Upper bound on the mass of the lightest neutralino in a general supersymmetric theory with grand unification

P. N. Pandita
NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark
and Department of Physics, North Eastern Hill University, P.O. Box 51, Laitumkhrah, Shillong 799 003, India
(Received 18 October 1994; revised manuscript received 30 January 1995)

We derive an upper bound on the mass of the lightest neutralino in a supersymmetric theory containing an arbitrary number of singlet, doublet, and triplet Higgs superfields under the standard model gauge group. Assuming grand unification of the gauge couplings, whereby the theory reduces to the minimal supersymmetric standard model with an arbitrary number of singlets, the bound can be expressed in terms of the gluino mass. Including radiative corrections, the upper bound on the mass of the lightest neutralino is 62 GeV for a gluino mass of 200 GeV, which increases to 178 GeV for a 1 TeV gluino.

PACS number(s): 14.80.Ly, 12.60.Jv

In supersymmetric theories with R-parity conservation [1], the lightest neutralino state is expected to be the lightest supersymmetric particle (LSP). In the minimal supersymmetric standard model (MSSM) at least two Higgs doublets $H_1$ and $H_2$, with hypercharge $Y = -1$ and +1, respectively, are required to give masses to quarks and leptons, and to cancel triangle gauge anomalies. The fermionic partners of these Higgs doublets mix with the fermionic partners of the gauge bosons to produce four neutralino states $\chi^0_1, i = 1, \ldots, 4$, and two chargino states $\chi^\pm_1, i = 1, 2$. In the nonminimal supersymmetric model containing a Higgs singlet and two Higgs doublets of the minimal model, the mixing of fermionic partners of neutral Higgs and gauge bosons produces five neutralino states. The neutralino states of the minimal [2-4] and the nonminimal model [5-7] have been studied in great detail, because the lightest neutralino, being the LSP, is the end product of any process involving supersymmetric particles in the final state.

In this paper we consider the neutralino mass matrix in a general supersymmetric theory containing an arbitrary number of singlet, doublet, and triplet Higgs superfields under the standard model gauge group. We obtain an upper bound on the mass of the lightest neutralino, and a lower bound on the mass of the heaviest neutralino state in such a general supersymmetric theory. These bounds depend on the soft-supersymmetry-breaking gaugino masses and the vacuum expectation values of the doublet and triplet, but not the singlet, Higgs fields. This is in contrast with the situation that obtains in the Higgs sector of such a theory [8], where the (tree-level) upper bound on the lightest Higgs boson mass is controlled by the vacuum expectation value of the doublet Higgs fields and dimensionless parameters only. Nevertheless, if we assume the simplest form of grand unification of such a general supersymmetric theory, whereby the triplet and the extra doublet Higgs fields are eliminated, then these bounds are controlled by soft-supersymmetry-breaking gaugino mass parameters, and $M_Z$ and $\theta_W$. Since the latter are known, the former entirely determine the bounds.

We start by recalling the neutralino mass matrix in the minimal supersymmetric standard model [1]. In the basis

$$
\psi^0_j = (-i\lambda', -i\lambda^3, \psi^1_{H_1}, \psi^2_{H_2}), \quad j = 1, 2, 3, 4,
$$

we find

$$
M^2_{\chi^0_1} \leq \min(M^2_{\chi^0_1} + M^2_{\chi^0_2} \sin^2 \theta_W, M^2_{\chi^0_2} + M^2_{\chi^0_3} \cos^2 \theta_W).
$$

On the other hand, the bigger eigenvalue of submatrix (2) gives a lower bound on the squared mass of the heaviest neutralino:

$$
M^2_{\chi^0_3} \geq \max(M^2_{\chi^0_1} + M^2_{\chi^0_2} \sin^2 \theta_W, M^2_{\chi^0_2} + M^2_{\chi^0_3} \cos^2 \theta_W).
$$

Here $M_1$ and $M_2$ are supersymmetry-breaking gaugino masses, and $g'$ and $g$ the gauge couplings, associated with the $U(1)_Y$ and $SU(2)_L$ subgroups of the standard model, respectively. Furthermore, $\tan \beta = v_2/v_1$, where $v_1 = \langle H^0_1 \rangle$ and $v_2 = \langle H^0_2 \rangle$, and $M^2_Z = (g^2 + g'^2)(v_1^2 + v_2^2)/2$. 

*Permanent address.
Since $M_Z$ and $\theta_W$ are known, the bounds (3) and (4) are controlled by the soft-supersymmetry-(SUSY-)breaking gaugino masses, in contrast with the bounds on the (tree-level) masses of the lightest and the heaviest scalar Higgs bosons in MSSM, which do not depend on supersymmetry-breaking masses [11].

We now consider a general class of supersymmetric models based on a standard model gauge group with an arbitrary Higgs sector. We shall assume [8], in addition to two Higgs doublets $H_1^{(1)}$, $H_2^{(1)}$ (with $Y = \mp 1$) which are coupled to the quarks and leptons in the superpotential

$$W^0 = h_Q L U_L^* H_2^{(1)} + h_D Q L D_L^* H_1^{(1)} + h_E L U_E^* H_1^{(1)},$$

where

$$W_1(b_1, b_2, N, \Sigma, \Psi_1, \Psi_2) = f_{ij}^{(1)} H_1^{(1)} H_2^{(1)} N^{(i)} + f_{ij}^{(2)} H_1^{(2)} H_2^{(2)} H_3^{(1)} + g_{ijk} H_1^{(1)} H_2^{(1)} H_3^{(1)} + \eta_{ijk} H_1^{(1)} H_2^{(1)} H_3^{(1)} + \zeta_{ijk} H_1^{(1)} H_2^{(1)} H_3^{(1)} + \zeta_{ijk} H_1^{(1)} H_2^{(1)} H_3^{(1)}$$

$$+ \frac{1}{6} \chi_{abc} \text{tr}(\Sigma^{(a)} \Sigma^{(b)} \Sigma^{(c)}) + \frac{1}{6} \lambda_{\mu\nu} N^{(\mu)} N^{(\nu)} N^{(\sigma)}.$$  

Without loss of generality, we can choose our basis in the space of Higgs doublets $H_1^{(1)}$ and $H_2^{(1)}$ such that only the Higgs doublets $H_1^{(1)}$ and $H_2^{(1)}$ acquire [14] a nonzero vacuum expectation value (VEV). This requires [8] that some of the Yukawa couplings in (6) vanish:

$$f_{ij}^{(1)} = f_{ij}^{(2)} = g_{ijk} = g_{ijk} = 0 \ (j \neq 1).$$

Assuming (8), the superpotential (6) can be written as

$$W' = -f_{11}^{(1)} H_1^{(1)} H_1^{(1)} N^{(\sigma)} + \frac{1}{\sqrt{2}} f_{11}^{(1)} H_1^{(1)} H_2^{(1)} H_3^{(1)}$$

$$+ \frac{1}{\sqrt{2}} h_{\alpha j k} \text{tr}(\Sigma^{(\alpha)} \Sigma^{(j)} \Sigma^{(k)}) + \frac{1}{6} \lambda_{\mu\nu} N^{(\mu)} N^{(\nu)} N^{(\sigma)}$$

where we have not explicitly written those terms in (9) which involve fields which do not obtain vacuum expectation values. Choosing

$$\psi_{\frac{3}{2}} = (-i\lambda', -i\lambda^2, \psi_{\frac{1}{2}}^3, \psi_{\frac{1}{2}}^3, \psi_{\frac{1}{2}}^3, \psi_{\frac{1}{2}}^3, \psi_{\frac{1}{2}}^3),$$

$$\sigma = 1, ..., n_\sigma, \quad a = 1, ..., \sigma, \quad i, j = 1, ..., \tau_1,$$

$$I = 1, ..., (4 + n_\sigma + 4 + 2t),$$

as the basis, it is straightforward to write the neutralino mass matrix $M$ for the general class of supersymmetric models based on standard model gauge group with an arbitrary Higgs sector. We shall not display this mass matrix here, but shall content ourselves with examining the upper left $2 \times 2$ submatrix of $M^T M$ corresponding to this mass matrix $M$. This submatrix can be written as

$$M_{12}^2 + M_{22}^2 \sin^2 \theta_W + 6g^2 y_2^2 - M_{12}^2 \sin \theta_W \cos \theta_W - 2g g_2' y_2^2$$

where

$$u = \langle \langle \xi^{(a)} \rangle \rangle, \quad x^{(a)} = \langle N^{(a)} \rangle, \quad y_1^{(i)} = \langle \psi^{(1)} \rangle, \quad y_2^{(j)} = \langle \psi^{(2)} \rangle,$$

$$y^2 = \sum_{i} [(y_1^{(i)})^2 + (y_2^{(i)})^2], \quad u^2 = \sum_{a} (u^{(a)})^2.$$  

In (11a) the expressions for the $W$ and $Z$ masses are

$$M_W^2 = \frac{1}{2} (g^2 + g_2'^2) [v_1^2 + v_2^2 + 4y_2^2], \quad (12a)$$

$$M_Z^2 = \frac{1}{2} g^2 [v_1^2 + v_2^2 + 4y_2^2 + 4y_2'^2]. \quad (12b)$$

The vacuum expectation values that enter into the $W$ and $Z$ masses (12a) and (12b) are experimentally constrained by the $\rho$ parameter. From a recent global fit, which includes the Collider Detector at Fermilab (CDF) data, we have [15]

$$\rho = \frac{M_W^2}{M_{Z}^2 \cos^2 \theta_W} = 1.0004 \pm 0.0002 \pm 0.0002, \quad (13)$$

where the second error is due to the Higgs boson mass. This result is remarkably close to the expected standard model value of $\rho = 1$. Taking a value of $\rho = 1$, implies, through (12a) and (12b), the following constraint on the triplet vacuum expectation values:
\[ 4u^2 = 2g^2. \] (14)

We note that (14) implies a fine-tuning of the parameters of the triplet vacuum expectation values in order to maintain the experimental result (13) in the general supersymmetric theory that we are considering. However, we shall assume the constraint (14) to be true, although our final results do not depend on this constraint. With this constraint the W and Z masses can be written as [16]

\[
M_{\chi_1^0}^2 \leq \min (M_1^2 + M_2^2 \sin^2 \theta_W + 6g^2u^2, M_2^2 + M_3^2 \cos^2 \theta_W + g^2v^2),
\]

\[
M_{\chi_1^0}^2 \geq \max (M_1^2 + M_2^2 \sin^2 \theta_W + 6g^2u^2, M_2^2 + M_3^2 \cos^2 \theta_W + g^2v^2).
\]

The bounds (16) and (17) depend on \textit{a priori} unknown vacuum expectation values of the triplet Higgs fields and gaugino mass parameters. Nevertheless, as we shall see, these bounds can become meaningful in theories with gauge coupling unification [17]. It is important to note that the singlet vacuum expectation values decouple from these bounds.

In the general supersymmetric theory that we are considering, the renormalization-group equations (RGE’s) [8] for the standard SU(3) x SU(2) x U(1) gauge couplings can be written as \[ g_1^2, g_2^2, g_3^2, \tan \theta_W = g'/g, \]

\[ g_3 \text{ is the SU(3) gauge coupling constant] .} \]

\[
16\pi^2 \frac{dg_1}{dt} = \left[ \frac{33}{5} + \frac{3}{5}(6c_1 + d) \right] g_1^3,
\]

\[
16\pi^2 \frac{dg_2}{dt} = (1 + 2t_0 + 4t_1 + d) g_2^3,
\]

\[
16\pi^2 \frac{dg_3}{dt} = -3g_3^3.
\]

The RGE’s depend on the number and the type of the Higgs representations \( (d, t_0, t_1) \). We note that additional doublets increase the \( \beta \) functions of the gauge couplings, even though the VEV’s of these doublets have been rotated away. If we assume that the gauge couplings unify at some grand-unification scale \( M_\mu \), i.e., \( g_1(M_\mu) = g_2(M_\mu) = g_3(M_\mu) = g_U \), then the simplest choice is \( d = t_0 = t_1 = 0 \), (19)

with \( n_s \) arbitrary, i.e., the MSSM with an arbitrary number of singlet superfields. In other words only Higgs singlets, in addition to the two Higgs doublets of the MSSM, are consistent with unification [18] without fine-tuned cancellations [19]. The bounds (16) and (17) now reduce to

\[
M_{\chi_1^0}^2 \leq \min (M_1^2 + M_2^2 \sin^2 \theta_W + 6g^2u^2, M_2^2 + M_3^2 \cos^2 \theta_W),
\]

\[
M_{\chi_1^0}^2 \geq \max (M_1^2 + M_2^2 \sin^2 \theta_W, M_2^2 + M_3^2 \cos^2 \theta_W),
\]

which are the same as the bounds (3) and (4) in MSSM. With condition (19), the gaugino mass parameters satisfy the one-loop RGE's ([M_3] \equiv m_3, the gluino mass) [20]

\[ 16\pi^2 \frac{dM_1}{dt} = b_i M_i g_i^2, \quad b_i = (\frac{33}{5}, 1, -3). \]

Equations (18), (19), and (22) imply \( \alpha_i = g_i^2/4\pi, \alpha_U = g_U/4\pi \)

\[ M_{\xi}^2 = M_{\xi}^2 \cos^2 \theta_W = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4y^2). \] (15)

The combination \( (v_1^2 + v_2^2 + 4y^2) \) is constrained to be \( \approx (174 \text{ GeV})^2 \), but the ratio \( g/(v_1^2 + v_2^2) \) is unconstrained. In the general supersymmetric model the smallest and the largest eigenvalue of (11a) serve as the bound on the mass of the lightest neutralino \( (\chi_1^0) \) and the lower bound on the mass of the heaviest neutralino \( (\chi_1^0) \), respectively. Using (14) in (11a), we can write these bounds as

\[ M_{\xi}^2 = M_{\xi}^2 \cos^2 \theta_W = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4y^2). \] (15)

\[
M_{\xi}^2 = M_{\xi}^2 \cos^2 \theta_W = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4y^2).
\]

The combination \( (v_1^2 + v_2^2 + 4y^2) \) is constrained to be \( \approx (174 \text{ GeV})^2 \), but the ratio \( g/(v_1^2 + v_2^2) \) is unconstrained. In the general supersymmetric model the smallest and the largest eigenvalue of (11a) serve as the bound on the mass of the lightest neutralino \( (\chi_1^0) \) and the lower bound on the mass of the heaviest neutralino \( (\chi_1^0) \), respectively. Using (14) in (11a), we can write these bounds as

\[ M_{\xi}^2 = M_{\xi}^2 \cos^2 \theta_W = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4y^2). \] (15)

\[ M_{\xi}^2 = M_{\xi}^2 \cos^2 \theta_W = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4y^2).
\]

The combination \( (v_1^2 + v_2^2 + 4y^2) \) is constrained to be \( \approx (174 \text{ GeV})^2 \), but the ratio \( g/(v_1^2 + v_2^2) \) is unconstrained. In the general supersymmetric model the smallest and the largest eigenvalue of (11a) serve as the bound on the mass of the lightest neutralino \( (\chi_1^0) \) and the lower bound on the mass of the heaviest neutralino \( (\chi_1^0) \), respectively. Using (14) in (11a), we can write these bounds as

\[ M_{\xi}^2 = M_{\xi}^2 \cos^2 \theta_W = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4y^2). \] (15)

\[ M_{\xi}^2 = M_{\xi}^2 \cos^2 \theta_W = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4y^2).
\]

The combination \( (v_1^2 + v_2^2 + 4y^2) \) is constrained to be \( \approx (174 \text{ GeV})^2 \), but the ratio \( g/(v_1^2 + v_2^2) \) is unconstrained. In the general supersymmetric model the smallest and the largest eigenvalue of (11a) serve as the bound on the mass of the lightest neutralino \( (\chi_1^0) \) and the lower bound on the mass of the heaviest neutralino \( (\chi_1^0) \), respectively. Using (14) in (11a), we can write these bounds as

\[ M_{\xi}^2 = M_{\xi}^2 \cos^2 \theta_W = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4y^2). \] (15)

\[ M_{\xi}^2 = M_{\xi}^2 \cos^2 \theta_W = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4y^2).
\]

The combination \( (v_1^2 + v_2^2 + 4y^2) \) is constrained to be \( \approx (174 \text{ GeV})^2 \), but the ratio \( g/(v_1^2 + v_2^2) \) is unconstrained. In the general supersymmetric model the smallest and the largest eigenvalue of (11a) serve as the bound on the mass of the lightest neutralino \( (\chi_1^0) \) and the lower bound on the mass of the heaviest neutralino \( (\chi_1^0) \), respectively. Using (14) in (11a), we can write these bounds as

\[ M_{\xi}^2 = M_{\xi}^2 \cos^2 \theta_W = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4y^2). \] (15)

\[ M_{\xi}^2 = M_{\xi}^2 \cos^2 \theta_W = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4y^2).
\]
to the standard model gauge group (see [24]), our results (26) and (27) are valid in any such supersymmetric grand-unified theory.

Finally, we discuss the effect of radiative corrections to the upper and lower bounds on the lightest and heaviest neutralino masses derived above, which arise from the dominant top-quark $(t)$-top-squark $(\tilde{t})$ loops. For Higgsino-like neutralino, a simple estimate of the radiative corrections to its mass arising from top-quark-top-squark loops is given by

$$\Delta M_{\chi^0_1}^R \approx 3 \frac{\alpha_2^2}{16\pi^2} \ln \left( \frac{m_{\tilde{t}}^2}{m_t^2} \right) \approx 5\% , \quad (28)$$

where we have taken the stop mass to be equal to 1 TeV. A similar estimate holds for a gauginolike neutralino. These estimates are very close to the generic results which emerge from detailed calculations [28] carried out in the context of the MSSM, although, for some extreme values of parameters, the radiative corrections to the lightest neutralino mass can be as large as 20%. On the other hand, the radiative corrections to the heaviest neutralino mass in the minimal supersymmetric standard model do not exceed 5%. Taking these results as indicative of the radiative corrections in the general model we are considering, we estimate a conservative radiatively corrected upper bound on the mass of the lightest neutralino to be about 62 GeV for a 200 GeV gluino, which increases to 178 GeV for a gluino mass of 1 TeV [29]. Similarly, a conservative estimate for the lower bound on the heaviest neutralino mass becomes 101 GeV for a gluino mass of 200 GeV, the bound increasing to 290 GeV as the gluino mass increases to 1 TeV.

I would like to thank NORDITA, the Tata Institute of Fundamental Research, and the Indian National Science Academy for support while this work was carried out. This work was supported by the Departments of Science and Technology and Atomic Energy, India.


[9] Assuming CP invariance, the neutralino mass matrix is a real symmetric matrix.


[13] We do not consider more exotic Higgs representations which give lower values of the upper bound on the lightest Higgs boson mass in such theories.


[19] Gauge coupling unification can be achieved even when $d, t_0, t_1 > 0$ by adding colored multiplets or introducing an intermediate scale in the theory. U. Amaldi *et al.*, Phys. Lett. B 281, 374 (1992); W. de Boer, "Grand Unified Theories and Supersymmetry in Particle Physics and Cosmology," Institution Report No. IEKP-KA/94-01, 1994 (unpublished). We note that the fine-tuning involved to maintain gauge coupling unification in such models is much less objectionable as compared to the fine-tuning involved in Eq. (14). Since in this case the bounds (16) and (17) depend on the unknown triplet vacuum expectation values $y^2$, they are not calculable in terms of gluino mass only. We, therefore, consider the simplest choice (19) in this paper.