STUDY OF THE DYNAMICAL BEHAVIOUR OF CERTAIN MANYBODY SYSTEMS

DHRUBAJYOTI ROY CHOUDHURY
DEPARTMENT OF PHYSICS
SCHOOL OF PHYSICAL SCIENCES

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SYNOPSIS

The dynamical behaviour of manybody systems has been a subject of research for a long period. It provides a very good understanding of inter- and intra-particle interactions, which serve as the starting base for developing the microscopic model for the behaviour of a system and testing ground for different laws of nature. Over the past few decades, there has been a growing interest in the study of the dynamical behaviour of molecules, liquids and solids.

Liquid Helium is an interesting manybody system which does not solidify even at $T \approx 0K$. At $T_\lambda = 2.17K$ it undergoes a second-order phase transition and transforms from its normal phase (He-I) to its superfluid phase (He-II). Some of the important properties of He-II are: (i) Superfluidity, (ii) high thermal conductivity, (iii) negative volume expansion, (iv) singularity at $T = T_\lambda$ in the specific heat and related properties, (v) capacity to sustain the propagation of temperature waves, etc. Several efforts have been made to develop the theory of He-II [1] but the real microscopic picture of its unique behaviour has not emerged for a long time. The physics of He-II has, therefore, been understood in terms of two fluid phenomenology developed by
Landau [2]. Attempts have been made to strengthen this model by microscopic calculations of k=0 condensate [3] assuming that such condensate is related to the superfluid density. However, the existence of k=0 condensate has not been confirmed experimentally even after repeated efforts. In addition, the fact that 1-D and 2-D systems of He-II exhibit superfluid behaviour while the existence of k=0 condensate in these systems, is theoretically ruled out, indicates that k=0 condensation is not essential for superfluid behaviour.

He-II has always been presumed to have a random distribution of its atoms in normal and superfluid phase, while the fact, that the specific heat of He-II varies as $T^3$ at low temperature and its excitation spectrum resembles with that of a well ordered crystalline solid indicate that He-II has an ordered arrangements of atoms. Over the last few years, Jain [4] has been working on this problem and developed successfully its microscopic theory consistent with microscopic as well as macroscopic uncertainty, volume exclusion and modified volume exclusion conditions. His theory also provides microscopic basis for the system to behave as a mixture of two fluid as envisaged by Landau [2]. It is this microscopic theory which forms the basis of our present investigation.
Chapter-I of the thesis reviews the important aspects of theoretical formulations and experimental studies on liquid $^4$He.

Chapter-II describes the salient aspects of the microscopic theory of a system of interacting bosons such as liquid $^4$He, as developed by Jain [4]. Jain's theory reveals that: (i) the system below certain temperature $T_\lambda$ should behave as a homogeneous mixture of two fluid; (ii) the superfluid transition is identified as an order-disorder transition of particles in their phase space followed by B.E. condensation of particles as $(q,-q)$ pairs in a state of their centre of mass momentum $K=0$; individual atom retains $\hbar k = \hbar/2d$ momentum corresponding to zero-point energy. (iii) no state of the system corresponds to $k=0$ state of single particle hence the question of the existence of $k=0$ condensate in the system is far from reality. (iv) Particles in the phase below $T_\lambda$ represent 3-d network of stationary matter wave (SMW) pairs $(q,-q)$ that modulate the probability of finding the particles in phase space. Below $T_\lambda$ the SMW pair configuration is perturbed by the overlap of the wavepackets and the interatomic attraction and the particles develop a kind of collective binding energy identified as energy gap between normal and superfluid states and this gap is responsible for all the unique properties of superfluid. (v) The relative positions of particles in phase space
is found to be \( 2n \pi ( n = 1, 2, 3, \ldots ) \) and this fact is responsible for the phase coherence of particles and quantized vortices observed in \( \text{He-II} \) (vi) the system below \( T_\lambda \) defines a closed pack arrangement of the wavepackets of its particles that can be ascribed to have a symmetry such as hcp/fcc.

Chapter-III gives a detailed quantitative analysis of the thermal excitations of \( \text{He-II} \). Using the microscopic theory as proposed by Jain [4,5], it was noted that the system is isotropic and is equivalent to a crystal with one particle per unit cell. It was further noted that the shear force among its particles is negligible and thus the transverse modes are absent. The phonon energy was concluded to follow a dispersion relation for a single atomic chain. Since the interatomic separation \( (d) \) is not rigidly fixed and the size of the wavepacket of a particle is momentum dependent, it has been argued that it should be a function of wavevector \( q \) and thus obviously the force constant \( (C) \) is also a function of \( q \). Our study of the excitation spectrum based on these inferences is reported in this chapter. This includes the analysis of:

(i) the "\( q \)" dependence of \( (C) \) and \( (d) \) of the excited state configuration.

(ii) the anomalous nature of the thermal excitation spectrum
on a quantitative scale.

(iii) the inter-relationship between "C" and "d".

(iv) the pressure dependence of the roton energy and momentum.

(v) The interaction between the multimode and the single particle excitation, which leads to a hybridization.

(vi) The relationship between the Thermal Excitation spectrum and structure factor. Our theoretical results match with the experimental results to a very good approximation.

Chapter-IV gives the calculation of the first sound velocity as a function of temperature by assuming three possible structure of the system. The pressure dependence of the velocity of first sound at constant temperature has also been analysed. To understand the change in the strength of inter-particle interaction, we examined the variation of the force constant with pressure for wavevector $q = 0$. Calculation of the superfluid and normal density of He-II were done by using, what has been defined by Jain as the energy gap between the ground state of the system in superfluid and normal fluid configurations. These results have been used to obtain the velocity of II, III, IVth sound which have been found to agree with the experimental results to a good approximation.
Chapter-V presents a critical analysis of the observed Raman spectrum of He-II. The origin of a number of unexplained peaks in the high resolution Raman spectrum of He-II are reported.

Chapter-VI gives some important concluding remarks pertaining to our investigation and provides a line of action that we plan to follow for further investigation of the system.

2. (a) L.D. Landau, J. Phys. USSR 5, 71 (1941), reprinted in Ref. 1(b), pp. 191, and (b) L.D. Landau, J. Phys. USSR 11, 91 (1947), and reprinted in Ref. 1(b), pp. 243.


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