Confronting CERN LEP data, the proton lifetime, and small neutrino masses by threshold effects in SO(10) with $SU(2)_L \times U(1)_R \times SU(4)_C$ intermediate breaking

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We derive analytic formulas for mass scales and the GUT coupling constant in SO(10) with $SU(2)_L \times U(1)_R \times SU(4)_C$ ($\equiv G_{214}$) intermediate breaking including two-loops and threshold effects. We find that the mass-scale predictions are in agreement with the CERN LEP data and proton-lifetime limit provided threshold effects due to heavy Higgs scalars are included. In one case the predicted proton lifetime is close to the experimental limit, while the ν_e mass is small, the ν_μ mass is accessible to laboratory experiments, and the ν_τ mass is consistent with the 17 keV neutrino. In the most interesting case the predicted proton lifetime is accessible to the Superkamiokande experiments and the ν_e and ν_μ masses have the right values needed for understanding the solar neutrino problem using the Mikheyev-Smirnov-Wolfenstein mechanism. The ν_τ mass is found to be consistent with the requirement for the dark matter of the Universe.

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I. INTRODUCTION

Precision measurements from the CERN e^+e^- collider (LEP) have revived interest in grand unified theories (GUT's) [1] with or without supersymmetry (SUSY). Although the minimal SU(5) model has been decisively ruled out by the available data on $\sin^2\theta_W$ and the proton lifetime (τ_p), SUSY SU(5) and non-SUSY models with intermediate symmetries or additional degrees of freedom [2–5] are quite consistent with them. The SO(10) model has been investigated in detail before [6] and after [4,5,7–9] the LEP measurements. The GUT contains the left-right symmetric Pati-Salam gauge group [1] as its subgroup and undergoes spontaneous symmetry breaking to the standard model through various routes providing the option of breaking left-right discrete symmetry ($\equiv$parity) at the GUT scale or keeping it intact down to the intermediate scale. In addition to being consistent with the LEP data and the experimental limit on τ_p the model has the potential to explain neutrino masses over a wide range of values through the natural seesaw mechanism or induced contributions [10–12].

While analyzing the implications of LEP and proton lifetime measurements, recently, Deshpande, Keith, and Pal [9] have ruled out the chain

$$\begin{aligned} SO(10) &\xrightarrow{M_U} SU(2)_L \times U(1)_R \times SU(4)_C (\equiv G_{214}) \\ &\xrightarrow{M_C} SU(2)_L \times U(1)_Y \times SU(3)_C (\equiv G_{213}) \\ &\xrightarrow{M_Z} U(1)_{em} \times SU(3)_C \ . \end{aligned} \quad (1)$$

But threshold effects due to superheavy-Higgs-scalar masses have been found to modify the GUT prediction substantially in other models [5,7,8,12,14,15]. The purpose of this paper is to show that the model (1) is consistent with the available data from LEP and proton-lifetime measurements when threshold effects are included. An interesting feature of the model is that the proton lifetime imposes a lower bound on the heavy-Higgs-boson-mass-splitting coefficient $\beta \geq 7$. The masses of ν_e and ν_μ predicted by the model with such values of β are quite consistent with those needed for understanding the solar neutrino puzzle using the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism. The ν_τ mass is consistent with the requirement of the cosmological dark matter of the Universe.

In Sec. II we derive analytic formulas for mass scales and the GUT coupling constant including two-loop and threshold effects. The new method of computation has been also explained in this section. The numerical computation of threshold corrections is presented in Sec. III where the implications on the proton lifetime and neutrino masses are also discussed. Finally the summary and conclusion of this work are presented in Sec. IV.

II. ANALYTIC FORMULAS FOR MASS SCALES, GUT COUPLING, AND THRESHOLD EFFECTS

The Higgs representation **54** and **45** of SO(10) are used to break $SO(10) \to G_{214}$ in the first step. In the second step **126** is used to break $G_{214} \to G_{213}$ and generate Majorana neutrino masses by the seesaw mechanism. The representation **10** contains the standard Higgs doublet and is used to break the electroweak symmetry at the final stage. The renormalization group equation (RGE) for the gauge couplings $g_i(\mu)$ of the standard group G_{213} is written as

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$$\mu\frac{\partial\alpha_i}{\partial\mu}=\frac{a_i}{2\pi}\alpha^2(\mu)+\frac{1}{8\pi^2}\sum_{j=1}^{3}b_{ij}\alpha_i^2(\mu)\alpha_j(\mu)\ ,\qquad i,j=Y,2L,3C\ ,\tag{2}$$

where $\alpha_i=g_i^2/4\pi$ and $a_i(b_{ij})$ are one- (two-) loop coefficients [6]. For the evolution of the gauge couplings of the G_{214} group the same form of Eq. (2) is used with $a_i\rightarrow a_i'$ and $b_{ij}\rightarrow b_{ij}'$ where $i,j=2L,1R,4C$ [9]. Threshold effects at $\mu=M_U$ are taken into account through the matching functions $\lambda_i^u=\lambda_i(\mu=M_U)$ which modify the conventional GUT boundary conditions [13]:

$$\alpha_i^{-1}(M_U)=\alpha_G^{-1}-\lambda_i^U/12\pi\ ,\quad i=2L,1R,4C\ .\tag{3}$$

The threshold effects at $\mu=M_C$ are included using the corresponding boundary correction through the matching function $\lambda_i^C=\lambda_i(\mu=M_C)$. Assuming only the existence of 3 light fermion generations the λ functions are computed using Ref. [13].

Using the combination of $\alpha^{-1}(M_Z)-\frac{8}{3}\alpha_2^{-1}(M_Z)$ and $\alpha^{-1}(M_Z)-\frac{8}{3}\alpha_3^{-1}(M_Z)$ two simultaneous equations involving $\ln(M_U/M_Z)$ and $\ln(M_C/M_Z)$ are derived. These are solved to obtain the analytic formulas

$$\ln\frac{M_U}{M_Z}=\frac{4\Pi}{245\alpha}\left[16-13\frac{\alpha}{\alpha_S}-3\sin^2\theta_W\right]-\frac{1}{245}(6P_{1R}^U-9P_{4C}^U+3P_{2L}^U+10P_Y^C+3P_{2L}^C-13P_{3C}^C)+\Delta_U\ ,\tag{4}$$

$$\ln\frac{M_C}{M_Z}=\frac{3\pi}{245\alpha}\left[\frac{39}{2}+19\frac{\alpha}{\alpha_S}-71\sin^3\theta_W\right]-\frac{9}{1960}\left[13P_{1R}^U+\frac{64}{3}P_{4C}^U-\frac{103}{3}P_{2L}^U+\frac{65}{3}P_Y^C-\frac{103}{3}P_{2L}^C+\frac{38}{3}P_{3C}^C\right]+\Delta_C\ ,\tag{5}$$

$$\alpha_G^{-1}=\frac{1}{1470\alpha}\left[228+247\frac{\alpha}{\alpha_S}-1356\sin^2\theta_W\right]-\frac{1}{2940\pi}\times(114P_{1R}^U-171P_{4C}^U+792P_{2L}^U+190P_Y^C+792P_{2L}^C-247P_{3C}^C)+\Delta_G\ .\tag{6}$$

where the threshold contributions are

$$\Delta\ln\frac{M_U}{M_Z}\equiv\Delta_U=\frac{1}{735}(6\lambda_{1R}^U-9\lambda_{4C}^U+3\lambda_{2L}^U+10\lambda_Y^C+3\lambda_{2L}^C-13\lambda_{3C}^C)$$

$$\Delta\ln\frac{M_C}{M_Z}\equiv\Delta_C=\frac{3}{1960}\left[13\lambda_{1R}^U+\frac{64}{3}\lambda_{4C}^U-\frac{103}{3}\lambda_{2L}^U+\frac{65}{3}\lambda_Y^C-\frac{103}{3}\lambda_{2L}^C+\frac{38}{3}\lambda_{3C}^C\right]$$

$$\Delta\alpha_G^{-1}\equiv\Delta_G=-\frac{1}{8820\pi}(114\lambda_{1R}^U-171\lambda_{4C}^U+792\lambda_{2L}^U+190\lambda_Y^C+792\lambda_{2L}^C-247\lambda_{3C}^C)\ ,\tag{7}$$

where Eq. (6) has been obtained using (4) and (5) in the evolution equation for $\alpha^{-1}(M_Z)=5/3\alpha_Y^{-1}(M_Z)+\alpha_{2L}^{-1}(M_Z)$ [15]. In (4)–(6),

$$P_i^U=\sum_j B_{ij}'X_j^u\ ,\quad P_i^C=\sum B_{ij}X_j^C\ ,$$

$$X_j^U=\ln[\alpha_j(M_U)/\alpha_j(M_C)]\ ,$$

$$X_j^C=\ln[\alpha_j(M_C)/\alpha_j(M_Z)]\ ,\quad B_{ij}'=b_{ij}'/a_j'\ ,\quad B_{ij}=b_{ij}/a_j\ .$$

In the minimal SU(5) the solutions to the RGE's predict $\sin^2\theta_W$ and M_U in terms of α_s, α, and two-loop conributions and the corresponding analytic expressions have been obtained earlier [16]. In the single-intermediate-scale models, the mass scales are predicted in terms of $\sin^2\theta_W$, α, α_s, and two-loop terms. Derived here for the first time in model (1) the analytic formulas (4)–(6) provide M_U^0,M_C^0 and α_G^0 up to two-loop order in terms of α, α_s, and $\sin^2\theta_W$. With future improvement on precision measurements, these formulas can be used to make accurate predictions on the model. In addition, they contain analytic expressions for the threshold effects. Furthermore, the experimental uncertainties on M_U, M_C, α_G^{-1} are now readily evaluated from the most dominant one-loop terms in each case. We use the input parameters at the Z mass, $\sin^2\theta_W=0.2333\pm0.0008$, $\alpha_s=0.120\pm0.007$, $\alpha^{-1}=127.9\pm0.2$, and the limit on the proton lifetime for the $p\rightarrow e^+\pi^0$ mode [9]:

$$(\tau_p)_{\rm expt}\geq 3\times10^{32}\ {\rm yr}\ .\tag{8}$$

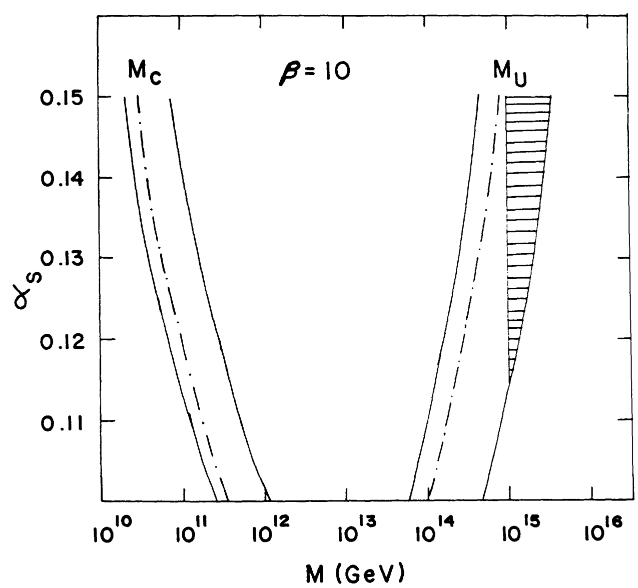

FIG. 1. Variation of mass scales (dot-dashed lines) and threshold effects (solid lines) as a functioin of α_s for $\beta=10$ with extremized $\Delta\ln(M_U/M_Z)$.

We adopt an improved procedure for computation where analytic formulas (4)–(6) are combined with the iterative convergence procedure for solutions to RGE's. In the first step the inputs M_U^0, M_C^0, and α_G^0 are estimated up to one loop through (4)–(6) to know the ranges of integration and the unknown coupling needed for the solutions to the RGE's. The values of the coupling constants are obtained for every μ by integrating (2) and its other form for $M_C^0-M_U^0$ when the solutions converge. These couplings are used to estimate the two-loop terms in (4)–(6) to obtain improved values of M_U^0, M_C^0, and α_G^0 in the second step. The steps are repeated until the values of M_U^0, M_C^0, and α_G^0 obtained in two sucessive steps are identical. In fact after five steps the solutions converge with

$$M_U^0=10^{14.5\pm0.2}\ \text{GeV}\ ,$$
$$M_C^0=10^{11\pm0.4}\ \text{GeV}\ ,\ \alpha_G^{0^{-1}}=44.32\pm0.35\ , \tag{9}$$

where the uncertainties have been computed from those of $\sin^2\theta_W$, α_s, and α at M_Z using the dominant one-loop contributions. The dot-dashed line in Fig. 1 expresses the variation of M_U^0 and M_C^0 with α_s. It is clear that although the model is consistent with the LEP data, M_U^0 is lower than the value necessary for the experimental limit of proton lifetime as noted in [9]. From Fig. 1 we also note an anticorrelation between M_U^0 and M_C^0 showing increase (decrease) of M_U^0 (M_C^0) with α_s.

III. HIGGS-SCALAR CONTRIBUTION TO THRESHOLD EFFECTS, PROTON STABILITY, AND NEUTRINO MASSES

Using extended survival hypothesis [17] the components of the Higgs representations present near M_C and M_U can be specified. To compute the matching functions we identify the G_{214} content near M_U and the G_{213} content near M_C corresponding to the relevant SO(10) representations.

$$\begin{aligned}
&\mu\simeq M_U:\\
&\quad 10\supset M_{H_1}(2,-1/2,1)+M_{H_2}(1,0,6)\ ,\\
&\quad 126\supset M_{H_1}(1,0,6)+M'_{H_2}(3,0,10)+M'_{H_3}(1,0,\overline{10})+M'_{H_4}(1,-1,\overline{10})+M'_{H_5}(2,1/2,15)+M'_{H_6}(2,-1/2,15)\ ,\\
&\quad 45\supset M'_{S_1}(3,0,1)+M'_{S_2}(1,0,15)\ ,\\
&\quad 54\supset M''_{S_1}(3,1,1)+M''_{S_2}(3,0,1)+M''_{S_3}(3,-1,1)+M''_{S_4}(1,0,20)N+M''_{S_5}(2,1/2,6)+M''_{S_6}(2,-1/2,6)\ , \qquad (10)\\
&\mu\simeq M_C:\\
&\quad 126\supset M_{C_1}(1,4/3,\overline{6})+M_{C_2}(1,4/3,\overline{3})\ .
\end{aligned}$$

Including these Higgs-scalar contributions gives [14]

$$\begin{aligned}
\Delta_U=\frac{1}{735}(&21+9\eta_{H_1}-18\eta_{H_2}-18\eta'_{H_1}-42\eta'_{H_2}-54\eta'_{H_3}+66\eta'_{H_4}-9\eta'_{H_5}-9\eta'_{H_6}+6\eta'_{S_1}\\
&-36\eta'_{S_2}+48\eta''_{S_1}+12\eta''_{S_2}+48\eta''_{S_3}-117\eta''_{S_4}+18\eta''_{S_5}+18\eta''_{S_6}-33\eta_{C_1}+3\eta_{C_2})
\end{aligned} \tag{11}$$

$$\begin{aligned}
\Delta_C=\frac{1}{1960}(&14/3-64\eta_{H_1}+128\eta_{H_2}+128\eta'_{H_1}-2968\eta'_{H_2}+64\eta'_{H_5}+64\eta'_{H_6}-206\eta'_{S_1}\\
&+256\eta'_{S_2}-178\eta''_{S_1}-412\eta''_{S_2}-178\eta''_{S_3}+832\eta''_{S_4}-128\eta''_{S_5}-128\eta''_{S_6}+398\eta_{C_1}+142\eta_{C_2})
\end{aligned} \tag{12}$$

$$\begin{aligned}
\Delta_G=-\frac{1}{8820}\pi(&4809+906\eta_{H_1}-342\eta_{H_2}-342\eta'_{H_1}+28602\eta'_{H_2}-1026\eta'_{H_3}+1254\eta'_{H_4}\\
&+10854\eta'_{H_5}+10854\eta'_{H_6}+1584\eta'_{S_1}-684\eta'_{S_2}+3852\eta''_{S_1}+3168\eta''_{S_2}+3852\eta''_{S_3}\\
&-2223\eta''_{S_4}+4752\eta''_{S_5}+4752\eta''_{S_6}+361\eta_{C_1}+57\eta_{C_2})
\end{aligned} \tag{13}$$

where $\eta_{C_i}=\ln M_{C_i}/M_C$, $i=1,2$ and $\eta_j=\ln M_j/M_U$, $j=$ all other indices, and η_i independent terms are due to degenerate superheavy gauge bosons at M_U or M_C.

It is easy to check that parity restoration for $\mu\geq M_U$ in the presence of SO(10) implies $\eta'_{H_i}\leq0$, $i=2,3,4$ and $\eta_{H_1}\leq0$, $\eta'_{S_1}\leq0$. Also $\eta'_{H_i}\leq0$, $\eta''_{S_i}\leq0$, $i=5,6$ or else $\eta'_{H_5}=\eta'_{H_6}\geq0$, $\eta''_{S_5}=\eta''_{S_6}\geq0$. Although these constraints do not give the actual values of the masses, they are found to be responsible for reducing threshold uncertainties in the mass scales.

Out of different possibilities investigated we find that M_U is consistent with $(\tau_p)_{\text{expt}}$ if Higgs scalars are nondegenerate and $M_C(M_U)$ has the lowest (highest) extremal value as a function of the Higgs-boson-mass-splitting factor β defined below and the corrections are of the form

$$\Delta_U=\frac{1}{735}[21+C_U \ln\beta] ,$$

$$\Delta_C=\frac{1}{1960}[14/3+C_c \ln\beta] ,$$

$$\Delta_G=-\frac{1}{8820\pi}[4809+C_G \ln\beta] . \quad (14)$$

The extremal decrease in $\Delta \ln(M_C/M_Z)$ is obtained with

$$\eta_{H_1}=\eta'_{H_2}=\eta'_{S_1}=\eta''_{S_1}=\eta''_{S_2}=\eta''_{S_3}=0 ,$$

$$\eta_{H_2}=\eta'_{H_1}=\eta'_{H_3}=\eta'_{H_4}=\eta'_{H_5}=\eta'_{H_6}=\eta'_{S_2}=\eta''_{S_4}=\eta''_{S_5}=\eta''_{S_6}=\eta_{C_1}=\eta_{C_2}=-\ln\beta$$

leading to

$$C_U=261 , \; C_c=-3816 , \; C_G=-9259 \quad (15)$$

and corresponds to an increase in the unification mass M_U^0 as shown in Table I. Keeping $\alpha_s=0.12$ we have plotted the ratio $R_p=(\tau_p)_{\max}/(\tau_p)_{\text{expt}}$ against β as shown by curve (a) in Fig. 2 where the dashed line represents the experimental limit. Throughout this paper we have used the formula

$$\tau_p=\frac{M_U^4}{\alpha_G^2 m_p^5}$$

for proton-lifetime calculations where m_p = proton mass. It is clear that we need $\beta \gtrsim 30$ in order to be consistent with the LEP data and the experimental limit on $(\tau_p)_{\text{expt}}$ if $\alpha_s \simeq 0.12$. In Fig. 3 we have presented the variation of R_p as a function of α_s for two values $\beta=10$ and $\beta=30$. If α_s is allowed to be 0.13, the value of $\beta=10$ is sufficient to be consistent with $(\tau_p)_{\text{expt}}$ but $\beta=30$ yields a proton lifetime nearly 6 times larger than the lower limit.

The other possibility under this category which gives the most interesting solution in the model is the maximal increase in $\Delta \ln(M_U/M_Z)$ obtained with

$$\eta_{H_1}=\eta'_{S_1}=\eta'_{H_4}=0 ;$$

$$\eta_{H_2}=\eta'_{H_1}=\eta'_{H_2}=\eta'_{H_3}=\eta'_{H_5}=\eta'_{H_6}=\eta'_{S_2}=\eta''_{S_4}=\eta_{C_1}=-\ln\beta ,$$

$$\eta''_{S_1}=\eta''_{S_2}=\eta''_{S_3}=\eta''_{S_5}=\eta''_{S_6}=\eta_{C_2}=\ln\beta ,$$

leading to

$$C_U=-147 , \; C_c=882 , \; C_G=-4793 \quad (16)$$

in (14). The increased (decreased) values of M_U (M_C) are shown by the positive (negative) exponents of M_U/M_U^0 (M_C/M_C^0) in Table I in the last three columns for values $\beta=\sqrt{10}-100$. As a function of α_s and fixed value of $\beta=10$ the allowed solutions including threshold effects are shown by the solid lines to the right- (left-) hand sides of the dot-dashed curves representing M_U^0 (M_C^0) in Fig. 1. Analogous solutions for $\beta=30$ are

TABLE I. Threshold corrections on mass scales and the GUT coupling constant as a function of heavy-Higgs-boson mass-splitting factor β in the case of extremized $\Delta \ln(M_C/M_Z)$ or $\Delta \ln(M_U/M_Z)$ for nondegenerate case.

	Extremization of Δ_C			Extremization of Δ_U		
β	$\frac{M_C}{M_C^0}$	$\frac{M_U}{M_U^0}$	$\Delta\alpha_G^{-1}$	$\frac{M_U}{M_U^0}$	$\frac{M_C}{M_C^0}$	$\Delta\alpha_G^{-1}$
$\sqrt{10}$	$10^{-0.97}$	$10^{+0.17}$	0.38	$10^{+0.33}$	$10^{-0.07}$	1.06
5	$10^{-1.35}$	$10^{0.25}$	0.53	$10^{+0.46}$	$10^{-0.09}$	1.47
10	$10^{-1.95}$	$10^{+0.36}$	0.77	$10^{+0.66}$	$10^{-0.14}$	2.11
30	$10^{-2.88}$	$10^{+0.52}$	1.13	$10^{+0.97}$	$10^{-0.20}$	3.13
50	$10^{-3.29}$	$10^{+0.60}$	1.30	$10^{+1.11}$	$10^{-0.23}$	3.58
100	$10^{-3.89}$	$10^{+0.71}$	1.53	$10^{+1.31}$	$10^{-0.27}$	4.23

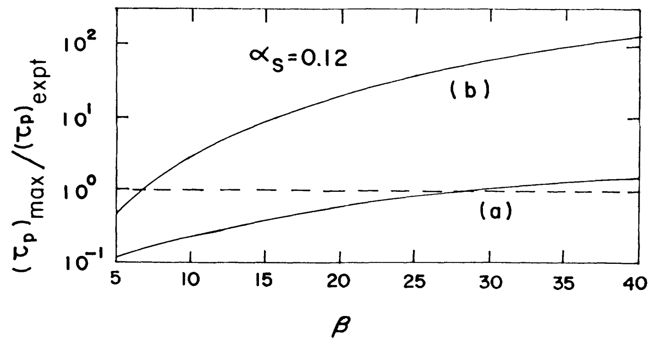

FIG. 2. Variation of the ratio of $R_p=(\tau_p)_{\max}/(\tau_p)_{\text{expt}}$ as a function of β for fixed $\alpha_s=0.12$ with extremized (a) $\Delta \ln(M_C/M_Z)$, (b) $\Delta \ln(M_U/M_Z)$.

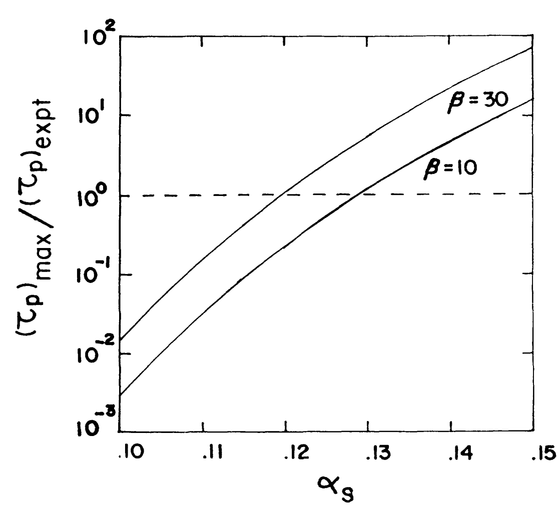

FIG. 3. Variation of R_p as a function of α_s for $\beta=10$ and $\beta=30$ with extremized $\Delta \ln(M_C/M_Z)$.

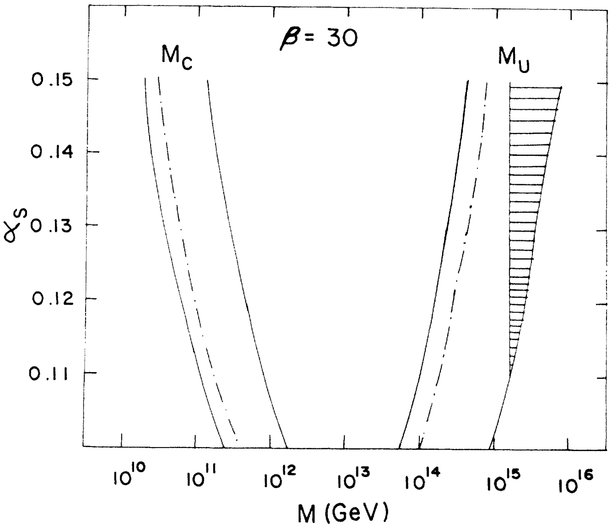

FIG. 4. Same as Fig. 1 but with $\beta = 30$.

shown in Fig. 4. These curves clearly exhibit decrease of M_C with increase in M_U as α_s increases. The shaded areas in Figs. 1 and 4 represent allowed values of the unification mass when threshold effects are included. In Fig. 5 we have plotted R_p against α_s for $\beta = 10$ and $\beta = 30$ where the horizontal line represents $(\tau_p)_{\rm expt}$. It is clear that $\beta = 10(30)$ ensures agreement with $(\tau_p)_{\rm expt}$ for all values of $\alpha_S \geq 0.114(0.101)$. Curve (b) in Fig. 2 shows the variation of R_p as a function of β for a fixed value of $\alpha_s = 0.12$ in the present case. We find that the experimental lower limit on τ_p corresponds to a lower bound

$$\beta \geq 7$$

which implies that the nondegenerate Higgs-scalar masses are split by a factor of nearly 50 among themselves being 7 times heavier or lighter on either sides of M_U or M_C.

It has been found by one of us (M.K.P.), Patra and Hazra [12] that the seesaw mechanism is natural in models even if parity and $SU(2)_R$ break at the same scale provided the neutral generator of $SU(2)_R$ breaks at a much lower scale. This latter situation is precisely the case in the present model. Using the radiative correction to the seesaw formula due to Bludman, Kennedy, and Langacker [18], the masses are represented as

$$m_{\nu_e} = 0.05 \frac{m_U^2}{M_N} , \quad m_{\nu_\mu} = 0.07 \frac{m_c^2}{M_N} , \quad m_{\nu_\tau} = 0.18 \frac{m_t^2}{M_N} . \tag{17}$$

It can be argued on the basis of vacuum stability that the right-handed Majorana neutrino mass $M_N \leq M_C$. In the nondegenerate case corresponding to (15) and with $\alpha_s = 0.12$ we find that for $\beta = 30$, $M_{\nu_e} = 10^{-5.4} - 10^{-4.6}$ eV, but m_{ν_μ} and m_{ν_τ} are larger than the values needed to solve the solar neutrino problem via MSW mechanism as shown in Fig. 6 where the shaded region is due to the experimental uncertainty in M_C computed from the dominant one-loop contribution in (5). While $m_{\nu_\mu} = 1 - 10$ eV is found to be consistent with the laboratory measurements, the value of m_{ν_τ} is consistent with the 17 keV neutrino if $\beta = 30$. In this case a mechanism is needed to make the ν_τ unstable to evade the cosmological bound.

With the other set of most interesting allowed solutions

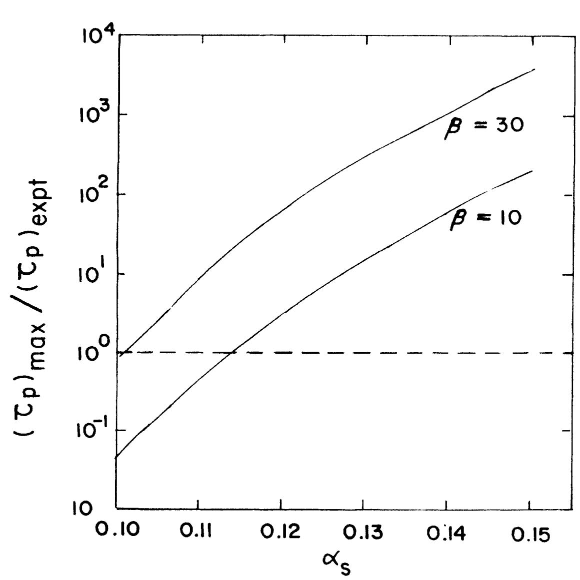

FIG. 5. Same as Fig. 3 but with extremized $\Delta \ln(M_U/M_Z)$.

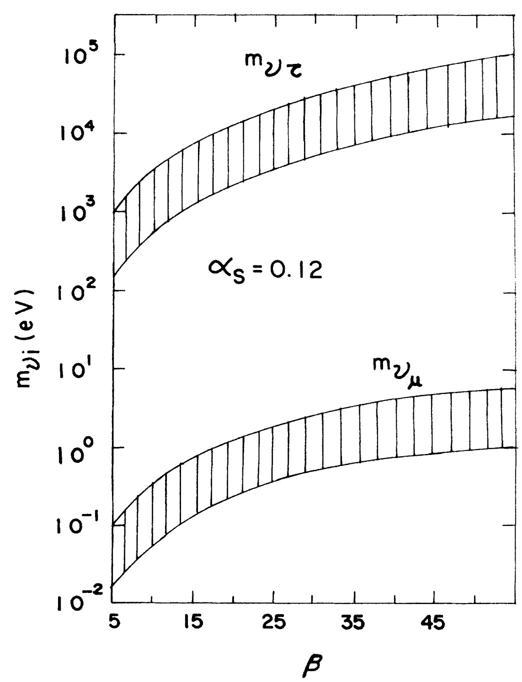

FIG. 6. Variation of m_{ν_μ} and m_{ν_τ} as a function of β for the case of extremized $\Delta \ln(M_C/M_Z)$ with fixed $\alpha_S = 0.12$.

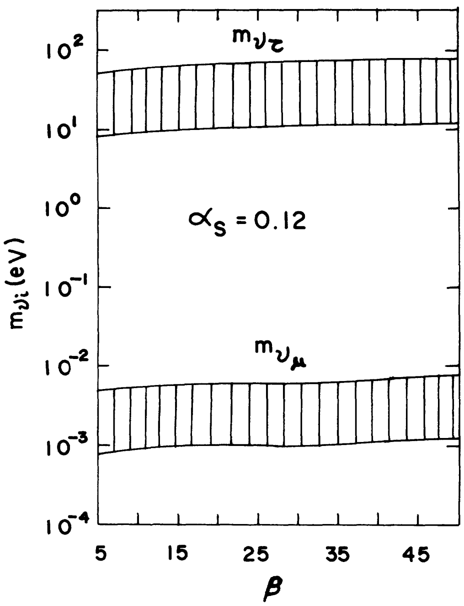

FIG. 7. Same as Fig. 6 but for extremized $\Delta \ln(M_U/M_Z)$.

in M_U leading to (16) we obtain, for $\beta=10$,

$$m_{\nu_e}=10^{-8}-10^{-7}\ \text{eV}\ ,\ m_{\nu_\mu}=10^{-3}-5\times10^{-3}\ \text{eV} \tag{18}$$

conistent with the values needed to solve the solar neutrino puzzle by MSW mechanism. The τ-neutrino mass for $\beta=10$ turns out to be

$$m_{\nu_\tau}=10-60\ \text{eV}\ . \tag{19}$$

While the lower limit of (19) is consistent with the cosmological dark matter of the Universe, the upper limit is consistent with the cosmological bound. With $\alpha_s=0.12$ the values of m_{ν_μ} and m_{ν_τ} are plotted against β as shown in Fig. 7. Interestingly the neutrino masses are not found to be sensitive to the parameter β. This is a reflection of the fact expressed in Table I where

$$\frac{M_C}{M_C^0}=10^{-0.07}-10^{-0.2}\ \text{as}\ \beta=\sqrt{10}-30\ .$$

IV. CONCLUSION AND DISCUSSION

In conclusion we have demonstrated that the SO(10) model with single G_{214} intermediate symmetry is not ruled out by the present LEP data and the experimental lower limit on the proton lifetime. In one type of solution obtained by extremizing the threshold effect on the intermediate scale, the nondegenerate heavy-Higgs-scalar masses have to differ by a factor corresponding to $\beta \gtrsim 30$ from M_U or M_C to guarantee $\tau_p \geq (\tau_p)_{\text{expt}}$. While the predicted value of the ν_e mass is small ($m_{\nu_e}=10^{-5}$ eV), the ν_μ mass ($m_{\nu_\mu}=3-5$ eV) could be accessible to measurements by laboratory experiments. The ν_τ mass ($m_{\nu_\tau}=5-30$ keV) obtained in the model is consistent with the 17 keV neutrino. Since the cosmological bound on the sum of neutrino masses ($\sum m_{\nu_i} \leq 60-100$ eV) is violated, a Majoron-like mechanism by the introduction of an additional global symmetry [19] is required to make ν_τ unstable for this class of solutions. Since the predicted τ_p is close to $(\tau_p)_{\text{expt}}$ for $\beta=30$, this class of solutions is likely to be ruled out by any reasonable improvement on the experimental limit on τ_p for the $p \to e^+\pi^0$ mode unless $\alpha_s > 0.12$. In the most interesting class of solutions obtained, even if the nondegenerate heavy-Higgs-scalar components are about 7 times heavier or lighter than the unification or the intermediate scale, all the available data can be easily accommodated by the model. The predicted values of proton lifetime saturate the Superkamiokande limit even if the heavy masses differ from the relevant scales by a factor corresonding to $\beta=40$ while the neutrino masses remain small and consistent with the values needed for the solar neutrino puzzle and the cosmological dark matter of the Universe. In this case the cosmological bound on the neutrino masses is always satisfied. Taking the heavy-Higgs-boson mass-splitting coefficient $\beta=10-20$ it is necessary to improve the available experimental limit on τ_p by a factor of at least 5–15 in order to verify or rule out this model.

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