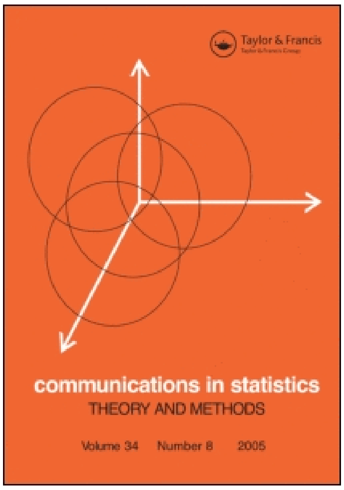

Communications in Statistics - Theory and Methods
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713597238

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Online Publication Date: 01 January 1995

To cite this Article Das, G. and Bez, K.(1995)'Preliminary test estimators in double sampling with two auxiliary variables',Communications in Statistics - Theory and Methods,24:5,1211 — 1226
To link to this Article: DOI: 10.1080/03610929508831548
URL: http://dx.doi.org/10.1080/03610929508831548

COMMUN. STATIST.—THEORY METH., 24(5), 1211-1226 (1995)

PRELIMINARY TEST ESTIMATORS IN DOUBLE SAMPLING WITH TWO AUXILIARY VARIABLES

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Key Words and Phrases:
auxiliary variable; regression estimator; double sampling; preliminary test estimator (PTE)

ABSTRACT

An attempt has been made to suggest some estimators for population mean in double sampling with two auxiliary variables, alternative to the usual regression estimator. When the experimenter has partial information about the mean of the auxiliary variable or variables, preliminary test estimators can be used. The bias, mean square error, relative efficiency and optimum allocation of sample sizes are obtained for the suggested estimators.

1. INTRODUCTION

It is a well known fact that for estimating the population mean of the variable Y, the precision of the estimator can be increased, when information on an auxiliary variable X, highly correlated with Y is readily available on all units of the population, incorporating the knowledge of μ_x, the population mean of X. When the relationship between Y and X is found to be approximately linear, but the line does not go through the origin, linear regression estimate may be used. To use the linear regression estimator it is usually assumed that the population mean μ_x is known. However in certain practical situations μ_x is not known a priori in which case the technique of double sampling is applied. Here one may take a preliminary sample to estimate it.

In certain situations the experimenter may have partial information about μ_x. Han (1973) has suggested the use of double sampling with partial information on the auxiliary variable. In order to utilise the partial information one can perform a preliminary test about the hypothesis that $H_o: \mu_x = \mu_o$ where μ_o is the value obtained from the partial information. After the preliminary sample is obtained, one can test $H_o: \mu_x = \mu_o$ against $H_1: \mu_x \neq \mu_o$. If H_o is accepted, μ_o will be used in the regression estimator; if H_o is rejected, the sample mean based on the preliminary sample is used. This estimator is usually called the preliminary test estimator (PTE).

In estmating the population mean μ_y of the random variable Y, suppose that in addition to information on an auxiliary variable X, information on yet another variable Z is available. When μ_x is not known, we can take a preliminary sample to estimate it. Again if μ_z is also not known, assume that Z is known over another large sample. In such situations an estimator using X and Z is being suggested by Mukerjee *et al* (1987).

2. PTE WITH PARTIAL INFORMATION ON ONE AUXILARY VARIABLE

Suppose, while considering regression estimators with two auxiliary variables in double sampling, partial information about one of the variables, say μ_z is available. In order to utilise the partial information, one can perform a preliminary test about the hypothesis that $H_0: \mu_z = \mu_0$ where μ_0 is the value obtained from the partial information. After the preliminary sample is obtained, $H_0: \mu_z = \mu_0$ can be tested against $H_1: \mu_z \neq \mu_0$. If H_0 is accepted, μ_0 will be used in the regression estimator; if H_0 is rejected, the sample mean based on the preliminary sample is used.

Now we proceed to construct a preliminary test estimator in double sampling with two auxiliary variables, having partial information on only one auxiliary variable. Let (X,Y,Z) have a trivariate normal distribution with mean (μ_x, μ_y, μ_z) and covariance matrix Σ in which the variances are denoted by σ_x^2, σ_y^2 and σ_z^2 and correlation coefficients by ρ_{yx}, ρ_{yz} and ρ_{xz}. The variables X and Z can be readily observed, while it is more expensive to observe the triplet (X,Y,Z). The problem is to estimate μ_y. In practice one can obtain a sample of size n' where both

X and Z are measured, then a subsample of size n is taken from the n′ observations and all three variables X,Y and Z are measured.

If Σ is known, we may let $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$ without loss of generality. The joint distribution of $(\bar{x}_n, \bar{y}_n, \bar{z}_n)$ is normal with mean (μ_x, μ_y, μ_z) and covariance matrix

$$1/n \begin{bmatrix} 1 & \rho_{yx} & \rho_{xz} \\ \rho_{yx} & 1 & \rho_{yz} \\ \rho_{xz} & \rho_{yz} & 1 \end{bmatrix}$$

When μ_z is unknown and the experimenter has partial information about it, a preliminary test $H_o : \mu_z = 0$ (letting $\mu_o = 0$, without loss of generality) can be employed. If H_o is accepted μ_o will be used in the regression estimator; if H_o is rejected the sample mean $\bar{z}_{n'}$ based on the preliminary sample consisting of n′ indepedent observations of Z is used. Since μ_x is totally unknown, it is estimated from another preliminary sample also of size n′. The suggested preliminary test estimator in double sampling with two auxiliary variables, having partial information on one auxiliary variable is defined as

$$t_1 = \begin{cases} \bar{y}_n + B_{yx}(\bar{x}_{n'} - \bar{x}_n) - B_{yz}\bar{z}_n & \text{if } |\bar{z}_{n'}| \le Z_\alpha / n'^{0.5} \\ \bar{y}_n + B_{yx}(\bar{x}_{n'} - \bar{x}_n) + B_{yz}(\bar{z}_{n'} - \bar{z}_n) & \text{if } |\bar{z}_{n'}| > Z_\alpha / n'^{0.5} \end{cases} \tag{2.1}$$

where $B_{yx} = (\rho_{yx} - \rho_{yz}\,\rho_{xz}) / (1 - \rho_{xz}^2)$

and $B_{yz} = (\rho_{yz} - \rho_{yx}\,\rho_{xz}) / (1 - \rho_{xz}^2)$

are population regression coefficients of Y on X and Z respectively, assumed to be known since Σ is known. Also Z_α is the $100(1 - \alpha/2)\%$ point of $N(0, 1)$, α being the level of preliminary test.

2.1 Bias of t_1

To evaluate the bias of t_1, we require the joint distribution of $(\bar{x}_{n'}, \bar{x}_n, \bar{z}_{n'}, \bar{z}_n, \bar{y}_n)$. It can be easily verified that the joint distribution of these is nothing but a multivariate normal with mean $(\mu_x, \mu_x, \mu_z, \mu_z, \mu_y)$ and variance covariance matrix

$$\begin{bmatrix} 1/n' & 1/n' & \rho_{xz}/n' & \rho_{xz}/n' & \rho_{yx}/n' \\ 1/n' & 1/n & \rho_{xz}/n' & \rho_{xz}/n & \rho_{yx}/n \\ \rho_{xz}/n' & \rho_{xz}/n' & 1/n' & 1/n' & \rho_{yz}/n' \\ \rho_{xz}/n' & \rho_{xz}/n & 1/n' & 1/n & \rho_{yz}/n \\ \rho_{yx}/n' & \rho_{yx}/n & \rho_{yz}/n' & \rho_{yz}/n & 1/n \end{bmatrix}$$

Therefore,

$$\begin{aligned} E(t_1) &= E(t_1 \mid |\bar{z}_{n'}| \le Z_\alpha/n'^{0.5}) P(|\bar{z}_{n'}| \le Z_\alpha/n'^{0.5}) \\ &\quad + E(t_1 \mid |\bar{z}_{n'}| > Z_\alpha/n'^{0.5}) P(|\bar{z}_{n'}| > Z_\alpha/n'^{0.5}) \\ &= \mu_y + \text{Bias}(t_1) \end{aligned} \tag{2.2}$$

where

$$\text{Bias}(t_1) = B_{yz}(\phi(A) - \phi(B))/n'^{0.5} - B_{yz}\,\mu_z\,(\Phi(A) - \Phi(B)) \tag{2.3}$$

where $\Phi(.)$ is the cumulative distribution function of $N(0, 1)$ and $\phi(.)$ is the density function and

$$A = Z_\alpha - \mu_z n'^{0.5}, \quad B = -Z_\alpha - \mu_z n'^{0.5}$$

As partial checks it can be seen that Bias $(t_1)=-B_{yz}\mu_z$ when $\alpha = 0$, i.e., when we always accept H_0. Also $Bias(t_1) = 0$ when $\alpha = 1$. Further, the value of $Bias(t_1)$ is symmetrical with respect to μ_z. Hence we need to consider only the behaviour of Bias (t_1) when $\mu_z \geq 0$. In order to get an idea about the behaviour of the bias with respect to μ_z, the values of $Bias(t_1)$ (in absolute values) can be computed for a set of values of n', α, and B_{yz}. We notice that $Bias(t_1)=0$ when $\mu_z=0$. Also, when μ_z increases from 0, $Bias(t_1)$ first increases to a maximum, then decreases to zero. The bias is very close to zero at $\mu_z=1$. The bias found here is quite small almost in all cases. The general behaviour of $Bias(t_1)$ with respect to μ_z is given in Figure 1.

2.2 MSE and Relative Efficiency of t_1

To obtain the mean square error (MSE) of t_1, we notice that

$$MSE(t_1^2)=E(t_1^2)-(E(t_1))^2+(Bias(t_1))^2 \quad (2.4)$$

By using multivariate normal distribution, $E(t_1^2)$ is found to be

$$\begin{aligned} E(t_1^2) &= (\mu_y^2+1/n) \\ &+(1/n-1/n')(B_{yx}^2+B_{yz}^2-2B_{yx}\rho_{yx}-2B_{yz}\rho_{yz}+2B_{yx}B_{yz}\rho_{xz}) \\ &+(\Phi(A)-\Phi(B))(B_{yz}^2(\mu_z^2+1/n')-2B_{yz}(\mu_y\mu_z+\rho_{yz}/n')) \\ &-(\phi(A)-\phi(B))(2B_{yz}^2\mu_z-2B_{yz}(\mu_y+\rho_{yz}\mu_z))/n'^{0.5} \\ &-(A\phi(A)-B\phi(B))(B_{yz}^2-2B_{yz}\rho_{yz})/n' \quad (2.5) \end{aligned}$$

Substituting (2.2), (2.3) and (2.5) in (2.4) we obtain

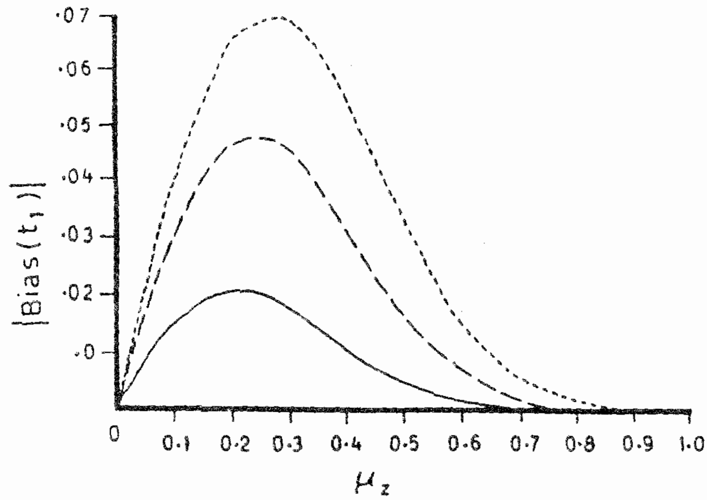

FIGURE 1.: Behaviour of Bias(t_1) with respect to μ_z for $n' = 30$, $\rho_{xz} = 0.6$, $\rho_{yx} = 0.7$, $\rho_{yz} = 0.8$.
......, $\alpha = 0.05$; ------, $\alpha = 0.10$; ———, $\alpha = 0.25$.

$$MSE(t_1) = g_1 + h_1 \qquad (2.6)$$

where $g_1 = (1-\rho^2_{y.xz})/n + \rho^2_{y.xz}/n'$

$$\rho^2_{y.xz} = (\rho^2_{yx}+\rho^2_{yz}-2\rho_{yx}\rho_{yz}\rho_{xz})/(1-\rho^2_{xz})$$

and $h_1 = (\Phi(A)-\Phi(B))(B^2_{yz}(\mu^2_z+1/n')-2B_{yz}\rho_{yz}/n')$

$$-(\phi(A)-\phi(B))(2B^2_{yz}\mu_z-2B_{yz}\rho_{yz}\mu_z)/n'^{0.5}$$

$$-(A\phi(A)-B\phi(B))(B^2_{yz}-2B_{yz}\rho_{yz})/n'$$

The quantity g_1 is the variance of the estimator t_2 (Mukerjee *et al*; (1987)), the linear regression estimator using two auxiliary variables in double sampling, which under the assumption that Σ is known and $\sigma^2_x=\sigma^2_y=\sigma^2_z=1$ is defined as follows:

$$t_2=\bar{y}_n + B_{yx}(\bar{x}_{n'} - \bar{x}_n)+B_{yz}(\bar{z}_{n'} - \bar{z}_n) \qquad (2.7)$$

The relative efficiency of t_1 to t_2 is defined as

$$e_1 = g_1 / (g_1 + h_1) \tag{2.8}$$

The values of e_1 can be easily computed for different values of μ_z. We notice that e_1 is symmetric about $\mu_z=0$, hence we need to consider only $\mu_z \geq 0$. In order to get an idea about the behaviour of the relative efficiency function with respect to μ_z, e_1 can be computed for a set of values of n, n', α and B_{yz}. It is found that in general e_1 has a maximum at $\mu_z = 0$. As μ_z increases, e_1 decreases to a minimum and then increases to unity. It is also found that e_1 is very close to 1 at $\mu_z = 1$. In general, the behaviour of e_1 is given in Figure 2.

2.3 Optimum allocation

Now we try to find out for a given cost function, what is the optimum allocation of sample sizes n' and n? Let the cost function be of the form

$$C = nc_1 + n'c_2 + n'c_3$$

where c_1, c_2 and c_3 are costs of observing Y, X and Z respectively. Or equivalently

$$C = nc_1 + n'c_1' \text{ where } c_1' = c_2 + c_3 \tag{2.9}$$

The values of n and n' are obtained by minimising $MSE(t_1)$ subject to the cost constraint (2.9). Following the derivation as in Han (1973), we obtain the optimum allocation when $\mu_z=0$ as follows

$$n = Ck^{0.5} / (c_1^{0.5} ((kc_1)^{0.5} + (k'c_1')^{0.5})) \tag{2.10}$$

and $$n' = Ck'^{0.5} / (c_1'^{0.5} ((kc_1)^{0.5} + (k'c_1')^{0.5})) \tag{2.11}$$

and the optimum value of $MSE(t_1)$ as

$$MSE_{opt}(t_1) = ((kc_1)^{0.5} + (k'c_1')^{0.5})^2 / C \tag{2.12}$$

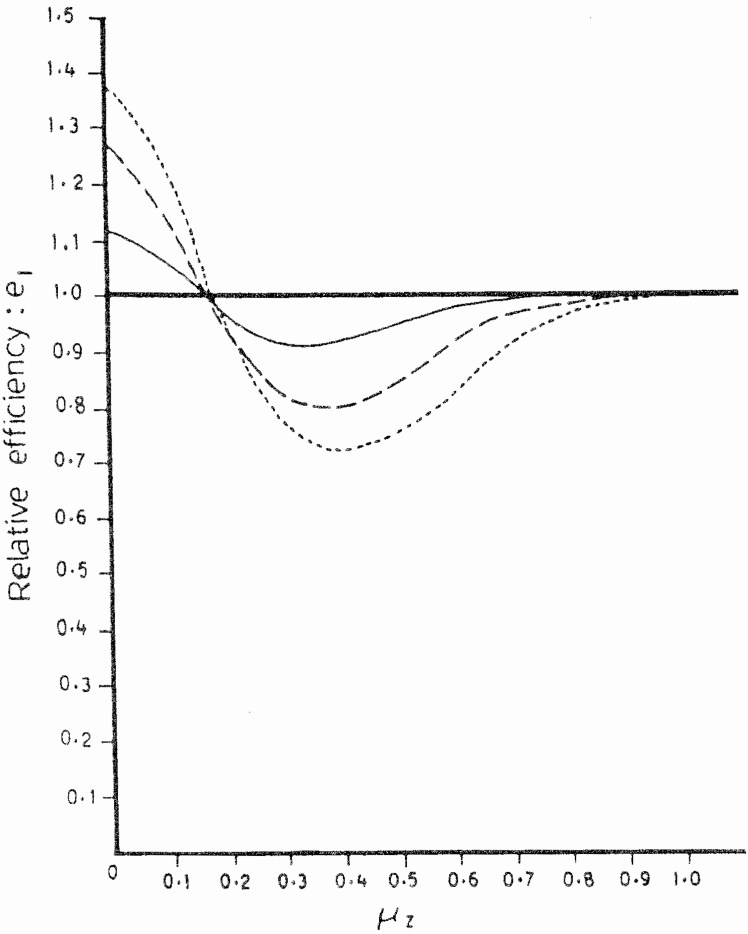

FIGURE 2.: Behaviour of Relative Efficiency e_1 with respect to μ_z for $n' = 30$, $n = 10$, $\rho_{xz} = 0.6$, $\rho_{yx} = 0.7$, $\rho_{yz} = 0.8$.
......, $\alpha = 0.05$, ------, $\alpha = 0.10$, ———, $\alpha = 0.25$.

where $k = 1 - \rho^2_{y.xz}$

and $k' = (\rho^2_{y.xz} - \rho^2_{yz} + (\alpha + 2Z_\alpha \phi(Z_\alpha))(\rho^2_{yz} - \rho^2_{y.xz}\rho^2_{xz}))/(1-\rho^2_{xz})$

We may now compare $MSE_{opt}(t_1)$ with the minimum variance of t_2, the regression estimator with two auxiliary varibles under double sampling without using preliminary test. From (2.6), the variance of t_2 is g_1 i.e.

$$V(t_2) = (1-\rho^2_{y.xz})/n + \rho^2_{y.xz}/n' \qquad (2.13)$$

and the minimum variance subject to the cost constraint (2.9)is

$$V_{opt}(t_2) = (c_1'^{0.5}\rho_{y.xz} + c_1^{0.5}(1-\rho^2_{y.xz}))^2 / C \qquad (2.14)$$

In order to compare (2.12) and (2.14), we observe that $\alpha+2Z_\alpha\phi(Z_\alpha)$ is a decreasing function of Z_α with a maximum equal to unity at $Z_\alpha=0$. Therefore we conclude that $MSE_{opt}(t_1) \le V_{opt}(t_2)$ provided $\rho^2_{yz} \ge \rho^2_{y.xz}\ \rho^2_{xz}$ with equality holding for $Z_\alpha=0$, which is the case when the two estimators coincide.

3.PTE WITH PARTIAL INFORMATION ON TWO AUXILARY VARIABLES

Next, we proceed to construct a preliminary test estimator in double sampling with two auxiliary variables having partial information on both the auxiliary variables. The assumption about the distribution of (X,Y,Z) is as in Section 2. The covariance matrix Σ is known with $\sigma^2_x = \sigma^2_y = \sigma^2_z = 1$.

When μ_x ,μ_z are unknown and the experimenter has partial information on them, say μ_{ox} and μ_{oz} respectively, he can employ preliminary tests for testing $H_{o1}: \mu_x = 0$ and $H_{o2}: \mu_z = 0$ (letting $\mu_{ox} = \mu_{oz} = 0$ without loss of generality). If H_{o1} is accepted, μ_{ox}

will be used in the regression estimator. However if H_{01} is rejected then the sample mean $\bar{x}_{n'}$ is used. Similarly if H_{02} is accepted, μ_{0z} will be used, otherwise the sample mean $\bar{z}_{n'}$ is used. Based on the above, the suggested preliminary test estimator is given by

$$t_s = \begin{cases} \bar{y}_n - B_{yx}\bar{x}_n - B_{yz}\bar{z}_n & \text{if } |\bar{x}_{n'}| \le Z_\alpha/n'^{0.5}, |\bar{z}_{n'}| \le Z_\alpha/n'^{0.5} \\ \bar{y}_n - B_{yx}(\bar{x}_{n'} - \bar{x}_n) - B_{yz}\bar{z}_n & \text{if } |\bar{x}_{n'}| > Z_\alpha/n'^{0.5}; |\bar{z}_{n'}| \le Z_\alpha/n'^{0.5} \\ \bar{y}_n - B_{yx}\bar{x}_n + B_{yz}(\bar{z}_{n'} - \bar{z}_n) & \text{if } |\bar{x}_{n'}| \le Z_\alpha/n'^{0.5}; |\bar{z}_{n'}| > Z_\alpha/n'^{0.5} \\ \bar{y}_n + B_{yx}(\bar{x}_{n'} - \bar{x}_n) + B_{yz}(\bar{z}_{n'} - \bar{z}_n) & \text{if } |\bar{x}_{n'}| > Z_\alpha/n'^{0.5}, \\ & \quad |\bar{z}_{n'}| > Z_\alpha/n'^{0.5} \end{cases} \tag{3.1}$$

3.1 Bias of t_s

To evaluate the bias of t_s, the joint distribution of $(\bar{x}_{n'}, \bar{x}_n, \bar{z}_{n'}, \bar{z}_n, \bar{y}_n)$ is required, which is same as described in Section 2 and we obtain

$$\begin{aligned} \text{Bias}(t_s) = {} & B_{yx}(\phi(a)-\phi(b))/n'^{0.5} - B_{yx}\mu_x(\Phi(a)-\Phi(b)) \\ & + B_{yz}(\phi(A)-\phi(B))/n'^{0.5} - B_{yz}\mu_z(\Phi(A)-\Phi(B)) \end{aligned} \tag{3.2}$$

where $a = Z_\alpha - \mu_x n'^{0.5}$ and $b = -Z_\alpha - \mu_x n'^{0.5}$

As partial checks it can be seen that $\text{Bias}(t_s) = -B_{yx}\mu_x - B_{yz}\mu_z$ when $\alpha=0$, that is when we always accept H_{01} and H_{02}; and $\text{Bias}(t_s)=0$ when $\alpha=1$. The values of $\text{Bias}(t_s)$ (in absolute values), computed for a set values of n', α, B_{yx} and B_{yz} show that $\text{Bias}(t_s)=0$ when $\mu_x=\mu_z=0$ and as (μ_x, μ_z) increases from $(0,0)$, $\text{Bias}(t_s)$ increases to a maximum, then decreases to zero. Bias is very close to zero when $\mu_x=\mu_z=1$. The general behaviour

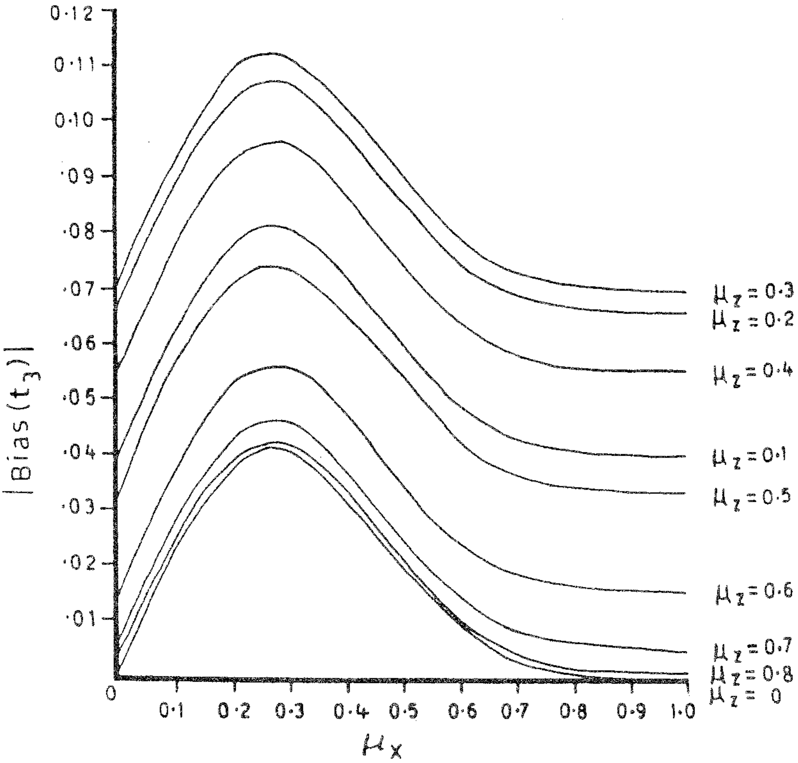

FIGURE 3.: Behaviour of Bias(t_3) with respect to μ_x and μ_z for $n' = 30$, $\alpha = 0.05$, $\rho_{xz} = 0.6$, $\rho_{yx} = 0.7$, $\rho_{yz} = 0.8$.

of Bias(t_3) with respect to μ_x and μ_z is shown in Figure 3 and is similar to that of Bias(t_1).

3.2 MSE and Relative Efficiency of t_3

The mean square error of t_3 is found to be

$$MSE(t_3) = g_1 + h_1' \tag{3.3}$$

where g_1 is same in (2.6) and

$$h_1' = (\Phi(a)-\Phi(b))B_{yx}(B_{yx}(\mu_x^2+1/n')-2\rho_{yx}/n'+B_{yz}(\mu_x\mu_z+\rho_{xz}/n'))$$
$$-(\phi(a)-\phi(b))B_{yx}(2\mu_x B_{yx}-2\mu_x\rho_{yx}+B_{yz}(\mu_z+\rho_{xz}\mu_x))/n'^{0.5}$$
$$-(a\phi(a)-b\phi(b))B_{yx}(B_{yx}-2\rho_{yx}+B_{yz}\rho_{xz})/n'$$
$$+(\Phi(A)-\Phi(B))B_{yz}(B_{yz}(\mu_z^2+1/n')-2\rho_{yz}/n'+B_{yx}(\mu_x\mu_z+\rho_{xz}/n'))$$
$$-(\phi(A)-\phi(B))B_{yz}(2\mu_z B_{yz}-2\mu_z\rho_{yz}+B_{yx}(\mu_x+\rho_{xz}\mu_z))/n'^{0.5}$$
$$-(A\phi(A)-B\phi(B))B_{yz}(B_{yz}-2\rho_{yz}+B_{yx}\rho_{xz})/n'$$

The relative efficiency of t_3 to t_2 is defined as:

$$e_2 = g_1 \;/\; (g_1 + h_1') \tag{3.4}$$

The value of e_2 can be easily computed for different values of n, n', α, B_{yx}, and B_{yz}. In general e_2 has a maximum at $\mu_x=\mu_z = 0$. It decreases to a minimum and then increases to unity as (μ_x, μ_z) increases from (0,0) It is also found that e_2 is very close to 1 at $\mu_x = \mu_z = 1$.The general behaviour of e_2 with respect to μ_x and μ_z is represented in Figure 4.

Again following the same derivation as in Han (1973) we obtain the optimum allocation when $\mu_x = \mu_z = 0$ as follows

$$n = CK^{0.5} / (c_1^{0.5} ((Kc_1)^{0.5}+(K'c_1')^{0.5})) \tag{3.5}$$

$$\text{and } n' = CK'^{0.5}/(c_1'^{0.5} ((Kc_1)^{0.5}+(K'c_1')^{0.5})) \tag{3.6}$$

and the optimum value of MSE(t_3) as

$$MSE_{opt}(t_3) = ((Kc_1)^{0.5} + (K'c_1')^{0.5})^2/C \tag{3.7}$$

where $K = 1 - \rho^2_{y.xz}$

and $K' = (\alpha + 2Z_\alpha\,\phi(Z_\alpha))\rho^2_{y.xz}$

In order to compare $MSE_{opt}(t_3)$ and $V_{opt}(t_2)$ we observe (2.14) and (3.7) and notice that since $\alpha + 2Z_\alpha\phi(Z_\alpha)$ is a decreasing function of Z_α with a maximum equal to unity

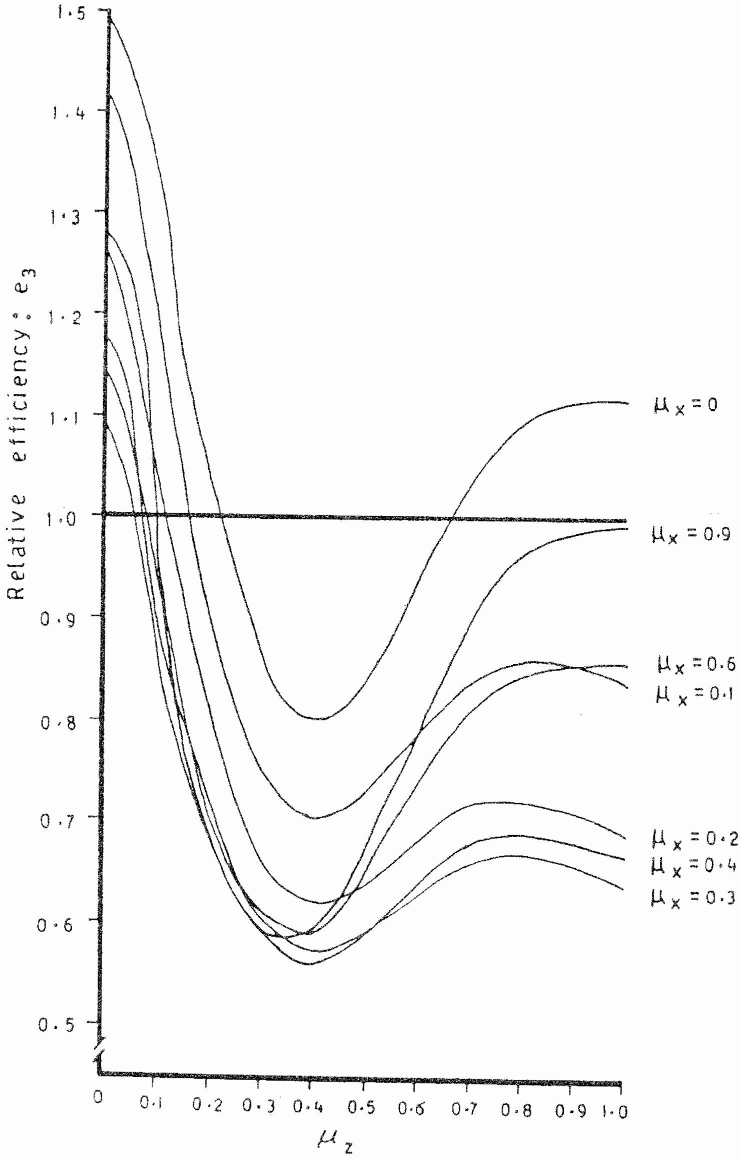

FIGURE 4.: Behaviour of relative efficiency e_z with respect to μ_x and μ_z for $n' = 80$, $n = 10$, $\alpha = 0.05$, $\rho_{xz} = 0.6$, $\rho_{yx} = 0.7$, $\rho_{yz} = 0.8$.

at $Z_\alpha = 0$, the $V_{opt}(t_2)$ is at least as large as that of $MSE_{opt}(t_8)$. Thus we conclude that

$$MSE_{opt}(t_8) \leq V_{opt}(t_2) \tag{3.8}$$

with equality holding for $Z_\alpha = 0$, which is the case when the two estimators coincide.

Similarly comparision can also be made between $MSE_{opt}(t_8)$ and $MSE_{opt}(t_1)$. One can easily observe from (2.12) and (3.7) that since $\alpha + 2 \ Z_\alpha \ \ \phi(Z_\alpha)$ is a decreasing function of Z_α with a maximum equal to unity at $Z_\alpha = 0$ and $\rho^2_{yz} \leq \rho^2_{y.xz}$, therefore,

$$MSE_{opt}(t_8) \leq MSE_{opt}(t_1) \tag{3.9}$$

4. CONCLUSION

We have shown in Section 2 that under certain condition PTE in double sampling with two auxiliary variables having partial information on only one auxiliary variable is more efficient than regression estimators in double sampling with two auxiliary variables. In Section 3 we also proved that under the optimum condition MSE of PTE in double sampling with two auxiliary variables is less than the MSE of usual regression estimator in double sampling with two auxiliary variables. Lastly from (3.9) we find that the PTE in double sampling with two auxiliary variables having partial information on both the auxiliary variables is better than the PTE in double sampling with two auxiliary variables having partial information on only one auxiliary variable. Hence from the proceedings of the previous two sections we may conclude that under the stated assumptions PTE is proved to be more efficient than the usual regression estimator.

ACKNOWLEDGEMENTS

The authors express their sincere thanks to the referees for their valueable comments which has helped to revise the manuscript.

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Received March, 1994; Revised December, 1994.