Low-mass right-handed gauge bosons, proton decay, and other observable predictions in $SU(8)_L \times SU(8)_R$ and SU(16)

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The prospects of an alternative method of decoupling the parity and $SU(2)_R$ breakings are reexamined under the constraints of CERN LEP measurements in the minimal chains of $SU(8)_L \times SU(8)_R$ and SU(16) grand unified theories. Including the uncertainties in the input parameters, one chain permits the $SU(2)_L \times SU(2)_R \times SU(4)_C$-breaking scale $M_C = 10^{5.6\pm0.6}$ GeV leading to the $\Delta(B-L) = -2$ nucleon-decay modes possibly accessible to the ongoing experiments and observable branching ratios for rare kaon decays. Uncertainties in the Higgs scalar masses near M_C result in wider ranges of predictions on the proton lifetime and mixing times for n-$\bar{n}$ and H-$\bar{H}$ oscillations. In a separate chain the $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$-breaking scale corresponding to the masses of right-handed gauge bosons could be $M_R = 10^{2.8\pm0.7}$ GeV, which seems to be the lowest value predicted so far in a non-SUSY GUT consistent with minimal fine-tuning of parameters.

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I. INTRODUCTION

Precision measurements at the e^+e^- CERN LEP have led to the revival of interest in grand unified theories (GUT's) [1,2]. In addition to examining the unification of gauge couplings at the GUT scale (M_U), the prediction of suitable intermediate scales signifying new physics beyond the standard model has also been emphasized [3,4]. The new physics associated with $V + A$ interactions in left-right gauge models can be experimentally tested provided the right-handed gauge bosons $(W_R^\pm, Z_R)$ resulting from the spontaneous breaking of $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ ($\equiv G_{2213}$) symmetry [5] are reasonably light ($M_R \leq 1$ TeV). Further, if the symmetry-breaking scale of the Pati-Salam gauge group $SU(2)_L \times SU(2)_R \times SU(4)_C$ ($\equiv G_{224}$) [1], $M_C = 10^5$–10^6 GeV, the relevant physical processes such as rare kaon decays and new modes of proton decay [6] might be accessible to verification by the ongoing experiments. In addition, n-$\bar{n}$ and H-$\bar{H}$ oscillations can be predicted with definite rates to be detected in the future. In all conventional methods of spontaneous breaking of left-right symmetric gauge structures, such as $SU(2)_L \times SU(2)_R \times SU(4)_C \times P$ ($\equiv G_{224P}$; P, parity, the left-right discrete symmetry)$\subset$SO(10), $SU(8)_L \times SU(8)_R$, or SU(16), be it via a single step or via multiple steps to the standard gauge group, the scales of parity breaking (M_P) and $SU(2)_R$ breaking (M_R) are identical ($M_P = M_R$). This has the consequence that the effective-gauge-coupling constants of $SU(2)_L$ and $SU(2)_R$ are equal to each other at low energies, barring small radiative corrections ($g_{2L} = g_{2R}$). Because of this, the GUT's such as SO(10), $SU(8)_L \times SU(8)_R$, and SU(16) require $\sin^2\theta_W$ to be too large ($\simeq 0.26$–0.28) compared to the observed value if $W_R^\pm$ and Z_R boson masses are to be near the TeV scale. But two different methods of decoupling of the $SU(2)_R$ breaking from the parity breaking have been proposed leading to $g_{2L}(\mu) > g_{2R}(\mu)$ for $\mu < M_P$ which significantly lower values of $M_R \ll M_P$ [7–10].

In SO(10) or SU(16) the eight left-handed fermions (ψ_L) and their charge conjugates $(\psi^c)_L$ are in the same spinorial representation **16**. There exists an element of gauge transformation, called D [7], which takes $\psi_L \rightarrow (\psi^c)_L$ and vice versa. The gauge transformation D acts like parity (P). The decoupling mechanism is implemented [7] in the first step by giving vacuum expectation values (VEV's) to the D-odd (or P-odd) neutral components of scalars which are singlets under G_{2213} or G_{224} leading to the spontaneous breaking of P but not $SU(2)_R$ which is broken in the usual manner at a lower scale. In this approach there is left-right asymmetry in the Higgs sector only for $\mu < M_P$. The lowest value of the G_{2213}-breaking scale obtained in this method in SO(10) has been found to be $M_R \simeq 10^7$ GeV [7–9].

The alternative method [10] of decoupling operates within all GUT's containing $SU(2)_L \times SU(4)_L^C \times SU(2)_R \times SU(4)_R^C$ possessing chiral SU(4) color in addition to the flavor subgroup $SU(2)_L \times SU(2)_R$. Therefore, it operates within $SU(8)_L \times SU(8)_R$, SU(16) [11,12] and also in $[SU(4)]^4$ but not in SO(10). The decoupling is implemented by following specific symmetry-breaking patterns where, after the first step, an asymmetry in the left- and right-handed gauge boson sectors is created. For example, the symmetry breaking SU(16), $SU(8)_L \times SU(8)_R \rightarrow SU(8)_L \times SU(2)_R \times SU(4)_R^C$ ($\equiv G_{824}$) yields the symmetry having different left- and right-handed gauge groups. The subsequent breaking of either $G_{824} \rightarrow G_{224}$ or $G_{824} \rightarrow G_{2213}$ yields the left-right gauge groups with $g_{2L}(\mu) > g_{2R}(\mu)$; the inequality in the two couplings being due to the asymmetry in the Higgs scalar as well as the gauge-boson sectors.

In SU(16) and $SU(8)_L \times SU(8)_R$ [11,12] there are triangle anomalies whose cancellation requires the introduction of the mirror family (ψ_L^m, ψ_R^m) corresponding to every standard family of quarks and leptons (ψ_L, ψ_R) and the imposition of mirror symmetry:

$$\psi_L \leftrightarrow \psi_R^m, \quad \psi_R \leftrightarrow \psi_L^m \ .$$

In order to achieve the standard model phenomenology at low energies, when an attempt is made to keep only the three standard families light and the mirror masses near M_U, the standard fermions also get bare Dirac masses of the order M_U. This is avoided by imposing an additional discrete symmetry:

$$\psi_L^m \leftrightarrow -\psi_L^m, \quad \psi_R^m \leftrightarrow -\psi_R^m \ .$$

The purpose of this paper is that the alternate decoupling mechanism proposed by one of the present authors (M.K.P.) and Pati [10] is capable of achieving M_R and M_C as low as $10^{2.8\pm0.7}$ and $10^{5.6\pm0.6}$ GeV, respectively, consistent with the CERN-LEP data. An approximate one-loop analysis in $SU(8)_L\times SU(8)_R$ and SU(16) was made in [10] using the old input data on $\sin^2\theta_W(M_W)$, $\alpha_s(M_W)$, and $\alpha^{-1}(M_W)$. In this work we carry out a more accurate analysis including two-loop effects and using the CERN-LEP meaurements and the improved estimtion of $\alpha^{-1}(M_Z)$ [13]:

$$\begin{aligned} \sin^2\theta_W(M_Z) &= 0.2324 \pm 0.0008 \ , \\ \alpha_s(M_Z) &= 0.12 \pm 0.007 \ , \\ \alpha^{-1}(M_Z) &= 127.9 \pm 0.1 \ . \end{aligned} \tag{1}$$

We examine the unification of gauge couplings by plotting the coupling-constant trajectories as a function of mass scale (μ) and following an improved procedure which simultaneously uses analytic formulas for mass scales derived here and the renormalization-group equations (RGE's). While qualitative estimates of the proton lifetimes and matter-antimatter oscillation rates were made in [10], quantitative evaluations including important uncertainties due to input parameters and the Higgs scalar masses have been caried out here for the first time. The uncertainties on the predicted values of mass scales arising out of the input parameters are evaluated and are noted to play a crucial role in bringing the $\Delta(B-L) = -2$ proton decay rates within the accessible range of the SuperKamiokande experiments, although central values are 4–5 orders larger than the present accessible limit. The uncertainty due to the Higgs scalar masses near the G_{224} breaking are found to widen the range of proton lifetime, further and predicted values of n-$\bar{n}$ and H-$\bar{H}$ oscillation mixing times.

II. MASS SCALES IN THE MINIMAL CHAINS

In the alternative method of decoupling the parity and $SU(2)_R$ breakings, the asymmetry $g_{2L}(\mu) > g_{2R}(\mu)$ for $\mu < M_U$ is created by a specific Higgs mechanism where SU(16) or $SU(8)_L\times SU(8)_R$ breaks spontaneosly to $SU(8)_L\times SU(2)_R\times SU(4)_R^C$ ($\equiv G_{824}$) at the GUT scale such that the residual symmetry $G_{824} \supset G_{224}$ or G_{2213} as its subgroup. The $SU(8)_L\times SU(8)_R$ Higgs representation (1,330) containing the G_{824} singlet achieves this breaking. In the second step the popular left-right gauge group G_{224} with $g_{2L} \neq g_{2R}$ is obtained by the spontaneous symmetry breaking of G_{824} through the G_{88} Higgs representation (36,36) containing the G_{224} singlet. However, to obtain G_{2213} from G_{824}, two of the G_{88} representations (36,36) and (1,63) are needed. Whereas the representation (36,36) contains the G_{224} singlet, the representation (1,63) contains the G_{224} submultiplet (1,1,15) whose vacuum expectation value in the G_{2213} singlet direction yields the desired left-right gauge group. In the third step the spontaneous symmetry breaking leads to the standard gauge group G_{213} by using the usual G_{224} submultiplet $\Delta_R(1,3,\overline{10}) \subset (1,36)$ of G_{88}. Finally the low-energy gauge group is obtained through the vacuum expectation value of the standard Higgs doublet contained in the G_{88} representation $(8,\bar{8})$.

Thus, in order to obtain the asymmetric groups G_{224} or G_{2213} with $g_{2L} \neq g_{2R}$ the new mechanism needs spontaneous symmetry breaking to the standard gauge group in at least two steps as illustrated in the minimal models (A) and (B):

$$\text{(A)} \quad SU(16), SU(8)_L\times SU(8)_R \xrightarrow[M_U]{(1.330)} G_{824} \xrightarrow[M_8]{(36,36)} G_{224} \xrightarrow[M_C]{(1.36)} G_{213} \xrightarrow[M_W]{(8,\bar{8})} U(1)_{\rm em}\times SU(3)_C \ ,$$

$$\text{(B)} \quad SU(16), SU(8)_L\times SU(8)_R \xrightarrow[M_U]{(1.330)} G_{824} \xrightarrow[M_8]{(1.63)+(36.36)} G_{2213} \xrightarrow[M_R]{(1.36)} G_{213} \xrightarrow[M_W]{(8,\bar{8})} U(1)_{\rm em}\times SU(3)_C \ .$$

Instead of $SU(8)_L\times SU(8)_R$ it is possible to break SU(16) directly to G_{824} since the G_{88} Higgs multiplets shown in models (A) and (B) are contained in the corresponding SU(16) reprsentations.

For the unbroken gauge symmetry G_i with coupling constants $g_i(\mu)$ and $\alpha_i(\mu) = g_i^2(\mu)/4\pi$ in the mass range $M_1 \le \mu \le M_2$ the renormalization-group equations (RGE's) are

$$\frac{1}{\alpha_i(M_1)} = \frac{1}{\alpha_i(M_2)} + \frac{a_i}{2\pi}\ln\frac{M_2}{M_1} + \frac{1}{4\pi}P_i(M_1,M_2) - \frac{C_i}{12\pi} \ , \tag{2}$$

where

$$P_i(M_1,M_2) = \sum_j B_{ij}\ln\frac{\alpha_j(M_2)}{\alpha_j(M_1)} \ .$$

The second (third) term on right-hand side (RHS) of (2) is the one- (two-)loop contribution. In the last term in (2) C_i is the mass-independent Dynkin index relative to the subgroup G_i of the effective gauge theory (EGT) multiplied by dimensions relative to other subgroups of the same EGT which result due to spontaneous symmetry breaking of the original theory at the higher scale M_2.

C_i is obtained by summing over all EGT representations of massive gauge bosons assumed to be degenerate at M_2 [14]. Even if mass-dependent terms in the threshold effect at M_2 are neglected, as in the present paper, the C_i terms are always present. The values of C_i at each boundary is given below for the two models. For the minimal choice of Higgs scalars necessary to implement the spontaneous symmetry breaking for $\mu \le M_8$, the loop coefficients have been derived earlier [8]. For $\mu \ge M_8$, the new coefficients are given below for models (A) and (B). In each chain at first the formulas for $\sin^2\theta_W(M_Z)$ and $\alpha(M_Z)/\alpha_s(M_Z)$ are obtained in terms of mass scales following the standard procedure. Using each of the two mass scales (M_U, M_C) or (M_U, M_R) is expressed analytically in terms of $\sin^2\theta_W(M_Z)$, $\alpha_s(M_Z)$ $\alpha(M_Z)$, and $\ln(M_8/M_Z)$.

Model (A): For the G_{824} group, $i, j = 8L, 2R, 4R$,

$$a_{8L} = -14, \quad a_{2R} = \tfrac{26}{3}, \; a_{4R} = 7\ ,$$

$$C^U_{8L} = 0, \quad C^U_{2R} = 30, \quad C^U_{4R} = 12\ ,$$

where the superscript U in C^U_i stands for the boundary at M_U. The other C_i coefficients at M_8 and M_C are

$$C^8_{2L} = 30, \quad C^8_{4L} = 12, \quad C^8_{4R} = 0, \quad C^8_{2R} = 0\ ,$$

$$C^C_{2L} = 0, \quad C^C_Y = \tfrac{14}{5}, \quad C^C_{3C} = 1\ ,$$

$$B_{ij} = \begin{pmatrix} -\frac{281}{4} & \frac{1845}{104} & \frac{435}{7} \\ -45 & \frac{904}{13} & \frac{660}{7} \\ -\frac{225}{2} & \frac{198}{13} & 136 \end{pmatrix},$$

leading to

$$\begin{aligned}\ln\frac{M_U}{M_Z} = {} & \frac{\pi}{2035\alpha}(154\alpha/\alpha_s - 114\sin^2\theta_W - 15) + \frac{2421}{2035}\ln\frac{M_8}{M_Z} \\ & + \frac{1}{8140}\{25P^C_Y + 129P^C_{2L} - 154P^C_{3C} + 15P^8_{2R} - 144P^8_{4C} + 129P^8_{2L} \\ & + 15P^U_{2R} - 144P^U_{4R} + 228P^U_{8L}\} - \frac{13}{407}\ ,\end{aligned} \tag{3}$$

$$\begin{aligned}\ln\frac{M_C}{M_Z} = {} & \frac{2\pi}{2035\alpha}(315 - 48\sin^2\theta_W - 792\alpha/\alpha_s) - \frac{4816}{2035}\ln\frac{M_8}{M_Z} \\ & - \frac{1}{4070}\{525P^C_Y + 267P^C_{2L} - 792P^C_{3C} + 315P^8_{2R} + 267P^8_{2L} - 582P^8_{4C} \\ & + 315P^U_{2R} - 582P^U_{4R} - 96P^U_{8L}\} - \frac{139}{407}\ .\end{aligned} \tag{4}$$

Model (B):

$$a_{8L} = -14, \quad a_{2R} = \tfrac{26}{3}, \quad a_{4R} = 7\ ,$$

C^U_{8L}, C^U_{2R}, and C^U_{4R} are the same as in model (A). The coefficients at M_8 and M_R are

$$C^8_{2L} = 30, \quad C^8_{3L} = 13, \quad C^8_{3R} = 1, \quad C^8_{(B\text{-}L)_L} = 16, \quad C^8_{(B-L)_R} = 4\ ,$$

$$C^R_{2L} = 0, \quad C^R_{3C} = 0, \quad C^R_Y = \tfrac{6}{5}\ ,$$

leading to

$$\begin{aligned}\ln\frac{M_U}{M_Z} = {} & \frac{\pi}{1688\alpha}(104\alpha/\alpha_s - 96\sin^2\theta_W - 3) + \frac{3811}{3376}\ln\frac{M_8}{M_Z} \\ & + \frac{1}{6752}\{125P^C_Y - 21P^C_{2L} - 104P^C_{3C} + 75P^8_{2R} - 21P^8_{2L} \\ & + 50P^8_{BL} - 104P^8_{3C} + 3P^U_{2R} - 102P^U_{4R} + 192P^U_{8L}\} + \frac{1035}{10\,128}\ ,\end{aligned} \tag{5}$$

$$\begin{aligned}\ln\frac{M_R}{M_Z} = {} & \frac{\pi}{844\alpha}(315 - 48\sin^2\theta_W - 792\alpha/\alpha_s) - \frac{5163}{1688}\ln\frac{M_8}{M_Z} \\ & - \frac{1}{3376}\{525P^C_Y + 267P^C_{2L} - 792P^C_{3C} + 315P^8_{2R} + 267P^8_{2L} \\ & + 210P^8_{BL} - 792P^8_{3C} + 315P^U_{2R} - 582P^U_{4R} - 96P^U_{8L}\} + \frac{703}{1688}\ .\end{aligned} \tag{6}$$

In (3)–(6) we use the notation $P_i^b = P_i(M_\alpha M_b)$ between two successive mass scales $M_b > M_\alpha$.

In models with two intermediate scales, one of the mass scales (M_8) is not determined in terms of the input parameters, but the consistency with the unification scheme is examined by imposing $M_U > M_8 > M_C$ or M_R. In addition to estimating values of mass scales one important advantage of formulas (3)–(6) is that uncertainties in M_U and M_C or M_R arising out of the input parameters of Eq. (1) are derivable in a straightforward manner using the dominant one-loop contributions.

The improved method of solutions for the mass scales consists of exploiting the analytic formulas and the renormalization-group equations (RGE's) together. For example, in model (A), the first step begins by dropping the two-loop contributions in (3) and (4) and obtaining an approximate set of values for (M_U, M_8, M_C). This set is used in (2) to obtain $\alpha_i^{-1}(\mu)$ as a function of μ by iterative convergence procedure. The values of $\alpha_i(M_U)$, $\alpha_i(M_8)$, and $\alpha_i(M_C)$ thus obtained are used to calculate all P_i functions and the two-loop contributions in (3) and (4) resulting in improved values of M_U and M_C in the beginning of the second step while M_8 remains fixed throughout. The second step and subsequent steps are repeated until the values of the mass scales and $\alpha_i(\mu)$ obtained in two successive steps converge. The process is repeated for another set of (M_U, M_8, M_C). The values of $\alpha_i(\mu)$ and mass scales of model (B) are obtained in the same manner as model (A). For every set of solutions in both of the chains the ratio σ is obtained using

$$\sigma = \frac{\alpha_{2L}(M_Z)}{\alpha_{2R}(M_I)}, \quad M_I = M_C \text{ or } M_R \ . \tag{7}$$

As noted in Sec. I, $\sigma \simeq 1$ in the conventional methods of left-right symmetry breaking where the scales of parity and $SU(2)_R$ breakings are identical. A substantial deviation of σ from unity signifies left-right asymmetry in G_{224} or G_{2213} and the possibility of a lower $SU(2)_R$-breaking scale. In the present model $g_{2L}(\mu)$ receives renormalization from the full $SU(8)_L$ gauge group as μ decreases from M_U to M_8 whereas g_{2R} is renormalized by $SU(2)_R \times SU(4)_R^C$. For values of $\mu < M_8$, the asymmetry in the Higgs sector also increases the difference between g_{2L} and g_{2R}. These lead to values of $\sigma \simeq 1.8$–2.3 as shown in Table I and it is this asymmetry which allows G_{224} breaking scale $M_C \ll M_P = M_U$.

The solutions for mass scales with the corresponding σ values are presented in Tables I and II for models (A) and (B), respectively. The unification of the gauge couplings in $SU(8)_L \times SU(8)_R$ in the cases of the two models are presented in Figs. 1 and 2 where α_G represents the GUT coupling constant.

TABLE I. Predictions on mass scales and the ratio of coupling constants (σ) in model (A) where the uncertainty factor in M_U (M_C) is $10^{\pm 0.05}$ ($10^{\pm 0.6}$) due to the input parameters. α_G is the $SU(8)_L \times SU(8)_R$ GUT coupling constant.

M_C (GeV)	M_8 (GeV)	M_U (GeV)	σ	α_G^{-1}
1.3×10^{6}	3.0×10^{18}	8.0×10^{18}	2.3	13.4
2.7×10^{8}	3.2×10^{17}	5.5×10^{17}	2.25	12.7
2.5×10^{11}	1.8×10^{16}	1.8×10^{16}	1.8	11.4

TABLE II. Same as Table I but for model (B) where the uncertainty factor in M_U (M_C) is $10^{\pm 0.06}$ ($10^{\pm 0.7}$).

M_R (GeV)	M_8 (GeV)	M_U (GeV)	σ	α_G^{-1}
6.0×10^{2}	5.5×10^{18}	9.6×10^{18}	1.24	13.0
6.7×10^{3}	2.5×10^{18}	4.0×10^{18}	1.22	12.5
1.1×10^{5}	1.0×10^{18}	1.4×10^{18}	1.26	12.5
1.26×10^{8}	1.0×10^{17}	1.0×10^{17}	1.28	11.6

Compared to SO(10) where the lowest value of the G_{2213} breaking scale achieved is $M_R \simeq 10^7$ GeV with two intermediate gauge symmetries, the alternate mechanism in the present case successfully achieves much lower values of $M_R = 10^{2.8\pm0.7}$ GeV in model (B). The low-mass $W_R^\pm$ and Z_R gauge bosons and the Higgs scalars Δ_R^0, Δ_R^+, and Δ_R^{++} associated with this scale lead to experimentally detectable $V + A$ currents in neutrinoless double β decay, muonium-antimuonium transitions, $K_L - K_S$ mass difference, electric dipole moments of the neutron, and electron 4–5 orders larger than the standard model, and neutrino mass spectrum of the eV-keV-MeV type [15]. Also such low-mass right-handed gauge bosons can be detected at accelerator energies. So far this appears to be the lowest value of M_R predicted in the context of any nonsupersymmetric (non-SUSY) GUT consistent with the CERN-LEP data and minimal fine-tuning of parameters. In the next section we analyze the implications of the G_{224} breaking scale $M_C = 10^6$ GeV.

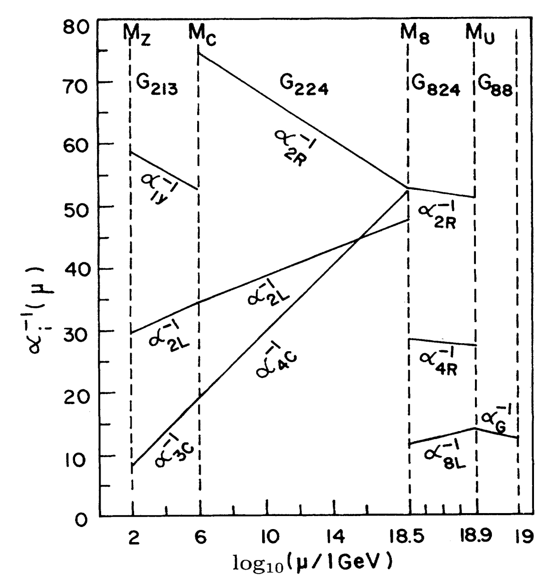

FIG. 1. Coupling-constant trajectories $\alpha_i^{-1}(\mu)$ as a function of mass scale μ in model (A) with two intermediate gauge groups. The scale for ln = 18.5–19 has been enlarged to guide the eye. α_G^{-1} shows the evolution of the GUT coupling constant.

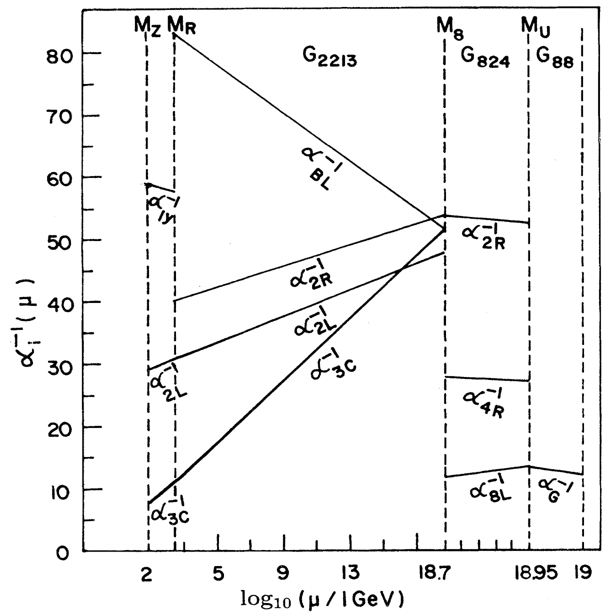

FIG. 2. Same as Fig. 1 but for model (b).

III. PREDICTIONS ON RARE DECAYS, n-$\bar{n}$ AND H-$\bar{H}$ OSCILLATIONS

Analytic expressions for observable quantities are given in the Appendix. Following the analogy of the standard-model Higgs boson whose mass, theoretically, could be a factor 10(1/10) above (below) M_W all the components of $\Delta_R(1,3,\overline{10})$ mediating n-$\bar{n}$ and H-$\bar{H}$ oscillations are now of the order $10^{\pm 1}M_C$. We take the masses to be $10^{\pm \log_{10}\beta}M_C$ with β = 1–10. Using the lowest allowed value of $M_C = 10^{6\pm 0.7}$ GeV in model (A), the predictions for the branching ratios in $K_L \to \mu^\pm e^\mp$ and the mixing times for n-$\bar{n}$ and H-$\bar{H}$ oscillations are [15–17]

$$B(K_L^0 \to \mu^\pm e^\mp) = 10^{-11.2\pm 2.8} \, ,$$

$$\tau_{n\text{-}\bar{n}} = 10^{8.8\pm 3.5\pm \log_{10}\beta} \text{ sec} \, ,$$

$$\tau_{H\text{-}H} = 10^{29.5\pm 5.6\pm \log_{10}\beta} \text{ yr} \, . \qquad (8)$$

The ongoing experiments have already probed into the limit [18]

$$B(K_L^0 \to \mu^\pm e^\mp)_{\text{expt}} \le 10^{-10.4} \, . \qquad (9)$$

In (8) the first (second) uncertainties are due to the input parameters (Higgs scalar masses). It is clear that the central value of the branching ratio predicted is nearly one order less than the current experimental limit, but the uncertainty in the input parameters alone makes it consistent with the limit. The second uncertainty widens the ranges of the predicted values which can be readily computed from (8). For example, with $\beta = 5$, $(\tau_{n\text{-}\bar{n}})_{\max} = 10^{15.8}$ sec and $(\tau_{H\text{-}\bar{H}})_{\max} = 10^{40.7}$ yr. This is for the first time that uncertainties of both types are taken into account in Higgs mediated processes. The gauge-boson-mediated rare decay is affected by the uncertainty in M_C only.

IV. $\Delta(B-L) = -2$ PROTON DECAY

The unification mass in both the chains A and B are high, $M_U \ge 10^{18}$ GeV. Although in the $SU(8)_L \times SU(8)_R$ model there is no gauge-boson-mediated proton decay, such decays in SU(16) are suppressed because of the large values of M_U. However, it has been suggested that in SO(10), even with the G_{224} breaking scale as high as $M_C \simeq 10^{13}$–10^{14} GeV, the new modes of Higgs scalar mediated proton decay corresponding to $\Delta(B-L) = -2$, $\Delta(B+L) = 0$, and $\Delta F = -2$ (F, fermion number) could be experimentally detectable provided the color-triple (ξ_3) and octet (ξ_8) components of $\xi(2,2,15) \subset$ **126** of SO(10) are light ($m_{\xi_9} \simeq m_{\xi_8} \simeq 10^2$ GeV) [6]. An attractive feature of the new model (A') is that the vacuum expectation value (VEV) of the neutral color-singlet (ξ_0) component of $\xi(2,2,15)$ combined with the standard Higgs scalar VEV cures the bad mass relation in SO(10). Since ξ_3 and ξ_8 acquire masses of the order of $M_C \simeq 10^{12}$–10^{14} GeV by extended survival hypothesis [16], a special mechanism has been devised in SO(10) [6] in the presence of the additional Higgs representation **945** to keep these masses light. With the addition of $\xi(2,2,15) \subset (8,\bar{8})$ to model (A) in $SU(8)_L \times SU(8)_R$ or SU(16) the masses of ξ_3 and ξ_8 in model (A) can be easily of the order 10^5–10^6 GeV naturally [16] without requiring the special mechanism. With the color-singlet component in $\xi(2,2,15)$ acquiring a vacuum expectation value $\simeq 10^2$ GeV, the loop coefficients for model (A) are the following.

$M_Z \le \mu \le M_C$:

$$a_i = \begin{pmatrix} \frac{21}{5} \\ -3 \\ -7 \end{pmatrix} ,$$

$$B_{i,j} = \begin{pmatrix} \frac{199}{205} & -\frac{81}{95} & -\frac{44}{35} \\ \frac{9}{41} & -\frac{35}{19} & -\frac{12}{7} \\ \frac{11}{41} & -\frac{27}{19} & \frac{26}{7} \end{pmatrix} .$$

$M_C \le \mu \le M_8$:

$$a_i = \begin{pmatrix} 2 \\ \frac{26}{3} \\ -\frac{7}{3} \end{pmatrix} ,$$

$$B_{ij} = \begin{pmatrix} -\frac{8}{3} & -\frac{9}{26} & -\frac{135}{34} \\ -1 & \frac{584}{11} & -\frac{2295}{46} \\ -\frac{3}{2} & \frac{459}{22} & -\frac{643}{46} \end{pmatrix} .$$

$M_8 \le \mu \le M_U$:

$$a_i = \begin{pmatrix} -\frac{38}{3} \\ 14 \\ \frac{29}{3} \end{pmatrix} ,$$

$$B_{ij} = \begin{pmatrix} -\frac{6407}{76} & \frac{663}{56} & \frac{1395}{29} \\ -\frac{1323}{19} & 48 & \frac{2340}{29} \\ -\frac{5103}{38} & \frac{78}{7} & \frac{3100}{29} \end{pmatrix} . \qquad (10)$$

The formula for the mass scales are

$$\ln\frac{M_U}{M_Z} = \frac{\pi}{1163\alpha}(74\alpha/\alpha_s - 66\sin^2\theta_W - 3) + \frac{1329}{1163}\ln\frac{M_8}{M_Z} + \frac{1}{4652}\{5P_Y^C + 69P_{2L}^C - 74P_{3C}^C + 74P_{2L}^8 + 3P_{2R}^8 - 72P_{4C}^8 + 3P_{2R}^U - 72P_{4R}^U + 132P_{8L}^U\} - \frac{249}{6978}, \quad (11)$$

$$\ln\frac{M_C}{M_Z} = \frac{\pi}{1163\alpha}(315 - 48\sin^2\theta_W - 792\alpha/\alpha_S) - \frac{2405}{1163}\ln\frac{M_8}{M_Z} - \frac{1}{4652}\{525P_Y^C + 267P_{2L}^C - 792P_{3C}^C + 315P_{2R}^8 + 267P_{2L}^8 - 582P_{4C}^8 + 315P_{2R}^U - 582P_{4R}^U - 96P_{8L}^U\} + \frac{2085}{2326}. \quad (12)$$

Following the procedure already described, we compute values of mass scales $\alpha_i^{-1}(\mu)$ and σ presented in Table III. The coupling-constant trajectories are shown in Fig. 3 for the lowest allowed value of M_C in model (A):

$$M_C = 10^{5.64\pm0.6}\ \text{GeV}. \quad (13)$$

Taking the masses of ξ_3 and ξ_8 and the diquark Higgs scalars in $\Delta_R(1,3,\overline{10})$ in the allowed range of $10^{\pm\log_{10}\beta}M_C$ and denoting the Higgs-Yukawa couplings of $\Delta_R(1,3,\overline{10})$ and $\xi(2,2,15)$ as h_Δ and h_ξ, respectively, with λ as the Higgs quartic coupling, the amplitude for $3q \to lq\bar{q}$ has the canonical strength

$$A(3q \to 1q\bar{q}) \simeq \frac{\lambda H_\Delta h_\xi^2 \langle\Delta_R^0\rangle}{M_\xi^4 M_\Delta^2} = 10^{-35.5\pm3.0\pm6\log_{10}\beta}\ \text{GeV}^{-5}, \quad (14)$$

where we have assumed $M_{\xi_3} = M_{\xi_8} = M_\xi$, $g_{2R}(M_C)\langle\Delta_R^0\rangle = M_C$, $\lambda \simeq h_\Delta \simeq 1$, $h_\xi \simeq 10^{-4}$ for the first generation of quarks. This leads to the lifetime predictions

$$\tau_P \simeq 10^{37.6\pm6.0\pm12\log_{10}\beta}\ \text{yr} \quad (15)$$

for the nucleon decay modes: $n \to e^- + (\pi^+\pi)$, $p \to \nu + (e^+\nu)$, $n \to (e^- \text{ or } \mu^-) + (e^+\nu)$, $n \to (e^- \text{ or } \mu^-) + e^+e^- + (\pi^+ \text{ or } K^+)$, etc. In (14) and (15), the second (first) uncertainty is due to those in the Higgs scalar masses (input parameters). We emphasize that even if the central value of τ_P is 4–5 orders larger, the uncertainty in the input parameters alone brings the lifetime well within the accessible range of SuperKamiokande measurements. However, because of large uncertainty in the predicted value of the Higgs scalar masses it is difficult to rule out the model on the basis of proton-lifetime measurements.

TABLE III. Same as model (A) but including $\xi(2,2,15)$ in case of model (A). The uncertainty factor in M_U (M_C) is $10^{\pm0.06}$ ($10^{\pm0.6}$).

M_C (GeV)	M_8 (GeV)	M_U (GeV)	σ	α_G^{-1}
4.4×10^5	3.0×10^{18}	6.3×10^{18}	2.35	6.4
1.8×10^7	5.0×10^{17}	8.2×10^{17}	2.21	7.0
1.7×10^{10}	1.8×10^{16}	1.8×10^{16}	1.85	8.2

The predictions for rare decays, n-$\bar{n}$ and H-$\bar{H}$ oscillations, and the Majorana neutrino masses [15,17,20,21] are

$$B(K_L \to \mu^\pm e^\mp) \simeq 10^{-10.5\pm2.4},$$
$$\tau_{n\text{-}\bar{n}} \simeq 10^{7.4\pm3.0\pm5\log_{10}\beta}\ \text{sec},$$
$$\tau_{H\text{-}\bar{H}} \simeq 10^{27.1\pm4.8\pm8\log_{10}\beta}\ \text{yr},$$
$$m_{\nu_e} \simeq 10^{-2.6\pm0.6}\ \text{eV},$$
$$m_{\nu_\mu} \simeq 10^{2.4\pm0.6}\ \text{eV},$$
$$m_{\nu_\tau} \simeq 10^{0.8\pm0.6}\ \text{MeV}. \quad (16)$$

As in model (A), the cosmological bound on neutrino masses requires that at least ν_τ is made unstable, for exmaple, by the introduction of an additional global lepton number symmetry [19]. The uncertainty in the predicted values increases with values of β. For example, with $\beta = 5$, $(\tau_{n\text{-}\bar{n}})_{\max} = 10^{14}$ sec. $(\tau_{H\text{-}\bar{H}})_{\max} = 10^{37.5}$

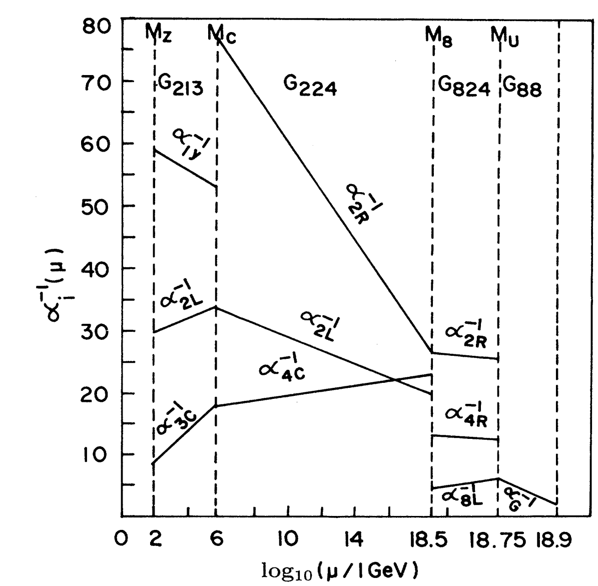

FIG. 3. Same as Fig. 2 but including $\xi(2,2,15)$ as in model (A).

yr and $(\tau_p)_{\max} = 10^{52}$ yr. Thus although consistency of the model predictions can be checked if such Higgs mediated processes are found experimentally, it is extremely difficult to rule out such models in the near future.

V. SUMMARY AND CONCLUSION

We have examined the predictions of SU(16) and $SU(8)_L \times SU(8)_R$ GUT's under the constraints of the CERN-LEP data using the alternative method of decoupling parity and $SU(2)_R$ breakings in the minimal chains. We find that right-handed gauge-boson masses as low as $10^{2.8\pm0.7}$ GeV are allowed in one of the models whereas in SO(10) models with two intermediate symmetries have high values, $M_R \geq 10^7$ GeV. This appears to be the lowest value of M_R predicted in any non-SUSY GUT so far consistent with the CERN-LEP data and minimal fine-tuning of parameters. In addition to manifesting in a number of physical processes such as neutrinoless double β decay, muonium-antimuonium transitions, and contributing significantly to $K_L - K_S$ mass difference, electric dipole moments of the electron and the neutron, and Majorana neutrino mass spectrum of the type eV-keV-MeV for the three generations, such low mass $W_R^{\pm}$ and Z_R bosons could be detected at accelerator energies.

The symmetry-breaking chain for $SU(2)_L \times SU(2)_R \times SU(4)_C$ gauge symmetry is permitted to be $M_C \simeq 10^{6\pm0.7}$ GeV leading to experimentally testable predictions on rare kaon decays, n-$\bar{n}$ and H-$\bar{H}$ oscillations. Including the Higgs scalar multiplet $\xi(2,2,15)$ cures the problem of bad fermion mass relation; at the same time all Higgs scalars mediating $\Delta(B-L) = -2$ proton decay naturally acquire the lighter masses $M_C \simeq 10^{5.6\pm0.6}$ GeV without the necessity of invoking a special mechanism as in the SO(10) model [6]. Although the central value of the proton lifetime is noted to be 4–5 orders larger than the current experimentally accessible limit, the uncertainty in the input values of $\alpha_S(M_Z)$, $\sin^2\theta_W(M_Z)$, and $\alpha^{-1}(M_Z)$ alone makes these predictions consistent with the limit. However, when uncertainties due to the Higgs boson masses mediating the proton decay and matter-antimatter oscillations are taken into account, the predicted values have a wider range. Although the relevant experiments might test the consistency of the models, it is extremly difficult to rule them out on the basis of matter-antimatter oscillations or Higgs mediated proton decay experiments.

In this analysis we have not included threshold effects due to superheavy Higgs scalars and mirror fermions and higher-dimensional operator effects scaled by the inverse powers off the Planck mass. Because of the use of larger Higgs representations of $SU(8)_L \times SU(8)_R$ and SU(16), the threshold uncertainties are expected to be larger. Also as M_U is near the Planck mass, the contribution of higher-dimensional operators could be more significant compared to may other grand unified theories. Both the threshold and higher-dimension operator effects could smear out the predictive power of the models analyzed here.

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APPENDIX

Here we provide analytic expressions for all relevant observable predictions relevant to Secs. III–V of this paper.

1. Rare kaon decays [15]

$$B(K_L \to \mu^{\pm}e^{\mp}) = \frac{\Gamma(K_L \to \mu^{\pm}e^{\mp})}{\Gamma(K^+ \to \mu^+\nu_\mu)} = \frac{4\pi^2 m_K^4 \alpha_S^2(M_C) R}{G_F^2 \sin^2\theta_C m_\mu^2 (m_e + m_d)^2 M_C^4} = 6 \times 10^{13} R \frac{\alpha_S^2(M_C)}{[M_C(\text{GeV})]^4}, \quad \text{(A1)}$$

where

$$R \simeq \left(\frac{\alpha_S(\mu = 1\ \text{GeV})}{\alpha_S(m_b)}\right)^{24/25} \left(\frac{\alpha_S(m_b)}{\alpha_S(m_t)}\right)^{24/23} \times \left(\frac{\alpha_S(m_t)}{\alpha_S(M_C)}\right)^{24/21}$$

Here m_K is the K^0 meson mass; m_μ , m_S, and m_d are the masses of μ^-, s quark, and d quark, respectively, at low energies ($\mu = 1$ GeV); R is the renormalization factor for the quark masses, $\sin\theta_C \simeq 0.22$, $G_F = 1.166 \times 10^{-5}$ GeV^{-2} and $\alpha_s(\mu)$ is the $SU(3)_C$ coupling at μ. We have used $m_t = 174$ GeV.

2. n-$\bar{n}$ and H-$\bar{H}$ oscillations [15-17]

These processes are dominated by the exchanges of the $G_{213}\Delta_R(1,3,10) \subset (1,36)$ of $SU(8)_L \times SU(8)_R$ or **136** of SU(16). The amplitudes for the two processes are

$$G_{n\text{-}\bar{n}} \simeq \frac{\lambda h_\Delta^3 \langle \Delta_R^0 \rangle}{M_\Delta^6},$$

$$G_{H\text{-}\bar{H}} \simeq \frac{\lambda h_\Delta^4}{M_\Delta^8}.$$

Here $\lambda(h_\Delta)$ is the Higgs quartic (Higgs-Yukawa) coupling and all the Higgs scalar masses exchanged are assumed to be degenerate. These lead to the canonical values of the mixing times:

$$\tau_{n\text{-}\bar{n}}(\text{sec}) = \frac{0.954 \times 10^{-21}}{\lambda h_\Delta^3} \frac{M_\Delta^6}{\langle \Delta_R^0 \rangle},$$

$$\tau_{H\text{-}\bar{H}}(\text{yr}) = 3.5 \times 10^{-19} \frac{M_\Delta^8}{\lambda h_\Delta^4}. \quad \text{(A2)}$$

In (A2) all Higgs scalar masses and the VEV $\langle \Delta_R^0 \rangle$ are in GeV.

3. $\Delta(B\text{-}L) = -2$ proton decay [16]

The G_{213} submultiplets of Higgs scalars contained in $\Delta_R(1,3,\overline{10})$ and $\xi(2,2,15)$ mediate this decay mode with the cannonical amplitude

$$A(3q \to 1q\bar{q}) \simeq \frac{\lambda h_\Delta h_\xi^2 \langle \Delta_R^0 \rangle}{M_\xi^4 M_\Delta^2} ,$$

where h_ξ is the Higgs-Yukawa coupling of ξ with mass M_ξ. The masses of the components of ξ and Δ have been assumed to be degenerate at M_ξ and M_Δ , respectively.

4. Neutrino masses [20,21]

The Majorana neutrino masses for the three generations in models with G_{224} breaking scale $M_C \simeq 10^6$ GeV have been computed using the seesaw formula including radiative corrections:

$$m_{\nu_i} \simeq C_{\nu_i} \frac{m_{q_i}^2}{M_C}, \quad i = 1,2,3 ,$$

where $m_{q_i} \simeq m_u$, $m_{q_2} \simeq m_c$, $m_{q_3} \simeq m_t$, $C_{\nu_i} = 0.05$, $C_{\nu_2} = 0.07$, and $C_{\nu_3} = 0.18$.

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