98435	104.4649	29.58705	11.61903	2.54643	0.1887
28	16:00	137.09218	11.701	45.59029	7.095052	16.83554	14.60323	1.267704	0.2371
27	17:00	208.29619	19.941	53.44922	134.2329	24.25605	21.81501	1.111897	0.3542
29	18:00	326.31404	17.857	78.63988	48.92584	16.81169	14.37931	1.169158	0.2335
29	19:00	339.5561	38.982	62.00727	90.81668	10.04322	9.672557	1.038321	0.1571
29	20:00	347.30854	66.266	8.996005	102.1142	5.51187	2.313164	2.382829	0.0376
29	21:00	180.14697	82.157	9.77402	71.54957	6.38768	3.438487	1.857701	0.0558
29	22:00	175.11865	50.694	39.3908	49.84037	5.79746	3.836376	1.511181	0.0623
29	23:00	169.79847	23.334	79.12719	87.72818	4.92165	4.807407	1.023765	0.0781

Variation of the VTEC over the entire day of 22nd July 2009 is shown in Figure 5. VTEC data of few satellites i.e. SV PRN 31, SV PRN 7, SV PRN 27, SV PRN 29, SV PRN 6 and SV PRN 28 is used to construct the plot shown in Figure 5.

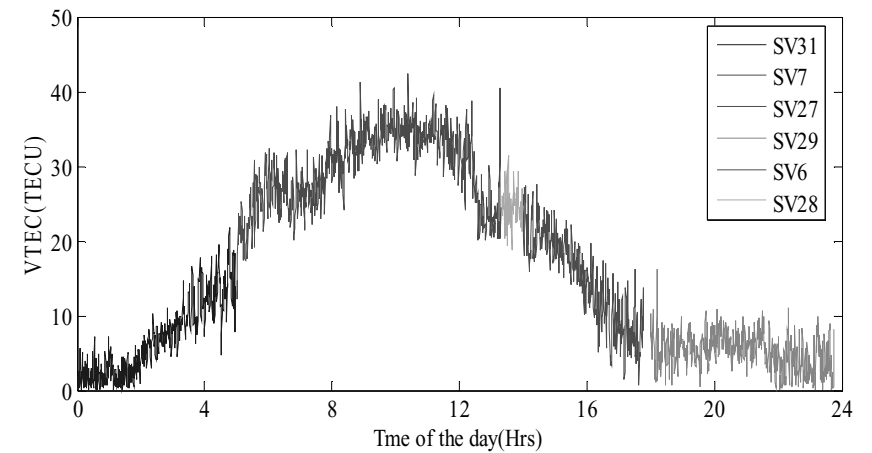


Figure 5 Total Electron Content computed using pseudorange measurements on 22nd July 2009

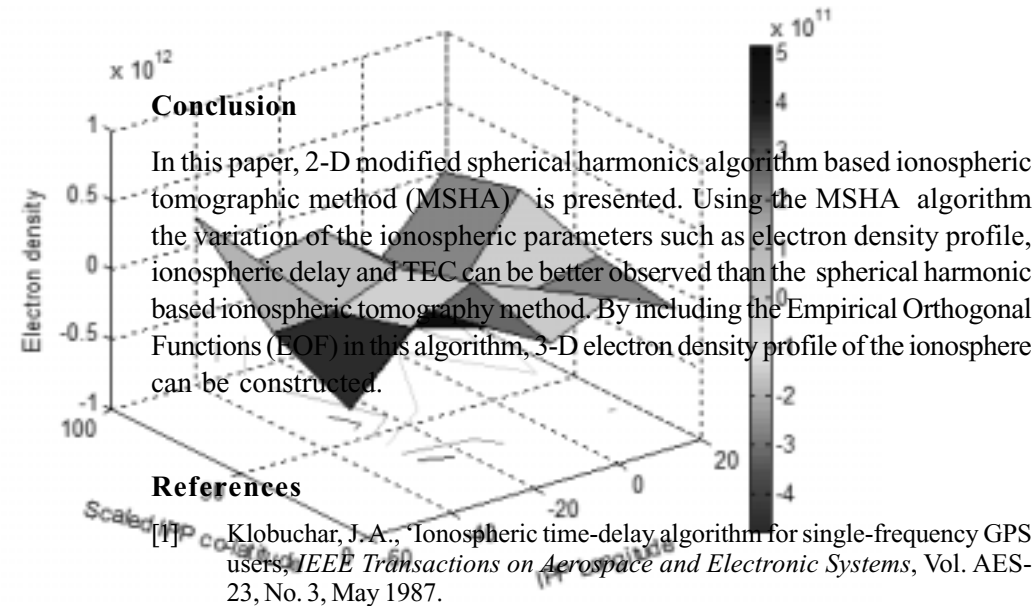
Using the details listed in Table 1, electron density profile is constructed at an epoch 11:15:00Hrs (LT) using the Modified Spherical Harmonic method (MSHA). Using this method, electron density profile for the Legendre function of degree 3, degree 10 and degree 15 are constructed and the corresponding Spherical harmonic coefficients are tabulated. For degree 3, number of SHF coefficients obtained is 20. The time taken for computation for degree 3 is 0.961830 sec. The Electron density values corresponding to these SHF coefficients are computed and are plotted over the IPP latitude and longitude. Figure 5 represents the electron density profile for Legendre function of degree 3 and the SHF coefficients are presented in Table 1.

Table 2 Spherical harmonic coefficients for Legendre function of degree 3.

Latitude/ Longitude	0	30	60	90
-46.0000	5.1375	-4.7700	-2.8474	3.4828
-31.1674	3.6724	3.2293	-1.7600	-1.4305
-16.3348	-2.9910	-2.3250	0	0
-1.5021	-0.2198	0	0.2835	0.5016
13.3305	0	-0.1104	-0.4646	-0.5472

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- [4] El-Rabbany A., *Introduction To GPS*, Artech House, 2002.
- [3] Y.Gao and Z.Z. Dio, Precise Ionosphere Modeling Using Regional GPS Network Data, *Journal of Global Positioning System* (2002), Vol. 1, No 1: 18-24.

Figure 6 Electron density profile for Legendre function of degree 3



- [2] Rishbeth, H. and O.K. Garriott, *Introduction to Ionospheric Physics*, Academic Press, New York, 1969.