SHOCK WAVES IN NON-IDEAL CONDUCTING AND NON-CONDUCTING FLUIDS

ABSTRACT

BINEETA NATH

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ABSTRACT

When the velocity of a fluid in motion becomes comparable with or exceeds that of sound, effects due to the compressibility of the fluid become of prime importance. Such motions are met with in gases. The flow of a gas is entirely different in nature according as it is subsonic or supersonic. One of the most important distinctive features of supersonic flow is the fact that there can occur in it what are called shock waves, across which the medium undergoes sudden and often considerable changes in velocity, pressure, density and temperature. These may result from various causes, for example, detonation of explosives, flow through rocket nozzles, supersonic flight of projectiles, electric discharges and so on.

Shock waves are the most conspicuous phenomena occuring in non-linear wave propagation. Even without being caused by initial discontinuities, they may appear and be propagated. The problem of shock waves has a bearing on many problems outside of supersonic aeronautics, for example, detonation waves, but also has great importance for several practical aeronautical problems. In fact, shock waves may cause sudden change in the aerodynamic behaviour of high speed aerocrafts affecting not only thier balance and stability but also control producing undesirable vibrations.

Since at high temperatures that prevail in the problems associated with shock waves a gas is ionized, electromagnetic effects may also be significant. A complete analysis of such a problem should therefore consist of study of

the gasdynamic flow and the electromagnetic fields simultaneously. By now, the theory of shock waves in non-conducting and conducting fluids has been much developed.

When a strong explosion takes place, the character of the motion of the substance depends essentially on its equation of state. The perfect gas law can often be applied to actual gases with sufficient accuracy. This approximation, may, however, be inadequate in a situation such as arrises in the case of an explosion. It is then necessary to take into account of the deviations of an actual gas from the ideal state which results from the interaction between its component molecules.

In this dissertation we have attempted to study some of the recent works done in the area of propagation of shock waves in non-ideal conducting and non-conducting fluids. The dissertation consists of four chapters.

The first chapter is introductory. It gives, in brief, an idea about Shock Waves, Shock Waves in the Magnetogasdynamics, Fundamental Equations of a Non-Ideal Gas, The Concept of Self-Similarity and The Whitham's rule, as used in the dissertation.

In chapter 2, we have studied two types of problem.

(i) Propagation of strong diverging shock waves in a non-ideal gas, and

(ii) Similarity solutions for the flow behind an exponential shock in a non-ideal gas.

In the first problem, Whitham's Method has been employed to determine the shock velocity and the other flow variables just behind the shock. The effects of non-idealness of the gas on the flow variables just behind the shock and on the density ratio across the shock are investigated.

In the second problem, we have studied similarity solutions for the flow of a non-ideal gas behind a strong exponential shock driven out by a piston (cylindrical or spherical) moving with time according to an exponential law. We have studied both the cases, when the flow between the shock and the piston is adiabatic or isothermal. The assumtion of isothermal flow is physically realistic, when radiation heat transfer effects are implicitly present. Effects of the non-idealness of the gas on the flow field between the shock and the piston are investigated. The variations of density-ratio across the shock and the location of the piston with the parameter of non-idealness of the gas $\bar{\alpha}$ are also investigated.

In the chapter 3, we have again studied two types of problem.
(i) Propagation of converging cylindrical shock waves in a non-ideal gas in the presence of of an azimuthal magnetic field, and
(ii) Propagation of shock waves generated by a piston moving in a non-ideal gas in the presence of azimuthal magnetic field.

In the first problem, Whitham's Method has been used to determine the shock velocity and other flow variables just behind the shock in the cases, when (I) the gas is weakly ionized before and behind the shock front, (II) the gas is strongly ionized before and behind the shock front, (III) the non-inonized gas undergoes intense ionization as a result of the passage of the shock. Here the initial magnetic induction (or magnetic field) is assumed to vary inversely as the spatial co-ordinate r. Effects of the non-idealness of the gas and the magnetic field on the shock propagation have been studied.

In the second problem, we have studied similarity solutions for the flow of a non-ideal gas behind a cylindrical or spherical shock wave under isothermal condition, in presence of azimuthal magnetic field, driven out by a piston moving with time according to power law. The gas is assumed to have infinite electrical conductivity and to obey a simplified van der Waals equation of state. The ambient azimuthal magnetic field is assumed to vary as some power of the distance from the axis or point of symmetry. Effects of the non-idealness of the gas and the Alfven-Mach number on the flow-field have been studied.

In the chapter 4, we have studied similarity solutions for one-dimensional adiabatic flow behind a magnetogasdynamic cylindrical shock wave propagating in a rotating non-ideal gas in presence of an azimuthal magnetic field. The density of the medium ahead of the shock is assumed to be constant. In order to obtain the similarity solutions the angular velocity of the ambient medium is assumed to be obeying a power law and to be decreasing as the distance from the axis increases. The effects of an increase in the value of the index for variation of angular velocity of the ambient medium, in the value of the parameter of the non-idealness of the gas and in the strength of the initial magnetic field are investigated.

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Math

CERTIFICATE

I certify that the dissertation entitled "SHOCK WAVES IN NON-IDEAL CONDUCTING AND NON-CONDUCTING FLUIDS" submitted by Ms. Bineeta Nath in partial fulfilment of the requirement of the degree of Master of Philosophy in Mathematics is the outcome of a study undertaken by the candidate.

I certify that the sources from which ideas have been borrowed have been duly referred to.

The material in this dissertation has not been presented for the award of a degree in any university before.

This dissertation may be placed before the examiners for evaluation and necessary formalities. I certify that this dissertation is worthy of consideration by the examiners.

K. K. Singh
Kaushal Kumar Singh

Dr.K.K. Singh
Associate Professor
Mathematics Department
N.E.H.U., Shillong - 793022.

Supervisor
Department of Mathematics
North-Eastern Hill University
Shillong – 793022

Place: Shillong.
20th January, 2010.

NORTH-EASTERN HILL UNIVERSITY

January, 2010

DECLARATION

I, Bineeta Nath, hereby declare that the subject matter in this dissertation is the record of work done by me, that the contents of this dissertation did not form basis of the award of any previous degree to me or to the best of my knowledge to anybody else, and that the dissertation has not been submitted by me for any research degree in any other university/institute.

This dissertation is being submitted to the North-Eastern Hill University for the degree of Master of Philosophy in Mathematics.

Bineeta Nath

Signature of the Candidate

Countersigned by:

Mangesh B. Rege 22-01-2010

Signature of the Head

Head,
Department of Mathematics,
North-Eastern Hill University,
Shillong-793022

K. K. Singh

Signature of the Supervisor

Dr.K.K. Singh
Associate Professor
Mathematics Department
N.E.H.U., Shillong - 793022.

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PREFACE

In this dissertation we have attempted to study some of the recent works done in the area of propagation of shock waves in non-ideal conducting and non-conducting fluids. The dissertation has been divided into four chapters.

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In the chapter 4, we have studied similarity solutions for one-dimensional adiabatic flow behind a magnetogasdynamic cylindrical shock wave propagating in a rotating non-ideal gas in presence of an azimuthal magnetic field. The density of the medium ahead of the shock is assumed to be constant. In order to obtain the similarity solutions the angular velocity of the ambient medium is assumed to be obeying a power law and to be decreasing as the distance from the axis increases. The effects of an increase in the value of the index for variation of angular velocity of the ambient medium, in the value of the parameter of the non-idealness of the gas and in the strength of the initial magnetic field are investigated.

Contents

Chapter 1

BASIC CONCEPTS AND FUNDAMENTAL EQUATIONS

In this chapter, we give, in brief, an idea about shock waves, shock waves in magnetogasdynamics, Fundamental equations of a Non- Ideal gas, the concept of self-similarity and the Whitham's rule which will be used in the forth coming chapters.

1.1 SHOCK WAVES

Shock waves are the most important distinctive features of supersonic flow of a gas, across which the medium undergoes sudden and often considerable changes in velocity, pressure, density and temperature. The occurance of shock waves is commonly associated with supersonic flight, explosions and electrical discharges. The formation of shock wave can be simply visualized by considering the uniform motion of a piston into an open ended tube filled with gas. A simple physical explanation of shock formation in this case is the following :-

Suppose the continuous motion of the piston is approximated by a set of forward moving pulses, each of short duration. When the piston makes the first short movement forward, a small disturbance is propagated forward into the gas at the speed of sound. This small amplitude wave (or sound wave) heats the gas slightly and since the square of the local speed of sound is proportional to the temperature, the second pulse will be propagated as another sound wave at a speed slightly in excess of the first one. Similarly the third pulse will be propagated at a speed slightly in excess of second and so on. Thus the discrete pulses cause a train of sound waves of ever increasing velocity to be propagated through the gas. The tendency is for faster moving rearmost waves to catch up with the slower moving foremost ones. In doing so the sound waves coalesce to form a more powerful shock front moving at a speed which is in excess of the local speed of sound.

Shock waves are the most conspicuous phenomena occuring in non-linear wave propagation. Even without being caused by initial discontinuities, they may appear and be propagated. The problem of shock waves has a bearing on many problems outside of supersonic aeronautics, for example, detonation waves, but also has great importance for several practical aeronautical problems. In fact, shock waves may cause sudden change in the aerodynamic behaviour of high speed aerocrafts affecting not only their balance and stability but also control producing undesirable vibrations.

It is true, of course, that a shock wave is not a discontinuity in the strict sense. It has a finite thickness across which the physical properties change continuously. The study of 'structure' of a shock wave involves the study of pressure, temperature, density, velocity etc. in small but finite thickness of shock wave. The thickness of the shock wave is of order of few molecular mean free path.

Relative to the shock wave, the flow on the upstream side must be supersonic, on the down stream side the flow relative to the shock wave may be either supersonic or subsonic, depending on the inclination to the incident stream of the normal to the wave. If the normal to the wave is parallel to the incident stream, the flow behind the wave is always subsonic relative to the wave. By now, the theory of shock waves in non-conducting and conducting media has been developed.([1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21])

1.2 SHOCK WAVES IN MAGNETOGASDYNEMICS

If a conducting fluid moves in a magetic field, electric fields are induced and electric currents flow. The magnetic field exerts forces on these currents which considerably modify the flow ([22]). In many problems the energy in the electric field is much smaller than that in magnetic field. In these cases, we may express all the electromagnetic quantities in terms of magnetic field([23]). As a result, we consider only the interaction between the magnetic field and the gas dynamic field. This analysis forms the subject matter of the well known 'magnetogasdynamics' and this interaction is of prime importance in most of astrophysical and geophysical problems and in behaviour of interstellar gaseous masses. As is done in many problems, we have ignored Maxwell's displacement current. We also assume as usual, that the dissipative mechanisms such as viscosity and thermal conductivity are absent.

The equations of motion for one dimensional magnetogasdynamic flow in a perfectly conductivity fluid are ([3, 4, 24, 25, 26, 27]) as under:
(i) The equation of continuity is

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\nu \rho u}{r} = 0 \, , \tag{1.2.1}$$

where $\nu = 0, 1, 2$ for planar, cylindrically and spherically symmetric flows, respectively; and u and ρ are fluid velocity and density at time t and at a distance r from the plane, axis or centre of symmetry.
(ii) The momentum equation in planar symmetry with magnetic field h per-

pendicular to the flow is

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu h}{\rho}\frac{\partial h}{\partial r}=0\,, \tag{1.2.2}$$

where μ is the magnetic permeability of the medium and p is the pressure.

The momentum equation in cylindrical or spherical symmetry with azimuthal magnetic field h, is

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu h}{\rho r}\frac{\partial (hr)}{\partial r}=0\,. \tag{1.2.3}$$

The momentum equation in cylindrical symmetry with axial magnetic field h, is

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu h}{\rho}\frac{\partial h}{\partial r}=0\,. \tag{1.2.4}$$

If the field is not perfectly conducting, but weakly conducting, the momentum equation in cylindrical symmetry takes the form

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\sigma\mu^2h^2u}{\rho}=0\,, \tag{1.2.5}$$

where σ is the electrical conductivity and h is axial or azimuthal magnetic field.

(iii) The magnetic field equation is

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial r}+h\frac{\partial u}{\partial r}+\frac{jhu}{r}=0\,, \tag{1.2.6}$$

where, for planar symmetry, $j=0$; for cylindrical symmetry with azimuthal magnetic field (h) , $j=0$; for cylindrical symmetry with axial magnetic field (h), $j=1$; and for spherical symmetry with azimuthal magnetic field (h),

$j = 1$.

(iv) The energy equation is

$$\frac{\partial(p\rho^{-\gamma})}{\partial t} + u\frac{\partial(p\rho^{-\gamma})}{\partial r} = 0 \, , \tag{1.2.7}$$

where γ is the ratio of specific heats of the fluid.

If the fluid is not perfectly conducting but weakly conducting, the energy equation takes the form

$$\frac{\partial(p\rho^{-\gamma})}{\partial t} + u\frac{\partial(p\rho^{-\gamma})}{\partial r} = \sigma(\gamma-1)\rho^{-\gamma}\mu^2 h_0^2 u^2 \, , \tag{1.2.8}$$

where h_0 is the initial axial or azimuthal magnetic field.

The relations connecting the flow variables on the two sides of the shock surface (the generalized Rankine-Hugoniot relations) in the co-ordinate system in which the velocity in front of the shock wave is zero, are as follows [3, 23]

$$h_2(U-u_2) = h_1 U \, , \tag{1.2.9}$$

$$\rho_2(U-u_2) = \rho_1 U \, , \tag{1.2.10}$$

$$p_2 + \frac{1}{2}\mu {h_2}^2 + \rho_2(U-u_2)^2 = p_1 + \frac{1}{2}\mu {h_1}^2 + \rho_1 U^2 \, , \tag{1.2.11}$$

$$\frac{1}{2}(U-u_2)^2 + \frac{\gamma p_2}{(\gamma-1)\rho_2} + \frac{\mu {h_2}^2}{\rho_2} = \frac{1}{2}U^2 + \frac{\gamma p_1}{(\gamma-1)\rho_1} + \frac{\mu {h_1}^2}{\rho_1} \tag{1.2.12}$$

where the subscripts '1' and '2' correspond to the values of the quantities just ahead and just behind the shock surface respectively, and U is the shock velocity.

If the fluid is weakly conducting, the magnetic field is continuous across the shock [4].

1.3 FUNDAMENTAL EQUATIONS OF A NON-IDEAL GAS

When a strong explosion takes place, the character of the motion of the substance depends essentially on its equation of state. Such a motion was studied originally for the case of an ideal gas; subsequently it was studied for imperfect media.

The equation of state at low density for a non ideal gas is obtained by considering an expansion of the pressure p in powers of the density ρ [28, 29]

$$p = \Gamma \rho T[1 + \rho C_1(T) + \rho^2 C_2(T) + \cdots] ,$$

where Γ is the gas constant and $C_1(T), C_2(T), \cdots$ are virial co-efficients. The first term in the expansion corresponds to an ideal gas. The second term is obtained by taking into account the interaction between pairs of molecules, and subsequent terms must involve the interactions between the groups of three, four, etc. molecules. In the high temperature range, the co-efficients $C_1(T)$ and $C_2(T)$ tend to constant values equal to b and $\left(\frac{5}{8}\right) b^2$, respectively. For gases $b\rho \ll 1$, b being the internal volume of the molecules, and therefore it is sufficient to consider the equation of state in the form [28, 30, 31]

$$p = \Gamma \rho T(1 + b\rho) , \tag{1.3.1}$$

In this equation the correction to pressure is missing due to neglect of second and higher powers of $b\rho$, i.e. due to neglect of interactions between groups of three, four, etc. molecules of the gas.

From thermodynamics, we have

$$\left(\frac{\partial e}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p \tag{1.3.2}$$

where e is the internal energy per unit mass of the gas and v is specific volume.

Using the equation of state (1.3.1) in equation (1.3.2), we get $\left(\frac{\partial e}{\partial v}\right)_T = 0$, which shows that the internal energy e is a function of temperature T only. Therefore,

$$e = C_v T\ , \tag{1.3.3}$$

where C_v is the specific heat at constant volume.

Using equation (1.3.2) in the first law of thermodynamics, we have

$$C_p - C_v = T\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p , \tag{1.3.4}$$

where C_p is the specific heat of the gas at constant pressure.

Using equation (1.3.1) in equation (1.3.4), we get

$$C_p - C_v = \frac{\Gamma(1+b\rho)^2}{1+2b\rho} \cong \Gamma\ , \tag{1.3.5}$$

neglecting second and higher powers of $b\rho$.

Equation (1.3.5) implies that

$$C_v = \frac{\Gamma}{(\gamma-1)}\ . \tag{1.3.6}$$

Then equations (1.3.1), (1.3.3) and (1.3.6) give the internal energy e as a function of p and ρ, in the form

$$e = \frac{p}{\rho(1+b\rho)(\gamma-1)}\ . \tag{1.3.7}$$

The isentropic speed of sound c may be calculated from equations (1.3.1) as follows:

$$c^2 = \frac{dp}{d\rho} = \frac{(1+2b\rho)}{(1+b\rho)}\frac{\gamma p}{\rho} \,. \tag{1.3.8}$$

The equation of the energy of the non-ideal gas whose equation of state is in the form of equation (1.3.1) and which is non-conducting or perfectly conducting, is given by ([17, 27, 30]),

$$\left(\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r}\right) - c^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \tag{1.3.9}$$

where c is given by equation (1.3.8).

If the gas is weakly conducting, it takes the form

$$\left(\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r}\right) - c^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = (\gamma-1)\sigma\mu {h_0}^2 u^2(1+b\rho) \tag{1.3.10}$$

where h_0 is the initail axial or azimuthal magnetic field.

At high pressure and density, the gas obeys a simplified Van der Waals equation of state of the form (Wu and Roberts [32], Roberts and Wu [33])

$$p = \frac{\Gamma\rho T}{1-b\rho}\,, \quad e = C_v T = \frac{p(1-b\rho)}{\rho(\gamma-1)}\,,$$

where, b is the 'Van der Waals excluded volume', it places a limit, $\rho_{max} = \frac{1}{b}$, on the density of the gas.

The equations of continuity, magnetic field and momentum are the same as for a perfect gas.

The fundamental equations and shock conditions for non-conducting fluids can simply be obtained by putting magnetic field terms to zero in the corresponding equations for conducting fluids.

1.4 THE CONCEPT OF SELF-SIMILARITY

The fluid motion is said to be one-dimensional when all its properties depend on only one geometric co-ordinate and on the time. The motion in which the distributions of the flow variables remain similar to themselves with time and vary only as a result of change in scale is called self-similar. Self-similar motion is of great importance in gas dynamics. In this case the flow variables do not depend on the co-ordinates and time separately, but depend only on particular combinations of them. The methods of dimensional analysis can be used to find exact solutions of certain problems of one dimensional unsteady motion of compressible fluid. The spherical, cylindrical and plane waves produce one dimensional motion.

In the Eulerian approach, the basic physical variables are the velocity u, the density ρ, and the pressure p. The characteristic parameters are the linear co-ordinate r, the time t and the constants that enter into the equations, the boundary and initial conditions of the problem. Since the dimensions of the quantities p and ρ contain the mass, at least one constant a', the dimensions of which also contain the mass, must be a characteristic parameter. Hence, as in [2], without any loss of generality, we can assume that

$$dim\, a' = ML^k T^s \,. \tag{1.4.1}$$

We can then write for the velocity, density and pressure as

$$u = \frac{r}{t}V, \;\; \rho = \frac{a'}{r^{k+3}t^s}G, \;\; p = \frac{a'}{r^{k+1}t^{s+2}}P, \tag{1.4.2}$$

where V, G and P are arbitrary quantities depending only on non-dimensional combinations of r, t and other parameters of the problem. In general, they are functions of two non-dimensional variables. But, if an additional characteristic parameter b' can be introduced with dimensions independent of those of a', the number of independent variables which can be formed by combining $r,\ t,\ a'$ and b' is reduced to one. Since, the dimensions of the constant a' contain the mass, we can choose the constant b' in such a manner that its dimensions do not contain the mass, that is

$$dim\, b' = L^m T^n \ . \tag{1.4.3}$$

The single non dimensional independent variable, in this case will be $\frac{r^m t^n}{b'}$ which can be replaced for $m \neq 0$ by the variable,

$$\eta = \frac{r}{b'^{\frac{1}{m}} t^{\delta}} \ , \tag{1.4.4}$$

where

$$\delta = -\left(\frac{n}{m}\right) \ .$$

If $m = 0$, $V, G,$ and P depend only on the time t and the velocity u is proportional to r.

The solution depending on the independent variables may contain a number of arbitrary constants.

This arguement shows that, when the characteristic parameters include two constants with independent dimensions in addition to r and t, the partial differential equation satisfied by the velocity, density and pressure in the one dimensional unsteady motion of a compressible fluid can be replaced by ordinary differential equations for V, G, and P. The solution of these ordinary differential equations can sometimes be obtained exactly in closed form and in other cases, approximately by using numerical integration.

1.5 WHITHAM'S METHOD

Whitham [4] has shown that the motion of the shock can be found in a simple way, without solving the equations for the flow behind the shock in detail. His method can be summarised as the following rule.

The relevant equations of motion are first written in characteristic form which is linear combination of two equations and contains the derivatives of u, p, ρ in only one direction in the (r, t) plane. Then the rule is to apply the differential relation which must be satisfied by the flow quantities along a characteristic to the flow quantities just behind the shock wave. Together with the shock relations this rule determines the motion of the shock wave.

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Chapter 2

PROPAGATION OF SHOCK WAVES IN A NON-IDEAL GAS

The assumption that the gas is ideal is no more valid when the flow takes place at high temperatures. Anisimov and Spiner [1] studied a problem of point explosion in a non-ideal gas by taking the equation of state in a simplified form, which describes the behaviour of the medium satisfactorily at low densities. Ranga Rao and Purohit [2] have studied the self-similar flow of a non-ideal gas driven by an expanding piston and obtained approximate analytical and numerical solutions by taking the equation of state suggested by Anisimov and Spiner [1]. In this chapter we shall study the following two types of problem by taking the same equation of state:

(i) Propagation of strong diverging shock waves in a non ideal gas, and

(ii) Similarity solutions for the flow behind an exponential shock in a non-ideal gas.

2.1 PROPAGATION OF STRONG DIVERGING SHOCK WAVES IN A NON-IDEAL GAS

This section is devoted to study the propagation of strong diverging shock waves in a non-ideal gas by using Whitham's Rule. We follow Singh and Singh [3] here.

2.1.1 FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

The fundamental equations for one-dimensional adiabatic unsteady planar, cylindrically or spherically symmetric flow of a non-ideal gas can be written as

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\nu \rho u}{r} = 0 , \tag{2.1.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 , \tag{2.1.2}$$

$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial r} - \frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 , \tag{2.1.3}$$

where $\nu = 0$, 1 *or* 2 corresponds to plane, cylindrical or spherical symmetry.

ρ is the density,

u the flow velocty,

p the pressure,

e the internal energy per unit mass,

r the distance,

t the time.

The equation of state for a non-ideal gas is borrowed from the statistical physics [4] which has been simplified by Anisimov and Spiner [1] in the form

$$p = \Gamma \rho T(1 + b\rho) \; , \tag{2.1.4}$$

where b is the internal volume of the molecules, Γ a gas constant and T the temperature of the gas. The internal energy per unit mass of non-ideal gas is given by Singh and Singh [3], Ojha [5], as

$$e = \frac{p}{(\gamma - 1)\rho(1 + b\rho)} \; , \tag{2.1.5}$$

which implies that

$$C_p - C_v = \Gamma \left(1 + \frac{b^2 \rho^2}{1 + 2b\rho}\right) \cong \Gamma \; , \tag{2.1.6}$$

neglecting the term $b^2\rho^2$.

By use of equation (2.1.5), equation (2.1.3) can be put in the form

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} - c^2 \left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \; , \tag{2.1.7}$$

where c is the speed of sound in the non-ideal gas, given by

$$c^2 = \left(\frac{1 + 2b\rho}{1 + b\rho}\right) \frac{\gamma p}{\rho} \; . \tag{2.1.8}$$

For diverging shocks, the characteristic form of the system of equations (2.1.1), (2.1.2) and (2.1.7) can be easily obtained by combining these equations in only one direction in the (r, t) plane. To find the characterictic form we proceed as follows :

Multiplying equation (2.1.2) by c, we get

$$\rho c\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}\right)+c\frac{\partial p}{\partial r}=0\,. \tag{2.1.9}$$

Then adding equations (2.1.7) and (2.1.9), we get

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}-c^2\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)+\rho c\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}\right)+c\frac{\partial p}{\partial r}=0\,. \tag{2.1.10}$$

Using equation (2.1.1) in (2.1.10) for the second term and then dividing the resulting equation by $u+c$, we get

$$\frac{1}{u+c}\frac{\partial p}{\partial t}+\frac{\partial p}{\partial r}+c\rho\left(\frac{1}{u+c}\frac{\partial u}{\partial t}+\frac{\partial u}{\partial r}\right)+\frac{\nu\rho c^2 u}{r(u+c)}=0\,. \tag{2.1.11}$$

Now, since

$$\begin{aligned}\frac{1}{u+c}\frac{\partial p}{\partial t}+\frac{\partial p}{\partial r}&=\frac{1}{u+c}\frac{dp}{dt}\,,\\ \frac{1}{u+c}\frac{\partial u}{\partial t}+\frac{\partial u}{\partial r}&=\frac{1}{u+c}\frac{du}{dt}\,,\end{aligned} \tag{2.1.12}$$

along the positive characteristic

$$\frac{dr}{dt}=u+c\,, \tag{2.1.13}$$

therefore equation (2.1.11) reduces to

$$dp+\rho c du+\frac{\nu\rho c^2 u}{(u+c)}\frac{dr}{r}=0 \tag{2.1.14}$$

which is the characteristic form of the system of equation (2.1.1) , (2.1.2) and (2.1.7) along the positive characteristic.

We consider that a strong plane, cylindrical or spherical shock wave is propagating into a medium (non- ideal gas) of constant density ρ_0 at rest

($u_0 = 0$) with negligibly small counter pressure $p_0 \cong 0$. The boundary conditions at the strong shock are as follows

$$\rho(U-u) = \rho_0 U \,,$$

$$p + \rho(U-u)^2 = \rho_0 U^2 \,,$$

$$e + \frac{p}{\rho} + \frac{1}{2}(U-u)^2 = \frac{1}{2}U^2 \,, \tag{2.1.15}$$

where U is the shock velocity and the quantities with suffix '0' and without zero denote the values of the quantities in front and behind the shock.

From the shock conditions (2.1.15), we get

$$\rho = \frac{\rho_0}{\beta} \,,$$

$$u = (1-\beta)U \,,$$

$$p = (1-\beta)\rho_0 U^2 \,, \tag{2.1.16}$$

where β is given by the relation

$$\beta = \frac{(\gamma - 1 - \alpha) + \sqrt{[(\gamma - 1 - \alpha)^2 + 4\alpha(\gamma+1)]}}{2(\gamma - 1)} \tag{2.1.17}$$

and

$$\alpha = (\gamma - 1) b \rho_0 \,.$$

2.1.2 SOLUTION OF THE PROBLEM

Now substituting equations (2.1.16) into the characteristic equation (2.1.14), we get

$$2\beta U dU + c dU + \frac{\nu c^2 U}{(1-\beta)U + c}\frac{dr}{r} = 0 \,. \tag{2.1.18}$$

Again substituting equations (2.1.16) into equation (2.1.8), we get

$$c^2 = \frac{(\beta + 2b\rho_0)\beta\gamma(1-\beta)U^2}{\beta + b\rho_0} \,. \tag{2.1.19}$$

Using this value of c in equation (2.1.18), we get

$$\frac{dU}{U} = -a_1\frac{dr}{r} \,, \tag{2.1.20}$$

where

$$a_1 = \frac{\nu\lambda^2}{\lambda^2 + (1+\beta)\lambda + 2\beta(1-\beta)} \,, \tag{2.1.21}$$

while

$$\lambda^2 = \frac{\gamma\beta(1-\beta)[2\alpha + (\gamma-1)\beta]}{\alpha + (\gamma-1)\beta} \,. \tag{2.1.22}$$

Integrating equation (2.1.20), we get

$$U = kr^{-a_1} \,, \tag{2.1.23}$$

where k is a constant of integration. Therefore, from equations (2.1.16), we get

$$\frac{u}{k} = (1-\beta)r^{-a_1}, \tag{2.1.24}$$

$$\frac{p}{\rho_0 k^2} = (1-\beta)r^{-2a_1} \,. \tag{2.1.25}$$

2.1.3 RESULTS AND DISCUSSION

The numerical values of $\frac{U}{k}$, $\frac{u}{k}$ and $\frac{p}{\rho_0 k^2}$ at different distances for various values of the non-ideal parameter α are given in Tables II, III and IV , For the purpose of calculations the following values of the parameters are taken, $\gamma = 1.4$ and $\alpha = 0.025$, 0.050 , 0.100 .

Further we cofined our study of solutions only for the case of cylindrical symmetry ($\nu = 1$) because of its application in aerodynamics. Equations (2.1.17), (2.1.21) and (2.1.22) will determine the values of a_1. Values of the density ratio β across the shock front and a_1 are shown in Table I.

TABLE - I

$\nu = 1, \gamma = 1.4$

α	β	a_1
0.025	0.2066558	0.2258203
0.050	0.2346268	0.2391637
0.100	0.2759781	0.2528743

TABLE - II

$\nu = 1, \alpha = 0.025$

r	$\frac{u}{k}$	$\frac{p}{\rho_0 k^2}$	$\frac{U}{k}$
0.01	2.2444103	6.3495486	2.8290498
0.1	1.3343875	2.2444103	1.681978
1.0	0.7933442	0.7933442	1.000000
10.0	0.4716733	0.2804277	0.5945381
10^2	0.2804277	0.0991243	0.3534755
10^3	0.166725	0.035038	0.2101547
10^4	0.0991243	0.012385	0.1249449
10^5	0.058933	0.0043778	0.0742843
10^6	0.0350379	0.0015475	0.0441649
10^7	0.0208266	0.0005470	0.02622517
10^8	0.01238509	0.000193347	0.015611247
10^9	0.00736341	0.000068343	0.009281481
10^{10}	0.0043793	0.0000242	0.00552

TABLE - III

$\nu = 1 \ , \ \alpha = 0.050$

r	$\frac{u}{k}$	$\frac{p}{\rho_0 k^2}$	$\frac{U}{k}$
0.01	2.3025053	6.9267267	3.0083433
0.1	1.3275074	2.3025054	1.7344576
1.0	0.7653732	0.7653732	1.0000000
10	0.4412752	0.2544168	0.5765491
10^2	0.2544167	0.0845704	0.3324088
10^3	0.1466837	0.0281119	0.1916500
10^4	0.0845703	0.0093447	0.1104956
10^5	0.0487589	0.0031062	0.0637061
10^6	0.0281119	0.0010325	0.0367297
10^7	0.0162078	0.0003432	0.0211764
10^8	0.0093446	0.0001141	0.0122092
10^9	0.0053875	0.0000380	0.0007039
10^{10}	0.0011063	0.000126	0.0040585

TABLE - IV

$\nu = 1$, $\alpha = 0.100$

r	$\frac{u}{k}$	$\frac{p}{\rho_0 k^2}$	$\frac{U}{k}$
0.01	2.3200658	7.434451	3.2044138
0.1	1.2960627	2.3200658	1.7900877
1.0	0.7240219	0.7240219	1.000000
10	0.4044616	0.2259451	0.5586318
10^2	0.2259451	0.7015106	0.3120695
10^3	0.1262201	0.0220042	0.1743319
10^4	0.0705106	0.0068668	0.0973874
10^5	0.393894	0.0021429	0.0544037
10^6	0.0220041	0.0006687	0.0303916
10^7	0.0122922	0.0002087	0.0169777
10^8	0.0068669	0.0000651	0.0094843
10^9	0.0038360	0.0000203	0.0052982
10^{10}	0.0021431	0.0000063	0.002960

Table I shows that as α increases, the density ratio β and a_1 increase. Tables II, III, and IV, show that the fluid velocity and the pressure behind the shock decrease as the shock diverges from the axis and this decrease becomes faster and faster as the value of α increases. The same is the case for shock velocity also. Thus we see that α influences the flow pattern.

2.2 SIMILARITY SOLUTIONS FOR THE FLOW BEHIND AN EXPONENTIAL SHOCK IN A NON-IDEAL GAS

In this section, we present similarity solutions for the flow of a non-ideal gas behind a strong exponential shock driven out by a piston (cylindrical or spherical) moving with time according to an exponential law. We follow Vishwakarma and Nath [6] here.

2.2.1 INTRODUCTION

Ragna Rao and Ramana [7] and Singh and Vishwakarma [8] obtained solutions for the problems of unsteady self-similar motion of a perfect gas displaced by a piston according to an exponential law. In the present study, we investigate the one-dimensional unsteady self-similar flow of a non-ideal gas behind a strong shock driven out by a cylindrical (or spherical) piston moving with time according to an exponential law, namely

$$r_p = Ae^{mt}, \ m > 0 , \tag{2.2.1}$$

where r_p is the radius of the piston, A and m are dimensional constants, and t the time.

Since we are concerned with self-similar motion, the shock will also be exponential. Thus

$$R = Be^{mt} , \tag{2.2.2}$$

where R is the radius of the shock, and B is a dimensional constant which is to be determined.

We study both the cases, when the flow between the shock and the piston is adiabatic or isothermal. The assumption of isothermal flow is physically realistic, when radiation heat transfer effects are implicitly present. As the shock propagates, the temperature behind it increases and becomes very large so that there is intense transfer of energy by radiation. This causes the temperature gradient to approach zero, that is the dependent temperature tends to become uniform behind the shock front and the flow becomes isothermal (Singh and Vishwakarma [8], Laumbach and Probstein [9], Sachdev and Ashraf[10]). With this assumption, we obtain the similarity solutions in sections 2.2.2 and 2.2.3 . In section 2.2.4 , we present the solutions for the flow taken to be adaibatic.

A comparative study between the solutions of isothermal and adiabatic flows will be made in section 2.2.5.

2.2.2 FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS - ISOTHERMAL FLOW

The fundamental equations for one dimensional unsteady and isothermal flow of a non-ideal gas can be written as [10, 11]

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}=0\ , \tag{2.2.3}$$

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\frac{\partial u}{\partial r}+\frac{\nu\rho u}{r}=0\ , \tag{2.2.4}$$

$$\frac{\partial T}{\partial r}=0\ , \tag{2.2.5}$$

where ρ is the density, p the pressure, u the velocity, t the time, r the distance, T the temperature, and $\nu = 1$ for cylindrical symmetry and $\nu = 2$ for spherical symmetry.

The equation of state for non-ideal gas is borrowed from the statistical physics [4] which has been simplified by Anisimov and Spiner [1] in the form

$$p = \Gamma \rho T(1 + b\rho) \ , \tag{2.2.6}$$

where $b(\ll 1)$ is the internal volume of the molecules and Γ is the gas constant.

The internal energy (e) per unit mass is given by (Singh and Singh [3], Ojha [5]),

$$e = \frac{p}{(\gamma - 1)\rho(1 + b\rho)} \ . \tag{2.2.7}$$

The laws of conservation of mass, momentum and energy across the shock front propagating with velocity $U(= \frac{dR}{dt})$ into the non-ideal gas at rest $(u_1 = 0)$ give the following shock conditions

$$\rho_2(U - u_2) = \rho_1 U = m_s \ (say) \ ,$$

$$p_2 + \rho_2(U - u_1)^2 = p_1 + \rho_1 U^2 \ ,$$

$$e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}(U - u_2)^2 - \frac{F_2}{m_s} = e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}U^2 - \frac{F_1}{m_s} \ , \tag{2.2.8}$$

where the suffixes '1' and '2' denote the quantities just ahead and just behind of the shock front, respectively and F is the radiation heat flux. If the shock is strong,

$$p_1 \cong e_1 \cong 0 \ . \tag{2.2.9}$$

Then the shock conditions (2.2.8) reduce to

$$\rho_2(U-u_2)=\rho_1 U=m_s\ (say)\ ,$$

$$p_2+\rho_2(U-u_2)^2=\rho_1 U^2\ ,$$

$$e_2+\frac{p_2}{\rho_2}+\frac{1}{2}(U-u_2)^2-\frac{F_2}{m_s}=\frac{1}{2}U^2-\frac{F_1}{m_s}\ . \tag{2.2.10}$$

Let β be the density ratio across the shock i.e., $\beta=\frac{\rho_1}{\rho_2}$. Then from first two equations of (2.2.10), we get

$$u_2=(1-\beta)U$$

and

$$p_2=(1-\beta)\rho_1 U^2\ .$$

Now substituting value of e_2 from equation (2.2.7) and above values of ρ_2, u_2 and p_2 in the third equation of (2.2.10), we obtain after some manipulation

$$2\beta[\beta\gamma+(\gamma-1)\overline{\alpha}]-(1+\beta)(\gamma-1)(\beta+\overline{\alpha})=\frac{2(F_2-F_1)[(\beta+\overline{\alpha})(\gamma-1)]}{p_2 U}\ ,$$

where

$$\overline{\alpha}=b\rho_1\ .$$

Thus the strong shock conditions (2.2.10) give

$$\rho_2=\frac{\rho_1}{\beta}\ ,$$

$$u_2=(1-\beta)U\ ,$$

$$p_2=(1-\beta)\rho_1 U^2\ , \tag{2.2.11}$$

where $\beta(0 < \beta < 1)$ is given by the relation

$$2\beta[\beta\gamma + (\gamma - 1)\overline{\alpha}] - (1 + \beta)(\gamma - 1)(\beta + \overline{\alpha}) = \frac{2(F_2 - F_1)[(\beta + \overline{\alpha})(\gamma - 1)]}{p_2 U}, \tag{2.2.12}$$

where

$$\overline{\alpha} = b\rho_1 . \tag{2.2.13}$$

As the shock is strong, we assume $F_2 - F_1$ to be negligible in comparison with the product of p_2 and U (Laumbach and Probstein [9]). Therefore, equation (2.2.12) reduces to

$$\beta^2(\gamma + 1) + \beta(\overline{\alpha} - 1)(\gamma - 1) - (\gamma - 1)\overline{\alpha} = 0 . \tag{2.2.14}$$

Equation(2.2.5) together with equation (2.2.6) gives

$$\frac{p}{p_2} = \frac{\rho(1 + b\rho)}{\rho_2(1 + b\rho_2)} . \tag{2.2.15}$$

2.2.3 SIMILARITY SOLUTIONS

To obtain similarity solutions, we write the unknown variables in the following form

$$u = UV(\eta) ,$$

$$\rho = \rho_1 D(\eta) ,$$

$$p = \rho_1 U^2 P(\eta) , \tag{2.2.16}$$

where V, D and P are the functions of the non-dimensional variable (similarity variable) $\eta = \frac{r}{R}$ only. The variable η assumes the value '1' at the shock front and η_p on the piston. Equations (2.2.1), (2.2.2) and the relation $\eta_p = \frac{r_p}{R}$ yield a relation between A and B in the form

$$A = B\eta_p \, . \tag{2.2.17}$$

Equation (2.2.15) with the aid of equations (2.2.16) and (2.2.11) yields a relation between P and D in the form

$$P(\eta) = \frac{\beta^2(1-\beta)[1+\overline{\alpha}D(\eta)]D(\eta)}{(\beta+\overline{\alpha})} \, . \tag{2.2.18}$$

By use of equations (2.2.16) and (2.2.18), equations (2.2.3) and (2.2.4) can be transformed to

$$\frac{dV}{d\eta} + \frac{V-\eta}{D}\frac{dD}{d\eta} + \frac{\nu V}{\eta} = 0 \, , \tag{2.2.19}$$

$$(V-\eta)\frac{dV}{d\eta} + V + \frac{\beta^2(1-\beta)}{D(\beta+b\rho_1)}(1+2b\rho_1 D)\frac{dD}{d\eta} = 0 \, . \tag{2.2.20}$$

Solving equations (2.2.19) and (2.2.20) for $\frac{dV}{d\eta}$ and $\frac{dD}{d\eta}$, we obtain

$$\frac{dV}{d\eta} = \frac{V(\eta - V)[\eta - \nu(V-\eta)]}{\eta D}L - \frac{\nu V}{\eta} \, , \tag{2.2.21}$$

$$\frac{dD}{d\eta} = \left[V - \frac{\nu V}{\eta}(V-\eta)\right] L \, , \tag{2.2.22}$$

where

$$L = \frac{D(\beta+\overline{\alpha})}{(V-\eta)^2[\beta+\overline{\alpha}] - \beta^2(1-\beta)\,(1+2\overline{\alpha}D)} \, ,$$

while the shock conditions (2.2.11) with the help of equations (2.2.16) take the form

$$V(1) = 1-\beta, \;\; P(1) = 1-\beta, \;\; D(1) = \frac{1}{\beta} \, . \tag{2.2.23}$$

In addition to the shock conditions (2.2.23), the condition to be satisfied at the piston surface is that the velocity of the fluid is equal to the velocity of the piston itself.

Now velocity of the fluid at piston surface,

$$u_p = mRV(\eta_p)$$

and velocity of the piston,

$$\frac{dr_p}{dt} = mr_p \ .$$

Therefore,

$$u_p = \frac{dr_p}{dt}$$

gives

$$V(\eta_p) = \eta_p \ . \tag{2.2.24}$$

Now, equations (2.2.21) and (2.2.22) can be numerically integrated to obtain the solution of the problem.

2.2.4 ADIABATIC FLOW

In this section, we present the similarity solution for the adiabatic flow behind a strong shock driven out by a cylindrical or spherical piston moving according to the exponential law (2.2.1), in the case of non-ideal gas.

The strong shock conditions, which serves as the boundary conditions for the probem will be same as the shock conditions (2.2.11) in the case of isothermal flow.

For the adiabatic flow, equation (2.2.5) is replaced by Vishwakarma [12]

$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial r} - \frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \tag{2.2.25}$$

By use of equations (2.2.16), equations (2.2.3), (2.2.4) and (2.2.25) can be transformed to

$$(V-\eta)\frac{dV}{d\eta}+V+\frac{1}{D}\frac{dP}{d\eta}=0\,, \tag{2.2.26}$$

$$\frac{dV}{d\eta}+\frac{(V-\eta)}{D}\frac{dD}{d\eta}+\frac{\nu V}{\eta}=0\,, \tag{2.2.27}$$

$$(V-\eta)\frac{dP}{d\eta}-\frac{P}{D}(V-\eta)k_1\frac{dD}{d\eta}+2P=0\,, \tag{2.2.28}$$

where

$$k_1=\frac{(\gamma-1)\overline{\alpha}^2D^2+\gamma(1+2\overline{\alpha}D)}{(1+\overline{\alpha}D)}\,. \tag{2.2.29}$$

Solving equations (2.2.26) to (2.2.28) for $\frac{dV}{d\eta}$, $\frac{dD}{d\eta}$ and $\frac{dP}{d\eta}$, we obtain

$$\frac{dV}{d\eta}=\frac{(V-\eta)}{\eta[Pk_1-(V-\eta)^2D]}\left[VD\eta-\nu VD(V-\eta)-\frac{2P\eta}{V-\eta}\right]-\frac{\nu V}{\eta}\,, \tag{2.2.30}$$

$$\frac{dD}{d\eta}=\frac{D}{\eta[(V-\eta)^2D-Pk_1]}\left[VD\eta-\nu VD(V-\eta)-\frac{2P\eta}{(V-\eta)}\right]\,, \tag{2.2.31}$$

$$\frac{dP}{d\eta}=\frac{Pk_1}{\eta[(V-\eta)^2D-pK_1]}\left[VD\eta-\nu VD(V-\eta)-\frac{2P\eta}{V-\eta}\right]-\frac{2P}{(V-\eta)}\,, \tag{2.2.32}$$

while the shock conditions (2.2.11) take the form (2.2.23). In addition to the shock conditions (2.2.23), the kinematic condition at the piston surface (2.2.24) must be satisfied.

The ordinary differential equations (2.2.30) to (2.2.32) can now be numerically integrated to obtain the solutions for the adiabatic flow behind the shock surface.

2.2.5 RESULTS AND DISCUSSION

Distribution of the flow variables between the shock surface ($\eta = 1$) and the piston ($\eta = \eta_p$) are obtained from equations (2.2.18), (2.2.21) and (2.2.22) for isothermal flow, and from equations (2.2.30) to (2.2.32) for adiabatic flow, by numerical integration. Runge-Kutta method of fourth order is used for numerical integration, starting from the shock front ($\eta = 1$) and continuing until a value η_p (the piston position) is reached such that $V(\eta_p) = \eta_p$. For the purpose of numerical calculations, the values of constant parameters are taken to be (Roberts and Wu[13]),

$$\nu = 2\,; \quad \gamma = 1.4,\ 1.66\,; \quad \overline{\alpha} = 0,\ 0.025,\ 0.050,\ 0.100\,.$$

The value $\nu = 2$ corresponds to spherical shocks and the value $\overline{\alpha} = 0$ to the perfect gas case which is discussed by Ranga Rao and Ramana [7].

Figures 1 and 2 show the variation of non-dimensional flow variables V, D and P in isothermal and adiabatic cases respectively. From these two figures it is clear that in both the cases all the flow variables increase as we move from the shock front to the piston. Figures 3 and 4 show the variation of piston position and of density ratio across the shock with the parameter of non-idealness $\overline{\alpha}$. Also, figure 3 shows that the distance of the piston from the shock front is less in the case of adiabatic flow in comparision with that in the case of isothermal flow.

Effects of an increase in the value of $\overline{\alpha}$ are
(i) to decrease the flow variables V, D and P in the flow field behind the shock in both the cases (see figures 1, 2),
(ii) to increase the distance of the piston from the shock front in both the cases (see figure 3) and
(iii) to increase the value of β (i.e. to decrease the shock strength) (see figure 4).

Effects of an increase in the value of γ are
(i) to decrease the flow variables V, D and P in both the cases (see figures 1, 2),
(ii) to increase the distance of the piston from the shock front in both the cases (see figure 3), and
(iii) to increase the value of β (i.e. to decrease the shock strength) (see figure 4).

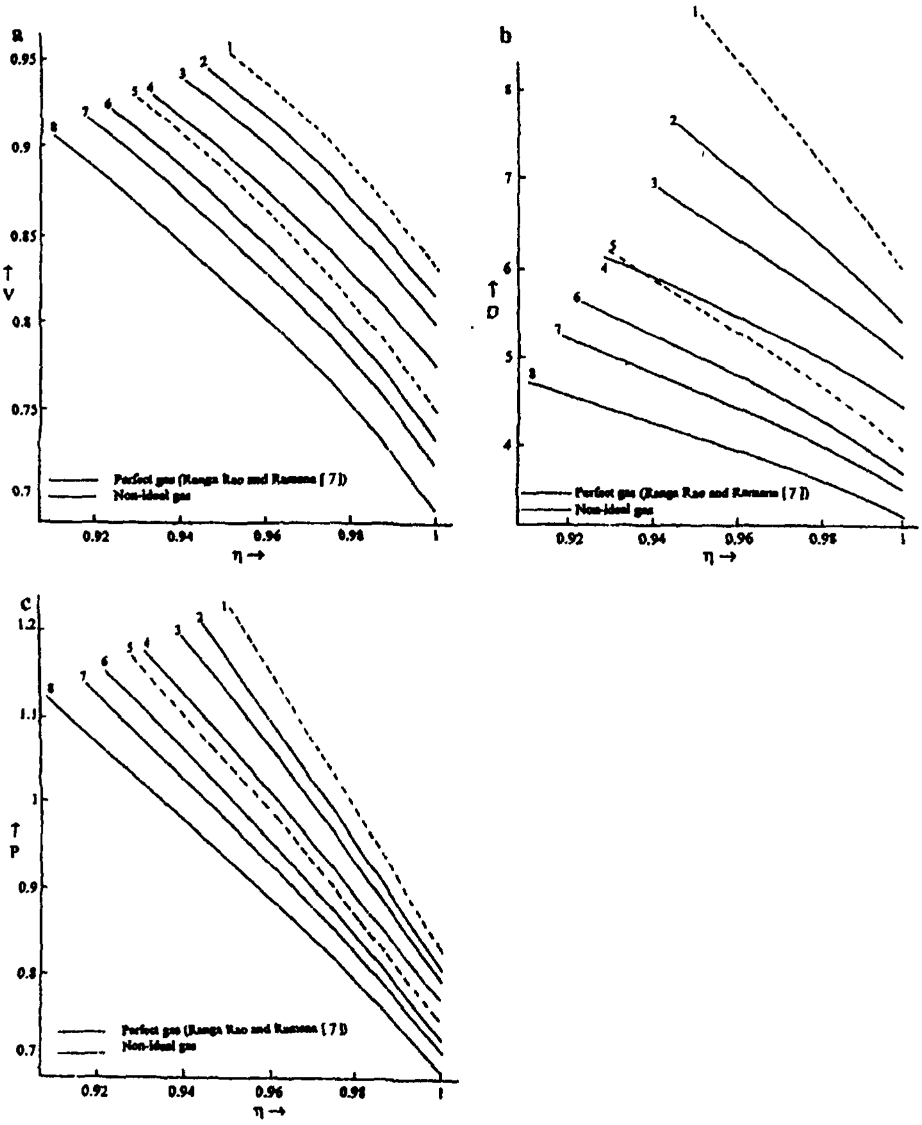

Figure 1. Variation of non-dimensional flow variables with non-dimensional distance η in the case of isothermal flow: (a) Non- dimensional velocity V, (b) Non-dimensional density D, (c) Non-dimensional pressure P.

1. $\gamma = 1.4$, $\bar{\alpha} = 0$; 2. $\gamma = 1.4$, $\bar{\alpha} = 0.025$; 3. $\gamma = 1.4$, $\bar{\alpha} = 0.05$; 4. $\gamma = 1.4$, $\bar{\alpha} = 0.1$; 5. $\gamma = 1.66$, $\bar{\alpha} = 0$; 6.$\gamma = 1.66$, $\bar{\alpha} = 0.025$; 7. $\gamma = 1.66$, $\bar{\alpha} = 0.05$; 8. $\gamma = 1.66$, $\bar{\alpha} = 0.1$;

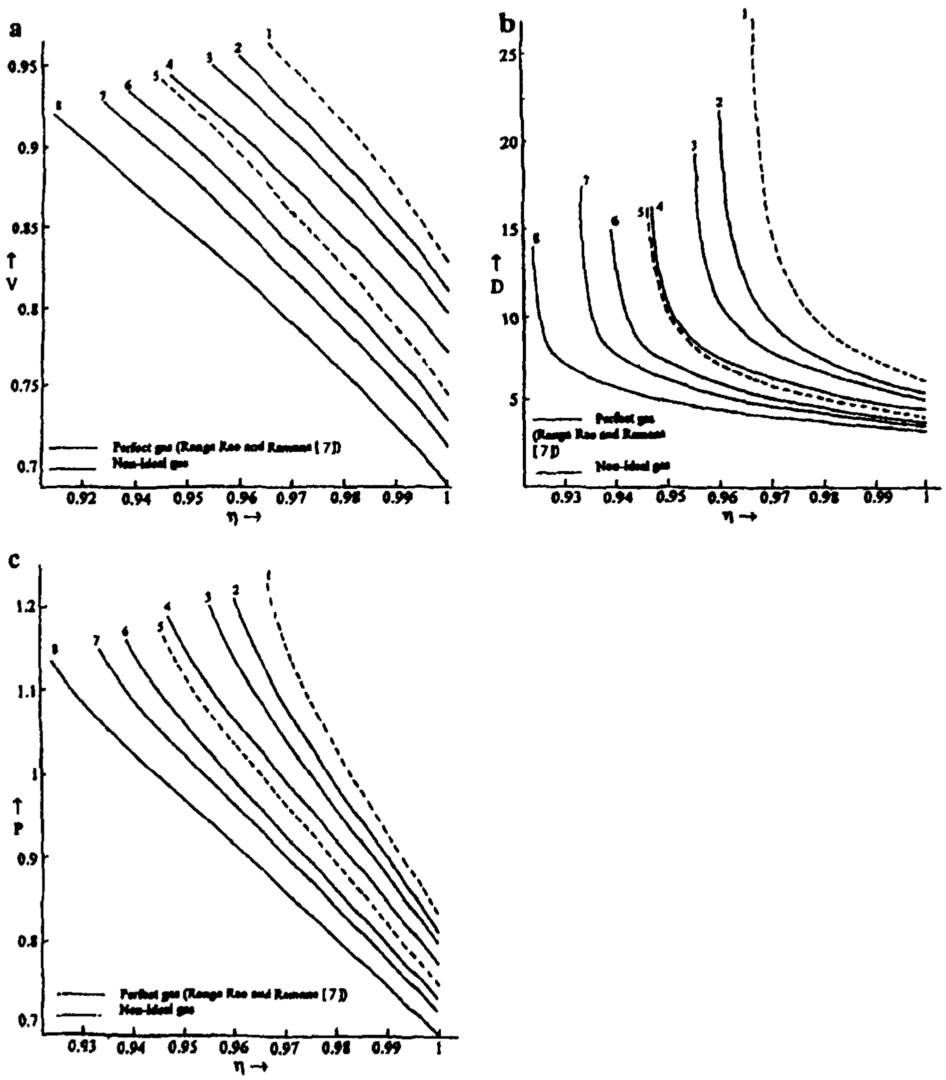

Figure 2. Variation of non-dimensional flow variables with non-dimensional distance η in the case of adiabatic flow: (a) Non-dimensional velocity V, (b) Non-dimensional density D, (c) Non-dimensional pressure P.

1. $\gamma = 1.4$, $\bar{\alpha} = 0$; 2. $\gamma = 1.4$, $\bar{\alpha} = 0.025$; 3. $\gamma = 1.4$, $\bar{\alpha} = 0.05$; 4. $\gamma = 1.4$, $\bar{\alpha} = 0.1$; 5. $\gamma = 1.66$, $\bar{\alpha} = 0$; 6.$\gamma = 1.66$, $\bar{\alpha} = 0.025$; 7. $\gamma = 1.66$, $\bar{\alpha} = 0.05$; 8. $\gamma = 1.66$, $\bar{\alpha} = 0.1$;

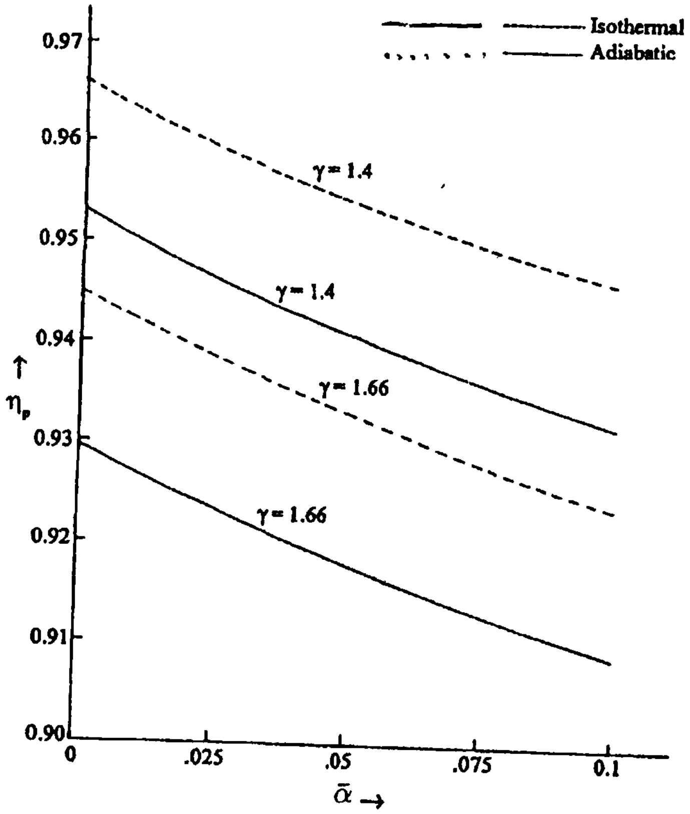

Figure 3. Variation of non-dimensional piston position η_p with parameter of non-idealness $\bar{\alpha}$ for γ = 1.4, 1.66 in isothermal and adiabatic cases.

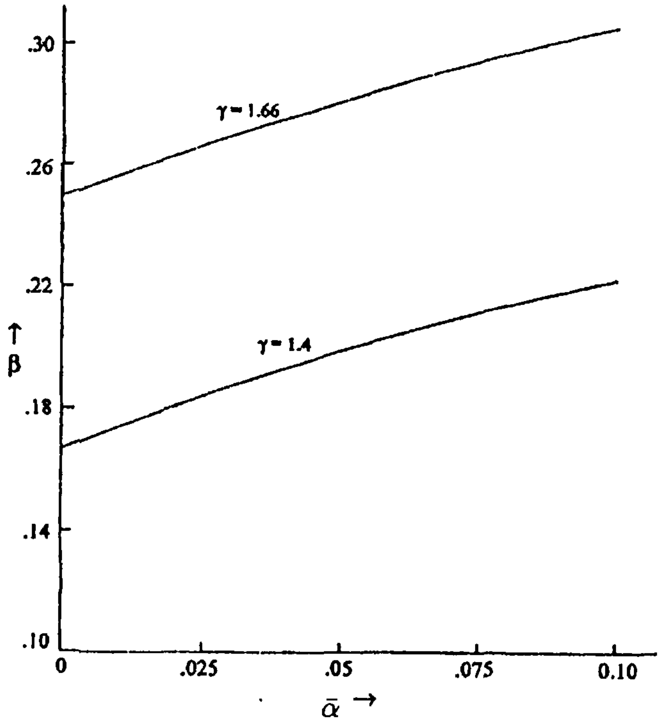

Figure 4. Variation of density ratio across the shock front β with the parameter of non-idealness $\bar{\alpha}$ for γ 1.4, 1.66.

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PROBLEM

Implosion of Cylindrical Shock Wave in a non-ideal gas.

The propagation of a converging cylindrical shock wave in a non-ideal gas can be studied by using Whitham's Rule.

Chapter 3

PROPAGATION OF MAGNETOGASDYNAMIC SHOCK WAVES IN A NON-IDEAL GAS

In the last chapter, we have studied propagation of shock waves in a non-ideal gas. In this chapter, we shall study propagation of shock waves in a non-ideal gas in presence of magnectic field. Since at high temperature that prevail in the problems associated with shock waves a gas is ionized, electromagnetic effects may also be significant. A complete analysis of such a problem should therefore consist of the study of the gas dynamic field and the electromagnectic field simultaneously. In this chapter we shall study the following two types of problem :

(i) Propagation of converging cylindrical shock waves in a non - ideal gas in the presence of an azimuthal magnetic field, and

(ii) Propagation of shock waves generated by a piston moving in a non-ideal gas in the presence of azimuthal magnetic field.

3.1 PROPAGATION OF CONVERGING SHOCK WAVES IN A NON-IDEAL GAS IN THE PRESENCE OF AN AZIMUTHAL MAGNETIC FIELD

This section is devoted to study the propagation of converging cylindrical shock wave in a non-ideal gas in the presence of an azimuthal magnetic field by using Whitham's Rule. We follow Singh and Vishwakarma [1] here.

3.1.1 Introduction

During the experiments involving the implosion of a shock wave in a gas, the following states may occur (Tyl [2]):

(I) The gas is weakly ionized before and behind the shock, i.e. $R_m \ll 1$, where R_m is the magnetic Reynolds number.

(II) The gas is strongly ionized before and behind the shock front, i.e. $R_m \gg 1$ or $\sigma \to \infty$, where σ is the electrical conductivity.

(III) Non-ionized (or weakly ionized) gas undergoes intense ionization as a result of the passage of the shock, i.e. σ increases in a jump like manner from 0 to ∞.

In our analysis, we study all the three cases. We assume the initial magnetic induction (or magnetic field) to vary inversely as the spatial co-ordinate r. Chester-Chisnell-Whitham's method (Whitham's Rule) is employed to determine the shock velocity and other flow variables just behind the shock.

3.1.2 FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

The equation of state for a non- ideal gas is borrowed from the Statistical Physics [3] which has been simplified by Anisimov and Spiner [4] in the form

$$p = \Gamma \rho T(1 + b\rho) \ , \tag{3.1.1}$$

where $b(\ll 1)$ is the internal volume of the molecules and Γ is the gas constant, and p, ρ and T are the pressure, the density and the temperature of the gas respectively.

The internal energy (e) per unit mass is given by (Singh and Singh [5], Ojha [6])

$$e = \frac{p}{\rho(\gamma - 1)(1 + b\rho)} \ , \tag{3.1.2}$$

which implies that

$$C_p - C_v = \Gamma \left(1 + \frac{b^2\rho^2}{1 + 2b\rho}\right) \cong \Gamma \ , \tag{3.1.3}$$

neglecting the term $b^2\rho^2$. Here, C_p and C_v are the specific heats of the gas at constant pressure and constant volume respectively.

We assume that the initial azimuthal magnetic induction depends on the spatial co-ordinate r as follows :

$$B_0 = \frac{k}{r} \quad (k = constant) \ . \tag{3.1.4}$$

The basic equations governing the unsteady and cylindrically symmetric motion of a weakly conducting non-ideal gas (Case I, $R_m \ll 1$) are given by (Tyl [2], Sakurai [7]) :

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0 \ , \tag{3.1.5}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}\right)+\frac{\partial p}{\partial r}=-\sigma {B_0}^2 u\,, \tag{3.1.6}$$

$$\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right)-c^2\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)=(\gamma-1)\sigma {B_0}^2 u^2(1+b\rho)\,, \tag{3.1.7}$$

where u is the velocity at a distance r from the axis of symmetry; γ is the ratio of specific heats; and 'c' the speed of sound in the non-ideal gas, is given by

$$c^2=\frac{(1+2b\rho)}{1+b\rho}\frac{\gamma p}{\rho}\,.$$

For converging shock, the characteristic form of the system of equations (3.1.5) to (3.1.7) can be easily obtained by combining these equations in only one direction in the (r,t) plane. To find the characteristic form we proceed as follows:

Multiplying equation (3.1.6) by c, we get

$$\rho c\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}\right)+c\frac{\partial p}{\partial r}=-\sigma c {B_0}^2 u\,. \tag{3.1.8}$$

Then subtracting equation (3.1.8) from equation (3.1.7), we get

$$\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right)-c^2\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)-\rho c\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}\right)-c\frac{\partial p}{\partial r}=(\gamma-1)\sigma {B_0}^2 u^2(1+b\rho)+\sigma c {B_0}^2 u\,. \tag{3.1.9}$$

Using equation (3.1.5) in (3.1.9) for the second term and then dividing the resulting equation by $(u-c)$, we get

$$\left(\frac{1}{(u-c)}\frac{\partial p}{\partial t}+\frac{\partial p}{\partial r}\right)-\rho c\left(\frac{1}{(u-c)}\frac{\partial u}{\partial t}+\frac{\partial u}{\partial r}\right)+\frac{\rho c^2 u}{r(u-c)}=\frac{(\gamma-1)\sigma {B_0}^2 u^2(1+b\rho)+\sigma c {B_0}^2 u}{(u-c)} \tag{3.1.10}$$

Now, since

$$\frac{1}{(u-c)}\frac{\partial p}{\partial t}+\frac{\partial p}{\partial r}=\frac{1}{(u-c)}\frac{dp}{dt}\,, \tag{3.1.11}$$

$$\frac{1}{(u-c)}\frac{\partial u}{\partial t}+\frac{\partial u}{\partial r}=\frac{1}{(u-c)}\frac{du}{dt}, \tag{3.1.12}$$

along the negative characteristic

$$\frac{dr}{dt}=u-c, \tag{3.1.13}$$

therefore equation (3.1.10) reduces to

$$dp-\rho c du+\frac{\rho c^2 u}{u-c}\frac{dr}{r}=\left[\frac{(\gamma-1)(1+b\rho)u^2+uc}{u-c}\right]\sigma {B_0}^2 dr \tag{3.1.14}$$

which is the required characteristic form of the system of equations (3.1.5) to (3.1.7) along the negative characteristic

The fundamental equations governing the unsteady flow behind a cylindrical magnetogasdynamic (Case II , $R_m \gg 1$) or gas ionizing (Case III, $\sigma : 0 \longrightarrow \infty$) shock are given by (Whitham [8], Tyl [2]):

$$\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial r}+\rho\frac{\partial u}{\partial r}+\frac{\rho u}{r}=0, \tag{3.1.15}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}\right)+\frac{\partial p}{\partial r}+\frac{B}{\mu}\frac{\partial B}{\partial r}+\frac{B^2}{\mu r}=0, \tag{3.1.16}$$

$$\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right)-c^2\left(\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial r}\right)=0, \tag{3.1.17}$$

$$\frac{\partial B}{\partial t}+\frac{\partial(Bu)}{\partial r}=0, \tag{3.1.18}$$

where μ is the magnetic permeability and B is the magnetic induction at a distance r from the axis of symmetry.

Equations (3.1.15) to (3.1.18) can be combined to obtain the characteristic equation as follows:

Multiplying equation (3.1.16) by a and then subtracting equation (3.1.17) from the resulting equation, we get after using equation (3.1.15) as

$$\left(\frac{1}{u-a}\frac{\partial}{\partial t}+\frac{\partial}{\partial r}\right)\left(p+\frac{1}{2}\frac{B^2}{\mu}\right)-\rho a\left(\frac{1}{u-a}\frac{\partial}{\partial t}+\frac{\partial}{\partial r}\right)u+\frac{\rho a^2 u}{r(u-a)}$$
$$-\frac{1}{u-a}\left[\left(\frac{\partial u}{\partial r}+\frac{\rho u}{r}\right)b^2\rho-\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)\frac{B^2}{2\mu}\right]\frac{B^2 a}{\mu r(u-a)}=0\,, \qquad (3.1.19)$$

where a is the effective speed of sound given by

$$a^2 = c^2 + {b_1}^2$$

and

$${b_1}^2 = \frac{B^2}{\mu\rho}\,.$$

Using equation (3.1.18) in (3.1.19), we obtain

$$\left(\frac{1}{u-a}\frac{\partial}{\partial t}+\frac{\partial}{\partial r}\right)\left(p+\frac{1}{2}\frac{B^2}{\mu}\right)-\rho a\left(\frac{1}{u-a}\frac{\partial}{\partial t}+\frac{\partial}{\partial r}\right)u+\frac{\rho a^2 u}{r(u-a)}-\frac{B^2(u+a)}{\mu r(u-a)}=0\,, \qquad (3.1.20)$$

which on using

$$\left(\frac{1}{u-a}\frac{\partial}{\partial t}+\frac{\partial}{\partial r}\right)p=\frac{1}{u-a}\frac{dp}{dt}\,, \qquad (3.1.21)$$

$$\left(\frac{1}{u-a}\frac{\partial}{\partial t}+\frac{\partial}{\partial r}\right)u=\frac{1}{u-a}\frac{du}{dt}\,, \qquad (3.1.22)$$

$$\left(\frac{1}{u-a}\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)B=\frac{1}{u-a}\frac{dB}{dt} \qquad (3.1.23)$$

reduces to characteristic equation

$$dp+\frac{B}{\mu}dB-\rho a\,du+\left[\frac{\rho a^2 u}{u-a}-\frac{B^2}{\mu}\frac{(u+a)}{u-a}\right]\frac{dr}{r}=0\,, \qquad (3.1.24)$$

along the negative characteristic

$$\frac{dr}{dt}=u-a\,.$$

The conditions to be satisfied on the surface of strong discontinuity the shock wave front are, in the case of $R_m \ll 1$ (CaseI) and if the electrical conductivity increases in a jump-like manner (Case III, $\sigma : 0 \longrightarrow \infty$) at the shock wave front, the same as in the case of $B = 0$ (Tyl [2], Tyl and Wlodarczyk [9]). The shock conditions in the cases I and II are, therefore

$$\rho_1(U - u_1) = \rho_0 U \ , \tag{3.1.25}$$

$$p_1 = p_0 + \rho_0 U u_1 \ , \tag{3.1.26}$$

$$e_1 + \frac{1}{2}(U - u_1)^2 + \frac{p_1}{\rho_1} = e_0 + \frac{1}{2}U^2 + \frac{p_0}{\rho_0} \ , \tag{3.1.27}$$

where U is the shock velocity and the indices '1' and '0' refer to the states just behind and just ahead of the shock front.

With the help of equations (3.1.25) and (3.1.26), equation (3.1.27) can be transformed to

$$e_1 = e_0 + \frac{1}{2}(p_0 + p_1)\left(\frac{1}{\rho_0} - \frac{1}{\rho_1}\right) \tag{3.1.28}$$

If the shock is strong,

$$p_0 \cong e_0 \cong 0 \tag{3.1.29}$$

Then the above shock conditions reduce to

$$\rho_1(U - u_1) = \rho_0 U \ ,$$

$$p_1 = \rho_0 U u_1 \ ,$$

$$e_1 = \frac{1}{2}p_1\left(\frac{1}{\rho_0} - \frac{1}{\rho_1}\right) . \tag{3.1.30}$$

From equation (3.1.30), we get the strong shock condition in the following form

$$u_1 = \frac{U(\beta - 1)}{\beta} \ ,$$

$$\rho_1 = \rho_0 \beta \ ,$$

$$p_1 = \rho_0 U^2 \frac{(\beta - 1)}{\beta} \ ,$$

$$c_1 = \left[\frac{\gamma(\beta - 1)(\gamma - 1 + 2\alpha\beta)}{\gamma - 1 + \alpha\beta} \right]^{\frac{1}{2}} \frac{|U|}{\beta} \ , \tag{3.1.31}$$

where

$$\frac{1}{\beta} = \frac{(\gamma - 1 - \alpha) + \sqrt{(\gamma - 1 - \alpha)^2 + 4\alpha(\gamma + 1)}}{2(\gamma + 1)} \tag{3.1.32}$$

and

$$\alpha = (\gamma - 1) b \rho_0 \ .$$

In the pure magnetogasdynamic case (Case II), the gas is strongly ionized, i.e. highly conducting, before and behind the shock front, upon which the magnetic induction may be discontinuous at the shock front. The shock conditions, in this case, may be written in the form (Whitham [8], Tyl [2]):

$$B_1 (U - u_1) = B_0 U \ ,$$

$$\rho_1 (U - u_1) = \rho_0 U \ ,$$

$$p_1 + \frac{{B_1}^2}{2\mu} + \rho_1 (U - u_1)^2 = p_0 + \frac{{B_0}^2}{2\mu} + \rho_0 U^2 \ ,$$

$$e_1 + \frac{1}{2}(U - u_1)^2 + \frac{p_1}{\rho_1} + \frac{{B_1}^2}{\mu\rho_1} = e_0 + \frac{1}{2}U^2 + \frac{p_0}{\rho_0} + \frac{{B_0}^2}{\mu\rho_0} \ . \tag{3.1.33}$$

From shock conditions (3.1.33), we get

$$\rho_1 = \rho_0 \xi \ ,$$

$$B_1 = B_0 \xi \ ,$$

$$|u_1| = \left[\frac{2(\xi - 1)^2}{[(\gamma + 1)\xi - (\gamma - 1)\xi^2 + \alpha\xi^2(1 - \xi)]} \left(A + {b_0}^2 \left\{ \left(1 - \frac{\gamma}{2}\right) \xi + \frac{\gamma}{2} + \frac{\alpha\xi}{2}(1 - \xi) \right\} \right) \right]^{\frac{1}{2}} ,$$

$$p_1 = p_0 + \frac{2\rho_0(\xi - 1)}{[(\gamma+1) - (\gamma-1)\xi + \alpha\xi(1-\xi)]}\left(A + \frac{{b_0}^2}{4}(\xi-1)^2\{\gamma - 1 + \alpha\xi\}\right) ,$$

$$U = \frac{u_1\xi}{\xi - 1} , \tag{3.1.34}$$

$$b_0 = \left(\frac{{B_0}^2}{\mu\rho_0}\right)^{\frac{1}{2}} ,$$

$$c_0 = \left[\frac{\gamma p_0}{\rho_0}\left(\frac{\gamma - 1 + 2\alpha}{\gamma - 1 + \alpha}\right)\right]^{\frac{1}{2}}$$

where

$$A = \frac{{c_0}^2}{\gamma(\gamma - 1 + 2\alpha)}\left[\gamma(\gamma-1) + \xi\alpha^2 + \gamma\alpha(1+\xi)\right] .$$

3.1.3 SOLUTION OF THE PROBLEM

The shock speed U and the flow variables just behind the shock are obtained by using the Whitham's rule in all the three cases. For converging shocks, the rule is to apply the characteristic equation (valid along the negative characteristic) to the flow quantities just behind the shock front.

CASE I Shock Wave in a Weakly Conducting Non-Ideal Gas, $R_m \ll 1$

Using the values of the flow variables just behind the shock, given by equations (3.1.31), into the charactersitic equations (3.1.14) bearing in mind that U and u are negative, we obtain

$$k_1\frac{d|U|}{dR} + k_2\frac{|U|}{R} = k_3\frac{\sigma {B_0}^2}{2\rho_0} , \tag{3.1.35}$$

where

$$k_1 = 1 + \frac{1}{2}k_4(\beta - 1) ,$$

$$k_2 = \frac{\gamma(\gamma - 1 + 2\alpha\beta)}{2(\gamma - 1 + \alpha\beta)(1 + k_4)} ,$$

$$k_3 = 1 - \frac{\gamma + \alpha\beta}{1 + k_4} ,$$

$$k_4 = \left[\frac{\gamma(\gamma - 1 + 2\alpha\beta)}{(\gamma - 1 + \alpha\beta)(\beta - 1)}\right]^{\frac{1}{2}} ,$$

and R is the shock radius.

Integrating equation (3.1.35), with the initial conditions $U = U_i$ at $R = R_i$, we find the solution as

$$\frac{U}{U_i} = (1 - k_5)\lambda^{-k_6} + k_5\lambda^{-1} , \qquad (3.1.36)$$

where

$$\lambda = \frac{R}{R_i} ,$$

$$k_5 = \frac{k_3 {B_{0i}}^2 R_m \beta}{(k_2 - k_1) 2\rho_0 \mu(\beta - 1) U_i^2} ,$$

$$k_6 = \frac{k_2}{k_1} ,$$

$$R_m = |u_{1i}| R_i \sigma \mu ,$$

and B_{0i} and u_{1i} are the values of B_0 and u_1 at $R = R_i$. Also, from the shock conditions (3.1.31),

$$\frac{u_1}{u_{1i}} = \frac{U}{U_i} ,$$

$$\frac{p_1}{p_{1i}} = \left(\frac{U}{U_i}\right)^2 , \qquad (3.1.37)$$

where p_{1i} is the value of p_1 at $R = R_i$.

CASE II Pure Magnetogasdynamic Shock Wave , $R_m \gg 1$

Using the values of flow variables just behind the shock, given by equations

(3.1.34), into the characteristic equation (3.1.24), we obtain after some simplifications,

$$\lambda^3\left[\left(\frac{2L}{\xi-1}+N\right)L_\xi-\frac{L^2}{\xi(\xi-1)^2}\right]\frac{d\xi}{d\lambda}=-\frac{\lambda^2LN^2}{L+N}+\frac{\xi{B_{0i}}^2}{\mu}\left(\frac{2L}{L+N}\right)$$
$$+\frac{(\xi-1)^2{B_{0i}}^2}{[(\gamma+1)\xi-(\gamma-1)\xi^2+\alpha\xi^2(1-\xi)]\mu}\times$$
$$\left[(\xi-1)(\gamma-1+\alpha\xi)+\frac{2N}{L}\left\{\left(1-\frac{\gamma}{2}\right)\xi+\frac{\gamma}{2}+\frac{\alpha\xi}{2}(1-\xi)\right\}\right]\ , \tag{3.1.38}$$

where

$$L(\xi,\lambda)=\left(\frac{2(\xi-1)^2}{(\gamma+1)\xi-(\gamma-1)\xi^2+\alpha\xi^2(1-\xi)}\left[A\rho_0+\frac{{B_{0i}}^2}{\lambda^2\mu}\left\{(1-\frac{\gamma}{2})\xi+\frac{\gamma}{2}+\frac{\alpha\xi}{2}(1-\xi)\right\}\right]\right)^{\frac{1}{2}}$$

$$N(\xi,\lambda)=\left[\frac{(\gamma-1+2\alpha\xi)}{(\gamma-1+\alpha\xi)}\frac{\gamma p_1}{\xi}+\frac{{B_{0i}}^2\xi}{\lambda^2\mu}\right]^{\frac{1}{2}}\ ,$$

$$p_1(\xi,\lambda)=p_0+\frac{2(\xi-1)}{[(\gamma+1)-(\gamma-1)\xi+\alpha\xi(1-\xi)]}\left[A\rho_0+\frac{{B_{0i}}^2}{4\mu\lambda^2}(\xi-1)^2(\gamma-1+\alpha\xi)\right]\ ,$$

$$L_\xi(\xi,\lambda)=\frac{(\xi-1)}{L[(\gamma+1)\xi-(\gamma-1)\xi^2+\alpha\xi^2(1-\xi)]}\left[\frac{(3-\gamma)\xi+(\gamma+1)+\alpha\xi(\xi^2-3\xi+2)}{(\gamma+1)\xi-(\gamma-1)\xi^2+\alpha\xi^2(1-\xi)}\right]\times$$
$$\left[\left(A\rho_0+\frac{{B_{0i}}^2}{\lambda^2\mu}\left\{(1-\frac{\gamma}{2})\xi+\frac{\gamma}{2}+\frac{\alpha\xi}{2}(1-\xi)\right\}\right)+\left\{\frac{p_0\alpha(\alpha+\gamma)}{\gamma-1+\alpha}+\frac{{B_{0i}}^2}{\lambda^2\mu}\left(1-\frac{\gamma}{2}+\frac{\alpha}{2}-\alpha\xi\right)\right\}(\xi-1)\right]$$

Also, from equations (3.1.34) and (3.1.38), we have

$$\frac{\rho_1}{\rho_{1i}}=\frac{\xi}{\xi_i}\ ,$$
$$\frac{B_1}{B_{1i}}=\frac{\xi}{\xi_i\lambda}\ ,$$
$$\frac{u_1}{u_{1i}}=\frac{L(\xi,\lambda)}{L(\xi_i,1)}\ , \tag{3.1.39}$$

$$\frac{U}{U_i} = \frac{\xi(\xi_i - 1)}{\xi_i(\xi - 1)} \frac{L(\xi, \lambda)}{L(\xi_i, 1)},$$

$$\frac{p_1}{p_{1i}} =$$

CASE III Gas ionizing Shock Wave ($\sigma : 0 \longrightarrow \infty$)

Since σ is zero ahead of the shock, the magnetic induction may be taken continuous across the shock in this case (Sakurai [7], Ranga Rao and Ramana [10]). Thus in this case, there is no jump of magnetic induction across the shock, therefore

$$B_0 = B_1 = \frac{k}{R}$$

which gives

$$\frac{dB_1}{B_1} = -\frac{dR}{R}. \tag{3.1.40}$$

Using the flow variables just behind the shock, given by equations (3.1.31) and (3.1.40) for the flow variables in the characteristic equation (3.1.24), we obtain after some simplifications,

$$\frac{d|U|}{d\lambda} = \frac{\beta |U|}{\lambda(2 + q\beta)(\beta - 1 + q\beta)} \left[\frac{2B^2}{\mu \rho_0 |U|^2} - \beta q^2\right], \tag{3.1.41}$$

where

$$q = \left\{\frac{\gamma(\gamma - 1 + 2\alpha\beta)(\beta - 1)}{\beta^2(\gamma - 1 + \alpha\beta)} \left[1 + \frac{1}{\gamma} \frac{(\gamma - 1 + \alpha\beta)}{(\gamma - 1 + 2\alpha\beta)} \frac{{B_1}^2 \beta}{\mu \rho_0 |U|^2 (\beta - 1)}\right]\right\}^{\frac{1}{2}}$$

and B_1 in terms of λ is given by

$$B_1 = B_{0i} \lambda^{-1}. \tag{3.1.42}$$

Further, from equations (3.1.31) and (3.1.42), we have

$$\frac{u_1}{u_{1i}} = \frac{U}{U_i},$$

$$
\begin{aligned}
\frac{p_1}{p_{1i}} &= \left(\frac{U}{U_i}\right)^2 , \\
\frac{B_1}{B_{1i}} &= \lambda^{-1} ,
\end{aligned}
\tag{3.1.43}
$$

where

$$U_i = \frac{u_{1i}\beta}{\beta - 1}$$

and

$$B_{1i} = B_{0i} .$$

By numerical integration of the differential equations (3.1.41), with initial conditions $U = U_i$ at $\lambda = 1$, we can calculate the values of $\frac{U}{U_i}$, $\frac{u_1}{u_{1i}}$, $\frac{p_1}{p_{1i}}$ in terms of λ.

3.1.4 RESULTS AND DISCUSSION

For the purpose of numerical calculations, the values of constant parameters are taken to be (Tyl [2])

$R_m = 0.001$ (*in the case I, only*) ;

$\alpha = 0 \ , \ 0.05 \ , \ 0.1;$

$\gamma = 1.4 \ ;$

$B_{0i} = 0.5 \ , \ 1.0 \ , \ 2.0\, Tesla \ ;$

$\mu = 4\pi \times 10^{-7} Henary/metre \ ;$

$\rho_0 = 1 kg/m^3 \ ;$

$|u_{1i}| = 4 \times 10^3 m/s$; *and*

$p_0 = 10^5 pascal$ (*in the case II, only*)

The value $\alpha = 0$ corresponds to the case of a perfect gas. In the case I, the values of $\frac{U}{U_i}$ are calculated from the equation (3.1.36) in terms of λ. It is found that an increase in the value of the parameter α characterising the non-idealness of the gas, accelerates the convergence of the shock (Fig.1). The effect of a change in the strength of the initial magnetic induction B_{0i} is negligible (not shown in the fig.). This is due to the fact that $R_m \ll 1$, in this case. In case II, ξ the density ratio across the shock varies as the shock propagates, which is in contrast with the cases using strong assumptions (Cases I and III), where it is equal to β, a constant given in the equation (3.1.32). Values of ξ are obtained by numerical integration of the differential equation (3.1.38) and then $\frac{p_1}{p_{1i}}$ and $\frac{U}{U_i}$ are calculated from equation (3.1.39). Variations of ξ with λ for different values of α and B_{0i} are shown in table I. Variations of $\frac{p_1}{p_{1i}}$ and $\frac{U}{U_i}$ versus λ for $\alpha = 0$,0.05 , 0.1 and for $B_{0i} = 0.5$, 1.0 , 2.0 are shown in figures (2a,b) and (3). It is found that the shock velocity increases very slowly in the begining, but near the axis it increases very rapidly. The non-idealness of the gas has small effect on the shock velocity, but an increase in the strength of the initial magnetic field (magnetic induction) accelerates the convergence of the shock [figures (2a, b)]. The pressure behind the shock is sufficiently affected by the non-idealness of the gas when the initial magnetic field is weak, but at strong initial magnetic field, this effect becomes insignificant (figure 3). Also, table-I shows that the density ratio across the shock, and therefore the shock strength decreases by an increase in the non-idealness of the gas and also by an increase in the strength of the initial magnetic field.

In the Case III, the values of $\frac{U}{U_i}$ and $\frac{p_1}{p_{1i}}$ are obtained in terms of λ, by numerical integration of the equation (3.1.41) and by use of equation (3.1.43). It is found that the shock speed and the pressure behind the shock first increase to a maximum and then decrease very fast as the shock converges towards the axis. The increase and decrease are faster when the initial magnetic field is weak or when the parameter of non-idealness of the gas is comparatively large. An increase in the strength of the initial magnetic field reduces the effect of the non-idealness of the gas (figures 4 and 5).

TABLE - I : Variation of ξ, the density ratio across the shock with λ at different values of B_{0i} and α, in the Case II, ($R_m >> 1$).

	ξ								
	$B_{0i} = 0.5$			$B_{0i} = 1.0$			$B_{0i} = 2.0$		
λ	$\alpha = 0$	$\alpha = 0.05$	$\alpha = 1.0$	$\alpha = 0$	$\alpha = 0.05$	$\alpha = 1.0$	$\alpha = 0$	$\alpha = 0.05$	$\alpha = 1.0$
1.0	5.34087	4.06416	3.51145	4.49820	3.77200	3.35780	3.28100	3.07880	2.91022
0.9	5.27133	4.04106	3.50066	4.35564	3.70400	3.32051	3.13176	2.96329	2.82538
0.8	5.18222	4.00992	3.48588	4.18838	3.61825	3.27195	2.96567	2.83208	2.72483
0.7	5.06604	3.96673	3.46499	3.99108	3.50874	3.20736	2.78309	2.68317	2.60547
0.6	4.91151	3.90474	3.43426	3.75716	3.36712	3.11942	2.58393	2.51457	2.46389
0.5	4.70120	3.81202	3.38671	3.47850	3.18194	2.99661	2.36787	2.32438	2.29659
0.4	4.40751	3.66651	3.30814	3.14525	2.93796	2.82092	2.13443	2.11101	2.1005
0.3	3.98579	3.42526	3.16664	2.74574	2.62626	2.56519	1.88297	1.87323	1.87348
0.2	3.36322	3.00360	2.88284	2.26664	2.19658	2.19400	1.61250	1.61024	1.61479
0.1	2.42499	2.25199	2.25860	1.69333	1.66283	1.67618	1.32072	1.32096	1.32470

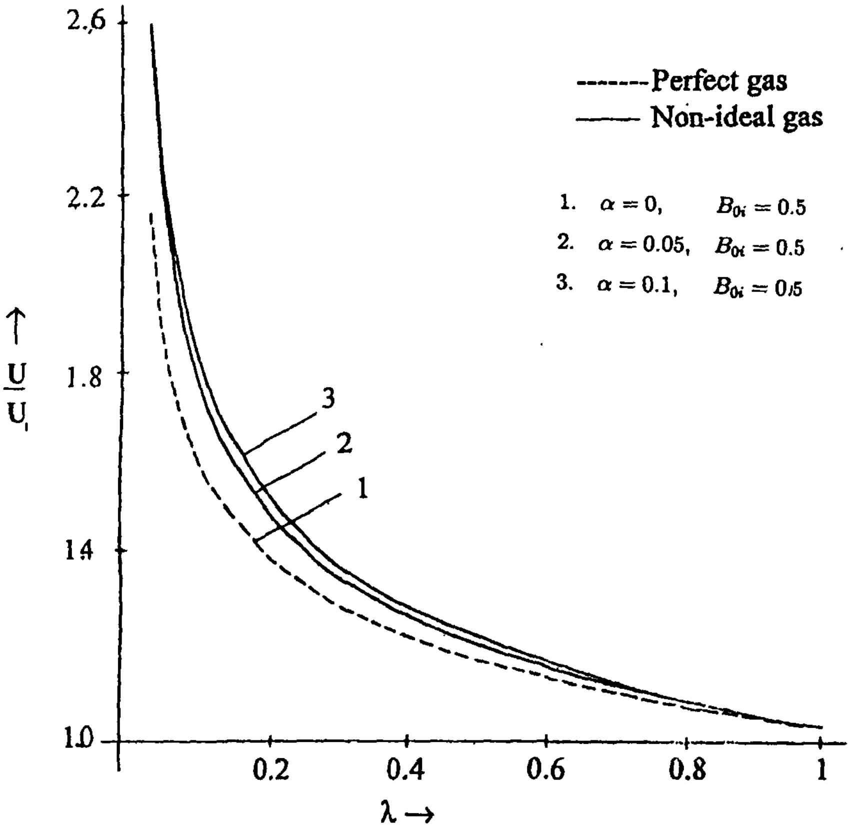

Figure . 1. Variation of shock velocity with distance in the case I ($R_m \ll 1$)

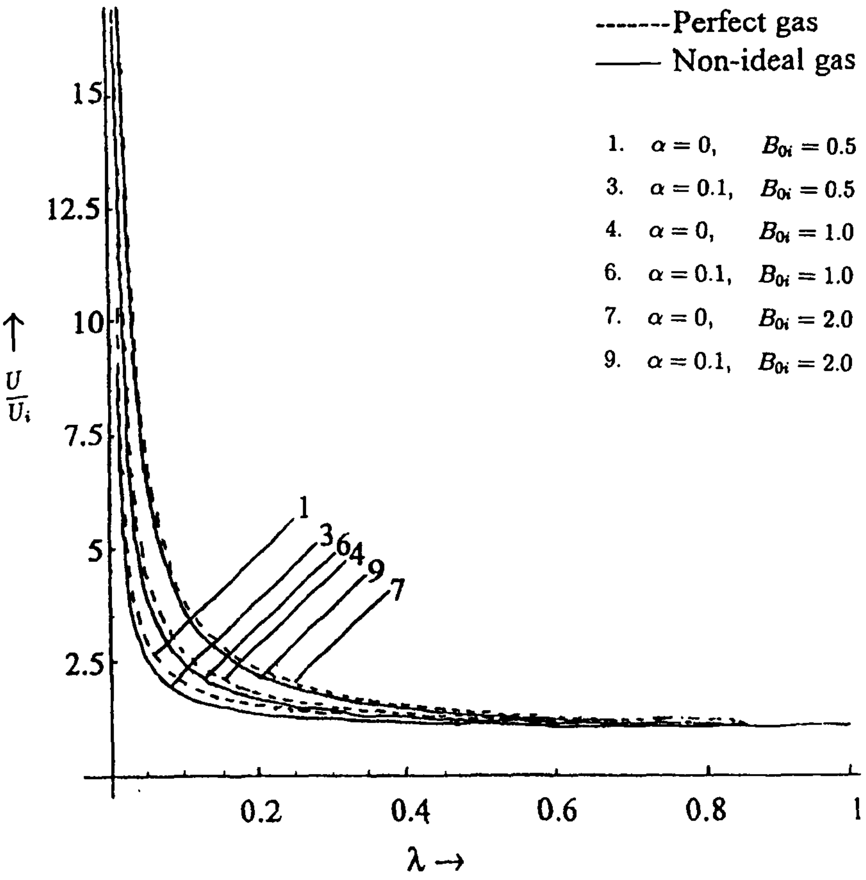

Figure 2(a). Variation of shock velocity with distance in the case II ($R_m >> 1$)

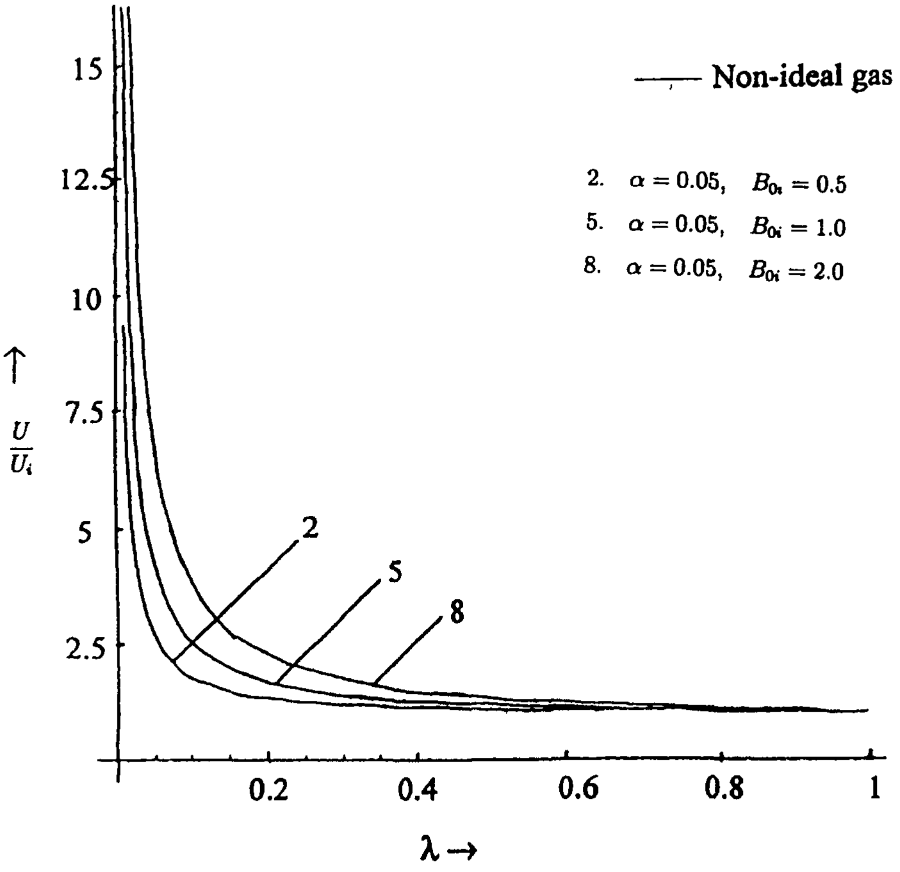

Figure 2(b). Variation of shock velocity with distance in the case II ($R_m >> 1$)

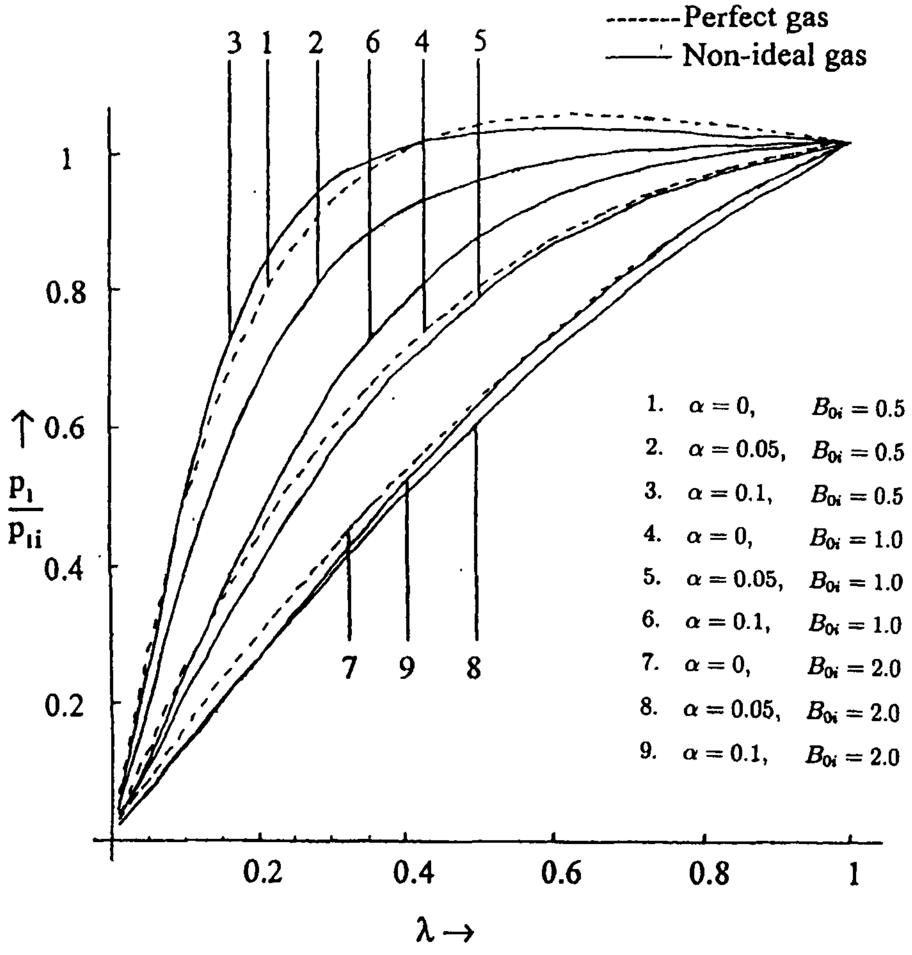

Figure 3. Variation of pressure with distance in the case II ($R_m >> 1$)

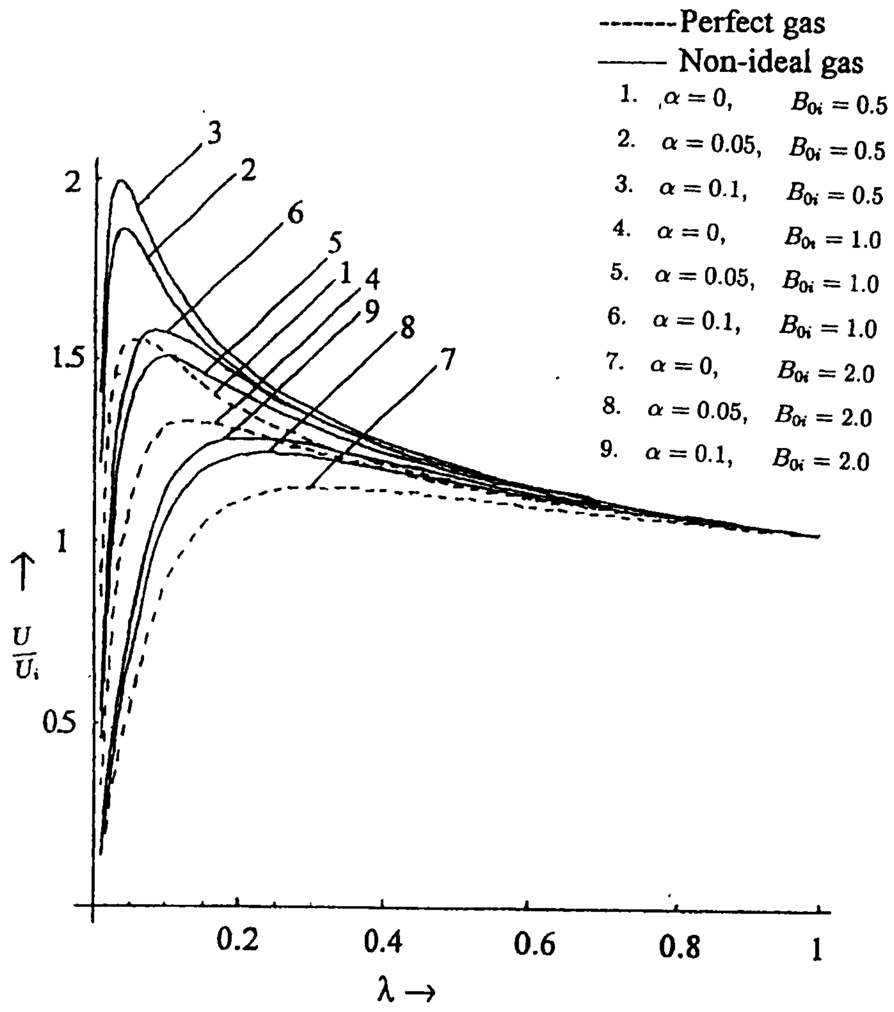

Figure 4: Variation of shock velocity with distance in the case III (σ : 0 → ∞ at the shock front)

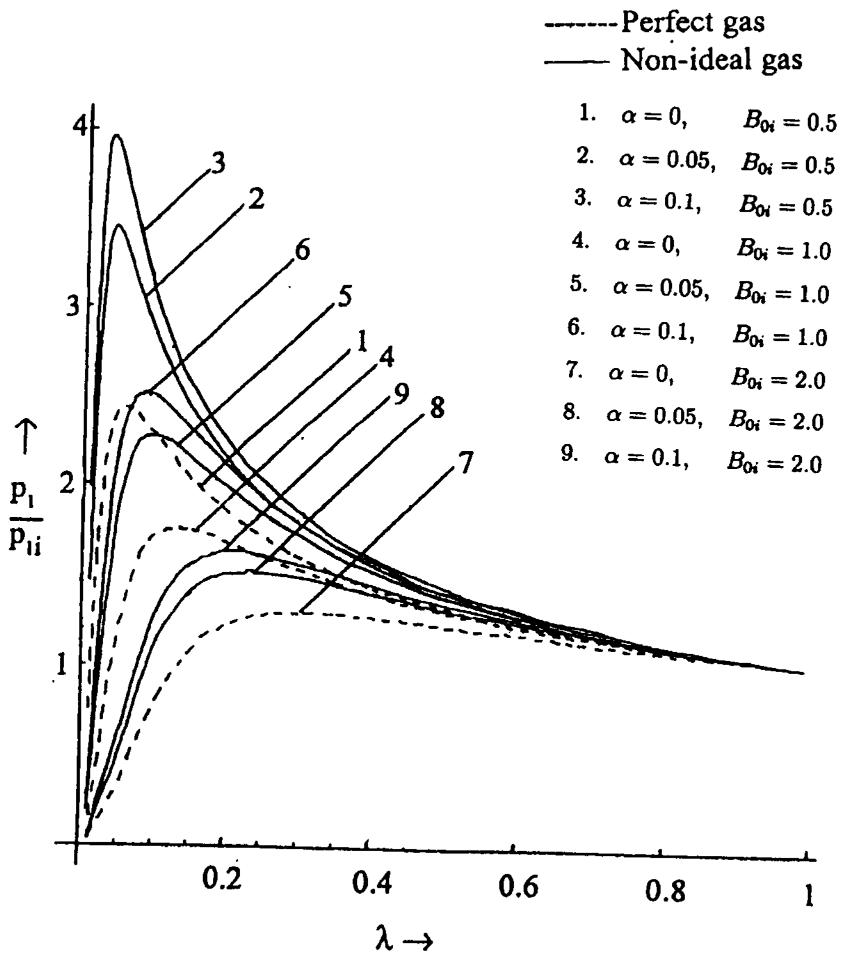

Figure 5. Variation of pressure with distance in the case III ($\sigma : 0 \rightarrow \infty$ at the shock front)

3.2 SHOCK WAVES GENERATED BY A PISTON MOVING IN A NON-IDEAL GAS IN THE PRESENCE OF AZIMUTHAL MAGNETIC FIELD

This section is devoted to study similarity solutions for the flow of a non-ideal gas behind a cylindrical (or spherical) shock wave, in the presence of azimuthal magnetic field, driven out by a piston moving with time according to power law. We follow Nath [11] here.

3.2.1 INTRODUCTION

Self-similar solutions for the propagation of a piston-driven shocks into an ideal or dusty gas have been obtained by Helliwell [12], Rosenau and Frankanthal [13], Steiner and Hirschler [14] and many others. In the present study, we investigate self-similar solutions for the propagation of a magnetogasdynamic shock wave in a non-ideal gas under isothermal condition, driven out by a cylindrical or spherical piston moving with time according to power law. The assumption of isothermal flow is physically realistic, when radiation heat transfer effects are implicitly present. The ambient azimuthal magnetic field is assumed to vary as some power of the distance from the axis or point of symmetry. In order to obtain similarity solutions of the problem it is necessary to take the density of the ambient non-ideal gas to be a constant. The gas ahead of the shock is assumed to be at rest.

3.2.2 FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

The fundamental equations for one-dimensional unsteady and isothermal flow of a perfectly electrically conducting and non-ideal gas in the presence of an azimuthal magnetic field may be expressed as (Sachdev and Ashraf [15], Zhuravskaya and Levin [16])

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\nu \rho u}{r} = 0 , \tag{3.2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left(\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right) = 0 , \tag{3.2.2}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + (\nu - 1)\frac{hu}{r} = 0 , \tag{3.2.3}$$

$$\frac{\partial T}{\partial r} = 0 , \tag{3.2.4}$$

where r and t are independent space and time co-ordinates, ρ is the density, p the pressure , u the flow velocity, h the azimuthal magnetic field, T the absolute temperature, μ the magnetic permeability, and $\nu = 1$ or 2 for line or point symmetry.

The above system of equations should be supplemented with an equation of state. We assume that the gas obeys a simplified Van der Waals equation of state of the form (Wu and Roberts [17], Roberts and Wu [18])

$$p = \frac{\Gamma T}{v - b} , \quad e = C_v T = \frac{(v - b)p}{\gamma - 1} , \tag{3.2.5}$$

where Γ is the gas constant, $v = \frac{1}{\rho}$ is the specific volume, $C_v = \frac{\Gamma}{\gamma - 1}$ is the specific heat at constant volume and γ is the ratio of specific heats. The constant b is the Van der Waals excluded volume, it places a limit, $\rho_{max} = \frac{1}{b}$, on the density of the gas. The above equation of state which is similar to equation of state suggested by Anisimav and Spiner [4] has been used by Wu and Roberts [17] and Roberts and Wu [18] to study the shock wave theory of sonoluminescence.

We assume that a shock (cylindrical or spherical) is propagating in the undisturbed non-ideal and perfectly conducting gas with constant density in the presence of azimuthal magnetic field which vary as $h = \frac{A}{r^m}$ (Rosenau [19]), where A and m are constants. The flow variables immediately ahead of the shock front are

$$u = u_1 = 0\,,$$

$$\rho = \rho_1 = \ constant\,,$$

$$h = h_1 = \frac{A}{r_s^m}\,,$$

$$p = p_1 = \frac{(1-m)}{2m}\frac{\mu A^2}{{r_s}^{2m}} \quad (0 < m < 1)\,, \tag{3.2.6}$$

where r_s is the shock radius, and the subscript '1' denote the conditions immediately ahead of the shock. The momentum equation (3.2.2) in the undisturbed state of the gas, gives the last equation of the equations (3.2.6). The jump conditions across the magnetogasdynamic shock are given by the principles of conservation of mass, momentum field and energy, across the shock, namely

$$\rho_2(U - u_2) = \rho_1 U\,,$$

$$h_2(U - u_2) = h_1 U\,,$$

$$p_2 + \frac{1}{2}\mu h_2^2 + \rho_2 (U - u_2)^2 = p_1 + \frac{1}{2}\mu h_1^2 + \rho_1 U^2\,,$$

$$e_2+\frac{p_2}{\rho_2}+\frac{1}{2}(U-u_2)^2+\frac{\mu h_2^2}{\rho_2}-\frac{F_2}{\rho_1 U}=e_1+\frac{p_1}{\rho_1}+\frac{1}{2}U^2+\frac{\mu h_1^2}{\rho_1}-\frac{F_1}{\rho_1 U}\,, \qquad (3.2.7)$$

where the subscript '2' denotes conditions immediately behind the shock front, and $U=\frac{dr_s}{dt}$ denotes the velocity of the shock front.

From equations (3.2.7), we get

$$u_2=(1-\beta)U\,,$$

$$\rho_2=\frac{\rho_1}{\beta}\,,$$

$$h_2=\frac{h_1}{\beta}\,,$$

$$p_2=\left[\frac{1}{\gamma M^2}+\left(1-\frac{1}{\beta^2}\right)\frac{M_A^{-2}}{2}+(1-\beta)\right]\rho_1 U^2\,, \qquad (3.2.8)$$

where $M=\left(\frac{\rho_1 U^2}{\gamma p_1}\right)^{\frac{1}{2}}$ is the shock - Mach number referred to frozen speed of sound $\sqrt{\frac{\gamma p_1}{\rho_1}}$ and $M_A=\left(\frac{\rho_1 U^2}{\mu h_1^2}\right)^{\frac{1}{2}}$ is the Alfven-Mach number. The quantity $\beta(0<\beta<1)$ is obtained by the relation

$$\beta^3-\beta^2\left(\frac{2}{(\gamma+1)M^2}+\frac{2\bar{\alpha}+\gamma(M_A^{-2}+1)-1}{(\gamma+1)}\right)+\beta\left(\frac{\bar{\alpha}-2+\gamma}{(\gamma+1)}\right)M_A^{-2}+\frac{\bar{\alpha}M_A^{-2}}{(\gamma+1)}=0\,, \qquad (3.2.9)$$

where F_2-F_1 is neglected in comparison with $\rho_1 U^3$, i.e. with $\rho_2 U$ (Laumbach and Probtein [20]), and $\bar{\alpha}=\rho_1 b$.

Equation (3.2.4) together with equation (3.2.5) give

$$\frac{p}{p_2}=\frac{\rho(1-b\rho_2)}{\rho_2(1-b\rho)}. \qquad (3.2.10)$$

3.2.3 SELF-SIMILARITY TRANSFORMATION

The inner boundary of the flow field behind the shock is assumed to be an expanding surface (piston). In the fromework of self-similarity (Sedov [21]) the velocity $u_p = \frac{dr_p}{dt}$ of the piston is assumed to follow a power law which reads (Steiner and Hirschler [14]).

$$u_p = \frac{dr_p}{dt} = U_0 \left(\frac{t}{t_0} \right)^n , \tag{3.2.11}$$

where r_p is the radius of the piston, t_0 denotes a reference time, U_0 is the piston velocity at $t = t_0$. The consideration of ambient pressure p_1 and ambient magnetic field h_1 imposes a restriction on "n" as $-\frac{1}{2} < n < 0$ (see equation (3.2.24)). Thus the piston velocity jumps at $t = 0$ from zero to infinite velocity leading to the formation of a shock of high strength in the initial phase. Concerning the shock boundary conditions self-similarity requires that the velocity of the shock $U = \frac{dr_s}{dt}$ in proportional to the velocity of the piston:

$$U = \frac{dr_s}{dt} = CU_0 \left(\frac{t}{t_0} \right)^n . \tag{3.2.12}$$

where r_s is the radius of the shock and C a constant. Using equation(3.2.12) the time and space co-ordinate can be transformed into a dimensionless self-similarity variable as follows :

$$\eta = \frac{r}{r_s} = \left[\frac{(n+1){t_0}^n}{U_0 C} \right] \left(\frac{r}{t^{(n+1)}} \right) . \tag{3.2.13}$$

Clearly, $\eta = \eta_p = \frac{r_p}{r_s}$ at the piston and $\eta = 1$ at the shock. To obtain the similarity solutions, we write the unknown variables in the following form

(Steiner and Hirschler [14], Ranga Rao and Purohit [22]):

$$u = \frac{r}{t}V(\eta)\ , \quad \rho = \rho_1 D(\eta)\ , \quad p = \frac{r^2}{t^2}\rho_1 P(\eta)\ , \quad \sqrt{\mu}h = \sqrt{\rho_1}\frac{r}{t}H(\eta)\ , \tag{3.2.14}$$

where V, D, P, and H are functions of the similarity variable η only.

Equation (3.2.10) with the aid of equations (3.2.14) and (3.2.8) yields a relation between P and D in the form

$$P(\eta) = \frac{L_1 D}{\eta^2(1-\bar{\alpha}D)}\ , \tag{3.2.15}$$

where,

$$L_1 = \left[\frac{1}{\gamma M^2} + \left(1 - \frac{1}{\beta^2}\right)\frac{M_A^{-2}}{2} + (1-\beta)\right](n+1)^2(\beta - \bar{\alpha})\ .$$

By use of equations (3.2.14) and (3.2.15), equations (3.2.1) , (3.2.2) and (3.2.3) can be transformed and simplified to

$$(V-n-1)\frac{dD}{d\eta} + D\frac{dV}{d\eta} + (\nu+1)\frac{DV}{\eta} = 0\ , \tag{3.2.16}$$

$$(V-n-1)\frac{dV}{d\eta} + \frac{L_1}{D\eta^2(1-\bar{\alpha}D)^2}\frac{dD}{d\eta} + \frac{H}{D}\frac{dH}{d\eta} + \frac{V(V-1)}{\eta} + \frac{2H^2}{D\eta} = 0 \tag{3.2.17}$$

$$(V-n-1)\frac{dH}{d\eta} + H\frac{dV}{d\eta} + \frac{H(V-1)}{\eta} + \frac{\nu HV}{\eta} = 0\ . \tag{3.2.18}$$

Solving equations (3.2.16) to (3.2.18) for $\frac{dD}{d\eta}$, $\frac{dH}{d\eta}$ and $\frac{dV}{d\eta}$, we get

$$\frac{dD}{d\eta} = \frac{-D}{(V-n-1)}\left(\frac{dV}{d\eta} + (\nu+1)\frac{V}{\eta}\right)\ , \tag{3.2.19}$$

$$\frac{dH}{d\eta} = \frac{-H}{(V-n-1)}\left(\frac{dV}{d\eta} + \frac{(\nu+1)V-1}{\eta}\right)\ , \tag{3.2.20}$$

$$\frac{dV}{d\eta} = \frac{1}{[(V-n-1)^2\eta^2 D(1-\bar{a}D)^2 - L_1 D - (1-\bar{a}D)^2\eta^2 H^2]\eta} \times$$
$$[-2(V-n-1)H^2\eta^2(1-\bar{a}D)^2 - V(V-1)(V-n-1)\eta^2 D(1-\bar{a}D)^2 +$$
$$(\nu+1)VL_1D + (\nu V + V - 1)\eta^2 H^2 (1-\bar{a}D)^2] \,. \qquad (3.2.21)$$

Using (3.2.14), the shock conditions (3.2.8) transform into

$$V(1) = (1-\beta)(n+1)\,, \quad D(1) = \frac{1}{\beta}\,, \quad H(1) = \frac{n+1}{\beta M_A}\,,$$
$$P(1) = \left\{\frac{1}{\gamma M^2} + (1-\beta) + \left(1 - \frac{1}{\beta^2}\right)\frac{{M_A}^{-2}}{2}\right\}(n+1)^2 \,. \qquad (3.2.22)$$

Because of the dependence of the boundary conditions (3.2.22) and the equations (3.2.19) to (3.2.21) on $\bar{a}$, M and M_A , similarity solutions exist only when these are constants. Thus a self similar solution of the piston problem in a non-ideal gas exist only when the initial density ρ_1 is constant, where as in the case of an ideal gas similar solutions exist even when the initial density is variable (see, for example, Helliwell [12], Rosenau and Frankenthal [13])

Since for the existance of similarity solutions M and M_A should be constants, therefore

$$m = -\left(\frac{n}{n+1}\right) \,. \qquad (3.2.23)$$

Thus,

$$M^2 = \frac{-2\rho_1(n+1)^{\frac{2n}{(n+1)}} n}{\gamma(2n+1)\mu A^2}\left(\frac{U_0 C}{t_0^n}\right)^{\frac{2}{(n+1)}} = \frac{-2n}{\gamma(2n+1)} M_A^{-2}\,, \qquad (3.2.24)$$

where $-\frac{1}{2} < n < 0$.

Also, the total energy of the disturbance is given by

$$E = 2\pi\nu \int_{r_p}^{r_s} \rho\left(e + \frac{\mu h^2}{2\rho} + \frac{1}{2}u^2\right) r^\nu dr \,. \qquad (3.2.25)$$

Using equations (3.2.5) and (3.2.14), equation (3.2.25) becomes

$$E = \frac{2\pi\nu\rho_1(U_0C)^{\frac{2}{(n+1)}}}{(n+1)^{\frac{2}{(n+1)}}t_0^{\frac{2n}{(n+1)}}} r_s^{\nu+\frac{3n+1}{n+1}} \int_{\eta_p}^{1} \left(\frac{(1-\bar{\alpha}D)P}{(\gamma-1)} + \frac{1}{2}DV^2 + \frac{1}{2}H^2 \right) \eta^{\nu+3} d\eta \, . \tag{3.2.26}$$

Hence, total energy of the shock wave is non-constant and varies as $r_s^{\nu+\frac{3n+1}{n+1}}$, where $\nu = 1$ or 2 for cylindrical or spherical shock.

The piston path coincides at $\eta_p = \frac{r_p}{r_s}$ with a particle path. Using equations (3.2.11) and (3.2.14) the relation

$$V(\eta_p) = n + 1 \, , \tag{3.2.27}$$

can be derived

In addition to the shock conditions (3.2.22), the kinematic condition (3.2.28) at the piston surface must be satisfied.

Now equations (3.2.19), (3.2.20) and (3.2.21) can be numerically integrated with boundary conditions (3.2.22) to obtain the solution of the problem.

Using the equations (3.2.22) the flow variables u, p, ρ and h can be related to thier corresponding values immediately behind the shock, as follows

$$\frac{u}{u_2} = \frac{V(\eta)}{V(1)}\eta \, , \quad \frac{p}{p_2} = \frac{P(\eta)}{P(1)}\eta^2 \, , \quad \frac{\rho}{\rho_2} = \frac{D(\eta)}{D(1)} \, , \quad \frac{h}{h_2} = \frac{H(\eta)}{H(1)}\eta \, . \tag{3.2.28}$$

3.2.4 Results and Discussion

The solutions of the equations (3.2.15), (3.2.19), (3.2.20) and (3.2.21) with boundary conditions (3.2.22) depends on five non-dimensional parameters - the wave geometry index ν, the adiabatic index γ, the piston velocity index

n, the Alfven's Mach number M_A and the parameter of non-idealness of the gas $\bar{\alpha}(= b\rho_1)$. For the purpose of numerical integrations, the following values of constant parameters are taken :

$$\nu = 2\ ; \ \ \gamma = 1.4\ ; \ \ n = -\frac{1}{4}\ ;$$

$$M_A^{-2} = 0\ , \ \ 0.005\ , \ \ 0.01 \quad and$$

$$\bar{\alpha} = 0\ , \ \ 0.05\ , \ \ 0.1\ .$$

The value $\nu = 2$ corresponds to spherical shocks, the value $\bar{\alpha} = 0$ to the perfect gas cese and the value $M_A^{-2} = 0$ to the non-magnetic case.

Figures 1 to 4 show the variation of the flow variables with η for various values of the parameters M_A^{-2} and $\bar{\alpha}$. It is clear from these figures that, as we move inward from the shock front towards the piston, the reduced fluid velocity $\frac{u}{u_2}$ increases and the reduced density $\frac{\rho}{\rho_2}$, the reduced magnetic field $\frac{h}{h_2}$ decreases where as the reduced pressure $\frac{p}{p_2}$ increases when $M_A^{-2} \neq 0$ decreases when $M_A^{-2} = 0$ (non - magnetic case).

The piston position η_p at which $V(\eta_p) = n + 1$, is obtained after integration of equations (3.2.19), (3.2.20) and (3.2.21) with the help of boundary conditions (3.2.22) and is tabulated in Table - I for dirrerent values of M_A^{-2} ans $\bar{\alpha}$. Values of the density ratio across the shock front $\beta = \frac{\rho_1}{\rho_2}$ are also shown in table - I for various values of M_A^{-2} and $\bar{\alpha}$. η_p is related with the velocity ratio of the shock and the piston, from equations (3.2.11), (3.2.12) and (3.2.13), as follows

$$\eta_p = \frac{1}{C} = \left(\frac{U}{u_p}\right)^{-1} . \tag{3.2.29}$$

The effects of an increase in the value of ${M_A}^{-2}$ (i.e. the effects of an increase in the strength of ambient magnetic field) are

(i) to increase the density ratio $\beta \left(= \frac{\rho_1}{\rho_2} \right)$ across the shock (see table - I), i.e. to decrease the shock strength,

(ii) to decrease η_p i.e. to increase the distance of the piston from the shock front (see table - I). Physically it means that the gas behind the shock becomes rarefied i.e. less compressed. This shows the same result as in (i), i.e. there is a decrease in the shock strength,

(iii) to decrease the flow velocity $\frac{u}{u_2}$, magnetic field $\frac{h}{h_2}$ and density $\frac{\rho}{\rho_2}$ in general. An increase in ${M_A}^{-2}$ causes a decrease in η_p (see table - I) which means that there is a decrease in piston velocity u_p. This decrease in piston velocity leads to a decrease in flow velocity $\frac{u}{u_2}$. Also in this case there is widening of disturbed region between the shock and the piston, causing less compression of magnetic flux and gas which leads to a decrease in magnetic field $\frac{h}{h_2}$ and density $\frac{\rho}{\rho_2}$.

The effects of an increase in the value of the parameter of non-idealness of the gas $\bar{a}$ are

(i) to increase the velocity $\frac{u}{u_2}$, density $\frac{\rho}{\rho_2}$ and magnetic field $\frac{h}{h_2}$ significantly,

(ii) to increase the pressure $\frac{p}{p_2}$ for $M_A^{-2} = 0$ (non-magnetic case) and to decrease it when $M_A^{-2} \neq 0$,

(iii) to increase the distance of the piston from the shock front (see table - I), and

(iv) to increase the density ratio $\beta \left(= \frac{\rho_1}{\rho_2} \right)$ across the shock (see table - I), i.e. to decrease the shock strength.

TABLE - I : Density ratio $\beta \left(= \frac{\rho_1}{\rho_2} \right)$ across the shock front and position of the piston η_p for different values of $\bar{\alpha}$ and M_A^{-2} with $\gamma = 1.4$, $n = -\frac{1}{4}$.

M_A^{-2}	$\bar{\alpha}$	β	η_p
0	0	0.166666	0.9397
	0.05	0.208333	0.9245
	0.1	0.250000	0.9087
0.005	0	0.182274	0.9226
	0.05	0.218590	0.9111
	0.1	0.265627	0.8947
0.01	0	0.196865	0.9107
	0.05	0.239991	0.8969
	0.1	0.278936	0.8840

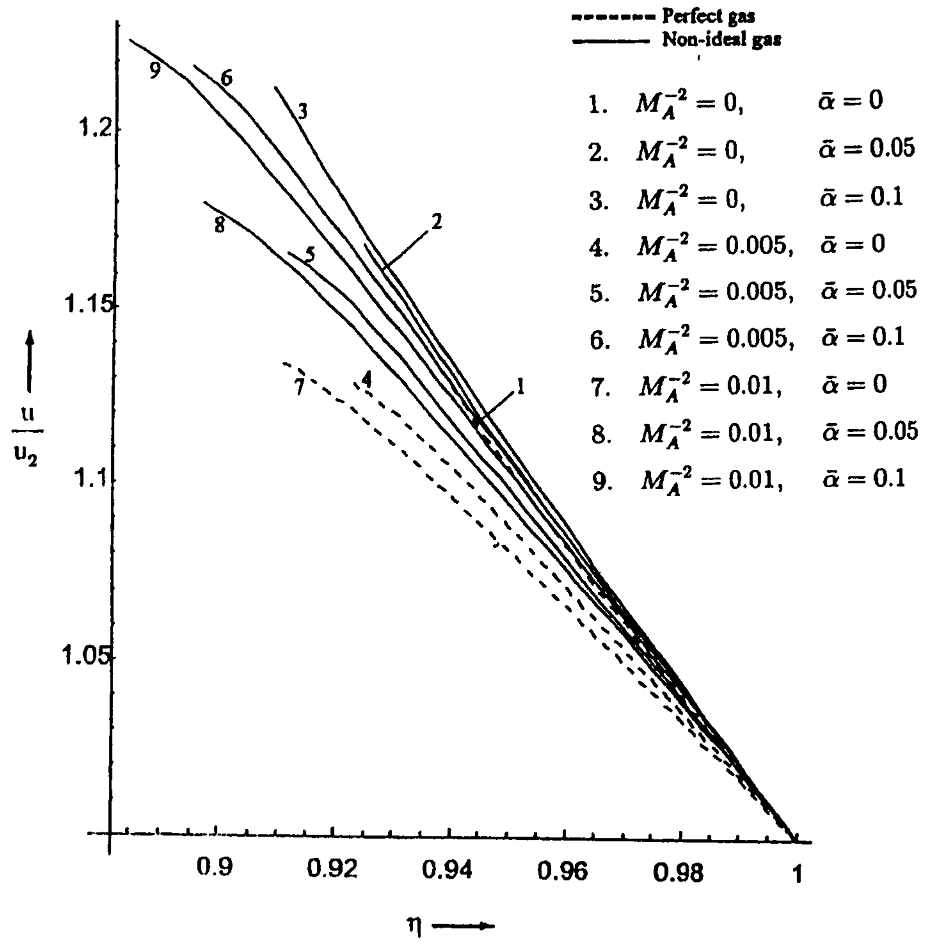

Figure 1. Distribution of the reduced velocity in the region behind the shock front.

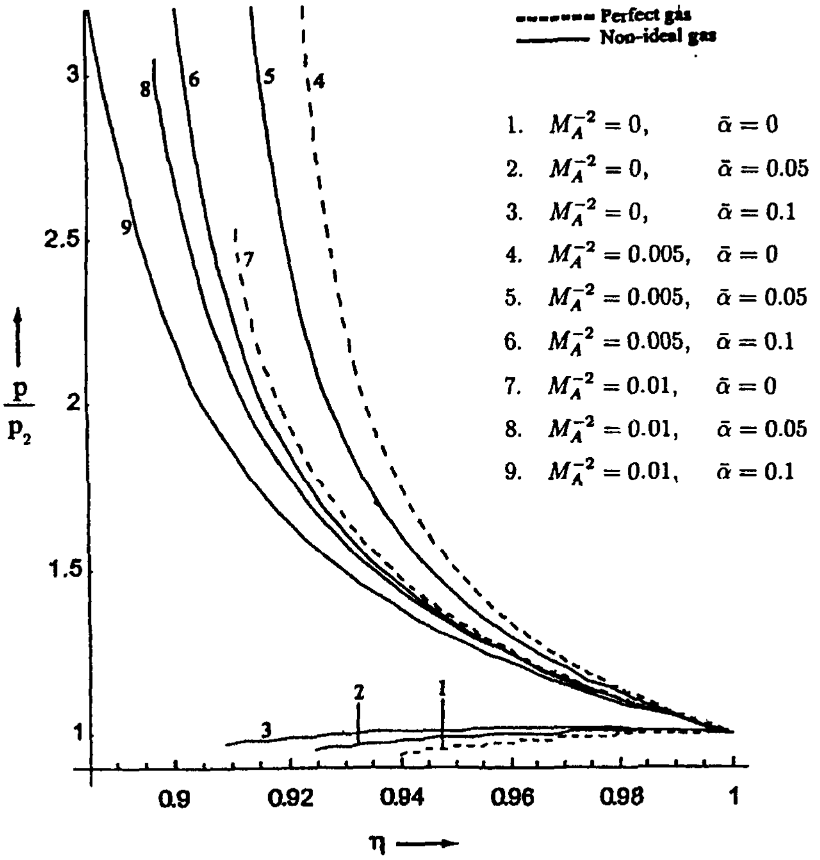

Figure 2. Distribution of the reduced pressure in the region behind the shock front.

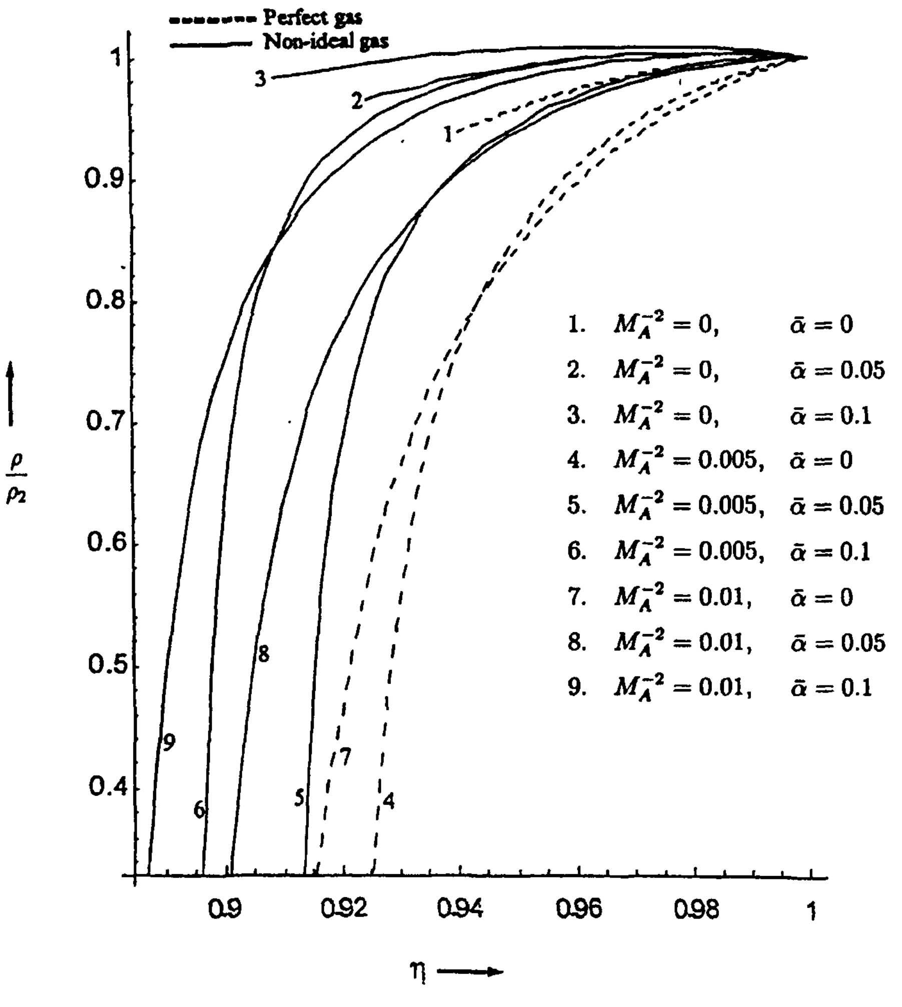

Figure 3. Distribution of the reduced density in the region behind the shock front.

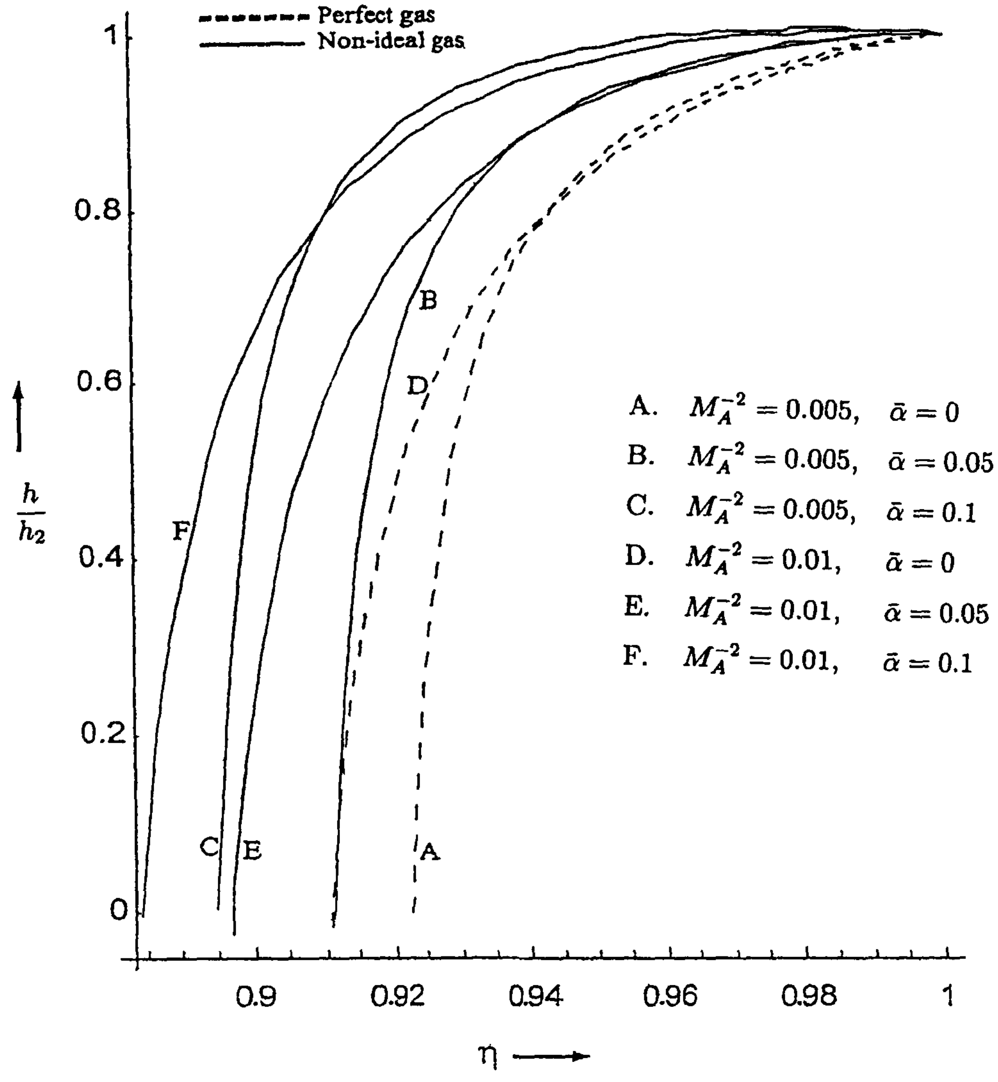

Figure 4. Distribution of the reduced magnetic field in the region behind the shock front.

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PROBLEM-I Implosion of cylindrical shock waves in a perfectly conducting non-ideal gas with radiation heat flux can be studied by using Whitham's Rule.

PROBLEM-II Propagation of shock waves generated by a piston (cylindrical or spherical) moving in a non-ideal gas in the presence of magnetic fieldin the adaibatic case can be studied.

Chapter 4

PROPAGATION OF MAGNETOGASDYNAMIC SHOCK WAVES IN A ROTATING NON-IDEAL GAS

At high temperatures, the normal gases like hydrogen and helium are ionized and the medium behaves like a medium of very high electrical conductivity, the electromagnetic effects may be significant. During study of gas motions at high temperatures one is thus led to consider the interaction of electromagnetic field with gasdynamic forces. There are many problems in which the energy in electric field is much smaller than that in the magnetic field. In these cases all the electromagnetic quantities may be expressed in terms of magnetic field.

Rotations of stars significantly affects the process taking place in their outer layers. Therefore question connected with the explosion in rotating gas atmospheres are of definite astrophysical interest.

Also in extreme conditions that prevail in most of the problems associated with shock waves, the assumption that the gas is ideal is no longer valid.

In this chapter, we shall study the following problem in a non-ideal gas:

4.1 SIMILARITY SOLUTIONS FOR A MAGNETOGASDYNAMIC CYLINDRICAL SHOCK WAVE IN A ROTATIONAL NON-IDEAL GAS FLOW

This section is devoted to the study of similarity solutions for adiabatic flow behind a magnetogasdynamic cylindrical shock wave propagating in a rotating non-ideal gas in presence of azimuthal magnetic field. We follow Vishwakarma et. al. [1] here.

4.1.1 INTRODUCTION

Because of high pressure and density that generally occur behind a shock wave, produced by an explosion, the assumption that the gas is ideal is no more valid. The popular alternative to the ideal gas is a simplified Van der Waals model. Roberts and Wu ([2],[3]) adopted this model to discuss the theory of sonoluminescence. In the present study, we adopt the same model of non-ideal gas to obtain the self-similar solutions for the flow behind a magnetogasdynamic cylindrical shock wave propagating in a rotating gas in the presence of an azimuthal magnetic field. The initial density of the medium is assumed to be constant. In order to obtain the similarity solutions, angular velocity of rotation of the ambient medium is assumed to be obeying a power law and to be decreasing as the distance from the axis increases.

Effects of a change in the strength of ambient magnetic field, in the non-idealness of the gas and in the index of variation of angular velocity of the ambient medium (or index of variation of ambient magnetic field) are investigated.

4.1.2 FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

The fundamental equations governing the unsteady adiabatic cylindrically symmetric motion of a non-ideal and perfectly conducting gas, which is rotating about the axis of symmetry and in which an azimuthal magnetic field is permeated and heat conduction and viscous stress are negligible (Whitham [4], Vishwakarma and Vishwakarma [5], Vishwakarma et. al. [1]), are

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0 \, , \tag{4.1.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left(\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right) - \frac{v^2}{r} = 0 \, , \tag{4.1.2}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} = 0 \, , \tag{4.1.3}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{uv}{r} = 0 \, , \tag{4.1.4}$$

$$\frac{de}{dt} + p\frac{d}{dt}\left(\frac{1}{\rho}\right) = 0 \, , \tag{4.1.5}$$

where ρ, p, h are the density, the pressure and the azimuthal magnetic field respectively; u and v are the radial and azimuthal components of the fluid velocity, μ is the magnetic permeability, r and t are the distance and time; and e is the internal energy per unit mass. Also, we write

$$v = Ar \, , \tag{4.1.6}$$

where A is the angular velocity of the medium at radial distance r from the axis of symmetry.

The gas is assumed to obey a simplified Van der Waals equation of state of the form (Wu and Roberts [2], Roberts and Wu [3])

$$p = \frac{\Gamma \rho T}{1 - b\rho} \, , \qquad e = C_v T = \frac{p(1 - b\rho)}{\rho(\gamma - 1)} \, , \tag{4.1.7}$$

where Γ is the gas constant, $C_v = \dfrac{\Gamma}{\gamma - 1}$ is the specific heat at constant volume and γ is the ratio of specific heats. The quantity b is the‘ Van der Waals excluded volumE’; it places a limit $\rho_{max} = \frac{1}{b}$, on the density of the gas.

We assume that a cylindrical shock is propagating outwards from the axis of symmetry in the non-ideal and perfectly conducting gas with constant initial density. The jump conditions across the magnetogasdynamic shock are

$$\rho_2(U - u_2) = h_1 U \, ,$$

$$h_2(U - u_2) = h_1 U \, ,$$

$$p_2 + \frac{\mu {h_2}^2}{2} + \rho(U - u_2)^2 = p_1 + \frac{\mu {h_1}^2}{2} + \rho_1 U^2 \, ,$$

$$e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}(U - u_2)^2 + \frac{\mu h_2^2}{\rho_2} = e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}U^2 + \frac{\mu h_1^2}{\rho_1} , \tag{4.1.8}$$

$$v_2 = v_1 ,$$

where U is the shock velocity and the suffixes '1' and '2' denote conditions immediately ahead and behind the shock front.

From equation (4.1.8), we get

$$u_2 = (1-\beta)U , \quad \rho_2 = \frac{\rho_1}{\beta} , \quad h_2 = \frac{h_1}{\beta} ,$$

$$p_2 = \left[\frac{1}{\gamma M^2} + \frac{2(1-\beta)}{\beta(\gamma+1) - (\gamma-1) - 2b\rho_1}\left\{\frac{1}{M^2} + \frac{\gamma-1}{4M_A^2}\left(\frac{1}{\beta} - 1\right)^2\right\}\right]\rho_1 U^2 ,$$

$$v_2 = v_1 , \tag{4.1.9}$$

where

$$\beta^3 - \beta^2 L + \left\{\frac{\gamma + b\rho_1 - 2}{(\gamma+1)M_A^2}\right\}\beta + \frac{b\rho_1}{(\gamma+1)M_A^2} = 0 , \tag{4.1.10}$$

$$L = \frac{\gamma-1}{\gamma+1} + \frac{2b\rho_1}{\gamma+1} + \frac{2}{(\gamma+1)M^2} + \frac{\gamma}{(\gamma+1)M_A^2} ,$$

$$M^2 = \frac{U^2}{\left(\frac{\gamma p_1}{\rho_1}\right)} , \tag{4.1.11}$$

$$M_A^2 = \frac{U^2}{\left(\frac{\mu h_1^2}{\rho_1}\right)} , \tag{4.1.12}$$

M and M_A being the shock Mach number referred to the frozen speed of sound $\left(\frac{\gamma p_1}{\rho_1}\right)^{\frac{1}{2}}$, and the Alfven - Mach number respectively.

Ahead of the shock the azimuthal magnetic field is assumed to vary as

$$h_1 = h_0 R^{\zeta} , \tag{4.1.13}$$

where h_0 and ζ are constants, and R is the shock radius.

In order to obtain the similarity solutions, it is assumed that the initial angular velocity A_1 varies as

$$A_1 = A_0 R^d \,, \tag{4.1.14}$$

where A_0 and d are constants. The assumption of varying initial angular velocity is necessary as $d = 0$ implies from relation (4.1.19) $\zeta = 1$, which is inconsistent with the relation (4.1.26).

The momentum equation (4.1.2) in the undisturbed state of gas, gives

$$p_1 = [\rho_1 A_0^2 - (1+\zeta)\mu h_2^0] \frac{R^{2\zeta}}{2\zeta} + constant \tag{4.1.15}$$

The total energy of the flow field behind the shock is not constant, but assumed to be time dependent and varying as (Roger [6], Freeman [7], Director and Dabora [8]).

$$E = E_0 t^w \,, \quad w \geq 0 \,, \tag{4.1.16}$$

where E_0 and w are constants. The positive values of w corresponds to the class in which the total energy increases with time. This increase can be achieved by the pressure exerted on the fluid by an expanding surface (a contact surface or a piston). Thus the flow is headed by a shock front and has an expanding surface as a inner boundary.

4.1.3 SIMILARITY SOLUTIONS

We introduce the following similarity transformations to reduce the equations of motion into ordinary differential equations:

$$u = UV(\eta)\ , \quad \rho = \rho_1 D(\eta)\ , \quad p = \rho_1 U^2 P(\eta)\ ,$$

$$v = UK(\eta)\ , \quad \sqrt{\mu}h = \sqrt{\rho_1}UH(\eta)\ , \tag{4.1.17}$$

where V, D, P, K, and H are functions of the non-dimensional variable $\eta = \frac{r}{R}$. The shock front is represented by $\eta = 1$.

The shock conditions (4.1.9) are transformed into

$$V(1) = 1 - \beta\ , \quad D(1) = \frac{1}{\beta}\ , \quad H(1) = \frac{1}{\beta M_A}\ ,$$

$$P(1) = \frac{1}{\gamma M^2} + \frac{2(1-\beta)}{\beta(\gamma+1) - (\gamma-1) - 2b\rho_1}\left\{\frac{1}{M^2} + \frac{(\gamma-1)}{4M_A^2}\left(\frac{1}{\beta} - 1\right)^2\right\}\ ,$$

$$K(1) = \left[\frac{2\zeta}{\gamma M^2} + \frac{1+\zeta}{M_A^2}\right]^{\frac{1}{2}}\ , \tag{4.1.18}$$

where

$$1 + d = \zeta\ . \tag{4.1.19}$$

The total energy of the non-ideal gas behind the shock is given by

$$E = 2\pi \int_{r_p}^{R} \left\{\frac{1}{2}\rho(u^2 + v^2) + \frac{p(1 - b\rho)}{\gamma - 1} + \frac{\mu h^2}{2}\right\} r dr = E_0 t^w\ , \tag{4.1.20}$$

where r_p is the radius of inner expanding surface. Applying the similarity transformations (4.1.17) to the relation (4.1.20), we find that the motion of the shock front is given by the equation

$$R^2 U^2 = \frac{E_0 t^w}{2\pi \rho_1 J}\ , \tag{4.1.21}$$

where,

$$J = \int_{\eta_p}^{1} \left[\frac{1}{2} D(V^2 + K^2) + \frac{P(1 - b\rho_1 D)}{\gamma - 1} + \frac{H^2}{2} \right] \eta d\eta \ , \tag{4.1.22}$$

in which η_p is the value of η at the inner ezpanding surface.

Equation (4.1.21) can be written as

$$R\frac{dR}{dt} = \left(\frac{E_0}{2\pi\rho_1 J} \right)^{\frac{1}{2}} t^{\frac{w}{2}} \ , \tag{4.1.23}$$

which on integration gives

$$R\left(\frac{8E_0}{\pi\rho_1 J} \right)^{\frac{1}{4}} \frac{1}{\sqrt{w+2}} t^{\frac{(w+2)}{4}} \ . \tag{4.1.24}$$

From equation (4.1.24), we get the shock velocity

$$U = \frac{dR}{dt} = \frac{(w+2)}{4} . \frac{R}{t} = \left(\frac{8E_0}{\pi\rho_1 J} \right)^{\frac{1}{w+2}} \frac{(w+2)^{\frac{w}{(w+2)}}}{4} R^{\frac{(w-2)}{(w+2)}} \ . \tag{4.1.25}$$

For similarity solutions, the shock-Mach number M and Alfven-Mach number M_A (which occur in the shock conditions (4.1.18)) must be constant parameters. Therefore, from (4.1.11) and (4.1.12), we have

$$\zeta = \frac{w-2}{w+2} \ . \tag{4.1.26}$$

To obtain the solution in a convenient form, we introduce the following transformations:

$$g = \frac{\rho}{\rho_2} \ , \quad y = \frac{p}{p_2} \ , \quad W = \frac{u}{U} \ , \quad Z = \frac{v}{U} \ , \quad s = \frac{h}{h_2} \ . \tag{4.1.27}$$

Using the transformations (4.1.27), the equations of motion (4.1.1) to (4.1.5) take the form

$$(W - \eta)\frac{dg}{d\eta} + g\frac{dW}{d\eta} + \frac{gW}{\eta} = 0 \ , \tag{4.1.28}$$

$$(W-\eta)\frac{dW}{d\eta}+\frac{\beta F}{g}\frac{dy}{d\eta}+\frac{s}{\beta M_A^2 g}\frac{ds}{d\eta}+\frac{s^2}{\beta M_A^2 g\eta}-\frac{Z^2}{\eta}+\zeta W=0\ , \qquad (4.1.29)$$

$$(W-\eta)\frac{ds}{d\eta}+s\frac{dW}{d\eta}+\zeta s=0\ , \qquad (4.1.30)$$

$$(W-\eta)\frac{dZ}{d\eta}+\frac{ZW}{\eta}+\zeta Z=0\ , \qquad (4.1.31)$$

$$2\zeta y+(W-\eta)\frac{dy}{d\eta}-\frac{\gamma y\beta(W-\eta)}{g(\beta-\bar{\alpha}g)}\frac{dg}{d\eta}=0\ , \qquad (4.1.32)$$

where

$$F=\frac{1}{\gamma M^2}+\frac{2(1-\beta)}{\beta(\gamma+1)-(\gamma-1)-2\bar{\alpha}}\left[\frac{1}{M^2}+\frac{\gamma-1}{4M_A^2}\left(\frac{1}{\beta}-1\right)^2\right] \qquad (4.1.33)$$

and $\bar{\alpha}=b\rho_1$ is the parameter of non-idealness of the gas.

In terms of the dimensionless variables η, W, y, g, s and Z the shock conditions take the form

$$\eta=1\ ,\quad W=(1-\beta)\ ,\quad g=1\ ,\quad s=1\ ,\quad y=1\ ,$$

$$Z=\left[\frac{(1+\zeta)}{M_A^2}+\frac{2\zeta}{\gamma M^2}\right]^{\frac{1}{2}}. \qquad (4.1.34)$$

Because of the dependence of the equations (4.1.29), (4.1.32), and (4.1.34) on $\bar{\alpha}$, similarity solutions exists only when on $\bar{\alpha}$ is constant, i.e. only when the initial density ρ_1 is constant.

In addition to the shock conditions (4.1.34), the condition to be satisfied at the inner boundary surface is that the velocity of the fluid is equal to the velocity of inner boundary itself. This kinematic condition, from equations (4.1.17) and (4.1.27), can be written as

$$W(\eta_p)=\eta_p\ . \qquad (4.1.35)$$

Solving equations (4.1.28) to (4.1.32) for $\frac{dW}{d\eta}$, $\frac{ds}{d\eta}$, $\frac{dg}{d\eta}$, $\frac{dy}{d\eta}$ and $\frac{dZ}{d\eta}$, we get

$$B\eta\frac{dW}{d\eta} = \frac{\gamma y\beta^2 FW}{\beta-\bar{\alpha}g} + 2\zeta\beta\eta yF - (W-\eta)\left[\frac{s^2}{\beta M_A^2} - Z^2 g + \zeta W g\eta\right] + \frac{\zeta s^2\eta}{\beta M_A^2}\,, \tag{4.1.36}$$

$$\begin{aligned}\beta\eta(W-\eta)\frac{ds}{d\eta} = -s\Big\{&\zeta\eta\left[(1-\zeta\eta)^2 g - \frac{s^2}{\beta M_A^2}\right] + (W-\eta)\frac{\gamma y\beta^2 FW}{\beta-\bar{\alpha}g}\\ &+ 2\zeta\beta\eta yF - (W-\eta)\left[\frac{s^2}{\beta M_A^2} - Z^2 g + \zeta W g\eta\right] + \frac{\zeta s^2\eta}{\beta M_A^2}\Big\}\,,\end{aligned} \tag{4.1.37}$$

$$\begin{aligned}\beta\eta(W-\eta)\frac{dg}{d\eta} = g\Big\{&-2\zeta\beta\eta yF + (W-\eta)\left[\frac{s^2}{\beta M_A^2} - Z^2 g + \zeta W g\eta\right]\\ &- \frac{\zeta s^2\eta}{\beta M_A^2} - W\left[(W-\eta)^2 g - \frac{s^2}{\beta M_A^2}\right]\Big\}\,,\end{aligned} \tag{4.1.38}$$

$$\begin{aligned}\beta\eta(W-\eta)\frac{dy}{d\eta} = \frac{\gamma y\beta}{\beta-\bar{\alpha}g}\Big\{&(W-\eta)\left[\frac{s^2}{\beta M_A^2} - Z^2 g + \zeta W g\eta\right] - \frac{\zeta s^2\eta}{\beta M_A^2}\\ &- W\left[(W-\eta)^2 g - \frac{s^2}{\beta M_A^2}\right]\Big\} - 2\zeta\eta y\left[(W-\eta)^2 g - \frac{s^2}{\beta M_A^2}\right]\,,\end{aligned} \tag{4.1.39}$$

$$\eta(W-\eta)\frac{dZ}{d\eta} = -Z(\zeta\eta + W)\,, \tag{4.1.40}$$

where

$$B = (W-\eta)^2 g - \frac{s^2}{\beta M_A^2} - \frac{\gamma y F\beta^2}{\beta-\bar{\alpha}g}\,. \tag{4.1.41}$$

Now, the ordinary differential equations (4.1.36) to (4.1.40) may be integrated, numerically, with the boundary conditions (4.1.34) to obtain the values of W, g, s, y, and Z.

4.1.4 RESULTS AND DISCUSSION

Similarity consideration led to the following relations among the constants ζ, d and w:

$$1 + d = \zeta \, , \quad \zeta = \frac{(w-2)}{(w+2)} \, . \tag{4.1.42}$$

Then the following two cases may exists:

(i) the constant velocity shock ($\zeta = 0$);

(ii) the decreasing velocity shock ($\zeta < 0$).

Therefore, for the purpose of numerical calculations $\zeta = 0 \; , \quad -0.5$ are chosen which correspond respectively to the following two sets of constants:

(i) $\zeta = 0 \; , \quad w = 2 \; , \quad d = -1$, and

(ii) $\zeta = -\frac{1}{2} \; , \quad w = \frac{2}{3} \; , \quad d = -\frac{3}{2}$.

Numerical integration of the set of differential equations (4.1.36) to (4.1.40) is performed to obtain the reduced variables W, Z, g, y, s, starting from the shock front to the inner expanding surface for the values of the constant parameters as $\gamma = \frac{5}{3}$; $M^{-2} = 0.01$; $M_A^{-2} = 0.02 \; , \quad 0.1$; $\bar{\alpha} = 0 \; , \quad 0.05 \; , \quad 0.1$; $\zeta = 0 \; , \quad -0.5$ (Rosenau and Frankenthal [9], Roberts and Wu [2], Wu and Roberts [3], Vishwakarma and Yadav [10]). The results are shown in figures 1-5. Values of η_p (the reduced position of the inner expanding surface) and the density ratio across the shock front $\beta = \dfrac{\rho_1}{\rho_2}$ are shown in tables I and II for different cases.

TABLE - I : Position of inner expanding surface η_p for $\gamma = \frac{5}{3}$, $M^{-2} = 0.01$ and various values of M_A^{-2}, $\bar{\alpha}$ and ζ.

M_A^{-2}	$\bar{\alpha}$	ζ	η_p
0.02	0	0	0.820
		−0.5	0.656
	0.05	0	0.806
		−0.5	0.628
	0.1	0	0.788
		−0.5	0.612
0.1	−0	0	0.727
		−0.5	0.473
	0.05	0	0.714
		−0.5	0.446
	0.1	0	0.711
		−0.5	0.456

TABLE - II : Density ratio β across the shock front for $\gamma = \dfrac{5}{3}$, $M^{-2} = 0.01$ and various values of M_A^{-2}, and $\bar{\alpha}$.

$\bar{\alpha}$	$\beta = \frac{\rho_1}{\rho_2}$	
	$M_A^{-2} = 0.02$	$M_A^{-2} = 0.1$
0	0.278962	0.355192
0.05	0.310485	0.379733
0.1	0.343672	0.393009

Figure 1 shows that the reduced radial velocity W increases from the shock front to the inner expanding surface when $\zeta = 0$; where as it decreases when $\zeta = -0.5$. Figures 2 and 4 show that the reduced density g and the reduced pressure y decrease rapidly behind the shock front. Figure 3 show that the reduced azimuthal magnetic field s increases rapidly from the shock front to the inner expanding surface, and this increase becomes slower when ζ is decreased or when M_A^{-2} is increased. Also, figure 5 shows that the reduced azimuthal velicity Z decreases rapidly behind the shock front when $\zeta = 0$ and it decreases slowly when $\zeta = -0.5$.

From tables I and II and figures 1-5, it is found that the effects of an increase in the strength of ambient magnetic field(i.e. the effects of an increase in the value of M_A^{-2}) are:

(i) to decrease the value of η_p, i.e. to increase the distance of inner expanding surface from the shock front. Physically, it means that the gas bahind the shock is less compressed, i.e. the shock strength is reduced;

(ii) to increase the value of β, i.e. to decrease the shock strength, which is the same as given in (i). Therefore the presence of the magnetic field has decaying effect on the shock wave;

(iii) to decrease the radial velocity and to increase the azimuthal velocity at any point in the flow field behind the shock ; and

(iv) to decrease the slopes of the profiles of density, pressure and azimuthal magnetic field.

The effects of an increase in the value of the parameter of the nonidealness of the gas $\bar{\alpha}$ are

(i) to decrease the value of η_p, i.e. to increase the distance of the inner expanding surface from the shock front (Table-I) ;

(ii) to increase the value of β, i.e. to decrease the shock strength (Table-II). Therefore the non-idealness of the gas has decaying effect on the shock wave ;

(iii) to decrease the radial velocity, in general; and to increase the azimuthal velocity slightly, at any point in the flow field behind the shock (figures 1 and 5) ;

(iv) to decrease the slope of the density profiles and to increase the slope of profiles of azimuthal magnetic field (figures 2 and 3).

The effects of an increase in the value of the index for variation of ambient azimuthal magnetic field ζ, i.e. the effects of an increase in the value of the index for variation of angular velocity of the ambient medium d are :

(i) to decrease the distance of inner expanding surface from the shock front. It means that the shock is stronger when the ambient magnetic field is uniform ($\zeta = 0$) in comparision with that when it is decreasing ($\zeta = -0.5$). It also means that the shock is stronger when the angular velocity of the ambient medium is slowly decreasing ;

(ii) to increase the radial velocity and azimuthal magnetic field at any point in the flow-field behind the shock front (figures 1 and 3) ;

(iii) to increase the tendency of rapid increase in azimuthal magnetic field and rapid decrease in azimuthal velocity, density and pressure.

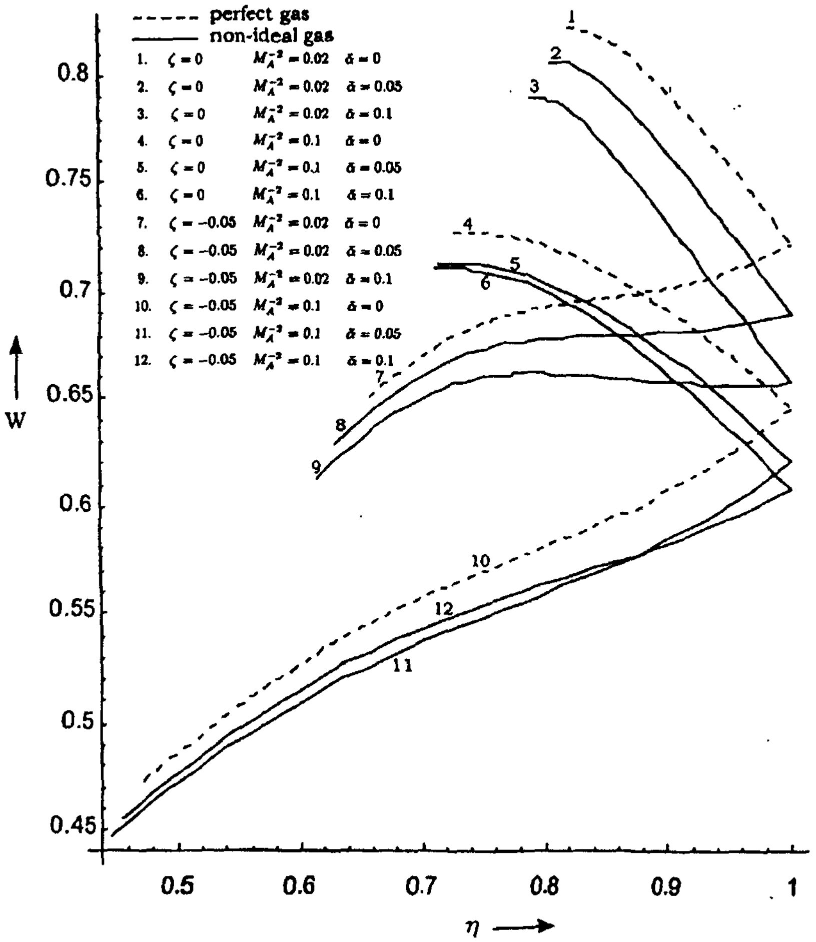

Figure 1. Variation of the reduced radial velocity W in the folw-field behind the shock front.

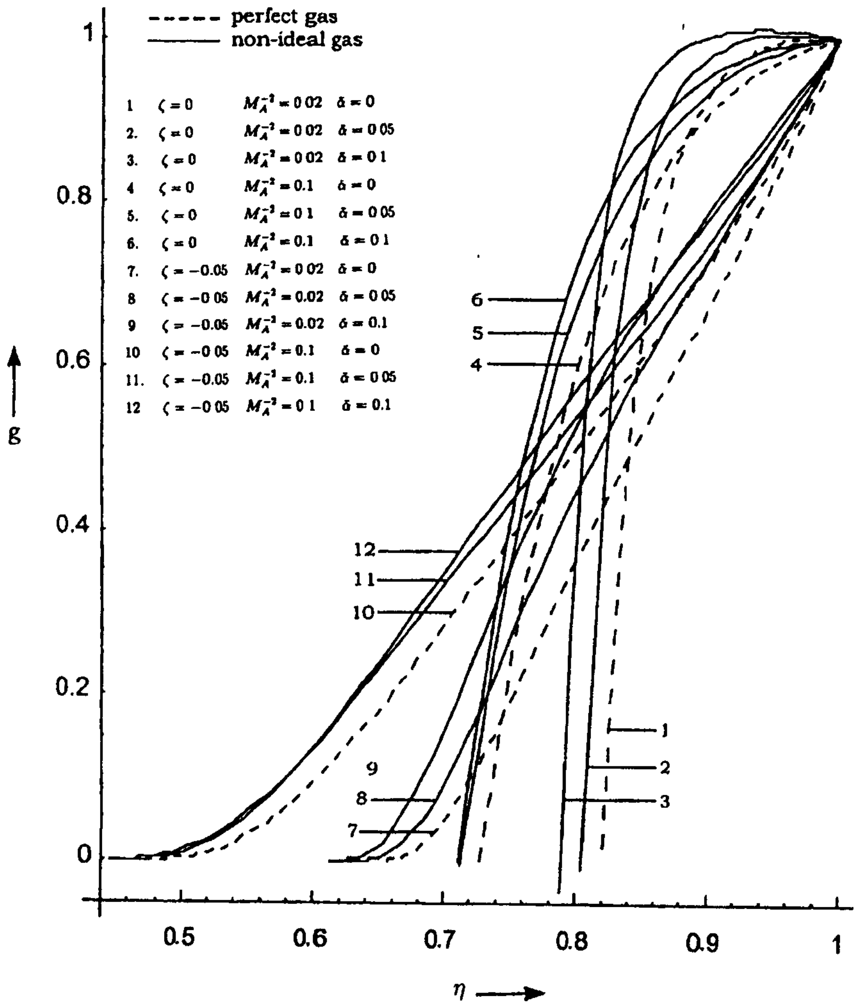

Figure 2. Variation of reduced density g in the flow field behind the shock front.

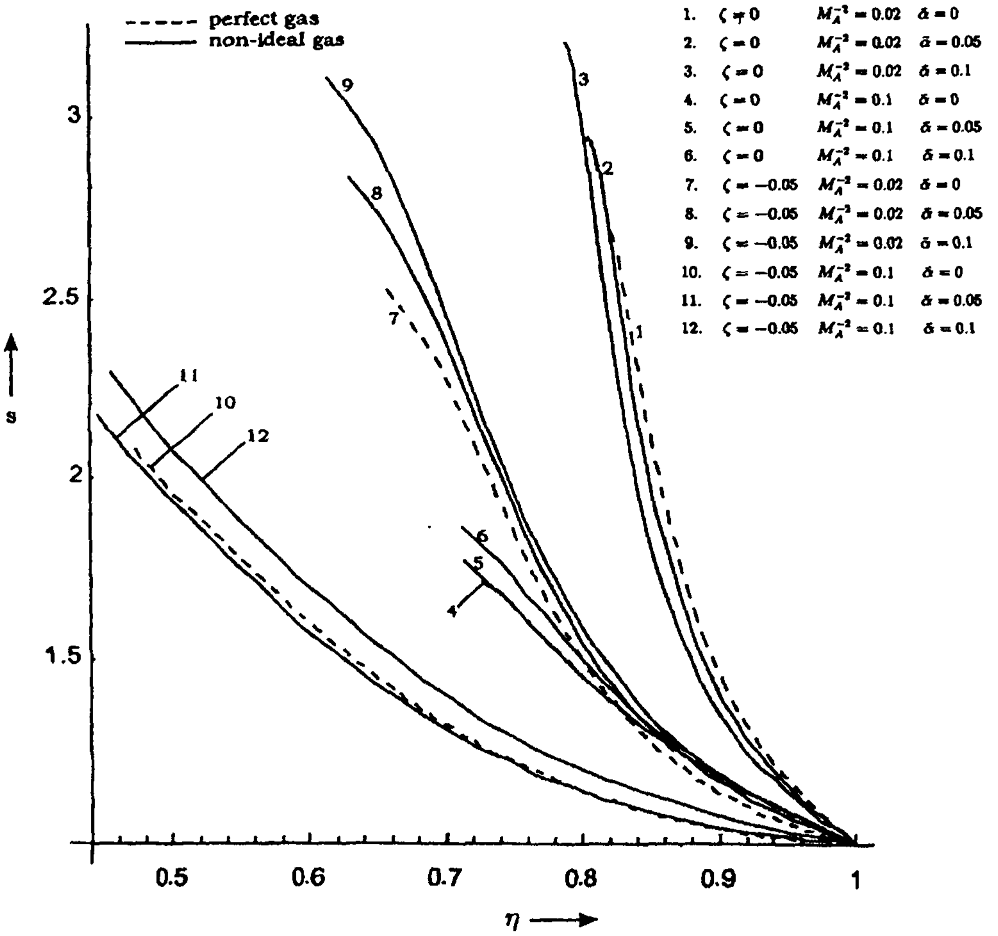

Figure 3. Variation of the reduced azimuthal magnetic field s in the flow-field behind the shock front.

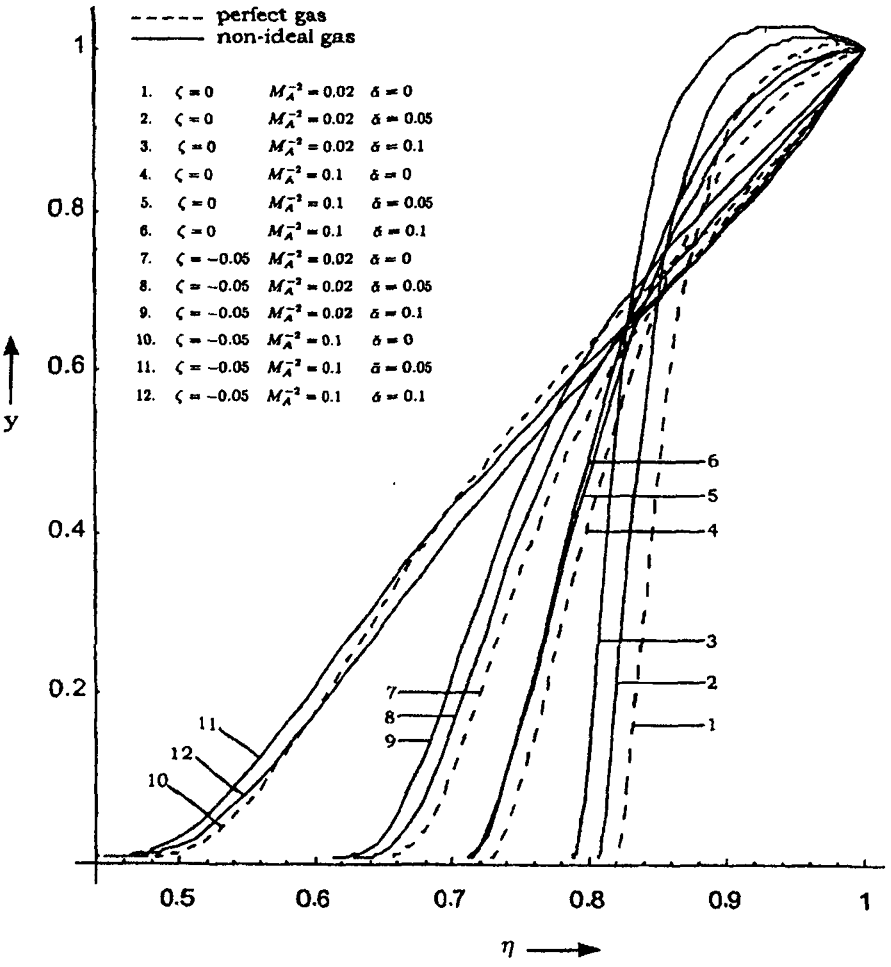

Figure 4. Variation of the reduced pressure y in the flow field behind the shock front.

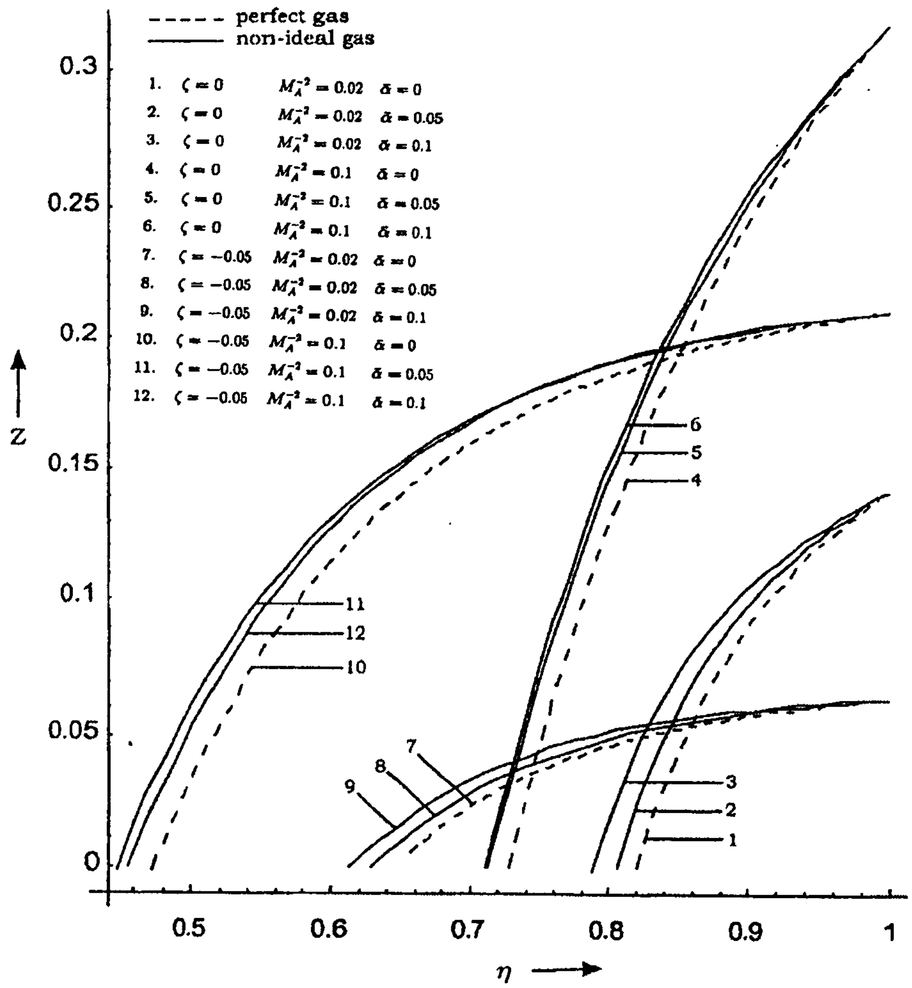

Figure 5. Variation of the reduced azimuthal velocity Z in the flow-field behind the shock front.

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BRIEF BIO - DATA

1.	**Name :**	BINEETA NATH
2.	**Sex :**	Femlale
3.	**Date of birth :**	$26^{th} February, 1982$
4.	**Father's Name :**	Shri. Bidhan Chandra Nath
5.	**Nationality :**	Indian
6.	**Permanent Adress :**	Tarun Nagar, By-Lane - 6 House no- 48, P.O. - Dispur Guwahati, Assam Pin- 781005
7.	**Academic Qualification :**	M. Sc. in Mathematics, Gauhati university.