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Generalized Second Law of Thermodynamics for Interacting Dark Energy in the DGP Braneworld

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Abstract In this paper, we investigate the validity of the generalized second law of thermodynamics (GSLT) in the DGP braneworld when the universe is filled with interacting two fluid system: one in the form of cold dark matter and other is holographic dark energy. The boundary of the universe is assumed to be enclosed by the dynamical apparent horizon or the event horizon. The universe is chosen to be homogeneous and isotropic FRW model and the validity of the first law has been assumed here.

Keywords DGP braneworld · Holographic dark energy · Generalized second law of thermodynamics (GSLT)

1 Introduction

Astrophysical observations made at the turn of the last century [1, 2] show conclusive evidence for acceleration in the late universe, which is still a challenge for cosmologists. It shows beginning of accelerated expansion in the recent past. It is found that cosmic acceleration is driven by some invisible fluid having its gravitational effect in the very late universe. This unknown fluid has distinguishing feature of violating strong energy condition (SEC) being called dark energy (DE) [3]. Various models have been proposed to solve this problem. A comprehensive review of these models is available in [4].

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In the race to investigate a viable cosmological model, satisfying observational constraints and explaining present cosmic acceleration, brane-gravity was introduced and brane-cosmology was developed. A review on brane-gravity and its various applications with special attention to cosmology is available in [5–8].

A simple and well studied model of brane-gravity (BG) is the Dvali-Gabadadze-Porrati (DGP) braneworld model [9–11]. In this model our 4-dimensional world is a FRW brane embedded in a 5-dimensional Minkowski bulk. It explains the origin of DE as the gravity on the brane leaking to the bulk at large scale. On the 4-dimensional brane the action of gravity is proportional to M_P^2 whereas in the bulk it is proportional to the corresponding quantity in 5-dimensions. The model is then characterized by a crossover length scale

$$r_c = \frac{M_P^2}{2M_5^2}$$

such that gravity is 4-dimensional theory at scales $a \ll r_c$ where matter behaves as pressureless dust but gravity *leaks out* into the bulk at scales $a \gg r_c$ and matter approaches the behavior of a cosmological constant.

In this conceptual set up, one of the important questions concerns the thermodynamical behavior of an accelerated expanding universe driven by DE. The first hint on the connection between general relativity and thermodynamics was given by Bekenstein in 1973 [12]. He outlined the laws of thermodynamics in the presence of black holes which turned out to be equivalent to the laws of black hole mechanics [13]. Study of gravitational thermodynamics in an accelerating universe has been a strong candidate and has been addressed to in many papers based on General relativity (GR) [14–19]. The reason of interest in this subject is two fold (i) it is natural to study thermodynamical aspect of accelerating universe and (ii) the astonishing result for phantom obtained in [20] which has either negative temperature or negative entropy. This is another problem of phantom cosmology like big-rip singularity for which some viable solution are proposed in [21]. The main problem of studying thermodynamics of the Universe is to define the entropy and temperature on the boundary of the universe. Generally the entropy and hence the temperature is taken from black hole physics but in other gravitational theories (such as $f(R)$ gravity) some correction terms may be needed.

Motivated by the profound connection between black hole physics and thermodynamics, in recent times there has been some deep thinking on the relation between gravity and thermodynamics. A pioneer work in this respect was done by Jacobson who disclosed that Einstein's gravitational field equation can be derived from the relation between horizon area and entropy together with Clausius relation $\delta Q = T\delta S$ [22]. Some recent discussion on the connection between gravity and thermodynamics on various gravity theories can be found on [23, 24]. Recently this connection between gravity and thermodynamics has been extended to braneworld scenarios [25–28]. In Ref. [29] it is shown that apparent horizon entropy extracted through connection between gravity and first law of thermodynamics satisfies the generalized second law of thermodynamics (GSLT) in DGP warped brane. In General Relativity (GR) frame work the authors of [30] have shown in contrast to the case of the apparent horizon, both first and second law of thermodynamics breakdown if one considers boundary of the universe to be the event horizon. Essentially the two horizons have significant difference both from geometrical and physical point of view. The cosmological event horizon exists in accelerating universe while it may not exist in the standard big bang model. From the thermodynamical point of view the universe bounded by the apparent horizon is a Bekenstien system having well defined entropy and temperature while universe bounded by the event horizon may not be a Bekenstein system and hence temperature and entropy are not well defined.

The other way to approach to the problem of DE arises from holographic principle which states that the number of degrees of freedom for a system within a finite region should be finite and is bounded by the area of its boundary. As in Ref. [31] one obtains holographic energy density as

$$\rho_D = 3c^2 M_P^2 L^{-2}$$

where L is an IR cut-off in units $M_P^2 = 1$. Li shows that [32] if we choose L as the radius of the event horizon we can get the correct equation of state and get the desired accelerating universe.

It may be noted that in literature, standard DGP model has been generalized to (i) LDGP model by adding a cosmological constant [33], (ii) QDGP model by adding quiescence perfect fluid [34], (iii) CDGP by Chaplygin gas [35] (iv) SDGP by a scalar field [36]. In [37] the DGP model has been analyzed by adding Holographic dark energy (HDE).

In a recent paper [38], validity of GSLT has been studied on event horizon for interacting DE. Assuming first law of thermodynamics on the event horizon, they have found conditions for validity of GSLT in both cases when FRW universe is filled with interacting two fluid system—one in the form of cold dark matter and the other is either holographic dark energy or new agegraphic dark energy. In [39], we have investigated the validity of GSLT of universe in the DGP braneworld. The boundary of the universe was assumed to be enclosed by the dynamical apparent horizon or the event horizon. In the present paper, we extend this investigation to interacting holographic dark energy model in DGP braneworld. The matter in the universe is taken in the form of interacting two fluid system—one component is cold dark matter (CDM) and the other is in the form of HDE.

The paper is organized as follows: Sect. 2 deals with interacting HDE in the DGP brane model while validity of GSLT has been examined for apparent and event horizon and conclusion are written pointwise in Sect. 3.

2 Interacting Holographic Dark Energy in DGP Model

In flat, homogeneous and isotropic brane the Friedmann equation [9] in DGP model is given by

$$H^2 = \left(\sqrt{\frac{\rho}{3} + \frac{1}{4r_c^2}} + \epsilon \frac{1}{2r_c}\right)^2 \tag{2.1}$$

or equivalently

$$H^2 - \epsilon \frac{H}{r_c} = \frac{\rho}{3} \tag{2.2}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, ρ is the total cosmic fluid energy density and $r_c = \frac{M_P^2}{2M_5^2}$ is the crossover scale which determines the transition from 4D to 5D behavior and $\epsilon = \pm 1$. (For simplicity we are using $8\pi G = 1$.)

For $\epsilon = 1$, we have standard DGP(+) model which is self accelerating model without any form of dark energy, and effective ω is always non-phantom. However for $\epsilon = -1$, we have DGP(–) model which does not self accelerate but requires dark energy on the brane. It experiences 5D gravitational modifications to its dynamics which effectively screen dark energy.

Here we take $\rho = \rho_m + \rho_D$ where ρ_m is the energy density of cold dark matter (CDM) and ρ_D is the energy density of HDE.

The Friedmann (2.2) can be written in following effective Einstein form

$$H^2 = \frac{1}{3}(\rho_m + \rho_{eff}) \tag{2.3}$$

$$\dot{H} = -\frac{1}{2}\{\rho_m + (\rho_{eff} + p_{eff})\} \tag{2.4}$$

where ρ_{eff} is the effective energy density given by

$$\rho_{eff} = \rho_D + \epsilon\frac{3H}{r_c} \tag{2.5}$$

and p_{eff} is the effective pressure given by

$$p_{eff} = p_D - \epsilon\frac{3H}{r_c} - \epsilon\frac{\dot{H}}{r_c H} \tag{2.6}$$

The individual conservation equation for effective DE and CDM are respectively given by

$$\dot{\rho}_{eff} + 3H(1+\omega_{eff})\rho_{eff} = -Q \tag{2.7}$$

and

$$\dot{\rho}_m + 3H\rho_m = Q \tag{2.8}$$

where $Q = \Gamma\rho_D$ is called interaction term [38] and the decay rate Γ corresponds to conversion of dark energy to dust (CDM). Following [38], if we define

$$\omega_{eff}^{(i)} = \omega_{eff} + \frac{\Gamma}{3H} - \epsilon\frac{\Gamma}{r_c\rho_{eff}} \quad \text{and} \quad \omega_m^{(i)} = -\frac{\Gamma u}{3H} \tag{2.9}$$

then the above conservation equations can be written in non-interacting form as

$$\dot{\rho}_{eff} + 3H(1+\omega_{eff}^{(i)})\rho_{eff} = 0 \tag{2.10}$$

and

$$\dot{\rho}_m + 3H(1+\omega_m^{(i)})\rho_m = 0 \tag{2.11}$$

where $u = \frac{\rho_D}{\rho_m}$ is the ratio of energy densities.

Also using (2.5) in (2.10) the actual energy conservation for DE is

$$\dot{\rho}_D + 3H(1+\omega_D^{(i)})\rho_D = 0 \tag{2.12}$$

where

$$\omega_D^{(i)} = \omega_D + \frac{\Gamma}{3H} \tag{2.13}$$

Combining (2.10) and (2.11), we get

$$\dot{\rho}_t + 3H(\rho_t + p_t) = 0 \tag{2.14}$$

where

$$\rho_t = \rho_m + \rho_{eff} \quad \text{and} \quad p_t = p_m + p_{eff} = \omega_m^{(i)}\rho_m + \omega_{eff}^{(i)}\rho_{eff} \tag{2.15}$$

3 The Generalized Second Law of Thermodynamics

In this section we examine the validity of GSLT on 3-DGP brane. Let us consider a region of FRW universe enveloped by the horizon and assume that the region bounded by the horizon act as a thermal system with boundary defined by the horizon and is filled with a perfect fluid of energy density ρ_t and pressure p_t given by (2.15).

Gravity on the brane does not obey Einstein theory, therefore usual area formula for the black hole entropy may not hold on the brane. So we extract the entropy of the horizon by assuming the first law of thermodynamics on the horizon [38].

The amount of energy crossing the horizon in time dt has the expression

$$-dE = 4\pi R_h^3 H(\rho_t + p_t)dt \tag{3.1}$$

where R_h is the radius of the horizon.

So from the first law of thermodynamics, we have

$$\frac{dS_h}{dt} = \frac{4\pi R_h^3 H}{T_h}[(1+\omega_m^{(i)})\rho_m + (1+\omega_{eff}^{(i)})\rho_{eff}] \tag{3.2}$$

where S_h and T_h are the entropy and temperature of the horizon respectively.

Using Gibb's equation [16],

$$T_h dS_I = dE_I + p_t dV$$

we obtain the variation of the entropy of the fluid inside the horizon as

$$T_h dS_I = d(\rho_m + \rho_D)V + (\rho_m + \rho_D + p_D)dV \tag{3.3}$$

where S_I and E_I are the entropy and energy of the matter distribution inside the horizon. Here we assume as in Refs. [40–42], the temperature of the source inside the horizon is in equilibrium with the temperature associated with the horizon. This assumption is certainly true at late times, when the universe fluids and the horizon will have interacted for a long time. But it may not be true at early or intermediate times, as the systems must interact for some length of time before they can attain equilibrium. However, in order to avoid complex calculation of non equilibrium thermodynamics, this assumption is widely followed in GSLT literature [40–45]. Therefore, it is to be noted that our result will hold only at late times of the universe evolution.

So starting with $E_I = \frac{4}{3}\pi R_h^3 \rho_t$ and $V = \frac{4}{3}\pi R_h^3$ and using (2.11), (2.12) and (3.3) and after some simplification one gets

$$\frac{dS_I}{dt} = -\frac{4\pi R_h^3 H}{T_h}[\{\rho_m(1+\omega_m^{(i)}) + \rho_D(1+\omega_D^{(i)})\rho_{eff}] + \frac{4\pi R_h^2}{T_h}\frac{dR_h}{dt}[\rho_m + \rho_D(1+\omega_D)] \tag{3.4}$$

Adding (3.2) and (3.4), one gets the resulting change of entropy

$$\frac{dS_h}{dt}+\frac{dS_I}{dt}=\frac{4\pi R_h^2}{T_h}\left[-\frac{\epsilon \dot{H} R_h}{r_c}+\{\rho_m+\rho_D(1+\omega_D)\}\dot{R}_h\right] \tag{3.5}$$

We shall now examine the validity of GSLT i.e.,

$$\frac{dS_h}{dt}+\frac{dS_I}{dt}\geq 0$$

for apparent and event horizon respectively.

3.1 Universe Bounded by Apparent Horizon

Here

$$R_h=R_A=\frac{1}{H} \tag{3.6}$$

So

$$\frac{dR_A}{dt}=-\frac{\dot{H}}{H^2}=\frac{1}{2H^2}\{\rho_m+(\rho_{eff}+p_{eff})\} \tag{3.7}$$

Hence (3.5) simplifies to

$$\frac{dS_{tot}}{dt}=\frac{4\pi R_A^2}{T_A}\left[\epsilon\frac{R_A}{2r_c}+\frac{1}{2H^2}\{\rho_m+\rho_D(1+\omega_D)\}\right]\{\rho_m+\rho_{eff}(1+\omega_{eff})\} \tag{3.8}$$

3.2 Universe Bounded by Event Horizon

In this case

$$R_h=R_E$$

Here due to holographic nature of the DE the energy density of the holographic matter can be written as

$$\rho_D=\frac{3c^2}{R_E^2} \tag{3.9}$$

So using the conservation equation (2.12) the time variation of the event horizon can be written as [43]

$$\dot{R}_E=\frac{3}{2}HR_E(1+\omega_D^{(i)}) \tag{3.10}$$

Hence (3.5) now becomes

$$\frac{dS_E}{dt}+\frac{dS_I}{dt}=\frac{4\pi R_E^2}{T_E}\left[\frac{\epsilon R_E}{2r_c}\{\rho_m+\rho_{eff}(1+\omega_{eff})\}\right.$$
$$\left.+\frac{3}{2}HR_E\left(1+\omega_D+\frac{\Gamma}{3H}\right)\{\rho_m+\rho_D(1+\omega_D)\}\right] \tag{3.11}$$

The following conclusion we can draw from the variation of the total entropy given by (3.8) or (3.11):

(a) The expression (3.11), which shows the total variation of matter as well as horizon entropy, is very complicated and it is hard to speculate the validity of GSLT. However, GSLT depends crucially on the interaction term Γ. Further it is to be noted that GSLT is an inherent property of a thermodynamical system and it should be satisfied throughout the evolution. Violation of GSLT means the present model may not be an thermodynamical system.
(b) The entropy variation does not depend on the interaction in the case of apparent horizon while in the case of event horizon there is dependence on interaction through $\dot{R}_E$ due to the choice of the energy density for holographic dark energy given by (3.9).
(c) If we put $r_c \to \infty$ (this happens at early epoch) i.e., neglect the brane effect then GSLT will always be satisfied for apparent horizon while GSLT will not be trivial for event horizon as it depends on the interaction term Γ.
(d) In deriving GSLT we do not need any specific choice for entropy and temperature at the horizon.

For future work it will be interesting to determine the form of entropy and temperature at the horizon in brane scenario.

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